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General Cosmological Constraints on the Masses of Stable Neutrinos and Other "Inos"

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ABSTRACT

The arguments favoring non-baryonic dark matter are summarized. In particular, if the cosmological density parameter $\Omega \gtrsim 0.15$, the universe must be dominated by non-baryonic matter. General cosmological constraints, independent of detailed galaxy formation scenarios, are presented on the masses of stable neutrinos and other "inos", where "ino" represents any candidate particle for the dark matter in the universe: 1. The requirement that the total mass density not exceed $\Omega \lesssim 4$ restricts neutrinos to two mass ranges, $\sum m_\nu \lesssim 400$ eV and $m_\nu \gtrsim 1$ GeV. (2) From age of the universe arguments, tighter constraints on the lower of the two mass ranges become $\sum m_\nu \lesssim 25$ eV ($\sum m_{ino} \lesssim 400$ eV) or $\sum m_\nu \lesssim 100$ eV ($\sum m_{ino} \lesssim 2$ keV) depending on age technique used. An actual determination of a neutrino mass puts an upper limit on the age of the universe, and in a neutrino-dominated universe $\Omega = 1$ is only possible for $\sum m_\nu \gtrsim 25$ eV. 3. From phase-space density arguments, a necessary (but not sufficient) condition for the clustering of neutrinos on large scales is that $m_\nu \gtrsim 3$ eV. 4. For the formation of large-scale structure, the maximum neutrino Jeans mass should not exceed supercluster scales and therefore $m_\nu \gtrsim 10$ eV. Three neutrinos of equal mass can be excluded if one uses globular cluster determinations of the age in (2) above. 5. If the formation of large-scale structure requires damping of small scales and hence a minimum value of the maximum neutrino Jeans mass, $m_{ino} \lesssim 200$ eV for dominant particles. In a universe with $\Omega \simeq 1$ and photon temperature $T_{\gamma_0} = 2.7$K, this also leads to the constraint that decoupling temperature T_D of the dominant "ino" is $T_D \lesssim 100$ MeV. 6. Big Bang Nucleosynthesis restricts the number of neutrino species to at most four, probably only three.

These arguments are then synthesized to show that all of the independent constraints can only be simultaneously met in a "best fit" model with $10\ \text{eV} \lesssim m_\nu \lesssim 25\ \text{eV}$ for the most massive neutrino eigenstate. Independently of galaxy formation arguments and with only the extremely conservative age limit, it can still be said that $3\ \text{eV} \lesssim m_\nu \lesssim 100\ \text{eV}$. Note also that if constraint (5) is valid then the dominant "ino" acts in every way like a massive neutrino and thus if (6) also holds, it probably is a massive neutrino. Differences in adiabatic and isothermal fluctuation models are discussed; in particular the GUTs preferred adiabatic mode is only consistent with limits on 3K anisotropies if non-baryonic matter dominates. Problems on small-scales with galaxy correlation studies and equilibrium time scales in a neutrino-dominated universe with adiabatic fluctuations are discussed. For $\Omega \sim 0.2 - 0.6$ cold matter such as axions or GeV mass "inos" could be the dominant matter, but in an $\Omega \simeq 1$ universe these as well as keV mass "inos" are not optimal for the dominant matter and the best fit is a neutrino of mass $m_\nu \simeq 25$ eV.

I. Introduction

For longer than a decade, astrophysicists have been investigating the cosmological implications of a nonzero neutrino rest mass [1-3]. If neutrino masses exceed a few eV, neutrinos are the dominant matter in the universe and hence play an important role in the formation and structure of galaxies. Marx and Szalay [2] and Cowsik and McClelland [3] suggested massive neutrinos as possible candidates for the "missing mass" in the haloes of galaxies. On larger and larger scales, less and less of the dynamically inferred mass can be accounted for by luminous matter [4]. Some form of dark matter must reside in the haloes of galaxies and clusters of galaxies. For many years, massive neutrinos were considered plausible but not compelling candidates for the dark matter, while the favored scenarios employed "ordinary" nucleonic matter (faint stars, gas clouds, etc.) as the missing mass. Primordial nucleosynthetic arguments (cf. [5] and references therein), however, have been strengthened to the point where it is now reasonably clear that $0.01 \leq \Omega_b \leq 0.14$ [6]. [Here Ω_b is the ratio of baryonic matter density ρ_b to critical density $\rho_c = \frac{3H_o^2}{8\pi G} = 1.88 \times 10^{-29} h_o^2\ \mathrm{gm/cm^3} = 8.1 \times 10^{-11} h_o^2\ \mathrm{eV}^4$ where the Hubble parameter $H_o = 100\ h_o\ \mathrm{kms^{-1}\ Mpc^{-1}}$. We will also use the notation Ω_ν to represent the ratio of neutrino density ρ_ν to critical density and simply Ω for the ratio of total energy density ρ of the universe to critical density.] Schramm and Steigman [7] pointed out that evidently some of the dark matter must be nonbaryonic if $\Omega \gtrsim 0.15$, as indicated by the matter on the scale of clusters of galaxies. Hence, even prior to experimental claims [8] of finite neutrino rest masses, the cosmological arguments implied the need for nonbaryonic dark matter. The previously favored nucleonic scenarios were thus excluded, leaving massive neutrinos as the least "ad hoc" possibility; other candidates for the dark matter include primordial black holes, axions, and supersymmetric particles like photinos. Assuming the isothermal mode of galaxy formation, Schramm and Steigman [7] deduced an upper limit on the neutrino mass of $m_\nu < 20$ eV; in this paper we find more general neutrino mass constraints from model-independent arguments.

Experimental evidence for neutrino masses [8] and neutrino oscillations [9] as well as theoretical work in grand unified theories (for a review, see P. Langacker [10]) further stimulated interest in massive neutrinos. The original Lubimov et al [8] result of $14 \text{ eV} \lesssim m_{\nu_e} \lesssim 42$ eV has been revised. Better line resolution of the apparatus means a lower subtracted line width and a neutrino mass that may be as low as $m_{\nu_e} = 0$. However refinement of the molecular corrections (the experiment uses tritium in a valine molecule) seems to cancel the above effect, and the group now claims $m_{\nu_e} = 35 \pm 5$ eV with these reduced uncertainties. In any case, these results have not been verified in other laboratories, and several experiments are in progress. The negative results from oscillation experiments place strict limits on the squared mass difference of neutrino species and the mixing angle.

In this paper we examine constraints on neutrino masses due to generalized cosmological arguments as well as specific models and look for masses consistent with all the arguments. In Section II we present the necessary background material, a discussion of mass-to-light ratios and the constraints on the baryonic and total energy density of the universe. We also mention the nucleosynthetic limits on the number of light weakly interacting particles. In Section III we discuss fermion densities appropriate to neutrinos and to supersymmetric "inos". Phase space arguments [11] will be presented to give necessary but not sufficient constraints on the masses of neutrinos which can cluster on various scales. The combined restrictions on the age of the universe, the Hubble parameter and the total energy density of the universe yield another mass constraint (cf. [12]). Since this constraint depends sensitively on the age, we will examine the age arguments and give two limits, one for the extreme lower age limits from the mean age of the elements, $t_u \gtrsim 8.7 \times 10^9$ yrs [13], and the other from globular clusters and big bang nucleosynthesis concordance, $t_u \gtrsim 13 \times 10^9$ yrs [14]. In Sections IV and V we discuss the evolution in a neutrino-dominated universe of the two independent modes of density perturbations that may be responsible for galaxy formation, adiabatic and isothermal. The neutrino Jeans mass, the minimum mass that can collapse under its own self-gravity, (cf. [15]) is discussed. We find the constraints on neutrino masses required for the success of each model, and discuss the difficulties in a neutrino-dominated universe with the hierarchical clustering usually associated with the isothermal picture. The adiabatic mode is also favored over the isothermal by grand unified theories and easily gives rise to the observed large-scale filaments and voids [16-18]. Several alternatives to massive neutrinos for the nonbaryonic dark matter have been proposed, among them supersymmetric particles [19,20], axions [21,22], and primordial black holes [23-25].

We will argue that if the role of the large-scale structure is dominant as implied by the large voids and large clusters, dark matter should not differ significantly from massive neutrinos, either in mass or decoupling temperature. Although we will argue that low mass neutrinos are preferred for the large scales we will also point out that they may encounter problems in understanding the smaller-scale structures as indicated by the correlation function [26,27] and the relative dynamical equilibration timescales of different size systems. Possible scenarios [28] which avoid these problems are discussed. Finally in Section VI we discuss experimental implications and summarize the results.

II. Cosmological Mass Densities

In this section we will summarize the cosmological density arguments. Although there are no direct observational density determinations yielding values of Ω greater than 1, one could argue that these techniques might be insensitive to a truly uniform density background. However, the deceleration parameter q_o in the standard hot big bang model with zero cosmological constant ($\lambda = 0$) is just $\Omega/2$; thus a limit on q_o does limit Ω independent of the background. Estimates of q_o range from 0 ± 0.5 to 1.5 ± 0.5 [29]. Thus an extreme upper limit to Ω with $\lambda = 0$ is $\Omega \lesssim 4$.

Dividing the mass of a bound system (obtained by application of the virial theorem) by its luminosity, one can obtain mass-to-light ratios (M/L) and estimates of matter contributions on different scales. Many authors (eg. [30]) find evidence for M/L increasing linearly with scale from $M/L \sim (1-2)$ for stars to $M/L \sim (300-800)h_o$ for rich clusters (see Table I, drawn largely from Faber and Gallagher [4]). Multiplying M/L on a given scale by an average luminosity density (uncertain by a factor of two) for the universe [31],

$$\mathcal{L} \simeq 2 \times 10^8 \; h_o (L_\odot / \mathrm{Mpc}^{-3}), \tag{2.1}$$

one obtains a mass density (also listed in Table I) implied by assuming M/L on that scale applies to the average light of the universe. Davis et al. [32] have suggested that the M/L curve may be approaching an asymptotic limit (perhaps at $\Omega = 1$) on the scales of superclusters, while other authors [33] believe that the curve flattens already on scales of binaries and small groups. Galaxy correlation studies [34], on the other hand, implicitly assume that the light is a good tracer of the mass, i.e. M/L does not increase on scales beyond the outermost haloes of galaxies. In any case the consensus is that some form of dark matter dominates the dynamics of objects on scales larger than 100 kpc and, as shown by flat rotation curves, may be important on scales larger than 10 kpc.

The above arguments are independent of whether or not the matter is baryonic. Big Bang Nucleosynthesis ([5] and references therein) provide density constraints on the baryonic components. A lower limit on the amount of baryonic matter in the universe can be derived from combined D and ^{3}He abundances [6]. This comes from the fact that no significant D has been produced since nucleosynthesis; much of the original deuterium has been converted to helium in stellar burning. The amount of ^{3}He, on the other hand, has increased in normal stellar processes. Hence the sum of their abundances $D + {}^3He$ today gives an upper limit on $D + {}^3He$ at nucleosynthesis and a lower limit on nucleonic matter, $\Omega_b h_o^2 \gtrsim 0.01$. The observed abundances of ^4He (mass fraction $Y \lesssim 0.25$), D, and ^7Li result in an upper limit to baryonic matter density, $\Omega_b\ h_o^2 \lesssim 0.034$. These arguments restrict baryonic matter to the range

$$0.01 \lesssim \Omega_b \lesssim 0.14. \tag{2.2}$$

Helium abundances from Big Bang Nucleosynthesis also constrain the number of neutrino species; at most four low mass ($\lesssim$ 1 MeV), long-lived neutrino species are compatible with $Y \lesssim 0.26$, and only three with the best observational limit of $Y \lesssim 0.25$ [35,36]. We know two of these experimentally, namely the ν_e where $m_{\nu_e} \lesssim$ 60 eV and the ν_μ where $m_{\nu_\mu} \lesssim$ 570 keV. The experimental mass limit on the ν_τ is $m_{\nu_\tau} \lesssim$ 250 MeV [37]. In this case we may already know all the neutrinos and other low mass "inos" which interact with the strength of neutrinos. Note, however, that the limit of three increases if the particle couples more weakly than the neutrino [38] and thus decouples in the early universe at a temperature $\gtrsim$100 MeV.

If M/L really keeps increasing on scales larger than binaries and small groups, then Ω exceeds the upper limit on Ω_b and we are forced to say that the bulk of the matter in the universe is non-baryonic. While the general trend towards larger Ω exists, including the recent determinations of Ω from the deviations in the Hubble flow toward the Virgo Cluster [39] arguing $0.25 \le \Omega \le 0.6$, the situation is by no means settled. In this paper we will assume that Ω is $\gtrsim 0.15$ but remember that this has not been unambiguously proven; there is still a possibility that $\Omega \leq 0.1$ and everything is baryonic, as emphasized by Gott et al. [40].

On the theoretical side, one should note that inflation (cf. [41] and references therein) explicitly predicts $\Omega = 1.000\ldots$ Thus from Eq. (2.2) inflation requires non-baryonic matter. We will see that for $0.2 \lesssim \Omega \lesssim 0.6$ several possibilities exist for the non-baryonic matter, but that for $\Omega \sim 1$ massive neutrinos will be the optimal case. On the basis of "simplicity" we believe that a nonbaryonic universe with $\Omega \gtrsim 0.15$ should satisfy $\Omega = 1$.

III. Neutrino (and other ino) Densities and Masses

The equilibrium number density of a relativistic fermionic species (subscript f) is given by

$$n_f = \frac{1}{2\pi^2} \int dp\; p^2 / [\exp(p - \mu_f)/T_f) + 1] \qquad (3.1)$$

(throughout we take $\hbar = c = k_B = 1$). The fermions fall out of chemical equilibrium at temperature T_D when the reaction rates for their production (e.g. $e^+e^- \rightarrow f\bar{f}$) can no longer keep up with the expansion of the universe. The fermion distribution continues to be described by Eq. (3.1), with momentum p and temperature T_f simply redshifting with the expansion. We shall consider only the case where chemical potential and hence lepton number are zero, as favored by the usual Grand Unified Theories (GUT s) scenarios (for a discussion of galaxy formation and neutrino mass limits with $\mu_\nu \neq 0$, see [42]). Since neutrinos decouple at $T_D \simeq 3$ MeV, they cannot participate in the heating due to e^+e^- annihilation at $T \simeq 0.5$ MeV. By entropy conservation it can be shown [43] that neutrino (T_ν) and photon (T_γ) temperatures after e^+e^- annihilation are related by $T_\nu = (4/11)^{1/3} T_\gamma$. Other "inos" decoupling earlier at higher temperatures may miss the heating due to annihilation of $\mu^+\mu^-$ or other species, and thus have lower temperatures given by $T_f = T_\gamma(3.9/g_{*f})^{1/3}$ for $T_\gamma << m_e$, where g_{*f} is the number of relativistic species at decoupling [38]. For particles which decouple while still relativistic (i.e., particle mass < decoupling temperature), the Fermi-Dirac number density is given by

$$n_f = \frac{3g_f T_f^3}{4\pi^2} \zeta(3) = \frac{3g_f T_\gamma^3}{4\pi^2} \zeta(3)(3.9/g_{*f}), \tag{3.2}$$

where g_f are spin degrees of freedom (2 for spin 1/2 particles like photinos and neutrinos) and $\zeta(3) = 1.020206$.

The value of the number density in the present epoch for a species of neutrinos which are relativistic at decoupling is given by $n_{\nu_i} = 109 \text{ cm}^{-3}(\frac{T_{\gamma o}}{2.7K})^3$ ($i = e, \mu, \tau$ and $T_{\gamma o}$ is photon temperature today), and the energy density in units of the closure density by

$$\Omega_{\nu_i} = \frac{\rho_{\nu_i}}{\rho_c} = \frac{m_{\nu_i}}{97 \text{ eV}} h_o^{-2}(\frac{T_{\gamma o}}{2.7K})^3 \tag{3.4}$$

where m_{ν_i} is the mass of a neutrino species. If the sum of the masses of different neutrino species exceeds $\sim 100 h_o^2$ eV the universe is closed. Requiring $\Omega \lesssim 4$ and $h_o \lesssim 1$ gives only the weak limit, $\sum m_\nu \lesssim 400$ eV; we will see that age of the universe constraints can strengthen this limit. The ratio of neutrino to baryonic matter is given by

$$\frac{\Omega_\nu}{\Omega_b} \gtrsim \frac{\sum_i m_{\nu_i}}{2.4 \text{ eV}} \tag{3.5}$$

where the equality sign corresponds to the largest value of baryonic matter density consistent with element abundances from primordial nucleosynthesis,

$\Omega_b h_o^2 \lesssim 0.034$. Hence if the sum of the neutrino masses exceeds a few eV, neutrinos are the dominant matter in the universe and must play an important role in galaxy formation.

The above discussion of fermion number densities assumes the fermions are relativistic at decoupling. For neutrinos more massive than a few MeV or for other fermions whose mass exceeds their decoupling temperature, this obviously does not hold. Lee and Weinberg as well as Dicus, Kolb, and Teplitz [44] showed that because of annihilation prior to decoupling, the mass density of very massive neutrinos would fall roughly as $m_\nu^{-1.85}$. Thus total density limits $\Omega \lesssim 4$ can be satisfied for sufficiently massive ($m_\nu \gtrsim 1$ GeV) particles, while if $\sum_i m_{\nu_i} \gtrsim 20$ GeV the density has fallen so low that neutrinos cannot be the dominant matter. Krauss [45] and Goldberg [46] have recently shown that for certain supersymmetric particles the annihilation rates can be slower than those for neutrinos. Thus the mass density corresponding to a given mass will be larger than the Lee and Weinberg value, and the mass limits are pushed to even higher values. It should be noted that contrary to the numerical limits given by Lee and Weinberg, Goldberg, and Krauss, but following Gunn et al. [47] for these very massive particles the appropriate mass density one has to worry about exceeding is the density of matter in groups of galaxies ($\Omega \leq 0.13$) not the total density of the universe (since as we will see these massive particles must cluster on small scales). This yields $m_\nu \gtrsim 6$ GeV for neutrinos and corresponding higher limits for photinos.

Figure 1 is a plot of total energy density in the universe ($\Omega \lesssim 4$) vs. Hubble parameter ($0.5 \lesssim h_o \lesssim 1$). The total energy density is the sum of baryon density ($\Omega_b h_o^2 \gtrsim 0.01$) and neutrino density ($\Omega_{\nu_i} h_o^2 \simeq m_{\nu_i}/97$ eV); we have plotted this sum for several values of neutrino mass. We have also plotted curves for several values of the age of the universe, which can be parameterized (for $\lambda = 0$) as $t_u = f(\Omega) H_o^{-1}$ (where $f(\Omega)$ is a monotonically decreasing function of Ω with values between 1 and ½ in the range of interest).

Several arguments [13] have been used to restrict the age of the universe: certainly it must exceed the age of the solar system $t_u > 4.6$ Gyr (1 Gyr = 10^9 yr), dynamical arguments ($h_o \gtrsim 0.5$) restrict $t_u \lesssim 20$ Gyr, the age of the globular clusters combined with an upper limit on ^{4}He fraction $Y \lesssim 0.26$ restricts 13 Gyr $\lesssim t_u \lesssim$ 19 Gyr [14], and nucleocosmochronology requires 8.7 Gyr $\lesssim t_u \lesssim$ 19 Gyr. Simple radioactive decay is not compatible with cases less than 12.5 Gyr. The range (13–16) Gyr is simultaneously consistent with all arguments, while the widest range allowed by the most stringent limits is (8.7 - 19)Gyr. Consistency with the widest range allowed as well as the restrictions $\Omega \lesssim 4$ and $h_o > \frac{1}{2}$ requires

$$\sum_i m_{\nu_i} \lesssim 100 \text{ eV} \qquad (8.7 \text{ Gyr} < t_u) \ . \tag{3.6}$$

Consistency with the "best fit" range of ages restricts

$$\sum_i m_{\nu_i} \lesssim 25 \text{ eV} \qquad (13 \text{ Gyr} < t_u) \tag{3.7}$$

(see also [12]), where $\Omega = 1$ is achieved only for $\sum_i m_{\nu_i} \simeq 25$ eV. As Schramm [14] noted, this best fit age also limits $h_o < 0.7$ to have concordance. By the inversion of this age argument, an actual neutrino mass gives an upper limit to the age of the universe. For example, if Lubimov et al. [8] are correct and $m_{\nu_e} \gtrsim 30$ eV, then the universe must be younger than 12 Gyr.

Similar constraints can also be found on the masses of other fermions which decouple while still relativistic and are candidates for the dark matter; the analysis differs only in the decoupling temperature and in the number of spin states available to the fermion [66]. The energy density of a fermion species of mass m_f is given by

$$\Omega_f = \frac{m_f}{97 \text{ eV}} h_o^{-2} (g_f/2) \left(\frac{10.75}{g_{*f}}\right) \left(\frac{T_{\gamma o}}{2.7\text{K}}\right)^3 \ . \tag{3.8}$$

For a particle of decoupling temperature $\gtrsim \begin{bmatrix} 100\text{ MeV} \\ 500\text{ MeV} \\ 100\text{ GeV} \\ 10^{14}\text{GeV} \end{bmatrix}$, $g_{*f} = \begin{bmatrix} 14.5 \\ 61.8 \\ 104 \\ 161 \end{bmatrix}$ and

$$\Omega_f = m_f \begin{bmatrix} 130 \\ 560 \\ 940 \\ 1450 \end{bmatrix}^{-1} h_o^{-2} (g_f/2)(T_{\gamma o}/2.7K)^3 eV^{-1}. \qquad (3.9)$$

Consistency with the widest range of ages requires $\sum_f m_f \leq \begin{bmatrix} 130 \\ 550 \\ 930 \\ 1440 \end{bmatrix} (2/g_f)$ eV, while consistency with the "best fit" range of ages restricts $\sum_f m_f \leq \begin{bmatrix} 32 \\ 140 \\ 230 \\ 360 \end{bmatrix} (2/g_f)$ eV. Notice that for all standard unified models with $g_{*f} \lesssim 161$, the limit from our best fit age argument does not allow m_f to exceed 400 eV, contrary to the limits in previous papers [19].

Tremaine and Gunn [11] have used phase space arguments to obtain a restriction on neutrino masses. The smaller the scale on which neutrinos are confined, the larger the velocity dispersion, and the easier it is for neutrinos to escape from the region. A necessary (but not sufficient) condition for trapping neutrinos on the scale of clusters is the requirement $m_\nu \gtrsim 5h_o^{1/2}$ eV, on the scale of binaries and small groups $m_\nu \gtrsim 14\, h_o^{1/2}$ eV, and in galaxies $m_\nu \gtrsim 20$ eV. If massive neutrinos are to solve the missing mass problem they must be trapped at least on scales of clusters of galaxies, i.e., $m_\nu \gtrsim 3$ eV. Of course to actually trap them requires some cluster formation scenarios. Possibilities will be addressed in the next two sections, where it will be shown that this lower limit can probably be strengthened for any realistic scenario.

IV. Adiabatic Perturbations

The formation of galaxies requires the clumping of baryons; i.e., enhancements $\delta_b = \frac{\delta\rho_b}{\rho_b}$ in the baryon density over the background value must grow from small values in the early universe to nonlinearity ($\delta_b > 1$) by the present-day to achieve the formation of bound structure. In the adiabatic mode the baryon perturbations δ_b are accompanied by radiation perturbations δ_γ, whereas in the isothermal mode initially $\delta_\gamma << \delta_b$.

In general any primordial fluctuation scheme for galaxy formation can be treated as a superposition of these two independent modes. Thus in the adiabatic theory of galaxy formation, initially $\delta_\gamma = \delta_\nu = \delta_{\bar{\nu}} = \frac{4}{3}\delta_b$ (where $\delta_i = \frac{\delta\rho_i}{\rho_i}$ describes the density enhancement of particle species i in a perturbation over the background value). These fluctuations grow together outside the horizon, and once inside the horizon their evolution depends on the value of the Jeans mass.

The Jeans mass is the smallest mass unstable to gravitational collapse. It is given by the rest mass of particles in a sphere of radius equal to the Jeans length λ_J, the scale on which radiation pressure forces just balance gravitational forces. The Jeans length is also the distance travelled by a sound wave of speed v_s in a collapse timescale: $\lambda_J \simeq v_s t \simeq v_s/(G\rho)^{\frac{1}{2}}$, where ρ is the total energy density of the universe [for a collisionless fluid like neutrinos the velocity dispersion plays the role of the pressure]. Objects larger than the Jeans mass are unstable to gravitational collapse while smaller ones are stable and merely oscillate as sound waves.

In Figure 2 we have plotted the evolution of neutrino ($M_{J\nu}$) and baryon (M_{Jb}) Jeans masses in a neutrino-dominated universe [15].
While the neutrinos are relativistic, the universe is radiation-dominated, $v_s \simeq \frac{c}{3^{\frac{1}{2}}}$, and the Jeans mass is simply (up to factors O(1)) the rest mass of the particle contained within the horizon, $M_J \propto (1+z)^{-3}$, where z is the redshift of the epoch. At the temperature where average neutrino momentum $\simeq$ rest mass, $\langle p_\nu \rangle \simeq m_\nu$, neutrinos become nonrelativistic and at about the same time begin to dominate the energy density of the universe. The neutrino Jeans mass reaches its peak value [20],

$$M_{\nu M} \simeq 1.8\ m_{pl}{}^3/m_\nu{}^2 \simeq 3 \times 10^{18}\ M_\odot/(m_\nu/\text{eV})^2 \qquad (4.1)$$

at $z_M \simeq 1900\ m_\nu(\text{eV})$, where $m_{pl} = G^{-\frac{1}{2}} = 1.2 \times 10^{19}$ GeV is the Planck mass.

Subsequently $v_s \simeq \langle p \rangle / m_\nu \propto T_\gamma$ and the neutrino Jeans mass falls as $M_{J\nu} \propto (1 + z)^{3/2}$. The baryon Jeans mass continues to rise although more slowly until recombination, where it drops from a peak value of $\simeq 10^{17}$ $M_\odot$ down to $\simeq 5 \times 10^5$ $M_\odot$. Prior to recombination the baryonic matter is ionized and hence closely coupled to the radiation. In the adiabatic theory, as photons diffuse out of overdense regions they drag the baryons with them, smoothing out any structure (Silk damping) on scales less than a critical mass [48,15]

$$M_s \simeq 3 \times 10^{13}\ \Omega_B^{-1/2} \Omega_\nu^{-3/4}\ h_o^{-5/2}\ M_\odot \tag{4.2}$$

It is the perturbations in the neutrinos, the dominant matter in the universe, that determine the formation of structure. Neutrino perturbations on scales $\gtrsim M_{\nu M}$ can grow once the neutrinos become the dominant matter. However, Bond <u>et al</u>. [15] have shown that neutrino perturbations on scales smaller than $M_{\nu M}$ are strongly damped by free-streaming of the neutrinos out of dense regions (Landau damping). Only perturbations on scales larger than $M_{\nu M}$ can survive and grow to nonlinearity [49]. To enable the formation of large-scale structure, we require this damping scale to be smaller than the largest structure observed, superclusters of mass $\simeq 10^{16}$ $M_\odot$ ([50] and references therein), i.e., $M_{\nu M} \lesssim M_{sc} \simeq 10^{16}$ $M_\odot$. Eq. (4.1) is only approximate; folding an initial power spectrum $|\delta_k|^2 \propto k^n$ with a transfer function to describe damping by neutrino diffusion, Bond, Szalay, and Turner [20] obtain an n-dependent power spectrum. Although the peak of the power spectrum is the scale on which perturbations first go nonlinear, significant power may exist on somewhat smaller or larger scales. We take the least restrictive limit, the smallest mass for physically plausible values of n that has significant power, and find that

$$M_{\nu M} \simeq \frac{9 \times 10^{17}\ M_\odot}{(m_\nu / \mathrm{eV})^2} \lesssim 10^{16}\ M_\odot\ . \tag{4.3}$$

In a universe with one massive neutrino species this requires

$$m_\nu \gtrsim 10 \text{ eV}.$$

If there are three species of neutrinos with equal mass, the mass of each species must satisfy $m_{\nu_i} \gtrsim 16$ eV, giving a sum of masses $\sum_i m_{\nu_i} \gtrsim 48$ eV. This is not compatible with the requirement $\sum_i m_{\nu_i} \lesssim 25$ eV from consistency of all the arguments restricting the age of the universe. A "best fit" model does not allow all the neutrino masses to be equal. Of course if we relax our age constraint to $t_u > 8.7$ Gyr then equal masses are allowed. If larger scales than $M_{\nu M}$ in Eq. (4.3) reach nonlinearity first and tidally strip the smaller scales, the limit on the masses only becomes more restrictive.

In this adiabatic picture with massive neutrinos, the smallest scales to form initially are large clusters. Zel'dovich [51] has argued that the gravitational collapse of these objects is likely to proceed faster in one dimension than in the other two, resulting in the formation of pancakes. To enable galaxies to fragment out of the larger structure, there must be sufficient cooling. Such cooling may occur most efficiently at the intersection of pancakes [28] . Thus galaxies might end up in filaments. In the adiabatic picture with massive neutrinos Jeans mass scales form first and smaller scales come from later cooling and fragmentation.

The alternative model of galaxy formation, where small scales form first and cluster hierarchically onto larger and larger scales, has not been shown to give rise to the observed structure on large scales. The observations of Kirshner et al. [16] show large voids in space as well as large clusters. Frenk, White, and Davis [27] have shown that models successful in achieving this large-scale structure seem to require the clumping of small scales before or during the initial collapse. So far all models attempted where small scales form first seem to result in a distribution of clumps on all scales and no extremely large voids. Also, dark matter which clusters first on small scales will yield a constant M/L; however $\Omega \simeq 1$ for the universe is possible only if M/L continues to increase with scale, or if there are regions with M/L larger than any measured value.

If, indeed, small scale damping is required for the formation of large-scale structure, we also get a lower limit on the Jeans mass. Because of the far reaching implications of this result it is important to be aware that the necessity for small-scale damping has not been rigorously proven. One can only say that small-scale damping does naturally give rise to the observed large-scale structure, whereas hierarchical models where small-scale structure forms first and clusters onto larger scales have not been successful. The failure of the attempts of Frenk, White, and Davis [27] to have hierarchical pictures work and the ease with which their small-scale damped models succeed is certainly suggestive if not compelling. Damping those scales smaller than cluster sizes,

$$M_{\nu M} \simeq \left(\frac{g_f}{2}\right)\left(\frac{10.75}{g_{*f}}\right)\frac{1.8\ m_{pl}^{\ 3}}{m_f^{\ 2}} \gtrsim 10^{14}\ m_\odot \qquad (4.4)$$

requires

$$m_f \lesssim 200\ \text{eV}\ (g_f/2)^{1/2}$$

for the mass of any non-interacting particle proposed for the dominant matter (for an alternative approach to similar results see [65]).
Note that for particles decoupling earlier than neutrinos the number density is lower, and hence to conserve the inequality in Eq. (4.4), the restriction on the mass only becomes tighter. If valid, this argument rules out the high mass branch, $m_f \gtrsim 6$ GeV for the dominant matter (if $\Omega \simeq 1$), since such high mass particles would cluster first on very small scales and would not

explain the large voids or large M/L (this does not mean such particles cannot exist; it merely means that their contribution to Ω must be small, $\lesssim 0.2$). This argument would also rule out gravitinos or other supersymmetric particles in the keV mass range.

Since our primary motivation for non-baryonic matter is to get a large Ω, this upper limit on the mass of our candidate particle becomes quite constraining. As we mentioned in Section III, the number density of any species, n_f, decreases roughly stepwise with increasing decoupling temperature T_f for that particle [38]. Given low particle masses m_f, in order to keep a high Ω, where $\Omega \simeq \frac{m_f n_f}{\rho_c}$, the decoupling temperature T_f must also be low. Quantitatively, the constraint $m_f \lesssim 200$ eV and $\Omega \sim 1$ argues that

$$n_f \gtrsim 61\ h_o^2\ (g_f/2)^{-\frac{1}{2}}\ \mathrm{cm}^{-3}.$$

Comparing this with Fermi-Dirac number density Eq. (3.2), we find that the number of relativistic species at decoupling must satisfy $g_{*f} \leq 16 h_o^{-2}\ (g_f/2)^{3/2} (\frac{T_{\gamma o}}{2.7K})^3$. Counting relativistic species, γ, $e^+ e^-$, ν at 1 MeV, adding $\mu^+ \mu^-$ at 100 MeV, and quarks and gluons at ~ 500 MeV (the quark-hadron phase transition), we find for $g_f \leq 2$ and $T_{\gamma o} = 2.7K$ that T_f must be $\lesssim 0(100$ MeV) or we exceed our limit to g_{*f}. In other words, in the favored model where Ω is large ($\Omega \simeq 1$) and large-scale structure forms first, the universe is probably dominated by particles which behave in all ways like massive neutrinos.

Any candidate particle for the dominant matter decoupling at $T_D \lesssim 100$ MeV is constrained by Big Bang Nucleosynthesis arguments in exactly the same way as neutrinos. We have seen in Section II that it is very difficult to add to the three known neutrino species a full additional neutrino type (or equivalently $g_* = 7/4$ additional degrees of freedom, where one bosonic spin state contributes $g_* = 1$ and one fermionic spin state contributes $g_* = 7/8$) without violating limits on the observed

^{4}He abundance. Thus it is probable that the dominant matter is a neutrino and not some other "ino". Axions have been proposed for the dark matter [21,22], but since they have a low Jeans mass they cannot be the dominant matter if the large voids require large scales forming first. These arguments would also prevent small (planetary mass) black holes [24] from being the dominant matter unless they were able to stimulate Ostriker-Cowie [52] explosions [25].

The formation of structure in the universe requires the growth of perturbations from small amplitudes in the early universe to nonlinearity ($\delta\rho/\rho > 1$) by the present day. In the standard adiabatic picture of galaxy formation without massive neutrinos, the coupling of baryons to photons before recombination damps the perturbations on scales smaller than the Silk mass and prevents perturbations from growing on larger scales. After recombination at $1 + z_R \simeq 1000$ the surviving perturbations in an Einstein-de-Sitter universe grow as $(1 + z)^{-1}$, allowing a growth factor of only about 1000 between recombination and the present day. To reach nonlinearity by today, the perturbations must have been $(\frac{\delta\rho}{\rho})_b \simeq 10^{-3}$ at recombination. However, adiabaticity implies a relationship between $(\frac{\delta\rho}{\rho})_b$ and $\frac{\delta T}{T}$ and hence a contribution to the microwave anisotropy $\frac{\delta T}{T} \simeq \frac{1}{3}(\frac{\delta\rho}{\rho})_b \simeq \frac{1}{3} \times 10^{-3}$, larger than the observed large-scale anisotropy, $\lesssim 2 \times 10^{-4}$. This dilemma is resolved in a universe dominated by massive neutrinos or other "inos" [53]. Initially, $\delta_\nu = \delta_\gamma = \frac{4}{3}\,\delta_b$, the perturbations grow together outside the horizon, and once inside the horizon in the photon-dominated era they oscillate like sound waves. After the neutrinos become the dominant matter at $1 + z_M \simeq 1900\ m_\nu$ (time t_M), neutrino perturbations grow as $(1 + z)^{-1}$ on scales larger than the maximum neutrino Jeans mass,

$M \geq M_{\nu M}$. Baryon perturbations are tied to the photons until after the time of recombination (t_R), when they can rapidly catch up to the neutrino perturbations. This extra growth period for neutrino perturbations allows nonlinearity by the present epoch with $\delta_\nu(t_R) \simeq 10^{-3}$ and

$$\delta_b(t_R) \simeq \delta_b(t_M) \simeq \delta_\nu(t_M) \simeq \delta_\nu(t_R)\left(\frac{1+z_R}{1+z_M}\right) \simeq \frac{10^{-3}}{1.9 m_\nu} \simeq 5 \times 10^{-4} m_\nu^{-1}. \quad (4.5)$$

The microwave anisotropy, now only $\frac{\delta T}{T} \simeq 2 \times 10^{-4} m_\nu^{-1}$, no longer violates the observed limit for cosmologically important neutrinos of mass $m_\nu \gtrsim 3$ eV.

Landau damping smooths out all neutrino perturbations on scales $M < M_{\nu M}$. Even after the baryons decouple from the photons at recombination, the growth of baryon perturbations is inhibited by the smooth neutrino background [15],

$$\delta_b \sim t^{\frac{1}{6}[-1 \pm (1+24R)^{\frac{1}{2}}]} \quad (4.6)$$

where

$$R = \rho_b/\rho = \left(1 + \frac{\Omega_\nu}{\Omega_b}\right)^{-1} \leq \left[1 + \frac{\sum_i m_{\nu_i}}{2.4\ \mathrm{eV}}\right]^{-1}. \quad (4.7)$$

Once the neutrino Jeans mass falls below the scale of interest the baryon perturbations can grow as $(1+z)^{-1}$, but this faster growth period is short. On galactic scales, for example, we find a growth factor since recombination of x 1000 only for $m_\nu = 0$, x 100 for $\sum_i m_{\nu_i} \lesssim 2$ eV, and x 10 for $\sum_i m_{\nu_i} \leq 10$ eV. Thus, on scales $M < M_{\nu M}$ in a neutrino-dominated universe, fluctuations in the baryon density could not grow sufficiently to lead to bound structure unless the amplitude of the perturbations at recombination were very large and thus would exceed the observed limits on the microwave anisotropy. Adiabatic perturbations with massive neutrinos can lead to galaxy formation only through the fragmentation of larger scales. We will return later to the problems of cooling rates necessary for fragmentation of collapsing neutrino pancakes.

Before leaving the adiabatic mode, it is worthwhile to remember that GUT s naturally allow adiabatic perturbations but not isothermal ones [54]. All attempts at producing isothermals in the context of GUT s have been very ad hoc and unnatural [54-56]. In fact inflationary scenarios seem naturally to give rise to adiabatic perturbations [57,58] with a Zel'dovich spectrum, i.e. fluctuations come within the horizon at constant although model-dependent [59,60] amplitude.

Isothermal Perturbations and Hierarchical Clustering

Isothermal perturbations are fluctuations in the entropy per baryon. Initially the photon and neutrino perturbations are very small: $\delta_b \neq 0$; δ_γ, $\delta_\nu << \delta_b$. Before recombination, the baryon perturbations are prevented from growing by the photon pressure. Even after recombination, baryon perturbation growth is inhibited by the smooth neutrino background. From Eq. (4.6) we find a growth factor of at most 100 for $\sum_i m_{\nu_i} \lesssim 2$ eV and x 10 for $\sum_i m_{\nu_i} \lesssim 10$ eV. Thus for galaxy formation in the isothermal picture with massive neutrinos to work one must impose large amplitude baryon fluctuations $\delta_b \gtrsim 0(0.1)$ as initial conditions. Such large amplitudes not only violate constraints on the microwave background but also seem unnatural - galaxies exist because they were put in as initial conditions. Neutrinos of mass $\gtrsim$ 3 eV allow limited growth (less than x 100) of baryon perturbations, and responding to the gravitational attraction of baryon overdensities, can serve as the dark matter in clusters of galaxies. Neutrinos of mass $\gtrsim$ 20 eV, however, would in the isothermal picture not be prevented from clustering on galactic scales and result in too much matter on small scales. Neutrinos of mass $\simeq$ 30 eV, for example, would contribute $\Omega_\nu \simeq 0.3$ to galaxies, more than the observed density $\Omega_{gal} \simeq 0.1$. This argument would obviously not apply if all the halos are stripped by tidal interactions as would occur in rich clusters. However most galaxies are in low density binaries and small groups like the Milky Way and M31 where the bulk of the 20 eV neutrino halos would be included within the orbit and would thus support the limit. Hence massive neutrinos in the purely isothermal picture are restricted to the range 3 eV $< m_\nu <$ 20 eV, as mentioned by Schramm and Steigman [7]. Gunn _et al_. [47] applied the same arguments in both isothermal and adiabatic models to the very heavy ($\gtrsim$ GeV) neutrinos (and other heavy "inos"), arguing that the Ω in such particles cannot exceed $\sim$ 0.1.

In addition to the above need for _ad hoc_ large amplitudes there is the fact that isothermals are not favored by GUT s generation of baryon number. Since most motivations for theoretical proposals for new exotic particles are GUT s - or super GUT s - related it would be somewhat inconsistent to use such particles in an isothermal model (although adiabatic hierarchical models are plausible). In the isothermal scheme small scales are not damped and thus they condense early and clusters are built up in a later hierarchical manner which makes it difficult to understand the large scale structure. However to be fair we should note that the isothermal picture, or at least its hierarchical consequence of building large scales from small scales has certain very attractive features. In particular the adiabatic pancake scheme does not by itself lead to the small scale structure. Such structure requires subsequent cooling and fragmentation of the baryons while leaving the non-interacting neutrinos on the large scale where they produce the large M/L. These

cooling and fragmentation processes are not well understood, so many ad hoc assumptions are required. On the other hand, isothermal perturbations naturally produce these small scales down all the way to the scale of globular clusters. The hierarchical picture, whether produced by isothermals or by adiabatics with a small "ino" Jeans mass so that small scales form first, does correctly fit the 2 and 3 point galaxy correlation functions ([26] and references therein) up to cluster scales of $\sim$ 5 Mpc. To fit this in the simple pancake picture (with no cooling or fragmentation assumptions) requires a fine tuning of the parameters or at least a very small Jeans mass that is not needed in a hierarchical scheme. In addition as Frenk, White, and Davis [27] point out, if neutrinos make the large scales first then galaxies might not form until redshifts $z < 1$, but quasars are seen with $z \sim 3.5$. In addition, the equilibration time scale can be determined from the dynamics of galaxies on various scales and it is found that the largest scales are not in dynamical equilibrium yet, whereas small scales are. For example the core of the Virgo Cluster as well as the Coma Cluster are well virialized, whereas the Virgo Supercluster is not.
On face value this might argue in favor of isothermals or at least a hierarchical picture with small ($\lesssim 10^8$ $M_\odot$) Jeans mass as given by axions, GeV mass photinos or planetary mass black holes. But as we've already seen GUTs argues against isothermals and the large scale voids, superclusters, and $\Omega \sim 1$ argue against any hierarchical picture on the largest scales (hierarchical models produce constant M/L.) What is the solution? Six possibilities immediately come to mind.

1) The isothermal picture holds, GUTs have nothing to do with cosmology, and the Davis [17] results become invalid in case (2) below.

2) $0.2 < \Omega < 0.6$, the universe is filled with roughly equal amounts of baryons and "cold" non-baryonic matter, and M/L is constant.

3) Adiabatics with a non-baryonic particle which has a low Jeans mass, some new hierarchical model with tidal stripping and a power spectrum which, contrary to previous attempts, does produce the large scale structure and enables $\Omega \sim 1$.

4) The adiabatic picture holds with a 10 to 25 eV neutrino and planetary mass black holes form at either the SU (2) x U (1) and/or quark-hadron phase transitions. Clumps of these black holes subsequently serve as seeds for early massive star formation in the neutrino pancakes in a manner similar to that proposed by Freese, Price and Schramm [25]. The explosions of these objects gives the small and intermediate scale structure as per Ostriker and Cowie [52]. [Note the Ostriker-Cowie method alone does not seem to produce the very largest scale structure since the energy required to move matter around on those scales exceeds the wildest speculations for such explosions (Ostriker 1983 , private communication).]

5) The adiabatic picture holds but there are two significant non-baryonic particles, the dominant one being the 10 to 25 eV neutrino described in the previous section and the second a particle such as the axion or massive gravitino which has a small Jeans mass and yields the small scale structure. Such a model is certainly possible but seems very ad hoc in that we require new particles to solve each problem.

6) The adiabatic picture holds and cooling and fragmentation of the baryons is enhanced at the intersection of pancakes [28]. In this case galaxy formation occurs in filaments. Fry [61] has shown that the galaxy correlation functions for filaments agrees with the observed 2 and 3 point gal-

axy correlation functions as well as the hierarchical case. To meet the time scale constraints one merely has to argue that within the filaments distant regions will take longer to equilibrate than central ones.

While all six of the above are in principle possible, it is our view that since calculations do exist which support (6) and no such calculations exist for (1) and (3) (in fact those that do exist argue against (1) and (2)) and since (4) and (5) may be somewhat ad hoc, and since (2) requires the peculiarity of large amounts of non-baryonic matter and yet still has $\Omega < 1$, we feel (6) is the most likely scenario with (2) and (3) the most reasonable alternatives. Of course, a 7th possibility with $\Omega < 0.15$ and only baryons is also still allowable with isothermals.

It is interesting to note that Szalay [62] found that only $\sim 3/8$ of the matter in the pancakes ended up in the filaments. This implies that the Ω inferred from even the largest clusters might only be $\sim 3/8$ of the total Ω. Thus Ω could easily be ~ 1 even though current observations on even the largest scales give $\Omega \lesssim 0.6$. It is also interesting that while the filaments form from cooling baryons they do trap some neutrinos even on relatively small scales. Thus on the scales of binaries and small groups one might have some neutrinos and so enable Ω_b to be safely below the Ω implied by the dynamics of binaries and small groups.

Scenarios of mixed isothermal and adiabatic components are also possible [63]; since the adiabatic mode grows more quickly, the resulting structure is essentially that of the adiabatic picture.

VI. Discussion and Summary

In this paper we have tried to be very explicit about the assumptions required for each cosmological "ino" mass argument, and then we synthesized the arguments to obtain some very powerful simultaneous constraints. Table II gives a summary of the arguments for different possible constituents of the universe. A universe with $\Omega \gtrsim 0.15$ must probably be dominated by non-baryonic matter, although the observational evidence pointing in this direction is not without loopholes. The galaxy formation mode compatible with GUT s, namely the adiabatic mode, produces disagreement with the 3 K background unless the universe is dominated by non-baryonic matter. We also mentioned that Big Bang Nucleosynthesis constrains the number of neutrinos to at most 4, probably only 3.

Merely from limits on the total mass density of the universe, neutrino masses are restricted to two ranges, $\sum_i m_{\nu_i} \lesssim 400$ eV or $m_{\nu_i} \gtrsim 6$ GeV (corresponding arguments for other "inos" depend on their decoupling temperatures). However, applying an additional age constraint at $t_u > 8.7$ Gyr, we find a stricter limit $\sum_i m_{\nu_i} \leq 100$ eV ($\sum_{ino} m_{ino} \lesssim 2$ keV) while the range $m_{\nu_i} \gtrsim 6$ GeV remains unchanged. If we further restrict the age by requiring consistency between globular cluster ages, observed helium abundances and Big Bang Nucleosynthesis, then $t_u > 13$ Gyr and $\sum_i m_{\nu_i} \leq 25$ eV ($\Sigma_{ino} m_{ino} \lesssim 400$ eV) (remember age arguments assume $\Lambda = 0$).

We mentioned that the Tremaine and Gunn [11] phase space argument gives a necessary (but not sufficient) limit of $m_\nu \gtrsim 3$ eV if neutrinos are to cluster on cosmologically significant scales. In the GUT s favored adiabatic scenario, we showed that a maximum neutrino Jeans mass small enough to allow the formation of superclusters requires $m_\nu > 10$ eV. For three neutrino species of equal mass, the limit becomes more restrictive, $m_{\nu_i} > 16$ eV ($\sum_i m_{\nu_i} > 48$ eV). Hence three equal neutrino masses are allowed only for a universe younger than 12 Byr, an age excluded by the best fit age arguments. Based on the conclusions of Frenk, White, and Davis [27] that the formation of large-scale structure required small-scale damping, a potentially far-reaching argument was presented that the mass of the dominant particle be less than $\sim$ 200 eV. In a high density universe of $\Omega \simeq 1$, with $T_{\gamma o}$=2.7K, a particle with this mass and $g_f \leq 2$ decouples at $T_D \lesssim 100$ MeV; in other words the dominant matter is similar to a massive neutrino. The nucleosynthetic constraint that there are probably only 3 neutrinos leads to the conclusion that not only does the dominant particle act like a massive neutrino but it probably is one. The mass for this best fit neutrino is $10 \lesssim m_\nu \lesssim 25$ eV. While the adiabatic picture with massive neutrinos successfully gives the large scale structure, some problems exist on smaller scales. Various speculative models are given which might solve the small scale problems while retaining a solution to the large scale. We feel the most promising is the scheme of Bond, Centrella, Szalay and Wilson [28], which investigates the cooling and fragmentation of pancakes into galaxies. It will be interesting to see whether subsequent work verifies this model.

In the most likely scenario the most massive neutrino would have a mass 10 eV $\lesssim m_\nu \lesssim$ 25 eV while the other neutrinos would have negligible masses. If the Lubimov et al. [8] result holds then we may have found the answer, massive ν_e with m_{ν_μ} and $m_{\nu_\tau} \lesssim 3$ eV. Although one might naïvely expect the ν_τ to be the most massive, because these are weak rather than mass eigenstates, depending on the mixing matrix any combination of leptons might be involved in the most massive eigenstate. Ushida et al. [64] in an experiment looking for $\nu_\mu - \nu_\tau$ oscillations sensitive to mixing angles $\sin^2 2\alpha \gtrsim 0.013$ found a limit $|m_{\nu_\mu}{}^2 - m_{\nu_\tau}{}^2| < 3.0$ eV^2 (90% confidence level). This suggests that either the mixing angle is very small, that limits on the age of the universe from globular clusters must be reevaluated (although all current possible corrections go towards longer, not shorter ages), or that there is some form of dark matter other than the three known neutrino species. Unfortunately, if the mass eigenstate is close to ν_τ and the mixing angle is small, the experimental detection of the dominant matter in the universe may take a long time.

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Table I

Mass-to-Light Ratios

Object	$\frac{M}{L} / (\frac{M}{L})_\odot$	Ω
stars	1-4	$(0.7-2.9) \times 10^{-3}\ h_o^{-1}$
spiral galaxies	$(8-12)h_o$	$(5.7-8.6) \times 10^{-3}$
elliptical and SO galaxies	$(10-20)h_o$	$(0.7-1.4) \times 10^{-2}$
binaries and small groups	$(60-180)h_o$	$(0.4-1.3) \times 10^{-1}$
clusters of galaxies	$(280-840)h_o$	0.2-0.6

Table II - Cosmological Matter

	Light emitting matter (glowing baryons)	Dark Matter (no dissipation into galactic cores)			
		COLD[1]		WARM[2]	HOT[3]
		Baryons	Others		
Examples	stars	Jupiters or stellar mass black holes[4]	Axions, (10-100)GeV mass "inos", planetary mass black holes	10eV-400 eV[7] "inos"	10 eV - 25 eV[7] "inos" (if $N_\nu=3$ then ν_e, ν_μ or ν_τ)
Maximum Jeans Mass	Irrelevant	Depends on fluctuation spectrum	$\lesssim M_\odot$	$\sim 10^{12} M_\odot - 10^{16} M_\odot$	$\sim 10^{15}-10^{16} M_\odot$
Ω_i	~ 0.01	$\sim 0.01 - 0.14$	$\lesssim 0.6$	$\gtrsim 1$ [6]	$\gtrsim 1$ [6]
M/L	$1 - 10 h_o$	$\lesssim 150\ h_o$	$\lesssim 800\ h_o$ [5]	increases with scale	
dark halos of dwarf spheroidals	--	O K		marginal on phase-space, unlikely from Jeans mass	NO
galaxy correlation function	observed	O K		requires special heating and cooling of baryons so mass anti-correlates with light	
large-scale filaments and voids	NO	no existing hierarchical model yields this structure		marginally natural	natural
$\frac{\delta T}{T} \lesssim 10^{-4}$ with adiabatic fluctuations	NO	NO	YES	YES	
galaxy formation epoch	$Z \sim 100$	$Z \sim 100$	$Z \sim (100-1)$	$Z \sim 10$	$Z \sim 1$

1 "cold": decouples while non-relativistic.
2 "warm": decouples while relativistic, present temperature $T_f < T_\nu \simeq 2K$.
3 "hot": decouples while relativistic, present temperature $T_f \simeq T_\nu \simeq 2K$.
4 stellar black holes were baryons at Big Bang Nucleosynthesis.
5 it is assumed that low Jeans mass will result in non-dissipative clustering with baryons; no self-consistent calculation shows otherwise.
6 since these don't fit small scales well, prime motivation is large scales and high Ω's.
7 assumes $t_u > 13$ Gyr or equivalently $\Omega h_o^2 < 0.25$, i.e. can have $\Omega = 1$ only for $h_o = 1/2$.

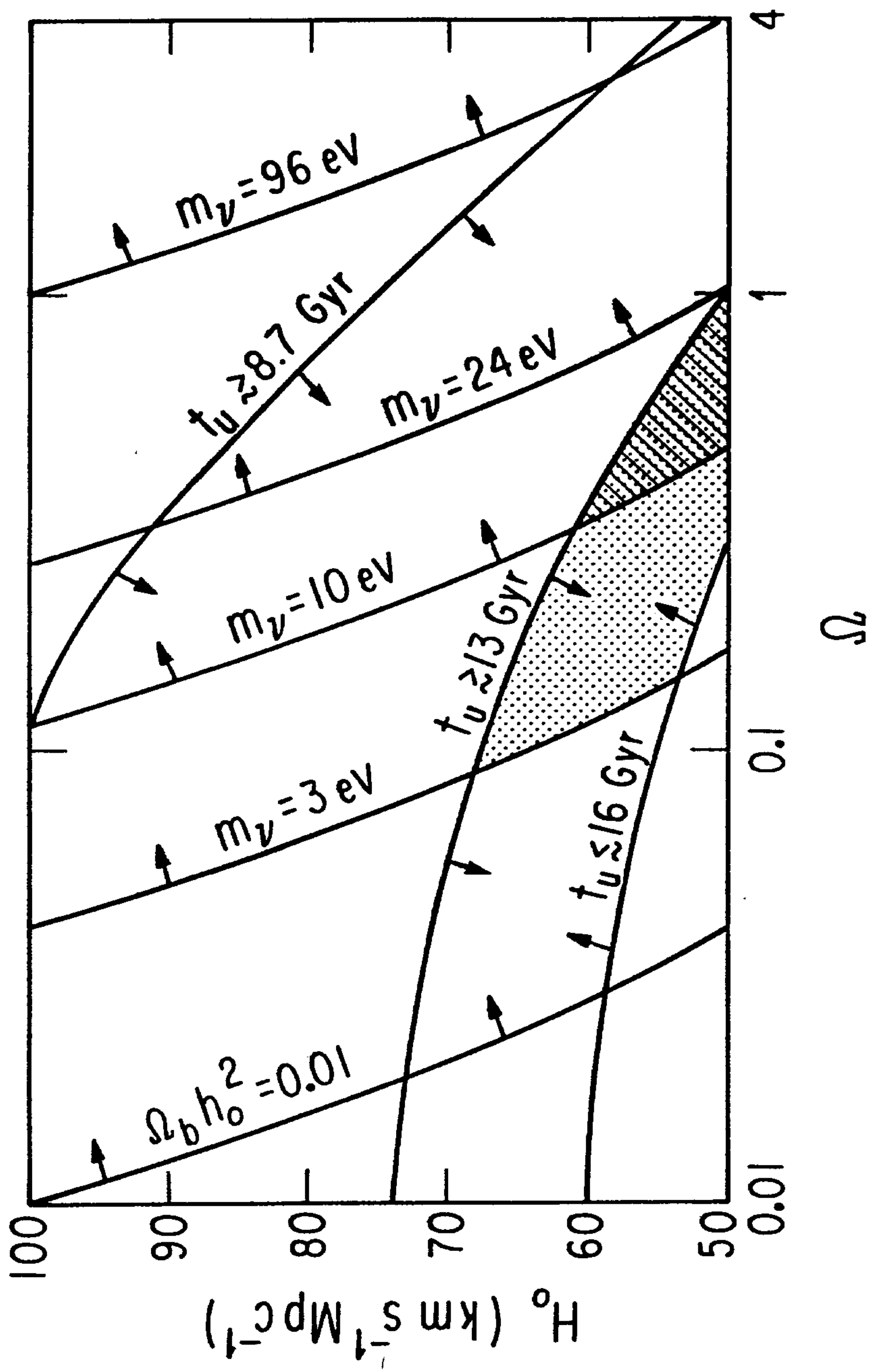
Figure Captions

Figure 1

On a plot of Hubble parameter H_o vs. energy density Ω we have drawn curves for several values of the age of the universe ($t_u = f(\Omega)H_o^{-1}$), for the energy density in baryons ($\Omega_b h_o^2 \geq 0.01$), and for the total energy density of the universe with several values of the neutrino mass $\Omega_{total} = \Omega_b + \Omega_\nu \geq (0.01 + \frac{\sum m_\nu}{97\ \mathrm{eV}})h_o^{-2}$ (for $T_{\gamma o} = 2.7\mathrm{K}$). The firm upper limit to the age of the universe $t_u < 8.7$ Gyr restricts $\sum m_\nu \lesssim 100$ eV, while an age range consistent with dynamics and globular clusters 13 Gyr $< t_u <$ 19 Gyr requires $\sum m_\nu \lesssim 25$ eV. The dotted region indicates the range of neutrinos massive enough to serve as the dark matter in clusters ($m_\nu > 3$ eV) yet consistent with all the age arguments. The smaller hatched region indicates the range of neutrinos which may be responsible for the formation of large-scale structure in the adiabatic picture ($m_\nu \gtrsim 10$ eV, cf. § 4).

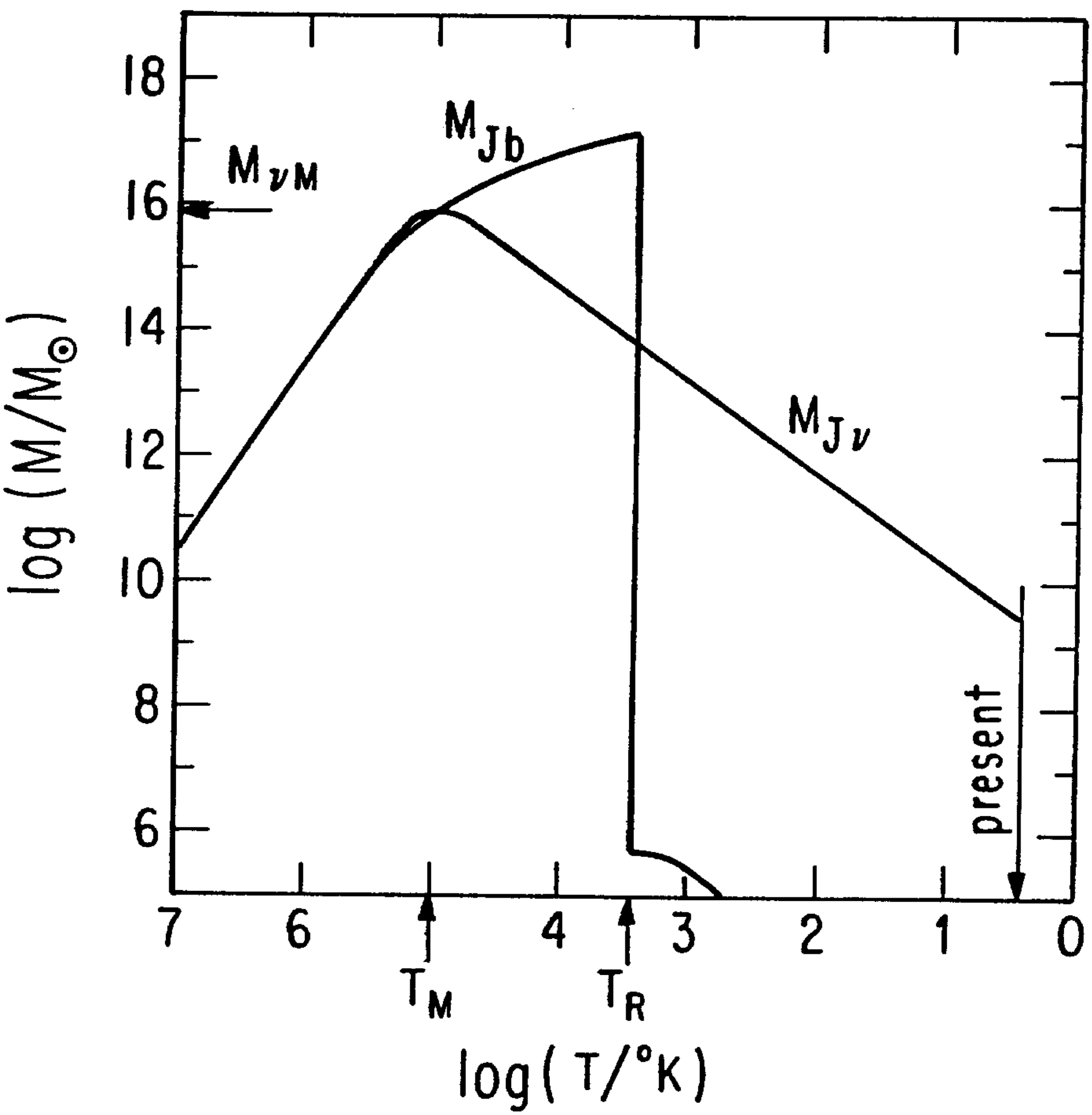
Figure 2

Neutrino and baryon Jeans masses as a function of temperature for $m_\nu \simeq 20$ eV. During the radiation-dominated era, the Jeans mass is approximately the comoving mass inside the horizon and grows as $(1 + z)^{-3}$. Once neutrinos become the dominant matter at $T_M \simeq 10^5$ K, the neutrino Jean mass peaks at $M_{\nu M} \simeq 1.8\ m_{pl}^3/m_\nu^2 \simeq 7.5 \times 10^{15}\ M_\odot$ for $m_\nu = 20$ eV and thereafter falls as $(1 + z)^{3/2}$. The exact shape of $M_{J\nu}$ near its peak value has been calculated by Bond, Efstathiou, and Silk (1980) and is merely approximated here. M_{Jb} drops at recombination ($T_R \simeq 2700$ K) to $\simeq 5 \times 10^5\ M_\odot$.



m_ν=96 eV
$t_u \gtrsim 8.7$ Gyr
m_ν=24 eV
m_ν=10 eV
$t_u \gtrsim 13$ Gyr
m_ν=3 eV
$t_u \lesssim 16$ Gyr
$\Omega_b h_o^2$=0.01
Ω
0.01
0.1
1
4
H_o (km s^{-1}Mpc^{-1})
100
90
80
70
60
50

Figure 1



18
16
14
12
10
8
6
log (M/M$_\odot$)
$M_{\nu M}$
M_{Jb}
$M_{J\nu}$
present
7
6
4
3
2
1
0
T_M
T_R
log(T/°K)

Figure 2