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INFLATION AND AXION COSMOLOGY

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Abstract

It is argued that the standard constraint $f \lesssim 10^{12} GeV$ in the axion theory and related constraints in the theories of other weakly interacting scalar fields do not apply in many versions of the inflationary universe scenario.

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In many realistic theories of elementary particles there exist some particles and fields which interact extremely weakly with each other and with other particles. Among such particles are axions [1], which are necessary to solve the strong CP-violation problem, and dilatons, which have been recently invocated in order to solve the cosmological constant problem [2]. Since such particles are extremely weakly interacting, the main constraints on the corresponding theories stem not from the accelerator data, but from cosmological considerations.

For example, in the theory of axion fields ϕ, which can be parameterized as $\phi = f\ sin\ \theta$, there exists the famous constraint $f \lesssim 10^{12} GeV$ [3]. This constraint leads to considerable cosmological difficulties in many superstring theories, in which the natural scale of f is of the order of $10^{16} - 10^{17} GeV$ [4]. Similar constraints do exist in the theories of dilatons, which was one of the main reasons for the authors of ref. [5] to reject the solution of the cosmological constant problem suggested in [2].

The main line of reasoning which leads to such constraints is very simple and apparently irrefutable. At sufficiently small ϕ (at $\phi \lesssim f$) the effective potential of a weakly interacting scalar field ϕ usually can be written as $V(\phi) = \frac{m^2}{2}\phi^2$. A typical initial value of the field ϕ in the early universe is $\phi = 0(f)$. At very early stages of the evolution of the universe, when the Hubble parameter H is much bigger than m, the amplitude of the field ϕ and its energy remain practically unchanged due to the large "friction term" $3H\dot{\phi}$ in the equation of motion of the field ϕ. Later, when the value of H(t) becomes smaller than m, the field ϕ oscillates near the point $\phi = 0$. Since this field practically does not interact with all other particles and fields, the amplitude of its oscillations decreases solely because of the expansion of the universe: $V(\phi) \sim a^{-3}(t)$, where $a(t)$ is the scale factor of the universe, see e.g. [6]. Note, that the energy density of massive nonrelativistic particles decreases in the same way, whereas the energy density of a hot dense gas of ultrarelativistic particles decreases much more rapidly, as a^{-4}. For this reason the energy density of the field ϕ, being originally rather small, may come to dominate at late stages of the evolution of the universe. In particular, in the axion theories with $\phi \sim 0(f) \sim 10^{12} GeV$ the energy density of the oscillating classical axion field at present should be two orders of magnitude greater than the energy density of protons. Since the energy density of protons now constitutes several percent of the overall energy density of the universe $\rho_o \sim 10^{-29} g \cdot cm^{-3}$, it was argued that in the theories with $f \gtrsim 10^{12} GeV$ the energy density of axions at present would be much greater than ρ_o, which would make our universe closed and would cause its premature collapse.

In the inflationary universe scenario the situation looks considerably different. During inflation long-wave perturbations of the axion field ϕ are generated. As a result, after inflation the universe becomes filled with the field ϕ gradually changing from $-f$ to f in its different domains [7]. In domains of the size of the same order as the size of the observable part of the universe, $\ell \sim 10^{28} cm$, the axion field ϕ typically changes by no more than $0(10)H$, where H is the Hubble parameter at the last stages of inflation [7]. For $H \ll f$ this means that our universe becomes divided into many mini-universes of a size $0(10^{28} cm)$ containing almost homogeneous field ϕ taking all values from $-f$ to f. In principle, we can safely live in a domain with $\phi \ll f$, so that in the observable part of the universe we will have no problems with axions mentioned above [8,7], and no constraints of the type $f \lesssim 10^{12} GeV$ will appear. However, at first glance such a possibility seems very unnatural. Indeed, it seems improbable that we were occasionally born in a non-typical domain of the universe with $\phi \ll f$, and just due to this happy occasion we see no contradiction between the theoretical predictions and the observational data.

This intuitive argument can be made slightly more precise. What we are speaking about is the ***conditional probability*** for an observer of our type to see the part of the universe with the given properties under an obvious and apparently trivial condition that the observer is a person who is alive during the process of making observations. Now, since all people are made out of baryons (and leptons), let us estimate the fraction of all baryons which exist in those parts of the universe in which the initial value of the field ϕ happened to be much smaller than f. The process of baryosynthesis in the inflationary universe occurs typically at $T \gtrsim 10^2 GeV$. At that time axions still have no effect on the evolution of the universe and the fraction of the volume of the universe in a state with the field in the internal $\phi_o \lesssim \phi \lesssim \phi_o + \Delta\phi$ was of the order of $\frac{\Delta\phi}{f}$ independent of the value of ϕ_o. In other words, the probability to find a given value of ϕ in some part of the universe at a given time was practically independent of ϕ for $\phi \lesssim f$. Therefore the total number of baryons in domains with $\phi_o \lesssim \phi \lesssim \phi_o + \Delta\phi$ does not depend on ϕ, $\frac{N_B(\Delta\phi)}{N_B(total)} \sim \frac{\Delta\phi}{f}$. For this reason one could argue that the main part of observers, which are made out of baryons, should be located in domains with $\phi \sim f$ rather than with ϕ close to 0. Thus, a typical observer would see himself not in a safe region with $\phi \ll f$, but in a typical domain of the universe with $\phi \sim f$, and this would contradict the observational data for $f \gg 10^{12} GeV$.

We see, that in the inflationary universe scenario there is *no rigorous proof* of the existence of the constraint $f \lesssim 10^{12} GeV$, and the derivation of this constraint actually is based on some intuitive anthropic arguments about the plausibility or implausibility of being born in a part of the universe with $\phi \ll f$. Now we are going to show that the arguments suggested above are insufficient to justify the validity of the constraint $f \lesssim 10^{12} GeV$ in the inflationary cosmology.

Indeed, let us consider an "improbable" domain with such a small value of the field $\phi = \phi_o \ll f$, that the present energy density ρ_ϕ of the field ϕ at present is one or two orders of magnitude greater than the energy density of protons ρ_ϕ, in agreement with observational data (we will call it "our" domain), and let us consider also another, ten times "more probable" domain, with $\phi \sim 10\phi_o \ll f$. The total energy density of matter inside this domain at the moment $t = 10^{10}$ years is $\rho_o \sim \rho_\phi \sim 10^{-29} g \cdot cm^{-3}$, just as in our domain ($\rho_o = \frac{M_p^2}{6\pi t^2}$ for a flat universe dominated by cold matter or axion fields). However, the *relative* energy density of axions as compared with the density of baryons $\frac{\rho_\phi}{\rho_B}$ increases as $(\frac{\phi}{\phi_o})^2 \sim 10^2$ times [3,7], which means that the density of protons in this domain is 10^2 smaller than the density of protons in the first domain. Since the ratio $\frac{n_\gamma}{n_B} = 10^{-9}$ usually does not depend on the value of the axion field, the density of photons in the second domain $n_\gamma \sim T^3$ will also be 10^2 times smaller than in the first one. At the early stages of the evolution of the universe the dominant contribution to the energy density of matter was given by hot relativistic matter. The growth of initial density perturbations $\frac{\delta\rho}{\rho} \sim 10^{-4}$ starts at the moment when the cold matter becomes dominant. As mentioned already this occurs due to the more rapid decrease of $\rho_\gamma \sim T^4 \sim a^{-4}$ as compared with the axion (or proton) density $\rho_\phi \sim a^{-3}$. In the hot universe the scale factor $a(t)$ is proportional to $\sqrt{t}$. Thus, the second domain will become dominated by cold matter 10^4 times earlier than the first one. Initial density perturbations of density in both domains will be the same, $\frac{\delta\rho}{\rho} \sim 10^{-4}$, at least if $\frac{H}{\phi} \lesssim 10^{-4}$ at the last stages of inflation [7]. The rate of growth of perturbations $\frac{\delta\rho}{\rho}$ is the same in both domains. Therefore the time t^*, at which these perturbations grow up to $\frac{\delta\rho}{\rho} \sim 0(1)$, in the second domain is 10^4 times smaller than in the first one. The density of galaxies (or galaxy-like objects) at present typically remains the same as at the moment when $\frac{\delta\rho}{\rho}$ becomes $0(1)$ and these objects become separated from the general process of the expansion of the universe. Consequently, the average density of matter inside galaxies in the second domain, which is proportional to $\frac{M_p^2}{6\pi(t^*)^2}$, is 10^8 times greater than in our domain.

Thus, we obtained a rather unexpected result. In the second domain, in which the value of the field ϕ is just one order of magnitude bigger than in our domain, the energy density inside galaxies would be 8 orders of magnitude greater! But it is well known, that the appearance of life of our type requires very special conditions, which in all likelihood are not satisfied in galaxies so radically different from ours. Therefore, even though ten times more baryons do exist in domains with the field ϕ which is ten times bigger than the field ϕ_o in our domain, it seems very plausible that from all these baryons it would be impossible to make one real man (i.e. an observer of our type). That is why we see ourselves in a domain filled with a sufficiently small field ϕ: We just cannot *see* ourselves in domains with large ϕ.

The argument we have used is based on the so-called anthropic principle, which, roughly speaking, says that a fish sees itself in the water not for the reason that all universe is filled with water, but just for the reason that it can live only in water and that there is enough water on the earth. Before the invention of the inflationary universe scenario the anthropic principle was considered as being rather metaphysical, since it implied that our universe was created many times before the final success. It was not clear also, why the conditions in the observable part of the universe are approximately the same everywhere, whereas it would be easier to make good conditions for life in a much smaller volume. In the context of the inflationary universe scenario this principle, being handled with care, becomes a very useful tool [9]. The inflationary universe itself produces domains of the universe filled with all possible values of the field ϕ, and the size of each domain filled with an almost homogeneous field ϕ after inflation becomes much greater than the size of the observable part of the universe. This opens new possibilities in understanding various properties of the part of the universe in which we live now. Simultaneously, this makes it possible to relax or remove some constraints on the elementary particle theories which were obtained in the context of the previous cosmological scheme. In this way we may also obtain an explanation of the small ratio $\frac{\rho_B}{\rho_c} \approx \frac{\rho_B}{\rho_\phi} \sim 10^{-2}$. Indeed, in our universe there exist domains in which the field ϕ initially was extremely small, so that at present $\frac{\rho_B}{\rho_\phi}$ is relatively large, say $\frac{\rho_B}{\rho_\phi} \sim 0(1)$. However, the number of baryons in such domains is suppressed by a small factor $(\frac{\phi}{f})^2$. On the other hand, there are much more domains with $\phi \sim f$ and $\frac{\rho_B}{\rho_\phi} \ll 1$. However, in such domains the existence of life of our type is improbable. This gives us the maximal probability to live in a domain in which $\frac{\rho_B}{\rho_\gamma}$ is small, but is not too small. For a certain choice of $f \gg 10^{12} GeV$ it may prove most probable to live in a domain with $\frac{\rho_B}{\rho_\gamma} \sim 10^{-1} - 10^{-2}$. This may make it possible to determine the value of f for which the observed ratio $\frac{\rho_B}{\rho_\gamma}$ would be the most probable one. However, this would require a much more detailed investigation of the galaxy formation, the origin of stars etc., as a function of the value of the field ϕ. We hope to return to a discussion of this question in a separate publication [10].

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