

AN INTERPRETATION OF THE FORWARD SLOPE  
IN PHOTOPRODUCTION OF  $\rho^0$  †

Y. Avni††

Stanford Linear Accelerator Center  
Stanford University, Stanford, California 94305

ABSTRACT

Forward photoproduction of  $\rho^0$  is studied in the framework of the Dual Absorptive Model. The observed data is shown to be consistent with the assumption that the pomeron and f amplitudes in  $\gamma p \rightarrow \rho^0 p$  are similar to those in  $\pi^\pm p \rightarrow \pi^\pm p$ . If this is true, f exchange has an important role in generating the slope of  $\gamma p \rightarrow \rho^0 p$ . Implications for electroproduction of  $\rho^0$  and  $\pi^0$  are discussed. The smallness of the slope of  $\gamma p \rightarrow \phi p$  as compared to  $\gamma p \rightarrow \rho^0 p$  is explained.

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†† Fulbright Scholar; On leave from the Weizmann Institute of Science, Rehovot, Israel.

The dual absorptive model<sup>1</sup> was recently shown to describe correctly the  $t$  dependence of the imaginary part of two-body meson-baryon scattering amplitudes,<sup>2,3</sup> for  $|t| \lesssim 1 \text{ BeV}^2$  and  $p_{\text{lab}} \gtrsim 3 \text{ BeV}/c$ . In this model, the imaginary part of a nondiffractive amplitude is peripheral when represented in the impact-parameter ( $b$ ) space. The  $t$ -dependence of such amplitudes is of the form " $J_{\Delta\lambda}$ " ( $R \sqrt{-t}$ ), where  $R$  is the (nondiffractive) interaction radius,  $\Delta\lambda$  is the net helicity flip,\* and " $J_{\Delta\lambda}$ " is a function which has the same gross behavior as  $J_{\Delta\lambda}$  (a Bessel function of order  $\Delta\lambda$ ). The exact functional form of " $J_{\Delta\lambda}$ " depends on the finer details of the impact parameter profile. It has become a common practice to parameterize " $J_{\Delta\lambda}$ " in the form  $e^{Bt} J_{\Delta\lambda}(R \sqrt{-t})$ , where  $2 \sqrt{B}$  is the width  $\sigma$  of the impact parameter profile. This choice seems to yield a nice description of experimental data.<sup>2,3</sup> The diffractive amplitudes are assumed to be central in  $b$ -space, and to have no structure (zeroes, dips, etc.) as functions of  $t$ . These amplitudes are assumed to be mainly nonflip, and mainly pure imaginary, and therefore parameterized by  $ie^{Bt} P$ .

In this note we wish to apply the dual absorptive model to the process  $\gamma p \rightarrow \rho^0 p$  near the forward direction. The exact values of the forward differential cross section,  $(d\sigma/dt)_0$ , and of the forward slope,  $S$ , vary from one experimental group to another, and also depend on the method of the analysis in which the  $\rho^0$  is extracted.<sup>4</sup> There are however two gross features of the data which seem to be commonly accepted: (a)  $(d\sigma/dt)_0$  decreases as a function of energy, (b)  $S$  has no strong energy dependence, and has typical values of  $6 - 8 \text{ BeV}^{-2}$  for  $4 \lesssim E_\gamma \lesssim 10 \text{ BeV}$ . These two properties exclude an explanation of the data for  $E_\gamma \gtrsim 4 \text{ BeV}$  using only pomeron exchange: pomeron amplitudes have a very weak energy dependence at  $t=0$ , and exhibit significant "shrinkage", i. e., increasing  $S$  with increasing energy. It is therefore of interest to try to explain

the data using P and f exchanges. \*\* The dual absorptive model makes such an attempt possible, providing the t-dependence of Im f.

In general, such an analysis should be done in a deductive manner by actually fitting the data with P- and f-amplitudes. This procedure is difficult to apply in our case, because the "data" is not defined unambiguously, as explained above. We therefore take an inductive approach--we will demonstrate that the observed  $(d\sigma/dt)_0$  and S may actually be obtained by a shrinking pomeron + peripheral f. We take the parameters  $B_p$ ,  $B_f$  and R of the P- and f-amplitudes from a recent fit to  $\pi p$  scattering,<sup>3</sup> and the only parameter which remains is the relative magnitude f:P. We will make several estimates of this ratio, all leading to approximately the same values of S. Our results show that 1 - 1.5  $\text{BeV}^{-2}$  out of 6 - 8  $\text{BeV}^{-2}$  are contributed to S by the f. \*\*\*

We assume the dominance of amplitudes which conserve helicity at the meson- and at the nucleon-vertex (there are two such independent amplitudes). We parameterize each such amplitude near  $t=0$  by

$$P = iA_p e^{B_p t} \quad (1)$$

$$\text{Im } f = \frac{a_f}{\sqrt{E_\gamma}} e^{\left(B_f + \frac{R^2}{4}\right)t} \quad (2)$$

We assume that  $A_p$  and  $a_f$  are (approximately) energy-independent, so that Im P(t=0) is constant, and Im f(t=0) has a Regge behavior. We have also used the approximation of  $J_0(R\sqrt{-t}) \approx e^{(R^2/4)t}$  for  $|t| \lesssim .1 \text{ BeV}^2$ . Neglecting terms of order  $|f|^2$  we find

$$\left(\frac{d\sigma}{dt}\right) = A_p^2 \left[ e^{2B_p t} + \frac{a_f}{A_p} \frac{2}{\sqrt{E_\gamma}} e^{\left(B_p + B_f + \frac{R^2}{4}\right)t} \right] \quad (3)$$

$$\left(\frac{d\sigma}{dt}\right)_0 = A_p^2 \left[ 1 + \frac{a_f}{A_p} \frac{2}{\sqrt{E_\gamma}} \right] \quad (4)$$

$$S = \frac{2B_p + \frac{a_f}{A_p} \frac{2}{\sqrt{E_\gamma}} (B_p + B_f + \frac{R^2}{4})}{1 + \frac{a_f}{A_p} \frac{2}{\sqrt{E_\gamma}}} \quad (5)$$

In these expressions,  $A_p$  and  $a_f$  are effective coefficients determined from the sum over the two helicity amplitudes.  $(d\sigma/dt)_0$  clearly decreases as a function of  $E_\gamma$ , and the two parameters  $A_p^2$  and  $a_f/A_p$  could be determined from a precise and unambiguous measurement of  $(d\sigma/dt)_0$  as a function of  $E_\gamma$ . The slope  $S$  depends only on  $a_f/A_p$  ( $B_p$ ,  $B_f$ , and  $R$ , are taken from  $\pi^\pm p \rightarrow \pi^\pm p$  at the corresponding value of  $s$ ). We make three estimates for the parameter  $a_f/A_p$ :

1. Use Vector Dominance to relate  $\gamma p \rightarrow \gamma p$  to  $\gamma p \rightarrow \rho^0 p$ , and take  $a_f/A_p$  from<sup>4</sup>

$$\sigma_T(\gamma p) \approx 100 + 53 E_\gamma^{-1/2} + 12 E_\gamma^{-1/2} (\mu b) \quad (6)$$

$$\sigma_T(\gamma p) = \text{Im } \tilde{P}(t=0) + \text{Im } \tilde{f}(t=0) + \text{Im } \tilde{A}_2(t=0) \quad (7)$$

which yields  $a_f/A_p = 0.53 \text{ BeV}^{1/2}$ .

2. Use the similarity between  $d\sigma/dt(\gamma p \rightarrow \rho^0 p)$  and  $d\sigma/dt(\pi^+ p) + d\sigma/dt(\pi^- p)$ , and assume that  $a_f/A_p$  has the same value as in  $\pi^\pm p$  elastic scattering.

We use the value found by Davier<sup>3</sup>  $a_f/A_p = 0.80 \text{ BeV}^{1/2}$ .

3. Parameterize the fit of Wolf<sup>4</sup> to  $(d\sigma/dt)_0$  in the form

$$\left(\frac{d\sigma}{dt}\right)_0 = A_p^2 \left[ 1 + \frac{a_f}{A_p} \frac{2}{\sqrt{E_\gamma}} + \frac{C}{E_\gamma} \right] \quad (8)$$

again we find  $a_f/A_p = 0.80 \text{ BeV}^{1/2}$ , however  $C$  turns out to be  $1.6 \text{ BeV}$ , while  $(a_f/A_p)^2 \approx .64 \text{ BeV}$ . In this case  $|\text{Re } f|^2$  cannot be neglected for

small values of  $E_\gamma$ , and has to be taken into account. The dual absorptive model does not specify the form of  $|\text{Re } f|^2$ , so we parameterized it in the form  $A_p^2 (.96/E_\gamma)^{2Dt}$  where  $D$  was assumed to be in the range  $-(B_f + \frac{R^2}{4}) \lesssim D \lesssim (B_f + \frac{R^2}{4})$ . The correct expression for  $S$  in this case is

$$S = \frac{2B_p + \frac{a_f}{A_p} \frac{2}{\sqrt{E_\gamma}} (B_p + B_f + \frac{R^2}{4}) + \left(\frac{a_f}{A_p}\right)^2 \frac{1}{E_\gamma} 2(B_f + \frac{R^2}{4}) + \left[ C - \left(\frac{a_f}{A_p}\right)^2 \right] \frac{1}{E_\gamma} 2D}{1 + \frac{a_f}{A_p} \frac{2}{\sqrt{E_\gamma}} + \frac{C}{E_\gamma}} \quad (9)$$

The resulting values of  $S$  for  $E_\gamma = 4, 9, 16$  BeV are summarized in Table I. Those values agree with the observed slope of  $6 - 8 \text{ BeV}^2$  (which has the ambiguities as mentioned above).

The main conclusion from our inductive approach is that the observed  $(d\sigma/dt)_0$  and  $S$  in  $\gamma p \rightarrow \rho^0 p$  are consistent with the dual absorptive model and that the parameters of the  $P$ - and  $f$ -amplitudes in  $\gamma p \rightarrow \rho^0 p$  are very similar to those in  $\pi^\pm p$ . The exact value of  $a_f/A_p$  does not have any crucial importance as long as it is not very far from the value given by Vector Dominance. We can thus understand--in the framework of the dual absorptive model--the weak energy dependence of the slope in the range  $E_\gamma = 4 - 9$  BeV even though the Pomeron itself is shrinking significantly, and in particular we find that approximately  $1 - 1.5 \text{ BeV}^{-2}$  are contributed to  $S$  by the presence of the  $f$ .\*\*\*

Our analysis has an interesting implication for electroproduction experiments. The idea that the interaction radius of a virtual photon depends on its virtual mass  $-Q^2$  (Ref. 5), has led to the suggestion that the process  $ep \rightarrow ep\pi^0$  may serve as a crucial test of dip mechanisms.<sup>6</sup> According to this suggestion, the slope of the process  $ep \rightarrow e\rho^0 p$  serves to measure the interaction radius,

and a dependence of the slope  $S$  on  $Q^2$  is interpreted as a varying radius of interaction. Rewriting the expression (5) for  $S$  in terms of radii we obtain

$$S = \frac{2 \frac{R_p^2}{4} + \frac{a_f}{A_p} \frac{2}{\sqrt{E_\gamma}} \left( \frac{R_p^2}{4} + \frac{\sigma_f^2}{4} + \frac{R^2}{4} \right)}{1 + \frac{a_f}{A_p} \frac{2}{\sqrt{E_\gamma}}} \quad (10)$$

$R_p$  ( $R$ ) is the diffractive (nondiffractive) radius,  $R_p = 2 \sqrt{B_p}$ , and  $\sigma_f$  is the width of the impact parameter profile of the  $f$ .<sup>1</sup> We note that a change of  $S$  as a function of  $Q^2$  may be caused by a change in  $a_f/A_p$  as a function of  $Q^2$ , even though  $R$ ,  $R_p$  and  $\sigma_f$  are constant. We have, however, an estimate of the possible variation in  $S$  due to variations in  $a_f/A_p$ : if  $a_f/A_p$  goes to zero, or increased by a factor of 2,  $S$  would change approximately by 1 - 1.5  $\text{BeV}^{-2}$ . Therefore, if  $S$  varies as a function of  $Q^2$  by less than that amount, one cannot yet conclude that the interaction radius varies. If, however,  $S$  varies by more than the above amount, it would probably indicate a change in the radius. The experimental situation to date is controversial.<sup>7</sup>

The possibility of a rapidly varying  $a_f/A_p$  as a function of  $Q^2$  is not at all radical. This ratio, in virtual Compton scattering, increases by a factor of 2 when  $Q^2$  is varied from 0 to 1  $\text{BeV}^2$ , if scaling persists down to  $Q^2=0$  when expressed in terms of the scaling variable  $\tilde{\omega}$ .<sup>8</sup> Similar strong dependence of  $a_f/A_p$  on  $Q^2$  in  $ep \rightarrow e\rho^0 p$  should not be surprising.

If, in fact,  $a_f/A_p$  increases as a function of  $Q^2$ , its effect on the slope  $S$  is in the opposite direction to the effect caused by decreasing radii. The two effects can partially mask each other. To separate the two effects one should measure  $S$  as a function of both  $Q^2$  and the energy of the photon.

We conclude with a few remarks:

1. One may question whether the numerical values of the various parameters which we have used are correct. Our main result does not depend crucially on them. The well-known similarity of  $d\sigma/dt(\gamma p \rightarrow \rho^0 p)$  and  $d\sigma/dt(\pi^+ p) + d\sigma/dt(\pi^- p)$  already indicates that similar dynamical mechanisms operate in both reactions. Once the role of the  $f$  in generating the slope of  $d\sigma/dt(\pi^\pm p)$  has been established,<sup>3</sup> a similar situation is to be expected in  $\gamma p \rightarrow \rho^0 p$ .
2. We suggest that  $\gamma p \rightarrow \rho^0 p$  at intermediate energies should not be associated with purely diffractive amplitudes. In particular,  $f$  exchange may have an important role in generating the forward slope. We urge experimental groups to consider this point while analyzing their data. Though no unique method of data analysis exists, some model-dependent analyses, incorporating  $f$  exchange, would be instructive. It would also be interesting to see whether more detailed predictions of the dual absorptive model are realized by the data.
3. Our arguments apply equally well to Compton scattering,\*\*\*\* if one includes also the  $A_2$  exchange (or if one considers  $(\gamma p \rightarrow \gamma p) + (\gamma n \rightarrow \gamma n)$ , or  $\gamma d \rightarrow \gamma d$ ). The general features of the data which we have used are known to be present in this process too.<sup>4</sup> We therefore suggest a non-negligible role of  $f$  and  $A_2$  in generating the forward slope also in this reaction.
4. Photoproduction of  $\phi$  is believed to be pure  $P$  exchange, since the  $f$  and  $A_2$  do not couple to the  $\phi$ . The slope of  $\gamma p \rightarrow \phi p$  should therefore be smaller than the slope of  $\gamma p \rightarrow \rho^0 p$  by approximately  $1.5 \text{ BeV}^{-2}$  for  $4 \lesssim E_\gamma \lesssim 16 \text{ BeV}$ . We thus provide a natural explanation to the well

known fact that the slope in  $\gamma p \rightarrow \phi p$  is smaller than in  $\gamma p \rightarrow \rho^0 p$ .<sup>4</sup>  
A recent measurement of  $S(\gamma p \rightarrow \phi p)$  at 8.5 BeV (Ref. 9) yields a value of  $5.4 \text{ BeV}^{-2}$ , which agrees very well with the predicted value of  $5.8 \text{ BeV}^{-2}$  (pure Pomeron) at 9 BeV.

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## FOOTNOTES

\* We use s-channel helicity amplitudes.

\*\*  $A_2$  exchange is presumably very small. Application of Vector Dominance to  $\gamma p \rightarrow \gamma p$  shows that the contribution of  $A_2$  to  $\gamma p \rightarrow \rho^0 p$  is  $\sim 1/9$  the contribution of f.

\*\*\* Actually by the interference of P with f.

\*\*\*\* Provided the possible  $J=0$  fixed pole has no important role in generating the forward slope.

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TABLE I

The slope of the forward differential cross section in  $\gamma p \rightarrow \rho^0 p$  as a function of  $E_\gamma$ , under three assumptions on the relative magnitude of P and f amplitudes (see text). The last column shows the slope expected when no f amplitude is present. The experimental values are 6 - 8  $\text{BeV}^{-2}$ , and they are not known uniquely as explained in the text.

|                     |    | $\frac{a_f}{A_p} (\text{BeV}^{1/2})$ |     |               |     |
|---------------------|----|--------------------------------------|-----|---------------|-----|
|                     |    | .53                                  | .80 | .80 + Ref     | 0   |
| $E_\gamma$<br>(BeV) | 4  | 6.4                                  | 6.9 | $6.7 \pm 1.6$ | 4.6 |
|                     | 9  | 7                                    | 7.6 | $7.5 \pm 1$   | 5.8 |
|                     | 16 | 7.8                                  | 8.2 | $8.1 \pm .7$  | 6.6 |