SLAC – PUB – 3386
July 1984
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GALAXY AND CLUSTER FORMATION IN A UNIVERSE DOMINATED BY COLD DARK MATTER*

Joel R. Primack
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305
and
Santa Cruz Institute for Particle Physics
University of California, Santa Cruz, California 95064

The dark matter (DM) that appears to be gravitationally dominant on all astronomical scales larger than the cores of galaxies[1] can be classified, on the basis of its characteristic free-streaming damping mass M_D, as hot ($M_D \sim 10^{15}M_\odot$), warm ($M_D \sim 10^{11}M_\odot$), or cold ($M_D < 10^8 M_\odot$). For the case of cold DM, the shape of the DM fluctuation spectrum is determined by (a) the primordial spectrum (on scales larger than the horizon), which is usually assumed to have a power spectrum of the form $|\delta_k|^2 \propto k^n$ (inflationary models[3] predict the "Zeldovich spectrum" $n = 1$); and (b) "stagspansion",[2] the stagnation of the growth of DM fluctuations that enter the horizon while the universe is still radiation-dominated, which flattens the fluctuation spectrum for $M \lesssim 10^{15}M_\odot$.[4–6]

An attractive feature of the cold dark matter hypothesis is its considerable predictive power: the post-recombination fluctuation spectrum is calculable, and it in turn governs the formation of galaxies and clusters. Good agreement with the data is obtained for a Zeldovich spectrum of primordial fluctuations.

1. WHY COLD DM?

There are strong astrophysical arguments that the DM does not consist of any form of ordinary matter ("baryons").[3] Although these arguments are not entirely compelling, they are sufficiently convincing to have motivated both astrophysicists and particle physicists to consider seriously the possibility that the DM consists of some other sort of matter.

If a species of neutrino is the gravitationally dominant component of the universe, [8,9] its mass $m_\nu = 100\ \Omega\ h^2 eV$ (where $h = H_0/100\ km\, s^{-1} Mpc^{-1}$ lies in the range $1/2 \leq h \leq 1$) implies a free-streaming damping mass $M_D \sim 10^{15}M_\odot$ corresponding to hot DM. Since fluctuations of galactic mass $\sim 10^{8-12}M_\odot$, much smaller than M_D, are strongly damped, galaxies can only form in a neutrino-dominated universe after fluctuations of supercluster mass $\sim 10^{15}M_\odot$ have col-

* Work supported in part by the Department of Energy, contract DE-AC03-76SF00515 and by the National Science Foundation, grant PHY-81-15541.

Invited talk presented at the 8th John Hopkins Workshop:
Particles and Gravity, Baltimore, Maryland, June 20-22, 1984

lapsed. Partly because this type of DM has been the most intensively studied, a number of potential problems have been identified — for example, the late formation of supercluster "pancakes", at $z_p \lesssim 2$,[10] which subsequently fragment into galaxies. However, the best limits on galaxy ages coming from globular clusters and other stellar populations, plus the possible association of QSO's with galactic nuclei, indicate that galaxy formation took place before $z = 3$.[11] This is inconsistent with the "top-down" neutrino theory, in which superclusters form before galaxies rather than after them.

Another problem with the neutrino picture is that large clusters of galaxies can accrete neutrinos more efficiently than ordinary galactic halos, which have lower escape velocities. One-dimensional numerical simulations predict that the ratio of total to baryonic mass M/M_b should be ~ 5 times larger for clusters ($\sim 10^{14}M_\odot$) than for ordinary galaxies ($M \sim 10^{12}M_\odot$).[12] While there is evidence that the mass-to-light ratio M/L does increase with scale, there is also considerable evidence that the more physically meaningful ratio of total to luminous mass M/M_{lum} remains constant from large clusters through groups of galaxies, binary galaxies, and ordinary spirals. (M_{lum}, which is the mass visible in galactic stars and gas plus hot, X-ray emitting gas, is $\leq M_b$, since an unknown fraction of the baryons is invisible—*e.g.,* in the form of diffuse ionized intergalactic gas at $T \sim 10^4$ K.)

This is illustrated in Fig. 1, which presents the available data for M/L and M/M_{lum}. The fact that the total-to-luminous mass of rich clusters is similar to that of galaxies including their massive halos, even though the clusters' mass-to-light ratio is larger, is due mainly to the different stellar population in the ellipticals, and the large contribution of X-ray emitting gas to M_{lum}, in rich clusters. (In very rich clusters such as Coma, there is $\sim 2 - 5$ times as much mass in hot gas as there is in stars.)

Finally, preliminary velocity dispersion data for Draco, Carina, and Ursa Minor as well as theoretical arguments[15] suggest that a significant amount of DM may reside in dwarf spheroidal galaxies. Because of the low velocity dispersion of dwarf galaxies, phase space constraints give a lower limit of $m >$ 500 eV for the mass of particles comprising this DM.[16] The present velocity dispersion estimates are uncertain owing to possible stellar oscillations, mass outflow, and binary motions, but these effects can be discovered and eliminated with careful monitoring. The mass limit of 500 eV would rule out neutrinos as the halo DM in dwarf spheroidal galaxies. If we assume that the DM has essentially the same composition everywhere, as is suggested by the constancy of M/M_{lum} in Fig. 1, then the DM is not neutrinos.[17]

The DM in the dwarf spheroidal halos is probably not warm DM either. Warm DM first collapses on a scale $\sim 10^{11}M_\odot$ with velocity dispersion $\sigma \sim$

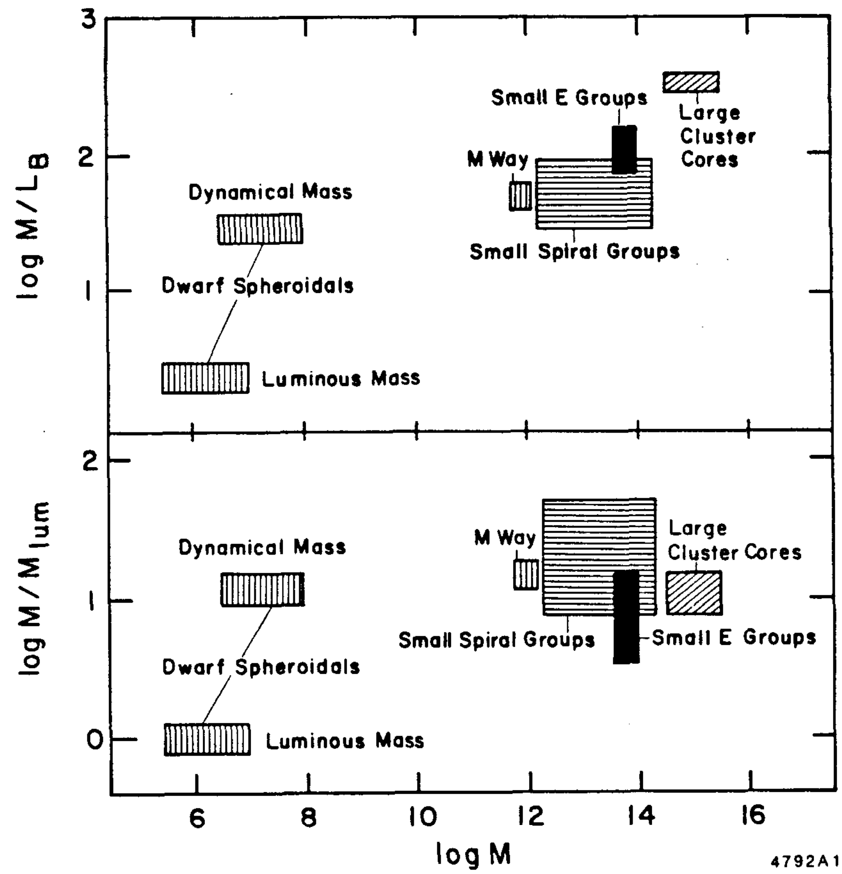

FIGURE 1. Mass-to-light ratio, M/L_B, and total-to-luminous mass, M/M_{lum}, for structures of various size in the universe. Although M/L_B increases systematically with mass, the more physically meaningful ratio M/M_{lum} appears to be constant on all scales within the errors.[18]

$10^2 km/s$, and too little could be captured by dwarf spheroidals, having $\sigma \sim 10\, km/s$, to form the heavy halos indicated by the observations.

Besides the evidence just summarized against hot and warm DM, a further reason to consider cold DM is the existence of several plausible physical candidates, including axions of mass $\sim 10^{-5}$ eV; [21,22] heavy stable particles, such as the photino, with a mass $\gtrsim 0.5$ GeV and very weak interactions;[23] and primordial black holes[24] with $10^{17} g \gtrsim m_{PBH} \gtrsim M_{\odot}$.[7] Still another exotic cold DM candidate has recently been proposed by Witten: "nuggets" of $u - s - d$ symmetric quark matter.[25] There is thus no shortage of cold DM candidate particles — although there is admittedly no direct evidence that any of them actually exists.

2. THE COLD DM FLUCTUATION SPECTRUM

We will follow the current conventional wisdom and assume that the primordial fluctuations were adiabatic. In the standard formulation, fluctuations $\delta \equiv \delta\rho/\rho$ grow as $\delta \sim a^2$ on scales larger than the horizon, where $a = (1+z)^{-1}$ is the scale factor normalized to 1 at the present. When a fluctuation enters the horizon in the radiation-dominated era, the photons (together with the charged particles) oscillate as an acoustic wave, and the non-interacting neutrinos freely stream away (they are still relativistic, since in the cold DM case their masses are $\ll 30$ eV). As a result, the main driving terms for the growth of δ_{DM} disappear and the growth accordingly stagnates ("stagspansion") until matter dominates; see Fig. 2. Matter domination first occurs at $z = z_{eq}$, where

$$\begin{aligned} z_{eq} &= 4.2 \times 10^4 h^2\, \Omega\, (1 + 0.68 N_\nu)^{-1} \\ &= 2.5 \times 10^4 h^2\, \Omega \text{ for } N_\nu = 3 \end{aligned} \tag{1}$$

The first study of the growth of cold DM fluctuations was the numerical calculations of Peebles,[4] who for simplicity ignored neutrinos: $N_\nu = 0$ in (1). Subsequent numerical calculations have included the effects of the known neutrino species ($N_\nu = 3$, $m_\nu \approx 0$) both outside and inside the horizon.[2,5,6,26,27,28] Numerically, the largest effect of including neutrinos is the change in z_{eq}.

It is instructive to make the further approximation of setting $\delta_{\gamma+b} = \delta_\nu = 0$ once a fluctuation is inside the horizon. Then one can analytically match the solution for $a > a_{\text{horizon}}$

$$\delta_{DM}(a) = A_1 D_1(a) + A_2 D_2(a)\ , \tag{2}$$

$$D_1 = 1 + 1.5y \quad \text{where} \quad y = a/a_{eq}\ , \tag{3a}$$

$$D_2 = D_1 \ell n \left[\frac{(1+y)^{1/2} + 1}{(1+y)^{1/2} - 1} \right] - 3(1+y)^{1/2}\ . \tag{3b}$$

to the growing mode $\delta_{DM} \sim a^2$ for $a < a_{\text{horizon}}$. Matching the derivatives requires $A_2 D_2$ comparable to $A_1 D_1$ but opposite in sign. For $a \gg a_{\text{horizon}}$ only the growing solution D_1 survives, which explains the moderate growth in δ_{DM} between horizon crossing and matter dominance. In the limit of large k, one finds $\delta_k \propto k^{n/2-2} \ell n\, k$. Correspondingly, for $M \ll M_{eq} \approx 10^{16} M_\odot$, the rms fluctuation in the mass within a random sphere containing average mass M is $\delta M/M \propto |\ell n\, M|^{3/2}$. Some authors have considered only the Meszaros solution (3a) and erroneously inferred that the fluctuation spectrum would be essentially flat for $M < M_{eq}$ for a Zeldovich primordial spectrum, which would then be inconsistent with observations.[29]

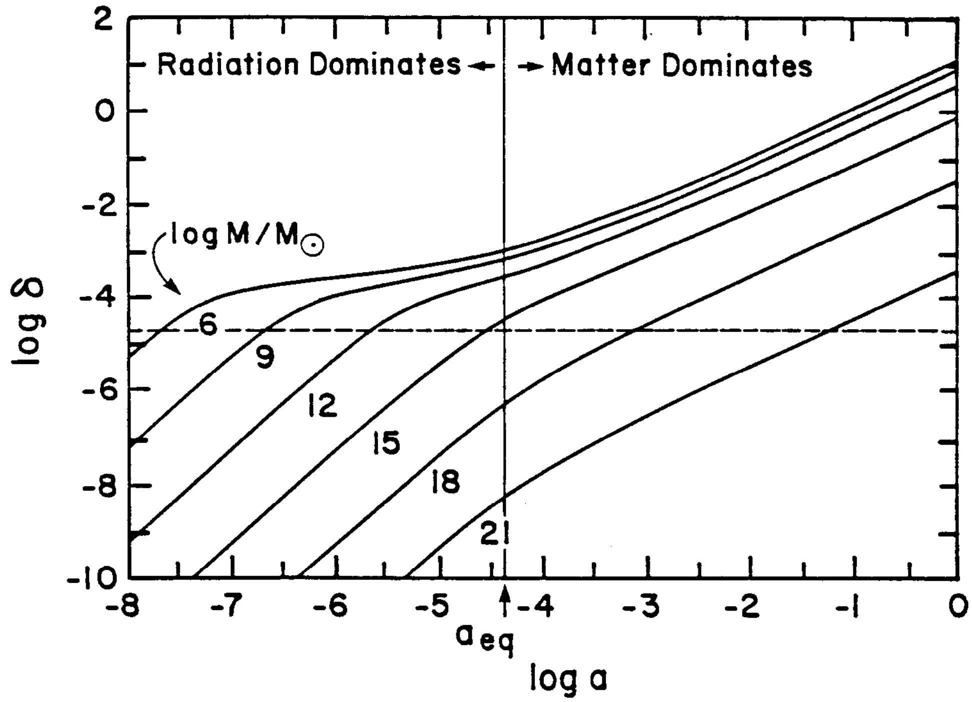

FIGURE 2. Numerical results for the growth of $\delta = k^{3/2}\delta_k$ versus scale factor a for fluctuations of various masses $M = \frac{4}{3}\pi^4 k^{-3}\rho_c$. The curves are drawn for $n = 1$, $\Omega = h = 1$, and a baryonic to total mass ratio of 0.1. The vertical line represents the value of a when the universe becomes matter dominated, and the dashed line shows the (constant for $n = 1$) value of δ when each mass scale crosses the horizon. These curves illustrate the stagnation of perturbation growth after small mass scales cross the horizon and show why at late times $\delta(k)$ is nearly flat for large k (small M). (From Ref. 5.)

Our numerical results for $\delta M/M$ are shown in Fig. 3 for $\Omega = h = 1$, assuming a Zeldovich ($n = 1$) spectrum (reflected in $\delta M/M \propto M^{-2/3}$ for $M > M_{eq}$). For either h or Ω less than unity, $\delta M/M$ is somewhat flatter.[7,6]

3. GALAXY AND CLUSTER FORMATION

The key features of galaxy formation in the cold DM picture are these: after recombination (at $z_{rec} \approx 1300$) the amplitude of the baryonic fluctuations rapidly grows to match that of the DM fluctuations; smaller-mass fluctuations grow to nonlinearity and virialize, and then are hierarchically clustered within successively larger bound systems; and finally the ordinary matter in bound systems of total mass $\sim 10^{8-12} M_\odot$ cools rapidly enough within their DM halos

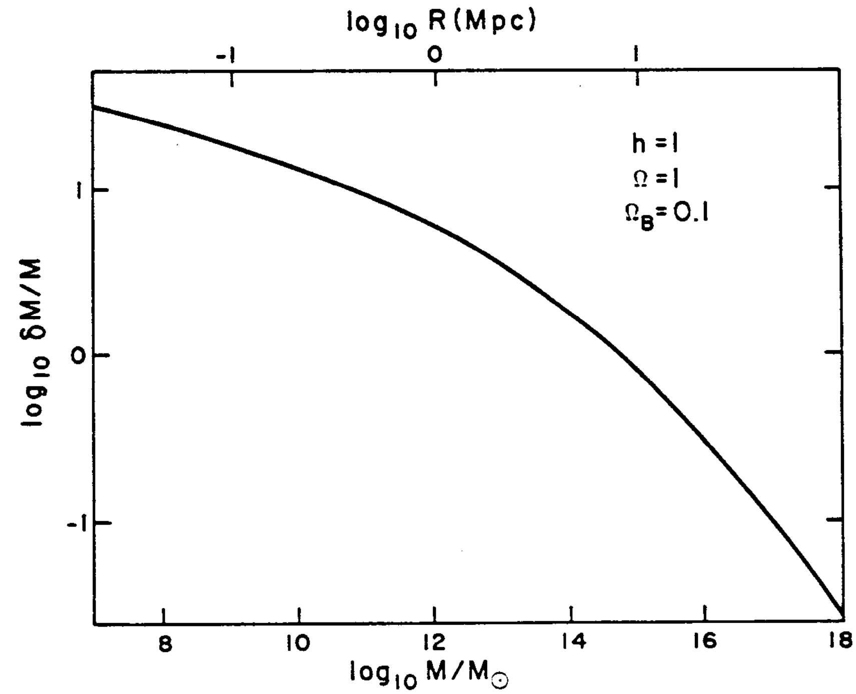

FIGURE 3. The rms mass fluctuations within a randomly placed sphere of radius R in a cold DM universe. The curve is normalized[4] at 8 Mpc and assumes a primordial Zeldovich ($n = 1$) fluctuation spectrum, and $h = \Omega = 1$. (From Ref. 2.)

to form galaxies, while larger mass fluctuations form clusters.

At any mass scale M, when the fluctuation $\delta M/M$ approaches unity, nonlinear gravitational effects become important. The fluctuation then separates from the Hubble expansion, reaches a maximum radius, and begins to contract. Spherically symmetric fluctuations, for example, contract to about half their maximum radii. During this contraction, violent relaxation[30] due to the rapidly varying gravitational field converts enough potential energy into kinetic energy for the virial theorem, $\langle PE \rangle = -2\langle KE \rangle$, to be satisfied. After virialization, the mean density within a fluctuation is roughly eight times the density corresponding to the maximum radius of expansion.[31]

Since the cold-DM fluctuation spectrum $\delta M/M$ is a decreasing function of M, smaller mass fluctuations will, on the average, become nonlinear and begin to collapse at earlier times than larger mass fluctuations. Small mass bound systems are subsequently clustered within larger mass systems, which go nonlinear at a later time. This hierarchical clustering of smaller systems into larger and yet larger gravitationally bound systems begins at the baryon Jeans mass ($M_{J,b} \sim 10^6 M_\odot$ at recombination) and continues until the present time. The baryonic substructures within larger mass clusters will be disrupted by subsequent virialization of the clusters unless significant mass segregation between baryons and DM has occured prior to cluster virialization. Hence, in order to maintain their existence as a separate substructure, the baryons must cool and gravitationally condense within their massive DM halos *before* virialization occurs on larger scales.[32]

Figure 4 shows the density of ordinary (baryonic) matter versus internal kinetic energy (temperature) of typical fluctuations of various sizes, just after virialization, calculated from $\delta M/M$ of Fig. 3. This is superimposed upon the Rees-Ostriker[33] cooling curves (for which cooling time equals gravitational free fall time) and data on galaxies (with kinetic energy determined from rotation velocity for spirals and velocity dispersion for ellipticals).[34]

Fluctuations that start with greater amplitude than average will turn around earlier, at higher density, and thus lie below the virialization curve on Fig. 4. As the baryons in a virialized fluctuation dissipate, their density will initially increase at constant T within the surrounding isothermal halo of dissipationless material (DM), and then T will increase as well when the baryon density exceeds the DM density, as suggested by the dashed line in the figure. The Zeldovich primordial spectrum is more consistent with the data on Fig. 4 than an $n = 2$ (or $n = 0$) primordial spectrum, which lies too low (too high) on the figure compared to the galaxies. With the Zeldovich spectrum, the important conclusion is that one should observe dissipated systems with large halos having total mass $10^8 M_\odot \lesssim M \lesssim 10^{12} M_\odot$. This is essentially the range of observed galaxy masses.

While the $n_b - T$ diagram (Fig. 4) is useful for comparing data and predictions with the cooling curves, it is also useful to consider total mass M versus T, as in Fig. 5. This avoids having to take into account the differing amounts of baryonic dissipation suffered by various galaxies. The heavy solid and dashed curves again correspond to the $n = 1$ cold DM spectrum, for ($\Omega = 1, h = 0.5$) and ($\Omega = 0.2, h = 1$) respectively. It is striking that the galaxies in the $M - T$ diagram lie along lines of roughly the same slope as these curves. This occurs because the effective slope of the $n = 1$ cold DM fluctuation spectrum in the galaxy mass range is $n_{eff} \approx -2$, which corresponds to the empirical Tully-Fisher and Faber-Jackson laws: $M \propto v^4$. The light dashed lines in Fig. 5 are the post-virialization curves for primordial fluctuation spectra with $n = 0$ (white noise) and $n = 2$. Again, the $n = 1$ (Zeldovich) spectrum is evidently the one that is most consistent with the data.

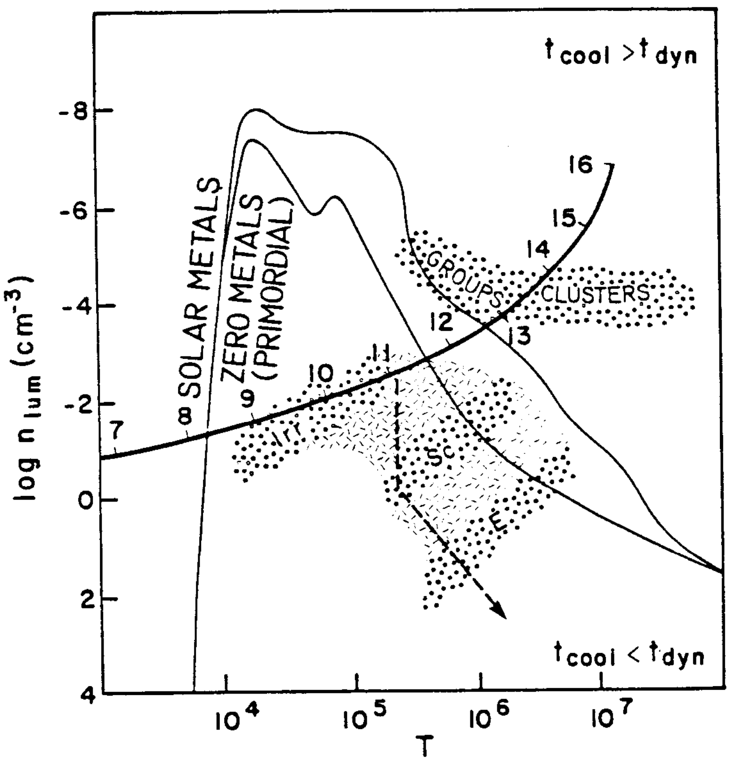

FIGURE 4. The baryonic density versus temperature as root-mean-square perturbations having total mass M become nonlinear and virialize. The numbers on the tick marks are the logarithm of M in units of $M_\odot$. This curve assumes $n = 1$, $\Omega = h = 1$, and a baryonic to total mass ratio of 0.07. The region where baryons can cool within a dynamical time lies below the cooling curves. Also shown are the positions of observed galaxies, groups, and clusters of galaxies. The dashed line represents a possible evolutionary path for dissipating baryons. (From Ref. 2.)

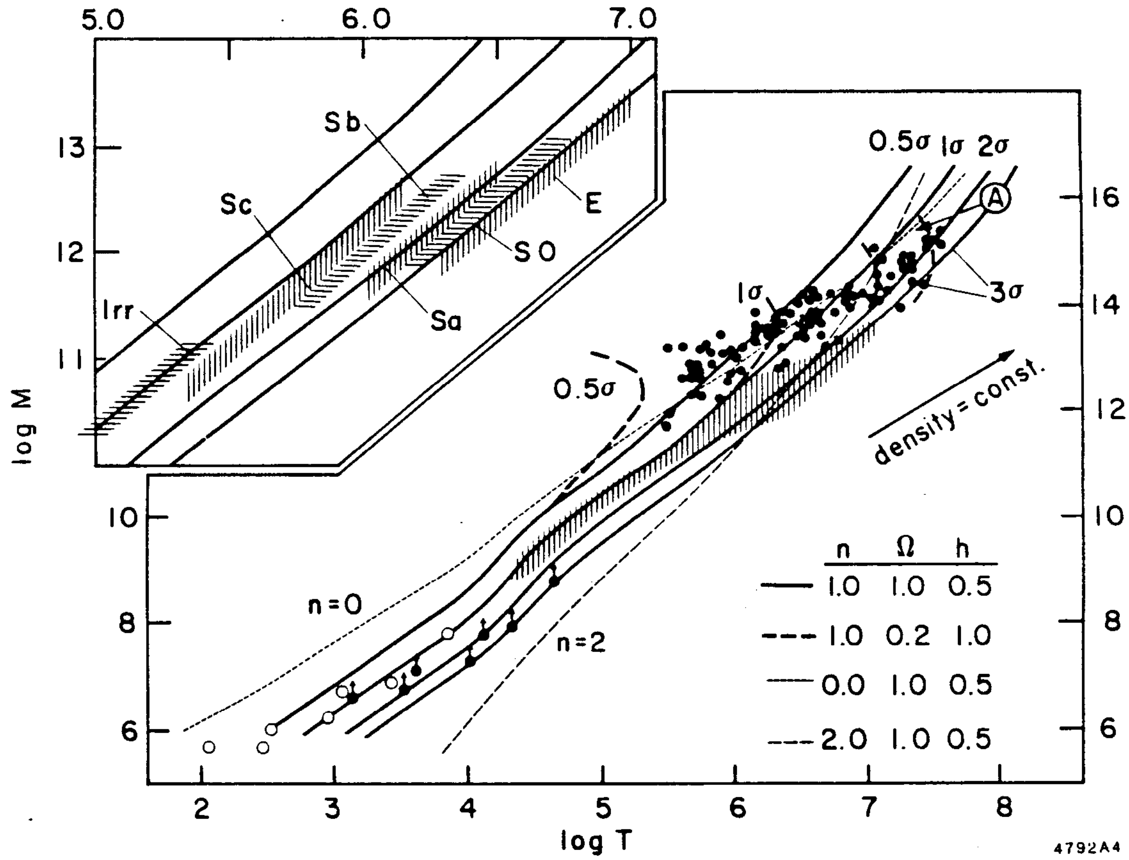

FIGURE 5. Total mass M versus virial temperature T. The quantity T is $\mu V^2/3k$, where μ is mean molecular weight (≈ 0.6 for ionized, primordial H + He) and k is Boltzmann's constant. M for groups and clusters is total dynamical mass. For galaxies, M is assumed to be 10 M_{lum} (corresponding to Fig. 1). If dwarf spheroidals actually have $M/L_B = 30$, they may have suffered baryon stripping[16], in which case M is a lower limit (arrows). Details of the region occupied by massive galaxies are shown in the inset in upper left.

Model curves represent the equilibria of structures that collapse dissipationlessly from the cold dark matter initial fluctuation spectra with $n = 1$. The curves labeled 1σ refer to fluctuations with $\delta M/M$ equal to the rms value. Curves labeled 0.5σ, 2σ, and 3σ refer to fluctuations having 0.5, 2, and 3 times the rms value. Heavy curves: $\Omega = 1$, $h = 0.5$; dashed curves: $\Omega = 0.2$, $h = 1$; these cases were chosen to span the astrophysically interesting range. In addition to the n = 1 curves, two 1σ curves for $n = 0$ and $n = 2$ are also shown (light dashes).

Major conclusions from the figure: 1) Either set of curves for $n = 1$ (Zeldovich spectrum) provides a good fit to the observations over 9 orders of magnitude in mass. Curves with $n = 0$ and $n = 2$ do not fit as well. 2) The apparent gap between galaxies and groups and clusters in Fig. 4 (which stems from baryonic dissipation) vanishes in this figure, and the clustering hierarchy is smooth and unbroken from the smallest structures to the largest ones. 3) The Fisher-Tully and Faber-Jackson laws for galaxies ($M \propto V^4$ or T^2) arise naturally as a consequence of the slope of the cold DM fluctuation spectrum in the mass region of galaxies. 4) Groups and clusters are distributed around the $n = 1$ loci about as expected. The apparent upward trend among the groups is not physically meaningful but arises from their selection as minimum-density enhancements (see constant-density arrow). 5) The exact locations of galaxies are somewhat uncertain. In particular, the temperatures of E's and S0's may be overestimated owing to the use of nuclear rather than global velocity dispersions. Taken at face value, however, the data suggest that early-type galaxies (E's and S0's) arise from high-$\delta M/M$ fluctuations, whereas late-type galaxies (Sc's and Irr's) arise from low-$\delta M/M$ fluctuations. 6) Groups and clusters appear to fill a wider band than galaxies. If real, this difference may indicate that very weak, low-$\delta M/M$ fluctuations on the mass scale of galaxies once existed but did not give rise to visible galaxies. This suggests further that galaxy formation, at least in some regions of the universe, may not have been fully complete and that galaxies are therefore not a reliable tracer of total mass. 7) There seems to be a real trend along the Hubble sequence to increasing mass among early-type galaxies. Neither this trend nor the rather sharp demarcation between galaxies and groups and clusters is fully understood. (This figure is from Ref. 7.)

The points in Fig. 5 represent essentially all of the clusters identified by Geller and Huchra[35] in the CfA catalog within 5000 km s^{-1}. The cluster data lie about where they should on the diagram, and even the statistics of the distribution seem roughly to correspond to the expectations represented by the $0.5, 1, 2,$ and 3σ curves.

Notice that spiral galaxies lie roughly along the 1σ curve while elliptical galaxies lie along the 2σ curve. Although this displacement is not large compared to the uncertainties, it is consistent with the fact that more than half of all galaxies are spirals, while only about 15 percent are ellipticals. In hierarchical clustering scenarios, it seems likely that the higher σ fluctuations will develop rather smaller angular momenta, as measured by the dimensionless parameter λ

($= JE^{\frac{1}{2}}G^{-1}M^{-\frac{5}{2}}$). There are two reasons for this: high-overdensity fluctuations collapse earlier than average fluctuations, and are thus typically surrounded by a relatively homogeneous matter distribution;[36] also, higher amplitude fluctuations are typically rounder[37] and consequently have lower quadrupole moments. Both effects result in less torque. This difference appears to exist with either white noise or a flatter spectrum, but to be somewhat larger in the latter case. If high σ fluctuations have little angular momentum, their baryons can collapse by a large factor in radius, forming high-density ellipticals and spheroidal bulges, as shown in Fig. 4. Since, with a flat spectrum, higher σ fluctuations occur preferentially in denser regions destined to become rich clusters, one expects[38] to find more ellipicals there—as is observed.[39] Note that the rich clusters lie along the same 2 and 3σ curves in Fig. 5 as do the elliptical galaxies.

Note also that while the galaxy data lies below the rms virialization curve, the data on groups and clusters of galaxies lies more or less evenly around it. This suggests that galaxy formation may be an inefficient process, with lower-amplitude fluctuations of galaxy mass not giving rise to visible galaxies.[40]

Presumably the collapse of the low-λ protoellipical galaxies is halted by star formation well before a flattened disk can form, yielding a stellar system of spheroidal shape. The mechanism governing the onset of star formation in these systems is unfortunately not yet understood, but may involve a threshold effect which sets in when the baryon density exceeds the DM halo density by a sufficient factor.[32,41] Disks (spirals and irregulars) form from average, higher-λ protogalaxies, which, for a given mass, are larger and more diffuse than their protoelliptical counterparts. The collapse of disks thus occurs via relatively slow infall of baryons from $\sim 10^2$ kpc, halted by angular momentum. Infall from such distances is consistent both with the extent of dark halos inferred from observations and with the high angular momenta of present-day disks ($\lambda \sim 0.4$).[42] The location of the galaxies in Fig. 4 is consistent with these ideas if the baryons in all galaxies collapsed by roughly the same factor, about an order of magnitude, but somewhat less for late-type irregulars and somewhat more for early-type E's and spheroidal bulges.

It has been theorized that the Hubble sequence originates in the distribution of either the initial angular momenta or else the initial densities[44] of protogalaxies. However, if overdensity and angular momentum are linked, with the high-σ fluctuations having lower λ, then these two apparently competitive theories become the opposite sides of the same coin.

It is interesting to ask whether the cold DM picture can account for the wide range of morphologies displayed by clusters of galaxies in X-ray[45] and optical-band[46] observations, ranging from regular, apparently relaxed configurations to complex, multicomponent structures. Preliminary results are encouraging. In particular, simulations show that large central condensations form quickly and can grow by subsequent mergers to form cD galaxies if most of the DM is in halos around the baryonic substructures, as expected for cold DM, but not if the DM is distributed diffusely.[47]

Consider finally the difference in Fig. 5 between the solid and dashed lines. The dashed lines, representing a lower-density universe ($\Omega = 0.2$), curve backward at the largest masses and lie far away from the circle representing the cores of the richest clusters, Abell classes 2 and 3. Since these regions of very high galaxy density contain at least several percent of the mass in the universe, the circle should lie between the 2 and 3σ lines (assuming Gaussian statistics). It does so for the solid ($\Omega = 1$) lines, but not for the dashed lines. At face value, this is evidence favoring an Einstein-de Sitter universe for cold DM. However, there are at least two reasons why this argument should probably not be taken too seriously. First, the velocity dispersions represented by the Abell cluster circle in Fig. 5 correspond to the cluster cores. The model curves on the other hand refer to the entire virialized cluster, over which the velocity dispersion is considerably lower (as indicated by the arrow attached to the circle in Fig. 5). Second, the assumption of spherical symmetry used in obtaining both sets of curves in the figure is only an approximation. The initial collapse is probably often quite anisotropic—more like a Zeldovich pancake than a sphere. It is therefore preferable to compare these data with N-body simulations rather than with the simple model represented by the curves in Fig. 5. This will require N-body simulations of large dynamical range, which can perhaps be achieved by putting many mass points into one cell of the P^3M−type simulations.[48] Until this becomes possible the data in the figure do not allow a clear-cut discrimination between the $\Omega = 0.2$ and $\Omega = 1$ cases, especially if the Hubble parameter h is allowed to vary simultaneously within the observationally allowed range, as has been assumed.

Other data are also relevant to the determination of Ω, of course.[7] For example, the latest observations of small-angle fluctuations in the cosmic background radiation[49] imply[26,27] $\Omega \geq 0.2h^{-4/3}$, unless there is significant reheating of the intergalactic medium after recombination.[50]

4. REMARKS

A universe with ~ 10 times as much cold dark matter as baryonic matter provides a remarkably good fit to the observed universe. This model predicts roughly the observed mass range of galaxies, the dissipational nature of galaxy collapse, and the observed Faber-Jackson and Tully-Fisher relations. It also gives dissipationless galactic halos and clusters. In addition, it may also provide natural explanations for galaxy-environment correlations and for the differences in angular momenta between ellipticals and spiral galaxies. Finally, the cold DM picture seems reasonably consistent with the observed large-scale clustering, including superclusters and voids.[51] In short, it appears to be the best model presently available and merits close scrutiny and testing in the future.

Acknowledgments

This paper is based on research done in collaboration with George Blumenthal, Sandra Faber, and Martin Rees[52] — all of whom I would like to thank for their efforts to teach me astrophysics! I am grateful to Sidney Drell for the hospitality of SLAC, and I acknowledge partial support from the National Science Foundation under grant PHY-81-15541.

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13. If the velocity dispersion data for the dwarf spheroidal galaxies are interpreted to imply heavy halos, the upper estimates in the figure result. The lower estimates follow from assuming that all the mass is visible. The former are probably more realistic, as discussed in Ref. 7, which is also the source of this figure and should be consulted for sources for the data displayed.

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17. It is also possible, of course, that there are two or more forms of DM or that the DM is unstable or a decay product (possibilities discussed further in Roncadelli, M. and Primack, J. R. (SLAC-PUB-3304, 1984) and references therein).

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