WEAK INTERACTIONS AT HIGH ENERGIES

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1. INTRODUCTION

These lectures will be devoted to a review of some open questions in the theory of the weak interactions, in particular such questions which stem from our ignorance about their high energy behaviour. I shall not discuss many of the better established facts about weak interactions which can be found in various survey articles [1-8]. I shall further confine myself to a few selected topics in the theory of leptonic phenomena. An attempt will be made to concentrate on such problems which are least obscured by guesses about the high energy behaviour of strong interactions and the corresponding form factors. In this spirit we shall discuss some reactions (such as weak lepton-lepton scattering) which are rather outlandish from an immediate practical point of view. However, the theoretical study of such processes is especially suited to bring out some of the most interesting general questions in weak interaction theory.

Quite recently strong evidence has been found [9] for the fact that the neutrino (ν_μ) which accompanies the μ^+ in π^+-decay is distinct from the neutrino (ν_e) which accompanies the positron in β^+-decay. ν_μ and ν_e have the same helicity (negative; left-handed). The probable upper limit on the mass of the ν_μ is about 5 electron masses [10]. (The limit on the ν_e-mass is 10^4 times smaller). In what follows we neglect any consequences of a possibly finite ν_μ-mass. (It would be surprising if this mass were non-zero). Thus we assume for both kinds of neutrinos the invariance under the transformation $K^+ \rightarrow \mu^+ + \nu_e$. As a result, neutrinos can have no (induced) magnetic moment. However, there does exist a non-vanishing electric charge form factor for neutrinos [11].

Earlier it had been noted as a theoretical possibility [12] that in strange particle decays the ν_μ and ν_e might change roles (so that $K^+_{\mu 2}$ decay would be $K^+ \rightarrow \mu^+ + \nu_e$). There is experimental evidence against this interchange [9]. In the following we assume throughout that there exist two distinct neutrinos and that in all processes μ^+ is paired with ν_μ, e^+ with ν_e.

There exists a body of evidence [7, 8] in support of the so-called μ-e universality. Prior to the two-neutrino discovery this principle could be stated as a substitutional invariance $\mu \leftrightarrow e$ in all interactions; μ and e are identically coupled and differ only in their mass. Now we must add: if we interchange μ and e, the same need be done for ν_μ and ν_e. If one believes that the μ-e difference is a secular effect of some interaction, then it would be hard to believe that there would not also exist non-secular μ-e distinctions. About this subject the last word has not been spoken by any means. It is going to be one of the main experimental problems for the future to find out how well this universality works in high energy leptonic processes. However, for the purposes of the present lectures, μ-e universality will be defined in the way just mentioned.

In Sec. II we consider pure leptonic processes, such as μ-decay and lepton-lepton scattering. The difficulties of the high frequency behaviour of the Fermi-interactions treated in lowest order are discussed as well as their modification if there exist intermediate vector bosons. Sec. III is devoted to a discussion of general aspects of weak radiative effects. Here the famous question is: If higher order corrections are small why is this so? A brief discussion is given of the high versus low cut off alternatives and of the difficulties connected with a power series expansion of these corrections. It is recalled that these problems present themselves whether or not there exist intermediate vector bosons.

In the now past one-neutrino days some prime examples for the discussion of higher order weak effects were the processes $\mu \rightarrow e\gamma$, 3e, etc. With the advent of the two neutrinos these questions are now happily solved - they don't exist to any order. But also in the two-neutrino theory there remain reactions of interest for the study of higher order problems, in particular some of those which cannot occur in lowest order and yet are not strictly forbidden. Such processes are mentioned in Sec. IV, devoted to some speculations about invariance groups for lepton problems which may be relevant to higher order weak effects.

In Sec. V we review the general structure of the heavy particle currents as they enter in the weak interactions with particular reference to CP-invariance and to a $|\Delta\vec{T}| = 1$ rule for the strangeness conserving processes. Finally we discuss in Sec. VI the principle of local action of lepton currents which may be of interest for an experimental exploration of weak radiative corrections.

The general spirit of what follows is to take the higher order weak effects seriously. This is not done because one can guarantee that they will produce observable effects in the foreseeable future (although one cannot assert the opposite either). Rather, the recent developments have served to bring to focus long known theoretical questions which now seem more immediate than before. Some of these questions may turn out to be ultraviolet herrings - like some of the problems posed in the early days of quantum electrodynamics. But even a proof of this would mean a distinct advance.

The topics to be discussed are all in the domain of high energy leptonic interactions. It is essential to the reasoning that one can isolate one dynamical factor, the lepton current, which can be studied independently of strong interaction effects. An approach of this kind cannot be followed for high energy non-leptonic interactions and this is the theoretical reason why such phenomena have not attracted much attention. The experimental reason is, of course, that one deals with tremendous background problems. In a sense the first experiments in this area have already been done [13]. At one time it was interesting to go below the associated production thresholds and see if single production is at all appreciable. We can now look upon such attempts as non-leptonic weak interaction experiments. However, since that time the nature of the problems has changed. From the theoretical weak interaction point of view, associated production thresholds do no longer form a particular point of interest (but from the experimental side the question will get progressively harder if one passes these thresholds). It is well to state, in the face of the experimental complexities, some qualitative questions one would like to ask. What about the $\Delta S = 1$ and $|\Delta \vec{T}| = \frac{1}{2}$ rules at high energies? What about the interesting parity properties found in Σ-decays, are they just low energy dynamical accidents or is something more subtle going on [14]? The hyperon and $K_{\pi 2}$-decays, being all of the two-body kind, give us a small number of "points" about non leptonic weak interactions but not distributions as in the 3-body leptonic decays. τ-decays give distributions of low Q-value only. It may well become necessary for the understanding of the non-leptonic weak interactions to face also the intricacies of high energy reactions in this domain.

It is appropriate to recall that several of the problems here discussed are "old" ones in the time scale of modern theoretical physics. Thus already in 1936, Heisenberg noted [15] that the n-th order weak interactions behave in the high energy region as $(\text{momentum})^{2n}$. This led him to speculate on the existence of a universal length [16]. I am told [17] that these considerations created a great stir when they were first presented at a Copenhagen Conference of that time. The first calculations on "weak" radiative corrections also were made in the mid-thirties. They were attempts to describe nuclear forces by lepton pairs [18]. These early explorations were all in the spirit of high cut offs (in the sense explained in Sec. III). Also the earliest calculation on high energy effects with low cut offs dates from this time, namely a study of the β-decay of fast protons with momentum transfer to a Coulomb field [19].

We have learned a lot more physics since then, but the high energy behaviour of weak interactions no doubt still has to yield most of its secrets.

I am indebted to Dr. G. Feinberg for stimulating discussions on many of the questions discussed here.

2. PURE LEPTONIC PROCESSES

These are the phenomena where to our knowledge strong interactions do not enter. An example is μ-decay. For this process the effective interaction is

$$L_{eff} = -\frac{G}{\sqrt{2}}\, j_\lambda^{(\mu)*}(x) j_\lambda^{(e)}(x) + \text{h.c.},$$

$$j_\lambda^{(\ell)}(x) = \bar{\ell}(x)\gamma_\lambda(1+\gamma_5)\gamma_\ell(x), \quad \bar{\ell} = \ell^\dagger\gamma_4 \tag{2.1}$$

$$j_\lambda^{(\ell)*}(x) = \eta_\lambda j_\lambda^{(\ell)\dagger}(x), \quad \eta_\lambda = \begin{cases} +1 & \lambda = 1,2,3 \\ -1 & \lambda = 4 \end{cases}.$$

$\dagger$ denotes hermitian conjugate. We put $\hbar = c = 1$ throughout. stands for e or μ. We introduce from the start the distinction between ν_μ and ν_e. (2.1) gives a good account of the decay $\bar{\mu} \to \bar{e} + \bar{\nu}_e + \nu_\mu$ with $G \approx 10^{-5} m_{\text{proton}}^{-2}$.

This coupling implies also the existence of the reactions

$$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_\mu + \mu^- \tag{2.2}$$

$$\nu_\mu + e^- \rightarrow \nu_e + \mu^- \tag{2.3}$$

and of the adjoint reactions $\nu_e + e^+ \rightarrow \nu_\mu + \mu^+$, $\bar{\nu}_\mu + e^+ \rightarrow \bar{\nu}_e + \mu^+$. These reactions are in accord with two conservation principles.

(1) The conservation of leptons;

(2) The conservation of μ-number [20].

These are additive laws for quantum numbers which may be assigned as follows:

Particle	Lepton number	μ-number
e^-	1	0
ν_e	1	0
μ^-	1	1
ν_μ	1	1

The quantum numbers for the corresponding anti-particles have opposite sign. An example of a forbidden reaction is

$$\nu_e + e^- \rightarrow \nu_\mu + \mu^- . \tag{2.4}$$

Remark. As long as weak interactions are treated only to lowest order, it is not wrong but a bit silly to introduce quantum numbers; what is allowed and what not is read off directly from the (effective) coupling. However, there has developed recently some interest in the possible observable higher order effects of weak interactions (see Sec. III). In this more general situation quantum numbers are helpful. Reaction (2.4) cannot go to any order in weak interactions.

We consider the cross-sections $d\sigma_{\bar{\nu}}$ and $d\sigma_\nu$ for the reaction (2.2) and (2.3) respectively. Let $\vec{k}$ and $\vec{k}'$ be the initial and final 3-momentum respectively in the centre-of-momentum system and θ the scattering angle. We have

$$d\sigma_{\bar{\nu}} = (G^2/\pi)[k'^2/\omega^2(k' + \omega_\mu)]\,(k\cos\theta + \omega_e)\,(k'\cos\theta + \omega_\mu)\,d\cos\theta, \tag{2.5}$$

$$d\sigma_\nu = (G^2/\pi)[(k' + \omega_\mu)/\omega_e]\,k'^2\,d\cos\theta, \tag{2.6}$$

where $\omega_e = (k^2 + m_e^2)^{1/2}$, $\omega_\mu = (k'^2 + m_\mu^2)^{1/2}$. $\bar{\nu}$- and ν-scattering are therefore different. This is due to a (V, A) interference effect as we shall see later in more detail. This interference vanishes in the forward direction, $d\sigma_{\bar{\nu}}(\theta = 0) = d\sigma_\nu\ (\theta = 0)$.

Eq. (2.6) shows that the scattering (2.3) goes via J = 0 only. The unitarity limit for total J = 0 scattering here is $\pi/2k^2$. The Fermi coupling

(1) therefore cannot possibly be correct [6] when we surpass this limit which is reached (neglect m_e and m_μ) for

$$k = (\pi^2/8G^2)^{1/4} \simeq 300 \text{ GeV} \tag{2.7}$$

corresponding to $\sigma \sim G \sim 10^{-33} \text{cm}^2$.

What damps the cross-section at these extremely high energies? Effectively it must be a mechanism which introduces some non locality. There are two suggestions which by no means mutually exclude each other; both of them may well be necessary to get a consistent picture.

(1) The Fermi interactions damp themselves. Because the cross-sections get so large at ultra high energies there is no reason to confine oneself to first order calculations, if one takes (2.1) seriously as a field theory coupling at such energies. The higher order terms will now also become important and one will guess largely on dimensional grounds at a damping factor [11] $\sim (1+G^2k^4)^{-1}$.

(2) The damping comes about by a physical mechanism that shows how the interaction (2.1) in itself is only an approximate description of the state of affairs which (no doubt) holds well at low frequencies. A natural guess here is [21, 22] that the effective weak interaction (2.1) is brought about by the coupling of the lepton current to an intermediate charged boson field W_λ

$$L = -g W_\lambda^* (j_\lambda^{(e)} + j_\lambda^{(\mu)}) + \text{h.c.} \tag{2.8}$$

G is then related to the dimensionless coupling constant g and to the boson mass m by

$$G^2/\sqrt{2} = g^2/m^2 . \tag{2.9}$$

Continuing to neglect lepton masses, the differential cross-section (2.6) is now damped by a factor $[1+2m^{-2}k^2(1+\cos\theta)]^{-2}$. We have therefore the high energy limit

$$\sigma_\nu \sim G^2 m^2 \sim g^2/m^2 . \tag{2.10}$$

(Note that this expression blows up for $m \to 0$ (for fixed g). This is as it should be - it is like the infrared catastrophe in electron-electron scattering treated to lowest order with neglect of the influence of soft radiation emission).

It should be noted that the expression (2.10) contains contributions from all J-values, not just from J = 0. If one projects out the J = 0 part of the amplitude in question to calculate the J = 0 scattering cross-section σ_0, the result is

$$\sigma_0 \sim (Gm^2/k^2) \lg^2 k/m . \tag{2.11}$$

This shows that it is not enough to take the intermediate boson effect to lowest approximation in order to avoid the conflict with unitarity at all energies [23]. If one now also takes into account self-damping one finds [23] $\sigma \sim Gm^2 k^{-2} \lg^{-1} g^{-2}$ and that the partial wave amplitudes decrease fast enough.

The interaction (2.8) obviously satisfies the conservations of leptons and μ-number. (2.4) remains forbidden. A new consequence of (2.8) is a second order (in g) coupling of the (e, ν_e) current to itself and likewise for (μ, ν_μ). It follows that the reactions (2.2) and

$$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^- \qquad (2.12)$$

should have the same cross-sections $\sim g^4$, apart from lepton mass effects. The Fermi interaction (2.1) allows the reaction (2.12) to happen as a higher order effect only, so there this equality does not hold.

The cross-sections involved here are presumably always $\lesssim 10^{-33}\mathrm{cm}^2$ and much smaller at low energies. It will be hard to observe them. But their study has taught us something about the high frequency behaviour of the weak interactions under conditions unobscured by strong coupling effects. By this the following is meant. If we study the scattering $\nu_e + \mu \rightarrow p + e^-$, say, using a point interaction of the Fermi type, then we would also get a cross-section $\sim k^2$ at high energies. However, the existence of strong couplings implies that the effective interaction for this process is certainly not a local interaction, as we shall discuss in considerable detail (Section V). Unlike the pure leptonic processes, we cannot use reactions involving strongly interacting particles to infer that unavoidably something new has to happen with weak interactions at high frequencies.

3. WEAK RADIATIVE CORRECTIONS

Higher order effects due to weak interactions are of interest because two opposing trends are at work. On the one hand we get higher powers of G which tend to make these effects insignificant. On the other hand, we saw in Sec. II that the local Fermi couplings are quite singular at high frequencies. (We leave the intermediate boson idea aside for a little while.) In the calculation of weak radiative corrections we meet of course integrations over all virtual frequencies and the question arises if high momentum contributions could perhaps be significant even though higher powers of G enter.

As an example, consider the corrections to the reaction (2.2). Note first of all that the order of the lowest non vanishing correction depends on the presence or absence of the self couplings of the (e, ν_e) current and of the (μ, ν_μ) current which we discussed in relation with Eq.(2.8). (The existence of such self couplings can of course be considered also in the absence of W-fields.) If these couplings are present, the lowest corrections are a result of the graphs in Fig. 1. As a result the ratio of the correction term to the leading term in the cross-section is proportional to the dimensionless quantity $G\Lambda^2$, where Λ is a cut off momentum. If there is no self-coupling, the lowest correction is due to the graph in Fig. 2. In this case the ratio just mentioned is proportional to $(G\Lambda^2)^2$. Similar consequences of self coupling effects enter in many problems [24].

In any case, we are faced with the important question, what should be our guess for the magnitude of $G\Lambda^2$, that is for the cut off. In a future theory this cut off must of course be connected with real physics. The interesting

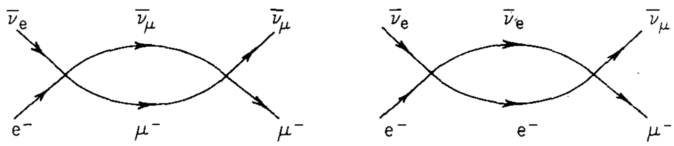

Fig. 1

Lowest corrections to (2. 2) in the presence of lepton current self-coupling.

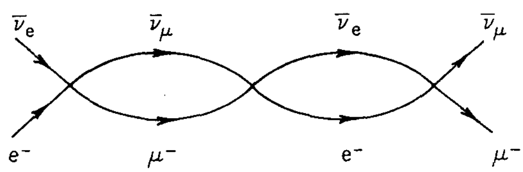

Fig. 2

Lowest corrections to (2. 2) in the absence of lepton current self-coupling.

thing is that which ever way we shall eventually get out of the dilemma something new happens.

(a) The cut off is low - weak radiative corrections are small. In other words, weak interactions are indeed weak at all frequencies, much weaker than electromagnetic effects. Thus the guess is now that $G\Lambda^2 \ll e^2$ or

$$\Lambda \lesssim 30 \text{ GeV}. \tag{3.1}$$

This characteristic energy is much lower than the "breakdown energy" (2. 7) which we discussed earlier. Thus if we believe all weak radiative corrections to be small, the physical mechanism which causes the cut off must set in long before we come close to the unitarity conflict discussed in Sec. II.

(b) The cut off is high. Now the interest lies in the possibility that weak radiative corrections might show up directly.

Whether or not these corrections will turn out to be observable, it does not seem sensible to treat them by a series expansion in G because higher powers in G are connected with increasing degrees of singularity. For example, the self-damping effect of Fermi interactions mentioned after Eq. (2. 7) would look silly if expanded in G^2.

Actually, there are some indications that the cut off is low, namely the smallness of the K_1^0 - K_2^0 mass difference and the present limits on the existence of parity non-conserving nuclear interactions [25]. It should be remembered, however, that in such cases the cut off due to strong interaction form factors may play an important role too. The example of lepton-lepton scattering is just so interesting because of the absence of strong interaction effects, but they are also pretty unrealistic for practical purposes. In Sec. VI we shall come back to the question how one may attempt to detect phenomena due to weak radiative corrections which cannot be confused with strong coupling effects.

Under any circumstances the weak radiative corrections pose interesting theoretical problems. If they are small, why are they small? Are they uniformly small in very high energy (real) phenomena? In these notes we keep an open mind about these questions. As we go along we will mention at times some items which bear on these corrections.

We saw in Sec. II that an intermediate vector boson field (if it exists) provides a mechanism to damp to lowest order the cross-section for reactions like (2.2) and (2.3). However, it does not follow by any means that the weak radiative corrections are small as well. The qualitative reason is the following. The propagator for a virtual vector boson is given by

$$- i (\delta_{\mu\nu} + m^{-2} q_\mu q_\nu)/(q^2 + m^2 - i\epsilon) .$$

Here q_μ is the four momentum transferred to the W-particle. When we calculate the cross-section for (2.3) to lowest order this propagator is sandwiched between free lepton spinors and the application of the Dirac equation to those spinors shows that the term $m^{-2} q_\mu q_\nu$ gives a contribution $\sim m_e m_\mu m^{-2}$ (which we actually neglected earlier). Thus in this case the propagator contributes a factor $\sim k^{-2}$ to the matrix element, as we saw earlier.

If the propagator is not taken between free particle states, its order may be $(\text{momentum})^0$ rather than $(\text{momentum})^{-2}$, unless some angular averaging (or a renormalization argument) reduces the order. Thus the question of weak radiative corrections remains critical even in the presence of a vector boson field.

In the language of field theory we can summarize the situation as follows. The four Fermi interaction is unrenormalizable, but so is the theory of a charged massive spin 1 field. (For a neutral field of this kind it was shown that the theory is renormalizable [26].) Whether or not there are W-fields, the theoretical study of weak radiative corrections is therefore important. In fact, they promise to be far more interesting than the electromagnetic corrections (for spin 0 and 1/2 interactions) just because the latter are much less singular. It has recently been shown by Lee [27] for electromagnetic interactions of charged massive vector mesons how one may attempt to obtain finite results by summing up the most singular parts of the higher order effects, assuming that there exists a finite limit as an effective cut off tends to ∞.

4. LEPTONIC SPIN

If we are only interested in μ-decay, it is possible to write down a more general interaction than Eq. (2.8), namely we could couple the electron- and μ-currents with distinct constants g_e and g_μ respectively, for in μ-decay only the product $g_e g_\mu$ enters. The equality $g_e = g_\mu = g$ means that we have chosen an interaction which satisfies μ-e universality (see Sec. I).

Actually, the recent discovery of the distinction between ν_e and ν_μ invites speculation about the existence of some sort of "spin" for leptons - in some ways similar to the isotopic spin for strongly interacting particles. In such an approach one may attempt to look upon the μ-e universality operation (see Sec. I) as a discrete element of some rotation group. Such a group can certainly manifest itself only where it makes sense to neglect the μ-e mass difference (or correct for it only in the kinematics). This may not be such a bad approximation in certain high energy neutrino experiments.

It may be somewhat early to pursue this subject at great length. It is another of those questions which would become of considerable interest if higher order weak effects were to be relevant in practice. Nevertheless, let us briefly state the general nature of the problem. There are two ways of approach which one may contemplate.

1. One groups the particles in terms of two "spinors" as follows [28]:

$$\psi_e = \binom{e}{\nu_e}, \quad \psi_\mu = \binom{\mu}{\nu_\mu}. \tag{4.1}$$

There is a Pauli-type spin operator $\vec{\zeta}$ acting on these spinors. The eigenvalues +1, -1 of ζ_3 refer to the upper and lower components of the spinors respectively. Consider structures of the type $\bar{\psi}_e \vec{\zeta} \psi_e$, $\bar{\psi}_\mu \vec{\zeta} \psi_\mu$. (For a while we neglect all ordinary spin factors, etc..) It is evident that we need the 1- and 2-components of these vectors in ζ-space to construct the currents $j_\lambda^{(e)}$ and $j_\lambda^{(\mu)}$. If we wish to impose a rotational invariance with respect to this space, it follows that we should also necessarily have to reckon with the occurrence of neutral lepton currents. And if we couple the lepton currents to the currents of the strongly interacting particles so as to describe β-decay, π-decay, hyperon and K-decays etc., we run therefore into the problem that no neutral leptonic decays of strongly interacting particles seem to exist. To be more precise, there is no evidence for the presence of neutral currents in strangeness changing leptonic decays [29]. For strangeness conserving decays it could easily be possible for neutral lepton currents to exist and yet to escape detection [30]. ($\pi^0 \rightarrow e^+ + e^-$ by this mechanism would be masked by electromagnetic processes, $\pi^0 \rightarrow \nu + \bar{\nu}$ is hopeless.)

It does not seem fruitful, therefore, to introduce a ζ-space invariance in the manner described. Note further that electromagnetic interactions also violate this invariance. Also one has to neglect the e- and the μ-mass completely to be able to rotate at all.

2. One groups the particles as follows [23]:

$$\psi_\ell = \binom{e}{\mu}, \quad \psi_\nu = \binom{\nu_e}{\nu_\mu}. \tag{4.2}$$

There is a spin $\vec{\rho}$, the "leptonic" acting on these spinors and $\rho_3 = +1, -1$ again refers to the upper and lower components respectively. We observe the following:

(a) Rotational symmetry in $\vec{\rho}$-space can be upheld if we neglect the μ-e mass difference.

(b) The conservation of leptons is a consequence of the gauge invariance of the first kind in leptonic spin space. This gauge group together with the leptonic spin group forms the group U(2). The μ-number introduced in Sec. II is the relevant eigenvalue of $\frac{1}{2}(1-\rho_3)$. μ-e universality follows as the consequence of the invariance under the unitary operation $S = i\rho_2$ (which is the analogue for this space of the charge conjugation operation).

(c) To construct the currents $j_\lambda^{(e)} + j_\lambda^{(\mu)}$ we need the combination $\psi_\ell \psi_\nu$ which is a scalar in leptonic spin space. No neutral lepton currents are necessary.

(d) To get the electromagnetic interactions we need $\psi_\ell \; \psi_\ell$ which is also a scalar in this space. Electromagnetism respects leptonic spin.

(e) If we couple a $\psi_\ell\ \psi_\nu$-structure to the strongly interacting particle current, the latter should be scalar too with respect to leptonic spin. Thus one can consider all strongly interacting particles individually as leptonic spin scalars. (The situation would be more complex in this respect if ν_e and ν_μ interchanged in strangeness violating decays, but as we said in the introduction, we assume that this interchange does not take place. Note further that the use of (4.1) would also necessitate to assign to the heavy particles a leptonic kind of quantum number.)

We give one example of the consequences of leptonic spin invariance in pure leptonic processes. Until further notice (to be given shortly) we neglect all electromagnetic effects. It was noted in Sec. II that in lowest order the reactions (2.2) and (2.12) have equal cross-sections (apart from the μ-e mass difference effects). Call these cross-sections σ_a and σ_b respectively. Next note that the reaction

$$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^- \tag{4.3}$$

is forbidden to lowest non vanishing order. But it is not forbidden rigorously as it satisfies conservation of leptons and of μ-number. See Fig. 3a for a typical graph. Let σ_c denote the cross-section of this last reaction.

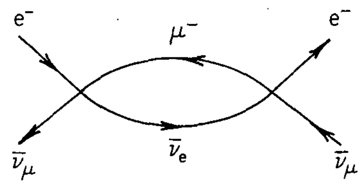

Fig. 3a

Reaction (4.3) as a second order weak effect.

If leptonic spin conservation applies, there exists a triangular inequality between σ_a, σ_b and σ_c namely [23]

$$\sqrt{\sigma_b} + \sqrt{\sigma_c} \geq \sqrt{\sigma_a} \geq \sqrt{\sigma_b} - \sqrt{\sigma_c}. \tag{4.4}$$

This is proved by the same methods as are used in isotopic spin discussions of nucleon-nucleon (and anti-nucleon) scattering.

It is also possible to apply related considerations to reactions involving heavy particles, for example to $\bar{\nu}_\mu + p \rightarrow \ell + \bar{\ell} + \ell + n$, where ℓ = e or μ and where such ℓ-combinations are chosen that conserve μ-number.

It will be clear from these examples that it may well take a long time before we will have proof of the validity of leptonic spin or related ideas. Perhaps more important than this invariance itself is the question what breaks it. What causes the μ-e mass difference? Here we cannot hide our ignorance behind strong interactions, as is often done with such abandon for the heavy particle mass differences. The μ-e difference is in fact our strongest present clue for the existence of something new at high frequencies [31]. Something clearly eludes us here.

In the previous discussion one point has emerged that is more general than leptonic spin itself, namely the possibility that higher order weak interactions might manifest themselves through processes that are forbidden if weak interactions are taken to lowest order only. A class of such phenomena are those where a neutral lepton pair is produced. It is clear that if we only insist on conservation of leptons and of μ-number, it is possible to have higher order processes of the kind

$$A \to B + \mu^+ + \mu^-, \tag{4.5}$$
$$A \to B + e^+ + e^-, \tag{4.6}$$
$$A \to B + \nu_e + \bar{\nu}_e, \tag{4.7}$$
$$A \to B + \nu_\mu + \bar{\nu}_\mu. \tag{4.8}$$

(Leptonic spin will relate rather than forbid such reactions.) Thus higher order weak interactions generate effective neutral lepton currents even if we assume that such currents do not appear in the primitive interactions.

An example of a reaction with a charged lepton pair is $K_2^0 \to \mu^+ + \mu^-$. This conceivable but not observed decay is just the one used earlier to find bounds on the coupling strength of a neutral lepton current. It can now also be used to set additional bounds on the cut off (see Sec. III) for weak radiative effects in strangeness changing processes.

Examples of neutrino pair production are $K^+ \to \pi^+ + \nu_e + \bar{\nu}_e$ or $\pi^+ + \nu_\mu + \bar{\nu}_\mu$. (These would be hard to entangle from $K^+ \to \pi^+ + 2\gamma$ without subsequent conversion.) In the spirit of Sec. III these reactions are mentioned here because their slow rate may be a further reflection of the cut off mechanism of the weak interactions.

We now ask what is the influence of electromagnetic effects on Eqs. (4.3 - 8). The following should be observed:

(1) The reaction (4.3) can go as a first order weak interaction provided we use a virtual photon. The mechanism is shown in Fig. 3b. This is the type of graph associated with the electric neutrino form factor. Also the reaction (2.12) gets a similar contribution. As electromagnetism respects leptonic spin, the relation (4.4) remains valid.

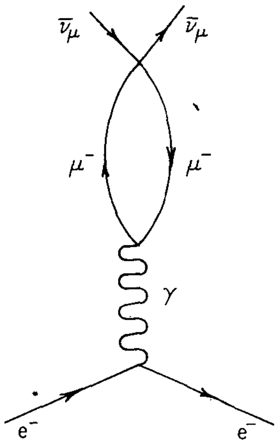

Fig. 3b

Reaction (4.3) as a first order weak effect generated by the neutrino form factor.

(2) Also the reaction $K_2^0 \to \mu^+ + \mu^-$ can proceed as a first order weak (non-leptonic) process, namely [32] $K_2^0 \to 2\gamma \to \mu^+ + \mu^-$.
This reaction, if ever found, does therefore not necessarily constitute evidence for (primitive) neutral lepton currents.
(3) Reactions like $K^+ \to \pi^+ + \nu_e + \bar{\nu}_e$ can also go as a first order weak process via the electric neutrino form factor, namely $K^+ \to \pi^+ + \gamma$, $\gamma \to \nu_e + \bar{\nu}_e$. (The first stage is allowed as a virtual process only.)

Finally we note that there is another way of introducing a μ-quantum number [33], namely a "μ-parity" instead of the μ-number introduced in Sec. II. The μ-parity rules are incompatible with leptonic spin.

5. THE STRUCTURE OF HEAVY PARTICLE CURRENTS

It is assumed in all current theories that the effective interaction for leptonic processes involving strongly interacting particles is of the form:

$$\mathcal{L}_{\rm eff} = -[\, J_\lambda^*(x)(j_\lambda^{(e)}(x) + j_\lambda^{(\mu)}(x)) + J_\lambda(x)(j_\lambda^{(e)*}(x) + j_\lambda^{(\mu)*}(x))] \,. \qquad (5.1)$$

$J_\lambda(x)$ is the effective heavy particle current. The *-notation is as in Eq. (2.1). (5.1) is in accordance with μ-e universality. J_λ contains one part which refers to strangeness conserving processes ($\Delta S = 0$) and a part for which $\Delta S = 1$. We like to think that J_λ does not have a $\Delta S = 2$ part even though the direct evidence for this is meagre where leptonic processes are concerned.

J_λ contains bilinear baryon terms and bilinear meson terms (also linear meson terms). The general form of <u>one</u> of the baryon terms is, as it appears

$$\text{in } J_\lambda : \bar{B}_1 O_\lambda B_2 , \quad \text{in } J_\lambda^* : \eta_\lambda \bar{B}_2 \bar{O}_\lambda B_1 , \qquad (5.2)$$

where

$$\bar{O}_\lambda = \gamma_4 O_\lambda^\dagger \gamma_4$$

and where the most general form of the vector operator O_λ is [34]

$$O_\lambda = \frac{i}{\sqrt{2}}\Big[\gamma_\lambda(g_V + g_A\gamma_5) + \Big(\frac{\overrightarrow{\partial}}{\partial x_\lambda} - \frac{\overleftarrow{\partial}}{\partial x_\lambda}\Big)(f_V + f_A\gamma_5)$$
$$+ \Big(\frac{\overrightarrow{\partial}}{\partial x_\lambda} + \frac{\overleftarrow{\partial}}{\partial x_\lambda}\Big)(h_V + h_A\gamma_5)\Big] \,. \qquad (5.3)$$

Therefore

$$\bar{O}_\lambda = \frac{i}{\sqrt{2}}\Big[\gamma_\lambda(g_V^* + g_A^*\gamma_5) + \Big(\frac{\overrightarrow{\partial}}{\partial x_\lambda} - \frac{\overleftarrow{\partial}}{\partial x_\lambda}\Big)(f_V^* - f_A^*\gamma_5)$$
$$- \Big(\frac{\overrightarrow{\partial}}{\partial x_\lambda} + \frac{\overleftarrow{\partial}}{\partial x_\lambda}\Big)(h_V^* - h_A^*\gamma_5)\Big] \,. \qquad (5.4)$$

The choice of (B_1, B_2) pairs is first of all dictated by charge conservation. For $\Delta S = 0$ we can take $(B_1, B_2) = (n, p)$, (Σ^0, Σ^+), (Σ^-, Σ^0) etc., for $\Delta S = 1$, $(B_1, B_2) = (\Lambda, p)$, $(\Sigma^- n)$, etc. The six quantities $g_V, \ldots, h_A$ are invariant operators, that is, they are functions of the $\Box$ operator (space-time), or equivalently of the invariant momentum transfer (momentum space). As we are discussing an effective interaction, these six functions must therefore be considered to describe the results of the iteration of all strong interactions. They are the unknown form factors or structure functions. We have a set of six such functions for each (B_1, B_2) pair. High energy leptonic reactions will largely be a gathering of information about the behaviour of the structure functions for large momentum transfers.

Similar considerations apply to those meson terms which are bilinear in the K- and π-fields [35]. Let us further assume that the K-particles are pseudoscalar, as is now pretty definite.

Then the meson terms have definite parity (they are all vector structures) and their general form is, as they appear

$$\text{in } J_\lambda : \bar{M}_1 O_\lambda M_2, \quad \text{in } J_\lambda^* : \eta_\lambda \bar{M}_2 O_\lambda M_1, \tag{5.5}$$

where

$$O_\lambda = i\,[f_V(\frac{\overrightarrow{\partial}}{\partial x_\lambda} - \frac{\overleftarrow{\partial}}{\partial x_\lambda}) + h_V(\frac{\overrightarrow{\partial}}{\partial x_\lambda} + \frac{\overleftarrow{\partial}}{\partial x_\lambda})], \tag{5.6}$$

$$O_\lambda = i\,[f_V^*(\frac{\overrightarrow{\partial}}{\partial x_\lambda} - \frac{\overleftarrow{\partial}}{\partial x_\lambda}) - h_V^*(\frac{\overrightarrow{\partial}}{\partial x_\lambda} + \frac{\overleftarrow{\partial}}{\partial x_\lambda})]. \tag{5.7}$$

Thus, for example, the $\Delta S = 1$ current contains $(\bar{K}, \pi^0)$ terms (K is the field of the charged K particles) which involve the two structure functions of $K_{\ell 3}^+$ decay. At present, the only direct experimental information on structure functions outside the non-relativistic domain stems in fact from the various $K_{\ell 3}$ models. It has been found that these functions vary quite slowly over the range covered by the spectra [36].

Form factors have been studied extensively within the framework of dispersion theory. We shall not review these calculations here, but rather concentrate on some general properties of structure functions. As we shall see presently, the number of independent structure functions becomes constrained when certain invariance arguments are used. Therefore, high energy experiments may provide tests of the validity in the high energy range of the symmetries in question. Before we discuss these problems, we first make two general remarks.

(1) The h-functions. These appear in conjunction with the sum of right and left derivatives which is the total derivative of the bilinear form on hand. By partial integration we can throw the total derivative over on the lepton terms. It follows from the application of the lepton Dirac equation that in any leptonic process the h-terms give contributions proportional to the lepton mass. Chances are therefore much better to observe such terms in μ- than in e-processes. Generally, the neglect of the lepton mass in any given process implies that we ignore the role of h-functions. This is true for all baryon and all meson terms in (5. 1). From Eqs. (5. 5 - 7) we see that wherever it is appropriate to neglect lepton masses, the effective meson current involves one single form factor only. This circumstance is of particular interest for K_{e3}-decay, for example [37].

(2) The f-functions. These appear together with the difference of right and left derivative. In the non-relativistic limit we retain only $\partial/\partial x_4$-terms which give contributions proportional to the heavy particle mass in question. The best known example is β-decay, $\Delta S = 0$ where we have:

$$(B_1, B_2) = (n, p) : O_\lambda = \gamma_\lambda (G_V - G_A \gamma_5), \text{ (N. R.)} \tag{5.8}$$

$$G_V = g_V(q^2 = 0) - 2mf_V(q^2 = 0) \cong 10^{-5} \text{ m}^{-2}, \tag{5.9}$$

$$G_A = -g_A(q^2 = 0) \cong 1,2\, G_V. \tag{5.10}$$

G_V and G_A are the Fermi and Gamow-Teller constants respectively. q^2 is the invariant momentum transfer. For $|\Delta S| = 1$ β-decays, the non-relativistic approximation may not be so good. For the e-mode in Λ-decay we have momentum transfers up to 175 MeV, for example [38].

Next we consider some invariance arguments.

(a) CP-invariance. It is a sufficient condition for CPT-invariance that we have a local theory invariant under the proper Lorentz group. Of course, the local property refers to primitive interactions and not to the effective interaction under discussion. The experimental situation with regard to CPT invariance has been discussed elsewhere [39] and it has been noted that more experiments are needed to verify its validity. We assume that CPT-invariance holds so that CP- and T-invariance imply each other [40]. We ask for the implications of CP-invariance.

Under the CP-transformation

$$(CP) : j_\lambda^{(\ell)}(x) \longleftrightarrow j_\lambda^{(\ell)*}(x). \tag{5.11}$$

For the heavy particle current we have

$$(C) : \frac{\overrightarrow{\partial}}{\partial x_\lambda} \longleftrightarrow \frac{\overleftarrow{\partial}}{\partial x_\lambda}, \tag{5.12}$$

$$(P) : \frac{\partial}{\partial x_\lambda} \to -\eta_\lambda \frac{\partial}{\partial x_\lambda}, \quad \text{for both } (\rightarrow) \text{ and } (\leftarrow). \tag{5.13}$$

It follows that CP-invariance implies that

$$(CP) : \text{all } f, g, h\text{-functions are real.} \tag{5.14}$$

This is true for all baryon and meson terms.

(b) $|\Delta\vec{T}| = 1$ for $\Delta S = 0$. Next we consider isotopic spin arguments for which we must treat $\Delta S = 0$ and 1 separately. This subsection is exclusively devoted to $\Delta S = 0$ currents. We inquire about the behaviour of these currents under the charge symmetry operation. This was first done by WEINBERG [41].

To begin with we note that in such $\Delta S = 0$ processes as neutron β-decay and π-decay we have $\Delta T_3 = 1$ and also $|\Delta\vec{T}| = 1$. One can imagine nuclear β-decays in which $|\Delta\vec{T}|$ could take different values, say 2. A current bilinear in nucleons cannot produce such a change, but there are other currents which could give such an effect, namely those bilinear in Σ's or in π's. It is most economical to assume that such currents are not there [42]. This is the origin of the $|\Delta\vec{T}| = 1$ rule which, it should be stressed, is a stronger statement than just the exclusion of $|\Delta\vec{T}| \neq 1$ terms in $\Delta S = 0$ currents.

$|\Delta\vec{T}| = 1$ rule. Not only do J and J* (for $\Delta S = 0$) each behave as components of an isovector, but they transform as components of the same isovector with the same phase relations as those which occur in the strong interactions.

Thus J and J^* are isotopically related to each other (with suitable conventions) in the same way as π^+ and π^-, not as π^+ and $-\pi^-$ or as a complex mixture of both. It should be noted that distinctions of this kind only make physical sense if the phase relations between π^+ and π^- (and likewise between other particle pairs related by charge symmetry) have been defined by another part of the interaction. In the present case the phase relations are of course defined by the strong interactions themselves. The weak couplings compatible with the $|\Delta\vec{T}| = 1$ rule are the first class couplings in the sense of WEINBERG [41].

The $|\Delta\vec{T}| = 1$ rule has consequences of two kinds. First, the fact that other $|\Delta\vec{T}|$ values than 1 are to be excluded affects specifically those terms in the currents which are bilinear in Σ, or in π, or in K. The pure Σ-part of J_λ is of the general form

$$(\Sigma, \Sigma) : \bar{\Sigma}^0 O_\lambda \Sigma^+ + \bar{\Sigma}^- O'_\lambda \Sigma^0. \tag{5.15}$$

and our restriction means that

$$O_\lambda = - O'_\lambda. \tag{5.16}$$

Eq. (5.16) similarly applies to $\pi\pi$- and to KK-terms.

Secondly, the specific connection between J and J^* implied by the $|\Delta\vec{T}|=1$ rule has two kinds of consequences. First, for all baryon and meson terms we have

$$\bar{O}_\lambda = O_\lambda. \tag{5.17}$$

Hence

$$|\Delta\vec{T}| = 1 : \begin{array}{ll} g_A, g_V, f_V, h_A & \text{are real,} \\ f_A, h_V & \text{are imaginary.} \end{array} \tag{5.18}$$

Next, consider the $\Sigma\Lambda$-terms in J_λ which are of the general form:

$$\bar{\Sigma}^- O_\lambda \Lambda + \bar{\Lambda} O'_\lambda \Sigma^+. \tag{5.19}$$

Each of these terms separately behaves like an isovector. Because of the conditions on phase relations we have (compare with the strong $\Sigma\Lambda\pi$ coupling!)

$$(\Sigma\Lambda) \ : \ O'_\lambda = O_\lambda. \tag{5.20}$$

Eqs. (5.17) and (5.20) imply that, apart from phase space corrections [43]

$$R(\Sigma^- \to \Lambda + e^- + \vec{\nu})/R(\Sigma^+ \to \Lambda + e^+ + \nu) = 1. \tag{5.21}$$

Other consequences of the $|\Delta\vec{T}| = 1$ rule are to be found in inelastic neutrino processes. Consider for example [22] :

$$\nu_\ell + p \rightarrow \ell^- + p + \pi^+ \qquad (R_1)$$
$$\nu_\ell + n \rightarrow \ell^- + n + \pi^+ \qquad (R_2) \qquad (5.22)$$
$$\nu_\ell + p \rightarrow \ell^- + p + \pi^0 \qquad (R_3)$$

The symbols in brackets denote the respective rates. The rule implies triangular inequalities like

$$\sqrt{R_2} + \sqrt{2R_3} \geq \sqrt{R_1}, \text{ etc.} \qquad (5.23)$$

It should be pointed out that the $|\Delta\vec{T}| = 1$ rule may be considered as a condition on primitive leptonic weak interactions because strong interactions respect the rule. It is clear that the rule is violated by electromagnetic corrections (and weak radiative corrections). These have therefore tacitly been ignored in the foregoing.

(c) Combined CP-invariance and $|\Delta\vec{T}| = 1$, $\Delta S = 0$. If both requirements are imposed it follows from Eqs. (5.14) and (5.18) that

$$f_A = h_V = 0 \qquad (5.24)$$

for all baryon and meson terms.

A consequence of $h_V = 0$ is the absence of an induced scalar term in μ-absorption by a nucleon.

(d) Conserved vector current, [21] $\Delta S = 0$. In this theory, CP-invariance and $|\Delta\vec{T}| = 1$ are incorporated. The basic idea is that the V-part of J_λ, J_λ^*, ($\Delta S = 0$) are proportional to the $T_3 = +1, -1$ components respectively of the isotopic spin current. This has two consequences. (1) These V-currents are conserved if we neglect electromagnetic effects. This explains the equality to a good approximation of the Fermi-constant in β-decay and the μ-decay constant (absence of renormalization effects). It is easily verified that our V-currents satisfy $\partial J_\lambda^V / \partial x_\lambda = 0$ and likewise for J_λ^{*V} as long as we neglect mass differences within any isotopic multiplet, because we may use (5.24). (We may apply the free particle wave equation to all field operators occurring in J_λ, J_λ^*.) Therefore, the fact that the V-current is conserved does not impose restrictions on the form factors stronger than the consequences of the (weaker) requirements of combined CP-invariance and the $|\Delta\vec{T}| = 1$ rule. (2) The V-parts in question have V-structure functions which are proportional to those structure functions which occur in the $T_3 = 0$ part of the isotopic spin current. That is, they are proportional to the corresponding electromagnetic isotopic vector form factors.

For the nucleon, for example, we have two form factors f_V and g_V (see also Eq. (5.24)) and the conserved current theory says that these can be expressed as follows (m = nucleon mass):

$$g_V = G_V \left[F_Q + (\mu_p - \mu_n) F_M \right],$$
$$f_V = \left[G_V / 2m \right] (\mu_p - \mu_n) F_M. \qquad (5.25)$$

F_Q and F_M are the isotopic vector electromagnetic form factors for charge and magnetic moment, normalized to unity at $q^2 = 0$. μ_p and μ_n are the proton and neutron moments in units $e/2m$. Thus Eq. (5. 25) allows us to use the information on the nucleon electromagnetic form factors in high energy lepton experiments [44]. In particular, if we neglect the lepton mass, we deal with only three form factors in the reaction ν + nucleon $\to \ell$ + nucleon, namely f_V, g_V and g_A. The first two are determined from high energy electron-nucleon scattering via Eq. (5. 25), if the conserved current idea is correct. There remains g_A as the only unknown structure function.

Another example where the proportionality to electromagnetic form factors is useful is the decay $\pi^- \to \pi^0 + e^- + \bar{\nu}_e$ which goes entirely via the V-current. This is practically a zero momentum transfer process so that the proportionality for the corresponding f_V, see Eq. (5. 6), is as $G_V/\sqrt{2}$ to e. Similarly for $K^0 \to K^- + e^+ + \nu_e$ [45].

(e) Isotopic spin properties, $|\Delta S| = 1$. It has been suggested [46] that the $|\Delta \vec{T}| = 1/2$ rule for non-leptonic processes should also apply to leptonic reactions $\Delta S = 1$. This rule implies the validity of the rule [21] $\Delta S/\Delta Q = +1$ (but not vice versa) and the latter seems to be violated [47]. There are certain theoretical ideas [48] which involve leptonic $\Delta S = 1$ couplings which are not exclusively of the $|\Delta \vec{T}| = 1/2$ type, but these fall outside the scope of the present survey and we shall not discuss them here.

(f) "Overall current $\times$ current coupling". It has been suggested [21] that all weak processes, leptonic and non-leptonic, follow from an effective interaction of the structure current times current. (In this scheme there appear also non-leptonic neutral currents.) The validity of this scheme (sometimes called universal Fermi interaction scheme) is tied to the applicability of a $|\Delta \vec{T}| = 1/2$ rule to both non-leptonic and leptonic interactions. In view of the preceding remarks it is not timely to discuss such proposals at the present stage of developments.

(g) Primitive interactions. We have exclusively dealt with effective interactions. One may ask about the structure of the primitive couplings which effectively lead to the currents here discussed. Suppose for example that we start from a theory with trilinear local couplings. Then the conserved vector current proposition implies that the primitive V-current for $\Delta S = 0$ is

$$\frac{iG_V}{\sqrt{2}}\Big[\bar{p}\gamma_\lambda n + \bar{\Xi}^0 \gamma_\lambda \Xi^- + \sqrt{2}(\bar{\Sigma}^0 \gamma_\lambda \Sigma^- - \bar{\Sigma}^+ \gamma_\lambda \Sigma^0) + \sqrt{2}\,\pi^+\Big(\frac{\overrightarrow{\partial}}{\partial x_\lambda} - \frac{\overleftarrow{\partial}}{\partial x_\lambda}\Big)\pi^0 + K^+\Big(\frac{\overrightarrow{\partial}}{\partial x_\lambda} - \frac{\overleftarrow{\partial}}{\partial x_\lambda}\Big)K_0\Big]. \tag{5.26}$$

So far, the study of such "basic" interactions has not yielded any useful results. Still, Eq. (5. 26) is at least interesting to look at, and to remind us that we are in need of arguments about the relative magnitude of the various terms in a current.

6. LOCAL ACTION OF LEPTON CURRENTS

As we discussed in Section V, the most general form of the terms which enter in the heavy particle current is an operator of the type O_λ or $\bar{O}_\lambda$ sandwiched between free fields. The finite distance character of these operators is the mathematical expression of the smearing out effects typical for strong coupling form factors. On the other hand, the lepton currents which appear in (5.1) are local, see their definition Eq. (2.1). This expresses the absence of strong interactions for leptons. This point structure of $j_\lambda^{(\ell)}(x)$ in Eqs. (2.1) and (5.1) is called the local action of lepton currents.

Schematically, the situation for a reaction of the type [49]

$$\nu + T \rightarrow F + \ell \tag{6.1}$$

is therefore as indicated in Fig. 4. The box represents the effects of strong interactions. T and F are attached at different points but the lepton pair

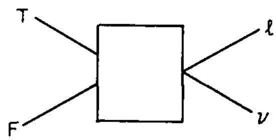

Fig. 4

The local action of lepton currents.

emerges at one point. The same picture applies, of course, also to decay reactions of the type

$$T \rightarrow F + \nu + \ell . \tag{6.2}$$

In Section III we surveyed some of the interesting theoretical questions connected with weak radiative corrections. In Eqs. (4.5 - 8) examples were given of reactions which are possible only via such higher order mechanisms if no neutral lepton currents exist. We now observe that if the weak radiative corrections play any observable role, deviations from the local action of lepton currents would be one possible way to find this out [50].

Consider for example the reactions

$$\bar{\nu}_\ell + p \rightarrow n + \ell^+ \tag{6.3}$$

$$\bar{\nu}_\ell + n \rightarrow p + \ell^- . \tag{6.4}$$

The character of the weak radiative corrections depends on the presence or absence of neutral lepton currents. If they are present (or absent) the lowest order corrections are as in Fig. 5 (or Fig. 6). (As was mentioned

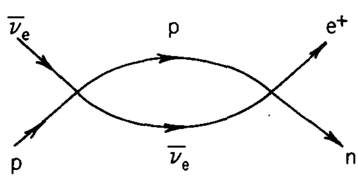

Fig. 5

Lowest weak radiative correction to (6.3) in the presence of a neutral lepton current.

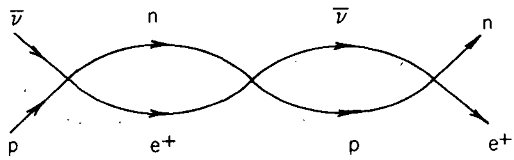

Fig. 6

Lowest weak radiative corrections to (6. 3) in the absence of a neutral lepton current.

in Section III one should not conclude from this that in the latter case the corrections are necessarily smaller than in the former.) In either case the leptons emerge from different points and we have an effective non-local action of the lepton current.

In order to judge whether non-local effects of this kind are present it is necessary to find out first what local action implies in practice. Before we turn to this question it seems worthwhile to observe the following.

(1) If deviations from local action are to turn up at all it is to be expected that the effect will be more manifest at high energies. In principle one can raise the question already for neutron β-decay but there, of course, one can not get much dynamical information anyway because the phase space is so small. The situation is more favourable for $K_{\ell 3}$ decays (and hyperon β-decays). The local action problem was first raised in a study of these modes [37]. Still, even here the momentum transfers are not very high. (The same is true for reactions (4. 5 - 8).) Thus, high energy lepton reactions are the best place to look for such effects.

(2) If deviations from local action are ever found it will be of particular interest to know if they satisfy μ-e universality.

(3) Also in the intermediate boson theories do we have the assumption of local action so that the whole question is independent of the existence of these bosons. To avoid confusion we note that in pure leptonic processes (Section II) an intermediate boson produces to lowest order a non-locality between two lepton pairs only [51] , but not between the members of each pair.

(4) The local action problem can be raised independently of strong interaction form factors.

(5) The problem is also independent of the existence of neutral lepton currents. If the latter were to exist (which at the moment does not look plausible) one would certainly assume that they were local to lowest order, on the same grounds as for the charged lepton currents.

(6) It is a hard question whether weak radiative corrections are the only conceivable source for possible deviations from locality.

(7) The first high energy neutrino experiment has shown [9] that a sizeable fraction of the events is inelastic. It is therefore of interest to look for such implications of local action which are valid also if F in Eq. (6. 1) represents an assembly of strongly interacting particles [52].

To get the results in their simplest form the choice of a co-ordinate system and of the variables in that system are important. We shall always work in the rest system of T. The various energy-momentum four vectors will be denoted as follows. $P_\lambda^{(0)} = (0, im_0)$ for T, $P_\lambda^\ell = (\vec{p}^\ell, i\omega)$ for ℓ, $P_\lambda^\nu = (\vec{p}^\nu, ip^\nu)$

for ν and $P_\lambda = (\vec{p}, iE)$ for F. Whenever F is an assembly of particles, $P^2 = -m^2$ is to be considered [52] as an independent variable along with $p = |\vec{P}|$. W will be either the differential cross-section for (6. 1) or the decay distribution for (6. 2), in either case summed over the lepton spin and over all intrinsic variables of F and T. Finally θ will be the angle between $\vec{p}^\nu$ and $\vec{p}$.

Theorem I [53]. Apart from a given kinematic factor, the local action of the lepton current implies that W is a quadratic function in each of the three variables $\cos\theta$, ω and p_ν:

$$K\,W\,(p, m, \cos\theta) = \alpha_0(p, m) + \alpha_1(p, m)\cos\theta + \alpha_2(p, m)\cos^2\theta, \tag{6.5}$$

$$K'\,W'\,(p, m, \omega) = \beta_0(p, m) + \beta_1(p, m)\omega + \beta_2(p, m)\omega^2, \tag{6.6}$$

$$K''W''(p, m, p_\nu) = \gamma_0(p, m) + \gamma_1(p, m)p_\nu + \gamma_2(p, m)p^2. \tag{6.7}$$

Here the K's denote the kinematic factors. The coefficients α, β, and γ depend on p and m only.

The proof of one of these relations implies that the other two hold as well because of [54] :

$$p = \pm (m^2 + m_0^2 - \mu^2 - 2m_0 E)/2(m_0 - E + p\cos\theta), \quad \omega = \mp p_\nu + m_0 - E. \tag{6.8}$$

We derive Eq. (6. 6). The transition probability for the reactions (6. 1) and (6. 2) are found by taking the appropriate matrix element of the space integral of (5. 1). Average the absolute square of the matrix element over the lepton spin. Because the lepton current is local, it follows that this average is of the form $A_{\alpha\beta} P^\nu_\alpha P^\ell_\beta$. Apart from a kinematic factor $\omega^{-1} p_\nu^{-1}$, $A_{\alpha\beta}$ depends on the heavy particle variables only and after performing all averages described above $A_{\alpha\beta}$ depends on P and P^0 only. The dependence on the (ℓ, ν) variables is therefore as follows. Either we get terms containing $P^\ell_\alpha P^\nu_\alpha$ which are independent of ω; or else we have to multiply each of the two factors:

$$\begin{aligned} P^0_\alpha P^\nu_\alpha &= \pm\, m_0(E + \omega - m_0), \\ P_\alpha P^\nu_\alpha &= \pm 1/2\,[\,2m_0\omega + m^2 - m_0^2 - \mu^2\,] \end{aligned} \tag{6.9}$$

with either of the factors:

$$P^0_\alpha P^\ell_\alpha = -m_0\omega, \quad P_\alpha P^\ell_\alpha = m_0(m_0 - \omega - E) + 1/2(m^2 + \mu^2 - m_0^2),$$

Under any circumstance we therefore get a quadratic function in ω. The scalar coefficients still depend on the residual independent heavy particle variables p and m. This proves Eq. (6. 6).

The proof of these relations evidently does not depend on the validity of CP-invariance nor on any of the $|\Delta\vec{T}|$ -rules for weak interactions. It has been shown [52] that if one considers the reactions (6. 1) and (6. 2) with specified lepton helicity, one obtains an expression with five structure functions [55] .

It should be noted that there exists one unavoidable deviation from local action, due to the electromagnetic coupling of ℓ with either T or F or both [56]. This effect should be small, especially at high energies.

For the "elastic" processes (6. 3) and (6. 4) it is of course possible to express the coefficients in Eqs.(6. 5 - 7) in terms of the nucleon structure functions, using Eqs.(5. 1 - 4) with $(B_1, B_2) = (p, n)$. Eq. (6. 6) then takes the following explicit form [53] :

$$d\sigma = [\beta_2(p)\omega^2 + \beta_1(p)\omega + \beta_0(p)]\,(md^3p)/[4\pi^2\omega E p_\nu dE_{tot}] \tag{6.10}$$

where the three structure functions β are given by [57]

$$\begin{aligned}(1/2)\beta_2 &= |g_A|^2 + |g_V - 2mf_V|^2 + 2m(E-m)(|f_V|^2 + |f_A|^2),\\ (1/2)\beta_1 &= (E-m)|g_A \pm g_V|^2 - (\mu^2/2m)(|g_A|^2 + |g_V|^2)\end{aligned} \tag{6.11}$$

$$\begin{aligned}&+ 2m(E - m - \mu^2/2m)[(E+m)|f_V|^2 + (E-m)|f_A|^2 - 2\mathrm{Re} f_V^* g_V]\\ &- \mu^2 \mathrm{Re}[h_V^* g_V - (E+m)f_V^* h_V - (E-m)f_A^* h_A]\end{aligned} \tag{6.12}$$

$$\begin{aligned}\beta_0 &= (E-m)(E - m - \mu^2/2m)|g_A \pm g_V|^2 + m(E - m + \mu^2/2m)\\ &(|g_A|^2 - |g_V|^2) - \left[2m^2(E-m) + (\mu^2/2m)(3E-m) - \mu^4/4m\right]\\ &\left[(E+m)|f_V^2| + (E-m)(f_A|^2 - 2\mathrm{Re} f_V^* g_V\right] + (\mu^2/2)(E - m + \mu^2/2m)\\ &\left[(E+m)(h_V|^2 + (E-m)|h_A|^2\right]\\ &- \mu^2(E-m)\mathrm{Re}\left[h_V^* g_V + h_A^* g_A - (E+m)f_V^* h_V - (E-m)f_A^* h_A\right]\\ &+ (\mu 4/2m)\mathrm{Re}\left[h_V^* g_V - h_A^* g_A - (E+m)f_V^* h_V - (E-m)f_A^* h_A\right]\end{aligned} \tag{6.13}$$

In Eqs.(6. 12 - 13) the upper and lower signs refer to the reactions (6. 4) and (6. 3) respectively. It follows from Eq. (6. 11) that the difference $d\sigma_\nu - d\sigma_{\bar\nu}$ is (apart from a kinematic factor) a linear function in ω. This difference depends on one combination of structure functions only, namely the V-A interference effect $\mathrm{Re}\, g_A^* g_V$. This is the same type of effect which we encountered in the comparison of Eqs.(2. 5) and (2. 6). It can be shown from quite general considerations that in these instances the differences between neutrino and anti-neutrino reactions must be due to interference between structure functions related to terms in the current of opposite parity [58]. The expressions for the β's simplify if both CP-invariance and the $|\Delta \vec{T}| = 1$ rule hold, on account of Eq. (5. 24).

Eq. (6. 7) takes the following form for the reactions (6. 3 - 4). Replace in Eq. (6. 10) the square bracket by $\gamma_2(p)p_\nu^2 + \gamma_1(p)p_\nu + \gamma_0(p)$ with

$$\begin{aligned}\gamma_2(p) &= \beta_2(p), \qquad \gamma_1(p) = 2(m-E)\beta_1(p) + \beta_0(p)\\ \gamma_2(p) &= (m-E)^2\beta_2(p) + (m-E)\beta_1(p) + \beta_0(p).\end{aligned} \tag{6.14}$$

This form is well suited to perform averages over a (known) incident neutrino spectrum for fixed p. After having done this, one can find one relation for fixed p between the three γ's. The theorem implies that there should in general exist a linear relation between four such measurements (that is, done with four distinct spectra).

Equations similar to (6.10 - 14) can also be written down [59] for the reactions ν + nucleon $\to$ hyperon + ℓ.

Finally we examine the structures which arise when deviations from local action due to weak radiative effects are taken seriously. We shall maintain the view that the primitive interaction is due to a local lepton current of the (V, A) type coupled to something else (be it a boson field or a heavy particle current).

When we take into account only those non-local effects induced by the strong interactions in the heavy particle current, then it follows from Lorentz invariance and from the just mentioned structure of the primitive interaction that the effective interaction is of the form: (V, A) heavy particle source x (V, A) lepton source. This is no longer true if the weak radiative corrections are included as well. It is instructive to distinguish between two general classes of such radiative effects.

(a) Lepton-lepton weak radiative corrections. These are schematically indicated in Fig. 7. A lepton pair is produced in point interaction with the

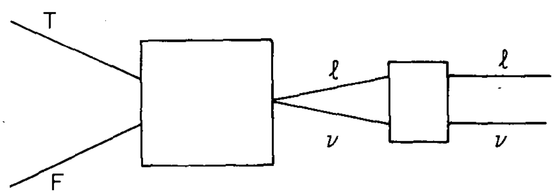

Fig. 7
General structure of lepton-lepton weak radiative corrections.

heavy particle source. The leptons then interact weakly with each other. In this special case the general interaction is still of the general form (5. 1) where J_λ is still the general current discussed in Section V. But $j_\lambda^{(\ell)}(x)$ is now no longer given by Eq. (2. 1). Instead, we must also admit the presence of induced leptonic terms. Thus $j_\lambda^{(\ell)}$ is now of the form:

$$j_\lambda^{(\ell)} = \bar{\ell}\, O_\lambda \nu, \tag{6.15}$$

where O_λ is given by Eq. (5. 3) with structure functions g, f, h appropriate to induced weak effects. However, we have assumed throughout that all basic neutrino reactions are of the two-component type. Hence the primitive interaction is invariant under the γ_5-transformation:

$$\nu' = \gamma_5 \nu, \qquad \bar{\nu}' = -\bar{\nu}\gamma_5 \tag{6.16}$$

and so, therefore, is the effective interaction. It follows that

$$(\ell, \nu) : g_A = g_V, \qquad f_A = f_V, \qquad h_A = h_V. \tag{6.17}$$

Let us next look at the expression (6. 15) in the zero lepton mass approximation which at high momentum transfers is certainly good for electrons and is not bad for μ-mesons. In this approximation we have the additional invariance for

$$l' = \gamma_5 l, \qquad \bar{l}' = -\bar{l}\gamma_5. \tag{6.18}$$

Whenever Eq. (6. 18) applies we have

$$h_A = h_V = f_A = f_V = 0. \tag{6.19}$$

The only structure function which then remains is g_ν which may now depend on the invariant momentum transfer. This does not change the situation insofar as the dependence on the individual four momenta p_λ^ν and p_λ^l is concerned. Thus the arguments used in the proof of Theorem I apply here too. Theorem II. In the zero lepton mass approximation the equations (6. 5 - 7) of the local action theorem are also valid if lepton-lepton weak radiative corrections are included.

(b) Lepton-heavy particle weak radiative corrections. These involve combinations of interactions between either ν or l and F or T. One example is drawn in Fig. 8. The effective interaction now contains in general the following kinds of terms:

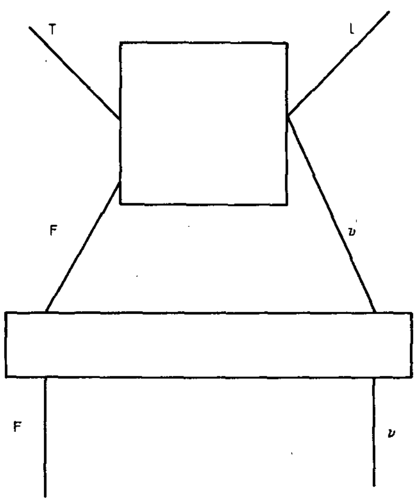

Fig. 8

A lepton-heavy particle weak radiative correction.

(α) Scalar terms of the form

$$S(x)\bar{l}(1+\gamma_5)\nu + \text{h.c.} \tag{6.20}$$

where S is a scalar /pseudoscalar function of the heavy particle fields. In Eq. (6. 20) the strict γ_5-invariance (6. 16) has been taken into account. A scalar structure function originating from the lepton part of (6. 16) may be thought to be absorbed in S. It follows from Eq. (6. 18) that the induced terms (6. 20) are zero in the zero lepton mass approximation.

(β) Vector terms. Their discussion is identical with the one given above for lepton-lepton corrections.

(γ) Tensor terms in the effective interaction of the form

$$J_{\mu\nu}(x)\bar{\ell}[g_T\sigma_{\mu\nu} + f_T\gamma_\mu(\frac{\overrightarrow{\partial}}{\partial x_\nu} - \frac{\overleftarrow{\partial}}{\partial x_\nu}) + h_T\gamma_\mu(\frac{\overrightarrow{\partial}}{\partial x_\nu} + \frac{\overleftarrow{\partial}}{\partial x_\nu})](1+\gamma_5)\nu \qquad (6.21)$$

with three lepton structure functions g_T, f_T, h_T. $J_{\mu\nu}$ is a heavy particle tensor source. Note that the h_T-terms can be brought to the form (V, A) x(V, A) by a partial integration, so for this term Theorem II applies forthwith. Moreover, in the zero lepton mass approximation the g-term goes to zero as well, so that in this case only the f_T-terms survive. If we now decompose $J_{\mu\nu}$ into its irreducible parts, its trace term does not contribute for zero lepton mass.

(δ) In the same way one can discuss tensors of higher rank. In the zero lepton mass approximation we thus find that the effective interaction can be written generally as:

$$J_\mu\bar{\ell}\gamma_\mu(1+\gamma_5)\nu + J_{\mu\nu}\bar{\ell}\gamma_\mu D_\nu(1+\gamma_5)\nu + J_{\mu\nu\rho}\bar{\ell}\gamma_\mu D_\nu D_\rho(1+\gamma_5)\nu + \ldots + \text{h.c.} \qquad (6.22)$$

where $D_\mu = \frac{\overrightarrow{\partial}}{\partial x_\mu} - \frac{\overleftarrow{\partial}}{\partial x_\mu}$. We have $J_{\mu\mu} = 0$, $J_{\mu\nu\rho} = J_{\mu\rho\nu}$, $J_{\mu\mu\rho} = 0$ etc. Each successive term raises by two the maximum power of $\cos\theta$, or ω, or p_γ which appears in the differential cross-section for any process of the type (6.1).

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[56] See ref. [37], Fig. 1 (b).
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