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On Confinement through Scalar Field Interactions

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Abstract

The scalar field plays a fundamental role in the investigation of confinement property characterising many particle physics models. This is achieved by coupling this particle directly with gauge fields at the lagrangian level. In this paper we follow the same approach to build an interquark potential [10].
In order to introduce the gravitational effects and inspired from bag models, we implement a scalar field interacting both with the vacuum and the electron field. In this context and with presence of the vacuum condensates, it is possible to derive a more accurate expression for the electron energy.

Keywords: Dilaton, Axion, Confinement

1 Introduction

Bag model is an interesting approach which provides an explanation of confinement property characterising quarks interactions. They are also used to

describe some physical phenomena related to the stability of some particles like electron. According to this approach, one supposes the existence of a cavity which is a space zone filled with scalar condensate. The scalar field appears in many theoretical physics models with different roles. It can be used either as a massless field or a massive one.
In QCD models, the scalar field appears like a pseudo-Goldstone boson[5]. Whereas, in cosmology, it is considered as an hypothetical particle which is a dark matter candidate that may explain the missing mass of universe [6].
The scalar field can interact in different ways. For example in string theory, it couples to super-Yang Mills gauge fields in curved space whereas in the Brans-Dicke model, it couples to the Ricci curvature[8].
For a long time the electron stability was studied using different approaches. These latters faced several difficulties related to the electron finite size, the origin of its mass and the problem with the relativistic transformation properties of the energy and momentum of the electron electromagnetic field. To overcome these difficulties, one introduces a model of an electron based on a charged conducting surface of a cavity with the presence of an electromagnetic field. In this way, we get a phenomenological solution for the electron stability problem. In fact, the cavity exhibits a surface tension due to the difference of the condensate density between its inside and its outside. The surface tension which depends on the the Higgs potential parameters of the electroweak gauge-model theory [9-15], allows us to establish the expression of the electron total energy. This Latter depends both on the vacuum expectation value of the Higgs field and the Ginzburg-Landau coherence length [16].

2 Confining potential from scalar field-gluon coupling

The confinement property means that the quarks and gluons cannot exist as separate objects. In order to investigate it, various QCD models were proposed to derive an interquark potential which exhibits confinement behavior. Such potential can be obtained by considering the coupling of a dilaton ϕ to the $4dSU(Nc)$ gauge field like in string theory models[8].
To this end, one suggests the following effective lagrangian:

$$L = -\frac{1}{4F(\phi)} G_{\mu\nu} G^{\mu\nu} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 + J^a_\mu A^\mu_a, \tag{1}$$

where the coupling $\frac{1}{4F(\phi)}$ is a function of the dilaton field ϕ and m is its mass. The form of this coupling can have several expressions according to the theoretical frameworks.
The current density $J^\nu_a = g\delta(r) C_a \eta^\mu$ appearing in the lagrangian is considered

as a point like static source. C_a is the expectation value of the $SU(N_c)$ generators for a normalized spinor in the color space, satisfying the algebra identity $\Sigma_a C_a^2 = \frac{N_c^2-1}{2N_c}$.
Using the lagrangian (1), one derives the following equations of motion corresponding to ϕ and A_μ fields:

$$\partial^2\phi - m^2\phi = \frac{1}{4}\frac{d(1/F(\phi))}{d\phi}G_{\mu\nu}G^{\mu\nu} \tag{2}$$

$$\partial_\mu(\frac{1}{F(\phi)}G_a^{\mu\nu}) + g\frac{1}{F(\phi)}A_\mu^b f_{ab}^c G_c^{\mu\nu} = -J_a^\nu, \tag{3}$$

which can be rewritten as:

$$\frac{d^2}{dr^2}(r\phi) - m^2(r\phi) = \frac{\mu^2}{2r^3}\frac{d(F(\phi))}{d\phi} \tag{4}$$

where $\mu = \frac{g}{4\pi}\sqrt{\frac{2N_c{}^2-1}{2N_c}}$.
The equation (4) may be solved for a given dilaton-gluon coupling $F(\phi)$ for which the interquark potential $V(r)$ is given by the following formula [10]:

$$V(r) = -\frac{g}{4\pi}C\int\frac{F(\phi(r))}{r^2}dr. \tag{5}$$

This form of the potential is very interesting because it generalizes the Coulombian potential formula

$$V_c(r) \sim \frac{1}{r}$$

obtained in the particular case: $F(\phi) = 1$.
By introducing the coupling between the dilaton field and the Yang Mills field strength, we derive more general potential which takes into account the confinement behavior of interquark interaction. Following this approach, one can get several forms of the interquark potential just by modifying the dilaton-gluon coupling $F(\phi)$.
Indeed, it was shown in [13,14] that the confining terms my take the following general form:

$$V = \sum_{n=0}^{+\infty}(-1)^n C_{2n+1}r^{2n+1}$$

in which the r powers coefficients are directly related to $< G_{\mu\nu}G^{\mu\nu} >$ and $< q\bar{q} >$, the QCD vacuum condensates describing the nonperturbative effects.

3 Scalar field condensate and electron energy

In order to investigate the electron stability, a model based on a scalar field ϕ and a $U(1)$ gauge field A_μ was suggested [10].

The electron charge is supposed to be uniformly distributed on of a spherical cavity immerged in a vacuum filled with the Higgs condensate. In this context, the lagrangian can be written as:

$$\mathcal{L} = \mathcal{L}^{\phi} + \mathcal{L}^{E} \tag{6}$$

the scalar part of the lagrangian is:

$$\mathcal{L}^{\phi} = \frac{1}{2} g_{\mu\nu} \partial\phi_{\mu} \partial^{\nu}\phi + V(\phi) \tag{7}$$

where $g_{\mu\nu}$ is the conventional Minkowski metric tensor: $g_{\mu\nu} = (1, -1, -1, -1)$. To ensure the spontaneous symmetry breaking, the potential $V(\phi)$ should take the form [11]:

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 \tag{8}$$

the second term in (6) depends on the $U(1)$ the field-strength as follows:

$$\mathcal{L}^{E} = -\frac{1}{16\pi} F_{\alpha\beta} F^{\alpha\beta} \tag{9}$$

where $F_{\alpha\beta} = \partial_{\alpha} A_{\beta} - \partial_{\beta} A_{\alpha}$.
In this framework, the stress energy tensor is defined by the relation:

$$T_{ij} = \frac{2}{\sqrt{|g|}} \frac{\partial}{\partial g^{ij}} (\sqrt{|g|}\mathcal{L}) \tag{10}$$

can be splitted into two main parts:

$$T_{ij} = T_{ij}^{H} + T_{ij}^{E} \tag{11}$$

where T_{ij}^{H} and T_{ij}^{E} are respectively the scalar and the gauge field contributions:

$$T_{ij}^{H} = \partial_i\phi\partial_j\phi - g_{ij}\mathcal{L}^{H} \tag{12}$$

and

$$T_{ij}^{E} = \frac{1}{4} g_{\beta j} F_{i\lambda} F^{\lambda\beta} + \frac{1}{4} g_{ij} F_{\mu\lambda} F^{\mu\lambda}. \tag{13}$$

Let's consider the electron charge as a static source confined in a spherical cavity of radius R and assume that the scalar field has a vanishing vacuum expectation value inside it. Whereas, its vacuum expectation value is supposed to have a non-vanishing constant value outside it.
In order to determine the transitional domains which separates between the two different vaccum expectation values of the scalar field, a coherent length δ should be defined.

This transitional zone can be seen as a domain wall separating the false vacuum and the true vacuum regions.
With these assumptions, the total energy takes the form [10].

$$E_{tot}(R) = \frac{e^2}{2R} + 4\pi\sigma R^2 \tag{14}$$

where σ is the surface tension due to the scalar field expectation value outside the cavity η given by: $\sigma = \frac{2\eta^2}{3\delta}$.
The first term appearing in (15) represents the colombian energy of the cavity surface whereas the second term depends on the the condensation energy and the coherent length.

4 Influence of space-time curvature on the electron energy

The two approaches mentioned above provide two different ways to derive an interaction potential which describes both the colombian interaction and take into account the confinement property.
In the first approach, a direct coupling between the scalar field and the field strength is necessary to obtain such interaction potential which depends on the form of the scalar field. Whereas, in the second approach, the interaction between the scalar field and the electric field was not taken into account.

Till now, we did not consider the gravitational effects to the total energy of the electron.
Let's now extend the two above approaches by adding a new term to the lagrangian:

$$\mathcal{L}^{\phi} = -\frac{1}{2}(\mu^2 + \xi R)\phi^2 - \frac{\lambda}{4}\phi^4, \tag{15}$$

Through this coupling, we can introduce the interaction between the scalar field and the space-time curvature, where the coupling between the scalar field ϕ and the space-time curvature R is parameterised by ξ and μ. The previous form of the potential can be retrieved if, $\xi = 0$[11].
In this framework, the scalar field vaccum expectation value obtained through the minimisation of the potential (15), is

$$\eta^2 = \frac{\mu^2 + \xi R}{\lambda}.$$

Using this value, the potential can be rewritten like:

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - \eta^2)^2 - \frac{\lambda}{4}\eta^4 \tag{16}$$

If we assume that the cavity radius is an approximation of the space-time curvature, then the total energy of the electron takes the following form:

$$E_{tot} = \frac{e^2}{2R} + \frac{\mu^2}{\delta\lambda}\pi R^2 + \frac{\xi}{\delta\lambda}R^3 \tag{17}$$

By analyzing (17), we remark that the spacetime curvature nature brings a new R-contribution to the confining part of the total electron energy.

Finally, due to the space-time curvature, the condensation energy value is R dependent:

$$\epsilon_{cond} = -\frac{1}{4\lambda^3}\left(\mu^2 + \xi R\right)^2 \tag{18}$$

In fact, the particles are not free but they are embedded in a small space time region, so any tentative to obtain an estimation of the electron radius must take into account not only the vaccum effects but also the gravitational ones.

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