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ESTIMATE OF NEUTRINO EVENT RATES AT SLAC - PHASE I

H. Pierre Noyes

When it became clear (cf. D. B. Lichtenberg, SLAC 25A - 1963) that the yield of forward pions from the 30-BeV proton machines is about the same as that predicted by the Drell-Ballam mechanism for 20-BeV electrons at SLAC, it was obvious that we could only hope to detect π-μ decay neutrinos at about the same level as already was being achieved at BNL and CERN, and that (since we could not exploit the fine structure of the machine) the background problem would be worse. For μ-e decay neutrinos, geometrical factors are roughly comparable (see below), and since the intensity of μ's is expected to be about the same (cf. D. Fries, TN-63-83) as for π's, the event rates in an unsophisticated experiment will be down by a factor of 100. It was therefore decided that neutrino studies were not suitable "first generation" experiments at SLAC, and that the question should be reopened only if higher energies (Phase II) were in prospect, or some type of neutrino process where SLAC would offer unique advantages were thought of. So far neither development has occured, but we occasionally get enquiries about neutrino event rates to be expected at SLAC. The following very crude calculations made in 1963, which essentially merely substantiate the conclusion drawn above, are presented "for the record." A critical review of these calculations has recently been made by Faissner (cf. Appendix A).

As is appropriate for a preliminary experimental design study of experiments which would be in serious financial competition with other areas of the project, and to which no existing experimental group is committed, the philosophy adopted is purposely conservative. We assume a detector of a size (50 tons) which has already been constructed, geometrically optimized but otherwise unspecified, and a shield thickness (20 m) known to be adequate. Further, we assume that low energy neutrinos can better be studied at other laboratories, and consider events due to neutrinos of energies greater than 1 BeV. Since preliminary surveys already exist, we assume that experiments will be made to study individually identified reactions, so adopt an average cross section of 0.6×10^{-38} cm^2 and quote event rates for this cross section rather than total event rates for all processes. (Our final results are therefore the same as quoting integral neutrino spectra above 1 BeV passing through the detector). For the same reason, we utilize only pions produced by the Drell-Ballam mechanism and ignore ordinary boson pair production, which contributes mainly to the low energy neutrinos. Since the overall production cross section to be expected is uncertain to at least a factor of 2, we make geometric approximations that do not significantly increase this overall uncertainty. To provide lower and upper limits, we compute either with the angular distribution of primaries given by the basic production mechanism, or with all primaries (within our energy limits) on axis, without specifying how the latter focusing is to be achieved. Under these drastic assumptions, the kinematics of the Lorentz transformation allows a very accurate treatment of the geometry, which we exploit. It should be emphasized that in the energy region of interest here, this approximation works only for π's and μ's, and the defocusing of K-μ decay neutrinos cannot be accommodated in this framework. Consequently a much more sophisticated treatment of the transport problem will be essential if it should turn out that K intensities are high enough to make such experiments interesting.

I. KINEMATICS

If a pion at rest decays to a neutrino of energy pc and a muon of energy $[(m_\mu c^2)^2 + (pc)^2]^{\frac{1}{2}}$, the energy of the neutrino is

$$pc = \frac{1}{2m_\pi c^2}\left[(m_\pi c^2)^2 - (m_\mu c^2)^2\right] = 0.0298150 \text{ BeV}$$

If the π has an energy $E = \gamma m_\pi c^2$ and the neutrino is emitted at an angle θ_p with respect to the pion direction, the energy of the neutrino P_c is uniquely related to the angle of emission by

$$pc = \gamma Pc\,(1 - \beta \cos \theta_p)$$

Since we have decided to consider only neutrinos of energy $Pc \geq 1$ BeV, these are confined to a cone of angle θ_p about the pion direction which is given by

$$\sin^2 \theta_p = \frac{1}{\gamma^2\beta^2}\left[\frac{2\gamma p}{P} - 1 - \frac{p^2}{P^2}\right]$$

To a high degree of accuracy (since $p/P < 0.03$)

$$\theta_p^\pi = \frac{7.99849^\circ}{E_\pi}\left(\frac{0.427149 E_\pi}{Pc} - 1\right)^{\frac{1}{2}} \tag{1}$$

where E_π is the energy of the pion in BeV. This limiting angle is plotted in Fig. 1, together with the angle subtended a 50-ton detector ("optimized" as discussed below) at different decay distances. Assuming for the moment that all neutrinos within this cone pass through the detector, we still need the fraction of the total number of π decays which lead to neutrinos at angles less than θ_p^π. Since the relation between the center-of-mass angle ϑ_{cm} and the lab angle is $(1 + \beta \cos \vartheta_{cm})(1 - \beta \cos \theta) = 1 - \beta^2$, and the angular distribution is uniform in the c.m. system, this fraction is

$$\frac{1}{2}\int_{\vartheta_{cm}}^{1} d\,(\cos\vartheta) = \frac{1-\beta^2}{2\beta}\int_{\theta_p}^{1} d\,\frac{1}{(1-\beta\cos\theta)} = \frac{1+\beta}{2\beta} - \frac{1-\beta^2}{2\beta}\,\frac{\gamma P}{p}$$

$$\doteq 1 - \frac{P}{2\gamma p} = 1 - 2.34110\ Pc/E_\pi = F_\pi\ (E_\pi, \theta_p^\pi) \tag{2}$$

[Note also $\frac{1+\beta}{2\beta} - \frac{1-\beta^2}{2\beta}\,\frac{1}{1-\beta\cos\theta_p} \doteq \frac{\gamma^2\theta_p^2}{1+\gamma^2\theta_p^2}$ cf. Eq. (14)]

For the $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$ decay, the situation differs in that p is no longer unique but can have any value between 0 and $m_\mu c/2$. However, Eq. (1) still holds, so the maximum energy of the neutrino varies between $E_\mu\ (1-\beta)/2$ at 180° and $E_\mu\ (1+\beta)/2$ at 0°. Again assuming that we are only interested in neutrinos of energy $Pc \geq 1$ BeV, these backward angles made no difference, and we are interested only in neutrinos within a cone of angle θ_p^μ about the μ direction given by

$$\theta_p^\mu = \frac{6.05381^\circ}{E_\mu}\left(\frac{E_\mu}{Pc} - 1\right)^{\frac{1}{2}} \tag{3}$$

which is plotted in Fig. 2. The only essential difference from the π cone is therefore that at any angle within this cone the neutrino energy is not unique, but has a spectrum of values only part of which are above Pc. To compute $F_\mu\ (E_\mu, \theta_p^\mu)$ we therefore need to integrate over the portion of this spectrum which lies above Pc as well as over the cone.

As is shown in Appendix B, the spectra of the ν_μ and the $\bar{\nu}_e$ are different and are given by

$$d\omega_{\nu_\mu} = \frac{4\pi^2 G^2 d^3P}{P_4 K_4 (2\pi)^6}\,(K\cdot P)\left[K^2 - \frac{4}{3}(P\cdot K)\right] \tag{4}$$

$$d\omega_{\bar{\nu}_e} = \frac{8\pi^2 G^2 d^3P}{P_4 K_4 (2\pi)^6} (K \cdot P)[K^2 - 2(P \cdot K)] \tag{5}$$

when P and K are the 4-momenta of the neutrino and the muon respectively. (For μ^+ decay these are the spectra of $\bar{\nu}_\mu$ and ν_e.) Since at any lab angle, the neutrino energy varies between 0 and $\mu/2\gamma\,(1 - \beta\cos\theta)$, we can check the overall normalization by

$$\begin{pmatrix} \omega_{\nu_\mu} \\ \omega_{\bar{\nu}_e} \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \frac{G^2\mu^2}{(2\pi)^3} \int_{-1}^{1} d(\cos\theta)(1 - \beta\cos\theta) \int_0^{\mu/2\gamma(1-\beta\cos\theta)} P^2 dP \left[1 - \begin{pmatrix} \frac{2}{3} \\ 1 \end{pmatrix} \frac{2\gamma P}{\mu}(1-\beta\cos\theta)\right]$$

or with $x = \frac{2\gamma P}{\mu}(1 - \beta\cos\theta)$, $y = 1 - \beta\cos\theta$

$$= \begin{pmatrix} 1 \\ 2 \end{pmatrix} \frac{G^2}{(2\pi)^3} \left(\frac{\mu}{2\gamma}\right)^3 \frac{1}{\beta} \int_{1-\beta}^{1+\beta} \frac{dy}{y^2} \int_0^1 dx \left[x^2 - \begin{pmatrix} \frac{2}{3} \\ 1 \end{pmatrix} x^3\right]$$

$$= \frac{G^2\mu^5}{3(4\pi)^3 y} = \frac{1}{\gamma\tau_\mu}$$

where τ_μ is the mean life in the rest system of the muon. If we now restrict ourselves to neutrinos of energy $Pc \geq 1\,\text{BeV}$, the lower limit on x is $\frac{2\gamma P}{\mu}\,y$, while restriction to the cone defined by Eq. (3) gives an upper limit on y of $\frac{\mu}{2\gamma P}$. Hence

$$\begin{pmatrix} F_{\nu_\mu}(E_\mu, \theta_\mu^P) \\ F_{\bar{\nu}_e}(E_\mu, \theta_\mu^P) \end{pmatrix} = \frac{1}{2\gamma^2\beta}\left[-\frac{1}{y} - \begin{pmatrix} 1 \\ 2 \end{pmatrix}\left(\frac{2\gamma P}{\mu}\right) y^2 + \begin{pmatrix} \frac{1}{3} \\ 1 \end{pmatrix}\left(\frac{2\gamma P}{\mu}\right)^4 y^3\right]_{1-\beta}^{\mu/2\gamma P}$$

$$= \frac{1+\beta}{2\beta}\left\{1 - \begin{pmatrix} \frac{5}{3} \\ 2 \end{pmatrix} \frac{1}{1+\beta}\left(\frac{2P}{\gamma\mu}\right) + \begin{pmatrix} 1 \\ 2 \end{pmatrix}\left[\frac{2P}{(1+\beta)\gamma\mu}\right]^3 - \begin{pmatrix} \frac{1}{3} \\ 1 \end{pmatrix}\left[\frac{2P}{(1+\beta)\gamma\mu}\right]^4\right\}$$

If we let $\beta = 1$ and $\epsilon = Pc/E_\mu$, these fractions are well approximated by

$$F_{\nu_\mu}(E_\mu, \theta^p_\mu) = 1 - 2\epsilon + 2\epsilon^3 - \epsilon^4 \tag{6}$$

$$F_{\bar{\nu}_e}(E_\mu, \theta^p_\mu) = 1 - \frac{5}{3}\epsilon + \epsilon^3 - \frac{1}{3}\epsilon^4 \tag{7}$$

which are plotted in Fig. 3.

Thus to the extent that we can say that all neutrinos within the cones defined by (1) or (3) have an equal chance of being detected, the only difference between π and μ decay is the different fraction of the decays which lead to neutrinos of energy $\geq Pc$ as given by Eqs. (2), (6), and (7). We discuss the geometrical corrections in the next section.

II. GEOMETRY

The geometry envisaged is sketched in Fig. 4. A (point) source emits pions or muons with some angular distribution $f(\theta)$, which decay over a distance D, and (together with other background) are stopped by a shield of thickness $S = 20$ m. The neutrinos enter a cylindrical detector of length L, and fixed volume $V = \pi r^2 L$. For $\rho = 1\ \text{gm/cm}^3 = 1\ \text{ton/m}^3$, which is representative of aluminum plate spark chambers, $V = 50\ \text{m}^3$. To the extent that the neutrinos follow the parent particle direction, the number of neutrino events will be proportional to $DLf(\alpha)$ with $\alpha = \dfrac{(V/\pi L)^{1/2}}{S + D + L}$. Hence, if we optimize simultaneously with respect to L and D

$$\begin{aligned} \frac{\partial}{\partial D}(DLf) &= 0 = Lf - \frac{DL\alpha}{D + S + L}\frac{\partial f}{\partial \alpha} \quad \text{or} \quad f = \frac{D}{D + S + L}\,\alpha\frac{\partial f}{\partial \alpha} \\ \frac{\partial}{\partial L}(DLF) &= 0 = Df - \frac{D + S + 3L}{2(D + S + L)}\,\alpha\frac{\partial f}{\partial \alpha} \quad \text{or} \quad f = \frac{D + S + 3L}{2(D + S + L)}\,\alpha\frac{\partial f}{\partial \alpha} \end{aligned} \tag{8}$$

Hence, independent of the angular distribution f (unless $\partial f/\partial \alpha = 0$)

$$D = S + 3L \tag{9}$$

optimizes the rate provided also

$$f(\alpha) = \frac{S + 3L}{2S + 4L} \alpha \frac{\partial F}{\partial \alpha} \quad \text{with} \quad \alpha = \frac{(V/\pi L)^{\frac{1}{2}}}{2S + 4L} \tag{10}$$

Thus (10) can be solved for L and D determined from (9). (This optimization is due to D. B. Lichtenberg.) For $S = 20$ m and $V = 50$ m^3 the Drell-Ballam $f(\alpha)$ gives $D \sim 40$ m for $E_\pi \sim 10$ BeV and is rather insensitive to E_π. However if we have complete focusing, $\frac{\partial f}{\partial \alpha} = 0$ and the optimization will depend on the defocusing due to decays rather than $f(\alpha)$. Since, in any case, we do not wish to optimize for a particular E_π, we consider three cases

$$\begin{aligned} D &= S + 2L \\ D &= S + 3L \\ D &= S + 4L \end{aligned} \tag{11}$$

and calculate event rates as a function of D. We find below that event rates are not particularly sensitive to the different detector shapes, so the results offer a reasonable guide to overall event rates. Clearly they are not optimized for particular neutrino energies — to do so would require also a discussion of focusing, and a much more complicated investigation of the neutrino spectrum than the present approach allows.

So long as $\theta_p < \alpha$, our assumption that the neutrinos follow the parent particle distribution includes too many events for $\alpha - \theta_p < \theta < \alpha$, and too few for $\alpha < \theta < \alpha + \theta_p$ (cf. Fig. 4). Rough calculations show that these "edge effects" introduce errors of less than 30%, so can be dropped within the accuracy aimed for here. However, if $\alpha < \theta_p$, not all the neutrinos within the cone pass through the detector, but only those within the cone $\delta(x)$ defined (for particles on axis) in Fig. 5. This gives a correction to the fraction of the decays [Eqs. 2, 6, 7] which give a neutrino of energy greater than Pc which pass through the detector. If we let X_o

be defined by $\theta_p = \frac{(V/\pi L)^{\frac{1}{2}}}{S + L + X_o}$ then for pions $F_\pi(E_\pi, \theta_p^\pi)$ is to be multiplied by

$$\frac{X_o}{D} + \frac{1}{D}\int_{X_o}^{D} \frac{\gamma^2\delta^2(x)}{1 + \gamma^2\delta^2(x)}\, dx \tag{12}$$

where

$$\delta(x) = \frac{(V/\pi L)^{\frac{1}{2}}}{S + L + X} \tag{13}$$

and we have used the approximation

$$\begin{aligned} \frac{1}{2}\int_{\cos\delta}^{1} \frac{1-\beta^2}{2\beta}\, d\, \frac{1}{1-\beta\cos\theta} &= \frac{1+\beta}{2\beta} - \frac{1-\beta^2}{2\beta(1-\beta\cos\delta)} \\ &\doteq 1 - \frac{1}{2\left(\frac{1}{1+\beta} + \frac{\beta}{2(1-\beta^2)}\frac{\delta^2}{2}\right)} \doteq \frac{\gamma^2\delta^2}{1+\gamma^2\delta^2} \end{aligned} \tag{14}$$

For μ decay, we must replace $F_{\nu_\mu}(E_\mu, \theta_p^\mu)$ by

$$\frac{X_o}{D}(1 - 2\epsilon + 2\epsilon^3 - \epsilon^4) + T(X_o) - 2\epsilon^3(2Q_1 + Q_2) + \epsilon^4(3Q_1 + 3Q_2 + Q_3) \tag{15}$$

and $F_{\bar{\nu}_e}(E_\mu, \theta_p^\mu)$ by

$$\frac{X_o}{D}(1 - \tfrac{5}{3}\epsilon + \epsilon^3 - \tfrac{1}{3}\epsilon^4) + T(X_o) - \epsilon^3(2Q_1 + Q_2) + \epsilon^4(Q_1 + Q_2 + \tfrac{1}{3}Q_3) \tag{16}$$

where

$$T(X_o) = \frac{1}{D} \int_{X_o}^{D} \frac{\gamma^2\delta^2(x)}{1 + \gamma^2\delta^2(x)} \, dx = \frac{\gamma}{D}\sqrt{\frac{50}{\pi L}} \left[\tan^{-1}\left(\frac{S + L + D}{\gamma\sqrt{\frac{50}{\pi L}}}\right) - \tan^{-1}\left(\frac{S + L + X_o}{\gamma\sqrt{\frac{50}{\pi L}}}\right)\right] \quad (17)$$

$$Q_n = \frac{1}{D} \int_{X_o}^{D} [\gamma\delta(x)]^{2n} \, dx = \frac{1}{(2n + 1)D}\left(\frac{50\gamma^2}{\pi L}\right)^n \left[\frac{1}{(S + L + X_o)^{2n+1}} - \frac{1}{(S + L + D)^{2n+1}}\right] \quad (18)$$

This "approximation" is correct for the completely focused case where all π's or μ's are assumed on axis, and neglects edge effects (which we already saw to be reasonably small) for other cases.

It is to be emphasized again that these corrections can be made in this simple form only because we take neutrinos greater than some energy and assume a constant neutrino cross section above that energy.

III. RATE CALCULATION

If $N(E)$ particles are produced in our source per (BeV day degree) with an angular distribution $f(\theta)$, and the mean free path of neutrinos in the detector is λ, the event rate per day is simply

$$\int_{E_{min}}^{E_{max}} dE \, \frac{D}{\gamma\beta C\tau} \frac{L}{\lambda} F(E, \theta_p) \int_0^{\alpha} f(\theta) \, d\theta \quad (19)$$

where for pions F is given by (2), and for muons giving ν_μ [or $\overline{\nu}_e$] by (6) [or (7)] if $\alpha > \theta_p$, and by the corrected expressions (2) $\times$ (12), (15) or (16) if the decay defocusing is important. Neglect of the exponential is justified by

$$\gamma\beta C\tau_\pi \doteq 54.77 \, E_\pi \text{ meters}$$
$$\gamma\beta C\tau_\mu \doteq 6246 \, E_\pi \text{ meters} \quad (20)$$

For density 1, assuming half the nucleons interact (i.e. considering only ν or $\bar{\nu}$ but not both) with a cross section of 0.6×10^{-38} cm^2, we have

$$\lambda = \frac{1.67 \times 10^{-24}}{0.3 \times 10^{-38}} = 5.56 \times 10^{12} \text{ m}$$

For pions we assume the Drell-Ballam distribution out to $2\theta_B = 2m_\pi/E_\pi = 2X(8^\circ/E_\pi)$, so normalizing $N(E)$ to this total amount

$$\int_0^\alpha f(\theta)\, d\theta = 1 \qquad \alpha > 2\theta_B$$

$$= \frac{\log[1 + (\alpha/\theta_B)^2] - \dfrac{(\alpha/\theta_B)^2}{1 + (\alpha/\theta_B)^2}}{\log 5 - 0.8} \qquad \alpha < 2\theta_B$$

Since the experimental angular distribution at CEA is somewhat sharper than the prediction, we also compute for $\theta_B = 6^\circ/E_\pi$. We assume 3×10^{14} 20 BeV electrons/sec or 2.592×10^{19} electron/day incident on a 1 radiation length H_2 target. Assuming the first half of the target produces thin-target bremsstrahlung and the second half pions, M. Thiebaux computed the numbers in Table I. If correction is made for thick target effects, he claimed these numbers are to be multiplied by the factor X given in the third column. Applying this correction, we obtain the event rates given in Figs. 6 and 7 for $Pc \geq 1$ BeV. The energy spectra are given in Figs. 8 and 9 for D = 35 m and 60 m.

For μ production we simply assume Bethe-Heitler, since most of the cross section is in the forward direction, where form factor effects can be ignored. If one compares the thin target approximation for 1 radiation length with the 10 radiation length calculation given by D. Fries (TN-63-83) one finds that the result is approximately the same as multiplying by 20-E where E is in BeV. This correction was applied to the unfocused μ pions and the $\bar{\nu}_e$ yield is compared with 10 times the thin target approximation in Fig. 10. This figure also gives the detector shapes to scale. For forward μ pions, since ~ 80% of the μ's lie within $2m_\mu/E_\mu$ of the axis [with an approximate distribution $\gamma^2\theta^2/(1 + \gamma^2\theta^2)$.] we assume this much can be put on axis and multiply the thin target intensity by 0.8 (20-E). This is also given in Fig. 8. Corresponding results for ν_μ are given in Fig. 11. Energy spectra for $\bar{\nu}_e$ are given in Figs. 12 and 13.

TABLE I

E_π(BeV)	π^-	π^+	X
		0	1.1
1.0	1.67, 15	1.39, 15	0.97
2.0	6.35, 14	6.23, 14	0.94
3.0	3.55, 14	3.67, 14	0.88
4.0	2.33, 14	2.46, 14	0.83
5.0	1.65, 14	1.77, 14	0.77
6.0	1.23, 14	1.32, 14	0.72
7.0	9.45, 13	1.02, 14	0.67
8.0	7.35, 13	7.93, 13	0.62
9.0	5.76, 13	6.22, 13	0.58
10.0	4.51, 13	4.87, 13	0.54
11.0	3.50, 13	3.79, 13	0.51
12.0	2.68, 13	2.91, 13	0.48
13.0	2.01, 13	2.19, 13	0.45
14.0	1.47, 13	1.60, 13	0.42
15.0	1.02, 13	1.12, 13	0.38
16.0	6.66, 12	7.29, 12	0.34
17.0	3.89, 12	4.23, 12	0.31
18.0	1.84, 12	1.94, 12	0.28
19.0	5.15, 11	4.85, 11	0.25

APPENDIX A

(Memo by H. Faissner, (October 19, 1964))

1. After having read some of the pertinent reports and having discussed with several people, I should like to make a few comments. I agree essentially with all conclusions arrived at, but there are a few points which could (and should) be exploited.

The numbers about expected neutrino fluxes at SLAC I got mainly from the report of H. P. Noyes of August 1963, and from some graphs he computed later. Most of the relevant data about the CERN PS neutrino facilities may be found in the yellow CERN report 63-37 (1963).

2. The neutrino fluxes obtainable respectively at SLAC and the CPS, without focussing the mesons, are expected to be equal (to within a factor of 2)

This is reasonable: one is exploiting peripheral processes in both cases; the expected SLAC intensity is 3×10^{14} e/sec as compared to 3×10^{11} P/sec in the CPS, one has to insert a factor of 1/137 for electromagnetic production, and the target efficiency is about a factor of 5 worse at SLAC. (Shielding thickness and decay distance are quite similar in both places.)

The numbers are summarized in Table I. (The CERN numbers are for 7×10^{11} circulating p's per pulse.) As far as I can see, Noyes' numbers are somewhat on the pessimistic side, neglecting π's below 1 GeV altogether and also the rise of the neutrino cross section above the asymptotic value in the VA-interference region, etc. Even with target absorption included, my best guess would be 1.5 to 2 events for SLAC.

TABLE I

EXPECTED AND OBSERVED NEUTRINO RATES

SLAC:	1	elastic event per day and ton (Noyes' report, Drell angular distribution)
	1.5	elastic event per day and ton (Noyes' report, Ballam angular distribution)
	0.5	elastic event per day and ton (Noyes' graph, π-absorption in target incl.)
CPS:	1.5	elastic event per day and ton (van der Meer prediction)
	2	elastic event per day and ton ("observed" without focussing)
	13	elastic event per day and ton (actually observed number with magnetic horn)

3. The gain by focussing the mesons is appreciable

For CERN this has been demonstrated experimentally, the gain being a factor of 5 to 7. Similar enhancements have been calculated by Noyes for the case of "ideal focussing," namely, into a pencil beam. For this type of focussing, CERN would expect enhancement factors of about 30.

It is difficult to judge if the true enhancement at SLAC could be expected to be as large as that: Noyes included only Drell pion production, and there is no theory to predict reliably the other processes. If one could believe in a "statistical multi-meson production," conditions were comparable to CERN. Presumably this is not quite true. My guess would be about 20 times enhancement by ideal focussing, which would lead one to hope for an enhancement around 4 for a practicable focussing device. This is large enough to warrant some effort. Furthermore, for quantitative experiments it is almost indispensable to work with a clean neutrino beam (i.e., with small anti-neutrino contamination).

4. For high energy neutrino physics, the contribution from K-mesons is of dominant importance

This is a trivial kinematical effect. I mention it explicitly because concentration on K-mesons has some consequences for the experimental design. Since the decay angle is large, and the decay length shorter than for pions, one gains nothing by lengthening the decay path; (as a matter of fact, with the CERN horn, one loses far lower momenta). But one gains appreciably if one shortens the thickness of the shielding (a factor of 2.5 if one cuts the shielding down to half of its present value of 25 m).

Nothing is known so far about K-production at SLAC. But I see no reason why conditions should be too different from CERN. (There at present 8% of the elastic reactions are due to neutrinos from $K\mu 2$ decays, the spectrum being quite flat up to 6 GeV, but falling down at about 9 GeV. By some changes in geometry, the relative contribution from K-meson neutrinos could easily be enhanced by a factor of 1.5.)

5. Conclusions

I appreciate the present attitude at SLAC, not to prepare a neutrino experiment now, since the expected flux is only comparable to the one obtained already at CERN, and the duty cycle is worse. But for phase II:

40 GeV primary energy, one should seriously reconsider the issue. It is clear that the most exciting part of neutrino physics will be done at higher energies than now available. Working with a focussing device is essential. Pion and, in particular, K-meson production data are badly needed.

APPENDIX B

$\bar{\nu}_e$ AND ν_μ SPECTRA FROM μ^- DECAY

(The following calculation was done by S. Berman, but carried through independently and checked by the author. Notation is from Berman's CERN 62-20 (1962) lecture notes on weak interactions.)

We assume a weak current

$$j_\lambda = \sqrt{\frac{G}{\sqrt{2}}}\left[\bar{\Psi}_e \gamma_\lambda (1+\gamma_5)\psi_{\nu_e} + \gamma_\lambda (1+\gamma_5)\psi_\nu + \bar{\Psi}_n \gamma_\lambda (1+\gamma_5)\psi_p\right]$$

with $G = 1.01 \times 10^{-5}\, M^{-2}$. Then the term from $j^+ j$ which gives μ decay is

$$j^+ j = \frac{G}{\sqrt{2}}\bar{\Psi}_{\nu_\mu}\gamma_\mu (1+\gamma_5)\psi_\mu \bar{\Psi}_e \gamma_\mu (1+\gamma_5)\psi_{\nu_e}$$

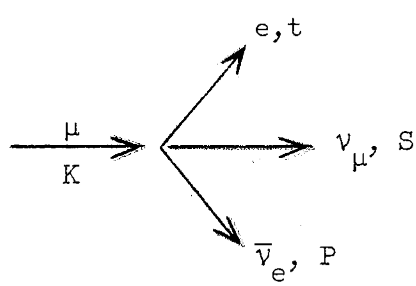
Since this is "V + A" we make a Fiertz transformation of the first kind to give

$$= -\frac{G}{\sqrt{2}}\bar{\Psi}_{\nu_\mu}\gamma_\mu (1+\gamma_5)\psi_\mu \bar{\Psi}_{\bar{\nu}_e}\gamma_\mu (1-\gamma_5)\psi_e$$

A Fiertz transformation of the second kind then gives

$$= \sqrt{2}G\, \bar{\Psi}_{\bar{\nu}_e}(1+\gamma_5)\psi_\mu \bar{\Psi}_{\nu_\mu}(1-\gamma_5)\psi_{\bar{e}}$$

Assuming the 4-momenta



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$$\frac{|M|^2}{2G^2} = T_r\left[(1-\gamma_5)\,\not{P}\,(1+\gamma_5)\,(\not{K}+m_\mu)\right] T_r\left[(1+\gamma_5)\,\not{S}\,(1-\gamma_5)\,(\not{t}-m_e)\right]$$

$$= 4\, T_r\,[\not{K}\not{P}]\, T_r\,[\not{S}\not{t}]$$

since $\quad (1\pm\gamma_5)^2 = 2\,(1\pm\gamma_5)$

$$|M|^2 = 2 \times 64G^2\,(K\cdot P)\,(S\cdot t) \times \tfrac{1}{2}$$

$$\omega = \frac{2\pi|M|^2}{2P_4\cdot 2S_4\cdot 2t_4\cdot 2K_4}\,\rho_f = 2\pi\,\frac{64G^2(K\cdot P)\,(S\cdot t)}{16P_4S_4t_4K_4}\,\rho_f = \frac{8\pi G^2(K\cdot P)\,(S\cdot t)}{P_4S_4t_4K_4}\,\rho_f$$

with

$$\rho_f = \frac{1}{(2\pi)^6}\,d^3S\;d^3t\;d^3K\;\delta^3(K-S-t-P)\;\delta(E)$$

using

$$\frac{1}{2t_4} = \delta\,(t^2 - m_e^2) \approx \delta\,(t^2)$$

$$\omega = \frac{32\pi G^2}{PK_4}\;\frac{(K\cdot P)\,(S\cdot t)}{(2\pi)^6}\;d^3P\;d^4S\;d^4t\;\delta(S^2)\;\delta(t^2)\;\delta^4(K-S-t-P)$$

Note that since S and t appear symmetrically (neglecting m_e), the ν_μ and the e spectrum are the same if S or t is integrated over

$$t = K-S-P$$

Doing the d^4t integration

$$S\cdot t = S\cdot(K-S-P) = S\cdot(K-P)$$

$$\omega = \frac{32\pi G^2}{PK_4} \frac{(K\cdot P)\ d^3P}{(2\pi)^6} \int d^4S \quad S\cdot(K-P)\ \delta\left((K-S-P)^2\right)\ \delta(S^2)$$

and

$$(K-S-P)^2 = (K-P)^2 - 2S\cdot(K-P)$$

$$= \frac{16\pi G^2\ (K\cdot P)\ (K-P)^2\ d^3P}{PK_4\ (2\pi)^6} \int d^4S\ \delta\Big((K-P)^2 - 2S\cdot(K-P)\Big)\ \delta(S^2)$$

Consider the covariant integral

$$I = \int d^4S\ \delta(S^2)\ \delta(Q^2 - 2Q\cdot S)$$

Since Q is time-like, we can evaluate in the C.S. where $\vec{Q} = 0$ then

$$I = 4\pi \int S^2 dS\ dS_4\ \delta(S_4^2 - S^2)\ \delta(Q_4^2 - 2Q_4 S_4)$$

and since $S = Q_4/2 \quad df/dS = 2Q_4$

$$I = 2\pi \int SdS\ \delta(Q_4^2 - 2Q_4 S) = \frac{\pi}{2}$$

Hence

$$\omega = \frac{8\pi^2 G^2\ (K\cdot P)\ (K^2 - 2P\cdot K)}{P_4 K\ (2\pi)^6}\ d^3P$$

which is the result quoted in Eq. (5).

For the ν_μ spectrum we have

$$\omega = \frac{32\pi G^2\ d^3S}{SK_4 (2\pi)^6} \int d^4P\ (K\cdot P)\ S\cdot(K-P)\ \delta(P^2)\ \delta\left((K-S-P)^2\right)$$

Consider first the term $(K\cdot P)\,(S\cdot K)$ which gives

$$K_\mu \int d^4P\; P_\mu\; \delta(P^2)\; \delta\,(K-S-P)^2 \;=\; K_\mu\; I_\mu$$

But $I_\mu = \int d^4P P_\mu \delta(P^2)\;\delta\Big((Q-P)^2\Big) = AQ_\mu$ since the only vector available is Q_μ. Hence

$$AQ^2 = \int d^4P\;(Q\cdot P)\;\delta(P^2)\;\delta\Big((Q-P)^2\Big) = \int d^4P\;(Q\cdot P)\;\delta(P^2)\;\delta(Q^2-2Q\cdot P)$$

$$= \tfrac{1}{2}\,Q^2 \int d^4P\;\delta(P^2)\;\delta(Q^2-2Q\cdot P) \;=\; \tfrac{\pi}{4}\,Q^2$$

as we showed above. Hence $K_\mu I_\mu = \frac{\pi}{4}\,K\cdot(K-S)$. Similarly the second term is of the form

$$I_{\mu\nu} = \int d^4P\;P_\mu P_\nu\;\delta(P^2)\;\delta\Big((Q-P)^2\Big) = A\delta_{\mu\nu}\,Q^2 + BQ_\mu Q_\nu$$

$$Q_\mu Q_\nu I_{\mu\nu} = \int d^4P\;(Q\cdot P)^2\;\delta(P^2)\;\delta\Big((Q-P)^2\Big) = (A+B)\;Q^4$$

$$= \tfrac{1}{4}\,Q^4 \int d^4P\;\delta(P^2)\;\delta(Q^2-2Q\cdot P) \;=\; \tfrac{\pi}{8}\,Q^4$$

so

$$A + B = \frac{\pi}{8}$$

Similarly

$$t_\mu t_\nu I_{\mu\nu} = \int d^4P\;(t\cdot P)^2\;\delta(P^2)\;\delta\Big((Q-P)^2\Big) = t^2\,Q^2A + (t\cdot Q)^2\,B$$

If we let $t = (t_4, 0)$ and $P_4 = \dfrac{Q^2}{2(q_4 - q\cos\theta)}$

$$At^2Q^2 + B(t\cdot Q)^2 = t_4^2 \int d^4P\ P_4^2\ \delta(P_4^2-P^2)\ \delta(Q^2-2q_4P_4 + 2qP\cos\theta)$$

$$= At_4^2\ (q_4^2-q^2) + Bt_4^2q_4^2 = \frac{t_4^2}{2} \int d\Omega_p \int dP\ \frac{Q^3}{8(q_4-q\cos\theta)^3}\ \frac{\delta(P_4-P)}{[-2q_4+2q\cos\theta]}$$

$$= \frac{\pi t_4^2Q^3}{16} \int_{-1}^{1} d(\cos\theta)\ \frac{1}{[q_4-q\cos\theta]^4} = \frac{\pi t_4^2Q^3}{48q} \int d\ \frac{1}{[q_4-q\cos\theta]^3}$$

$$= \frac{\pi t_4^2Q^3}{48q} \left[\frac{(q_4+q)^3 - (q_4-q)^3}{Q^3}\right] = \frac{\pi t_4^2}{8} \left(q_4^2 + \frac{1}{3} q^2\right)$$

$$= \frac{-\pi}{24}\ t_4^2\ \left(q_4^2 - q^2\right) + \frac{\pi}{6}\ t_4^2q_4^2$$

So $A = \frac{-\pi}{24}$ $B = \frac{\pi}{6}$; $A + B = \frac{\pi}{8}$ which checks. Hence

$$\int d^4P\ (K\cdot P)\ S\ \cdot\ (K-P)\ \delta(P^2)\ \delta\left((K-S-P)^2\right)$$

$$= (S\cdot K) \times \frac{\pi}{4}\ K\ \cdot\ (K-S)\ -\ \left[\frac{-\pi}{24}\ (K\cdot S)\ (K-S)^2 + \frac{\pi}{6}\ K\ \cdot\ (K-S)\ S\ \cdot\ (K-S)\right]$$

$$= (S\cdot K) \left[\frac{\pi}{12}\ K\ \cdot\ (K-S)\ +\ \frac{\pi}{24}\ (K-S)^2\right] = \frac{\pi}{8}\ (S\cdot K)\ \left[K^2 - \frac{4}{3}\ (K\cdot S)\right]$$

So

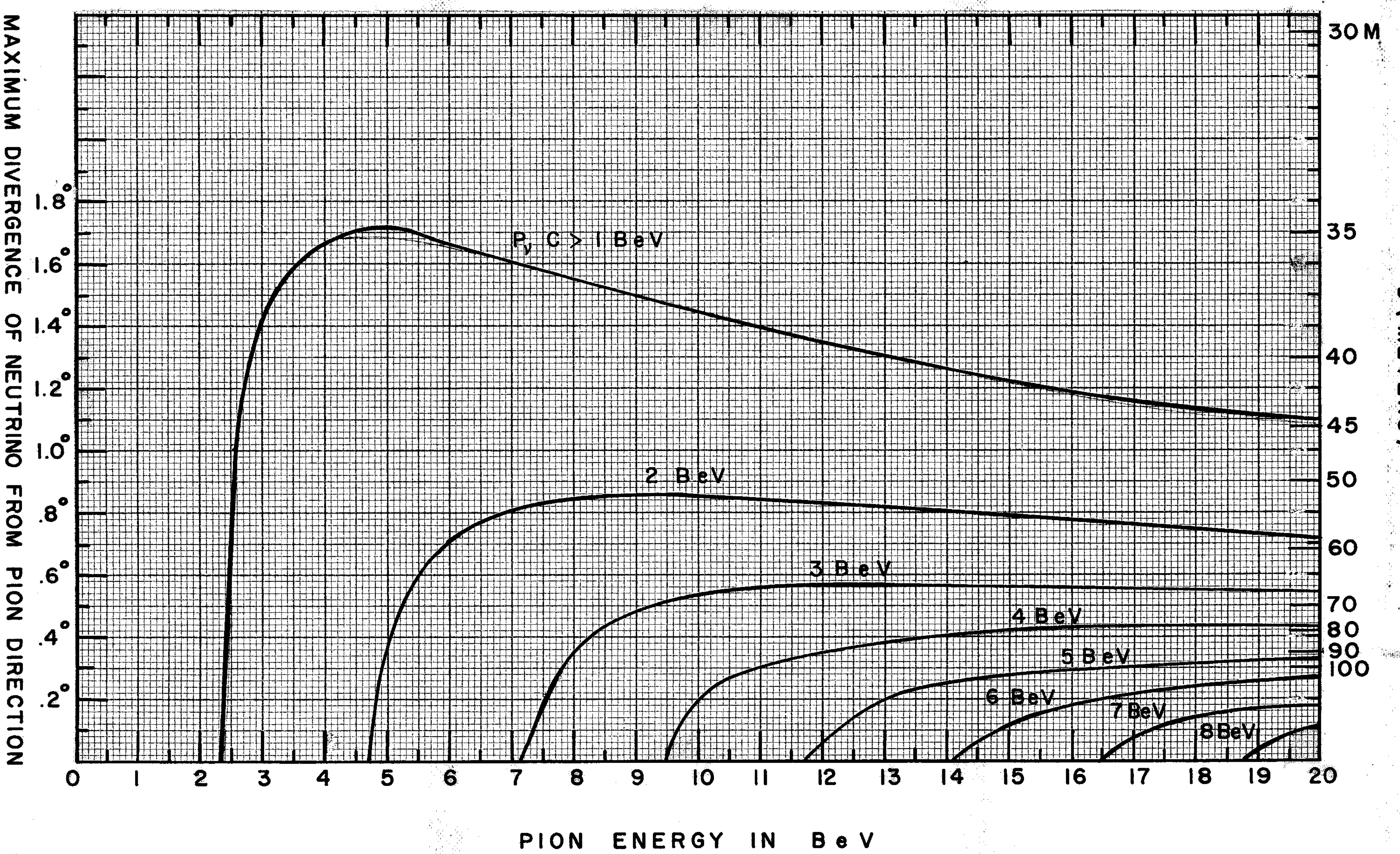
$$\omega = \frac{4\pi^2G^2d^3S}{SK_4}\ (K\cdot S)\ \left[K^2 - \frac{4}{3}\ (K\cdot S)\right]$$

as given in Eq. (4). As shown in the text $\omega = \frac{G^2\mu^5}{3(4\pi)^3\gamma}$, so for $G = 1.01 \times 10^{-5}\ M^{-2}$

$$\tau = \frac{3(4\pi)^3}{1.02 \times 10^{-10}} \; \frac{\hbar}{\mu c^2} \left(\frac{M}{\mu}\right)^4 = 2.306 \times 10^{-6} \text{ sec}$$

as compared to the experimental value of 2.212×10^{-6}. Since G^2 is taken from β decay the difference is presumably due to radiative corrections.

FIG. I ANGLE SUBTENDED BY A 50 TON DENSITY DETECTOR OF LENGTH (D-20)/3



LIMITING ANGLE FOR $\mu \to e$ DECAY NEUTRINOS OF ENERGY GREATER THAN P_ν

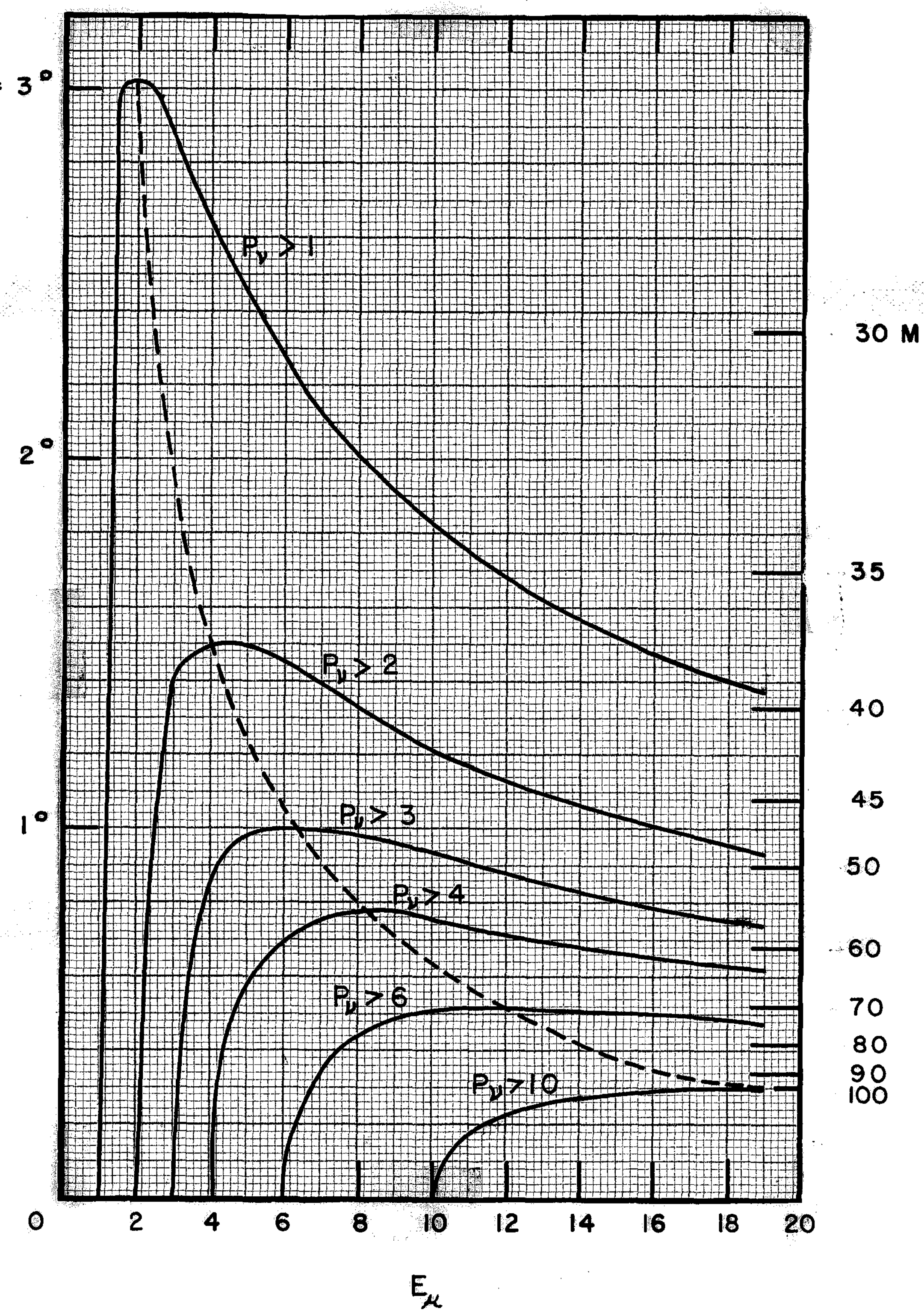
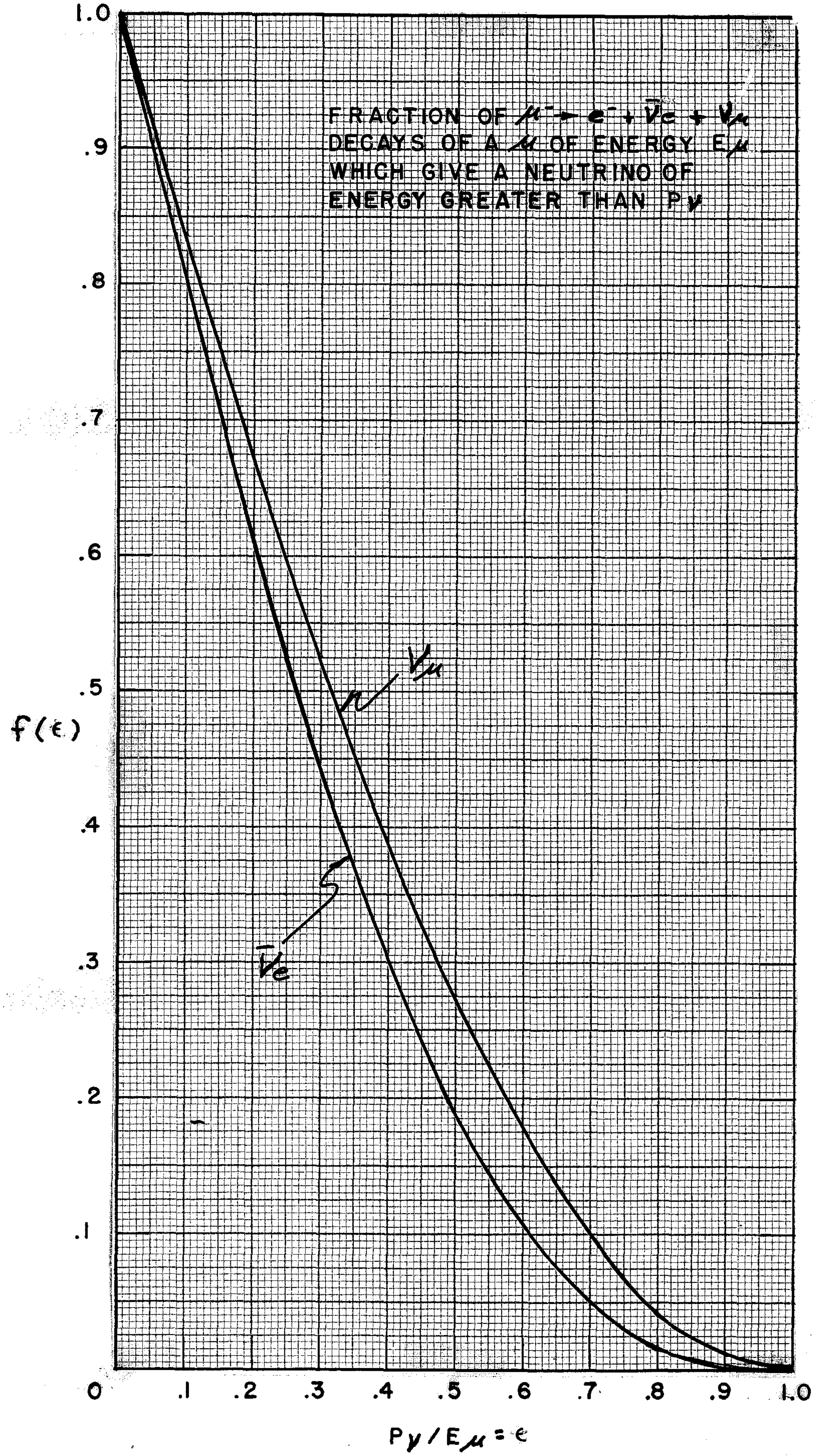


FIG. 2 ANGLE SUBTENDED BY A 50 TON DETECTOR OF LENGTH (D-20)/3

FIG. 3


FRACTION OF $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$
DECAYS OF A μ OF ENERGY E_μ
WHICH GIVE A NEUTRINO OF
ENERGY GREATER THAN P_ν
ν_μ
$\bar{\nu}_e$
$f(\epsilon)$
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
$P_\nu / E_\mu = \epsilon$

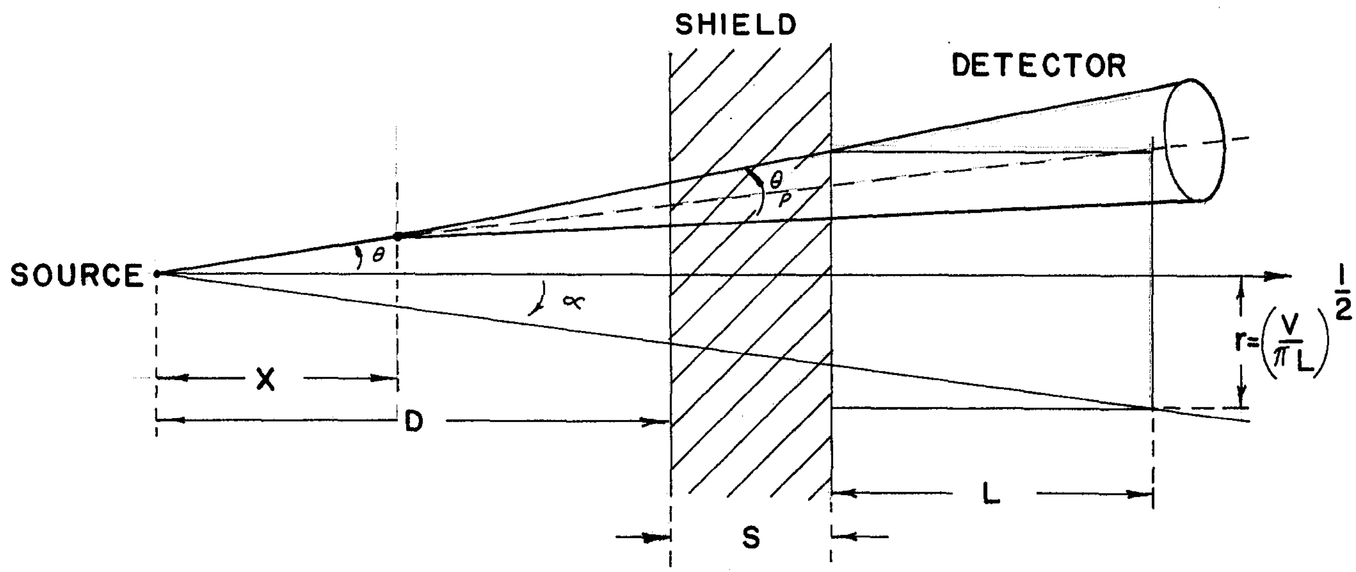


FIG. 4 SIMPLIFIED GEOMETRY FOR A NEUTRINO EXPERIMENT.

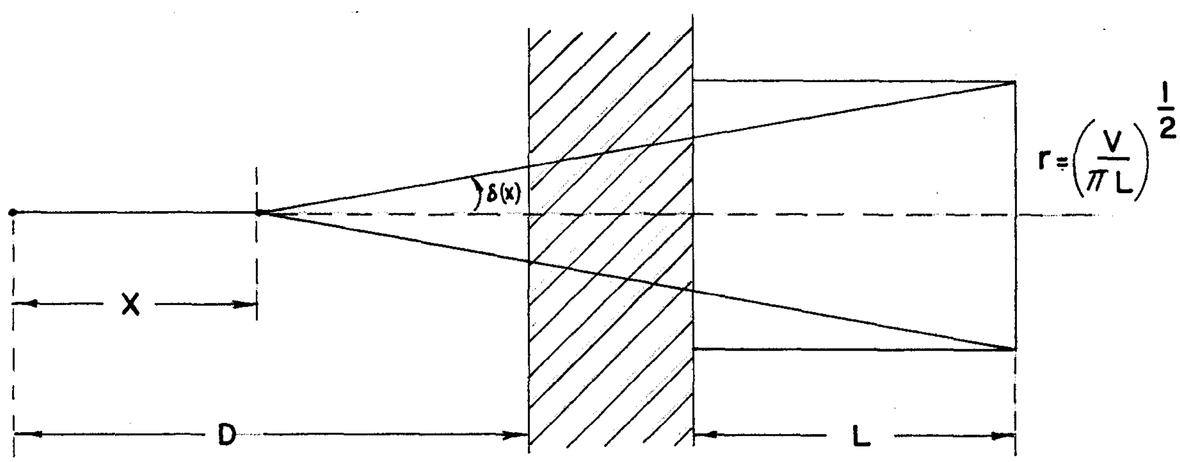


FIG. 5 DECAY DEFOCUSING OF THE NEUTRINO BEAM FOR ON AXIS DECAYS.

FIG. 6 π-μ DECAY NEUTRINO EVENT RATES FOR $\pi^- \rightarrow \mu^- + \bar{\nu}$

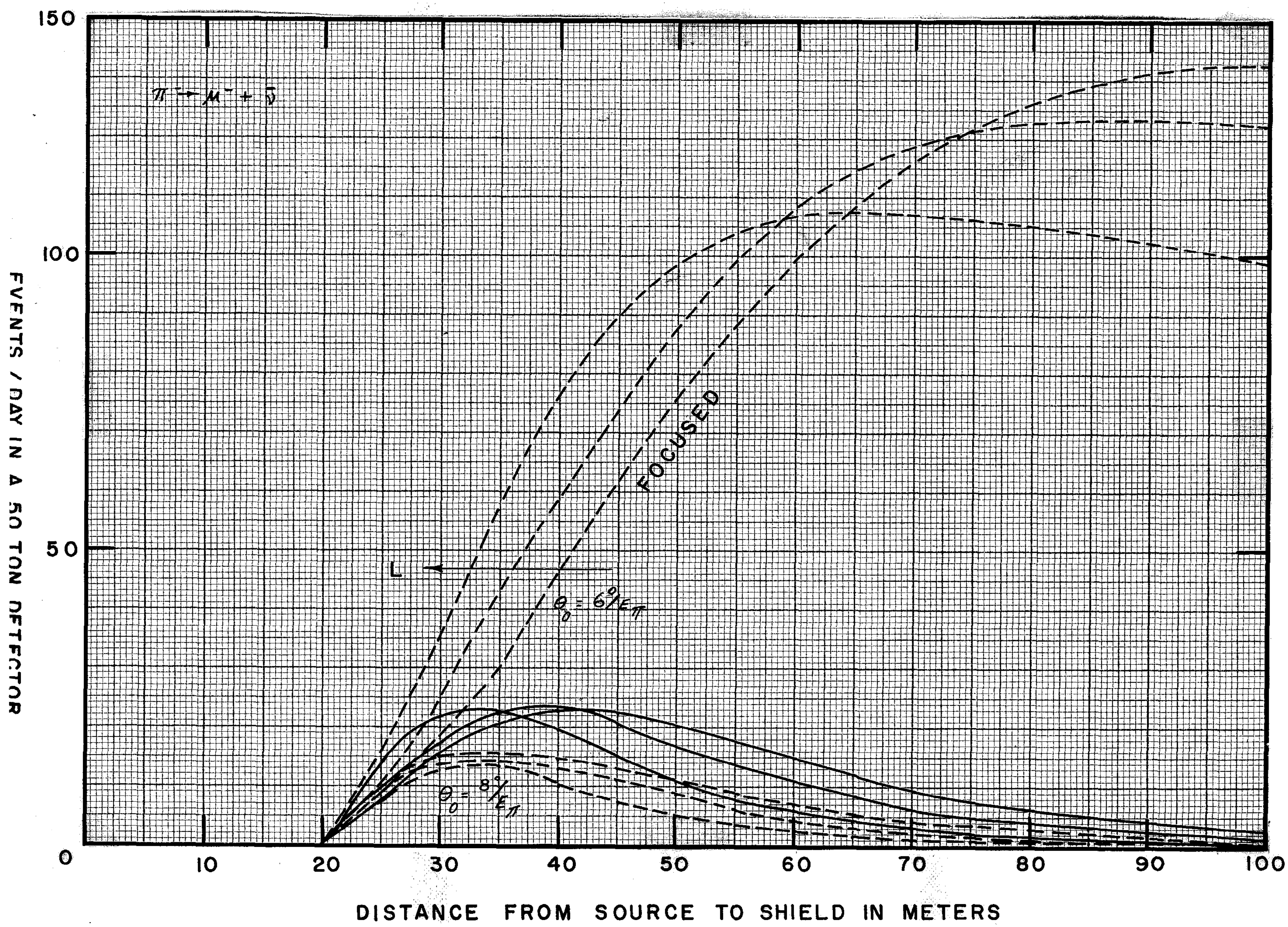


FIG. 7 π-μ DECAY NEUTRINO EVENT RATES FOR $\pi^+ \rightarrow \mu^+ + \nu$

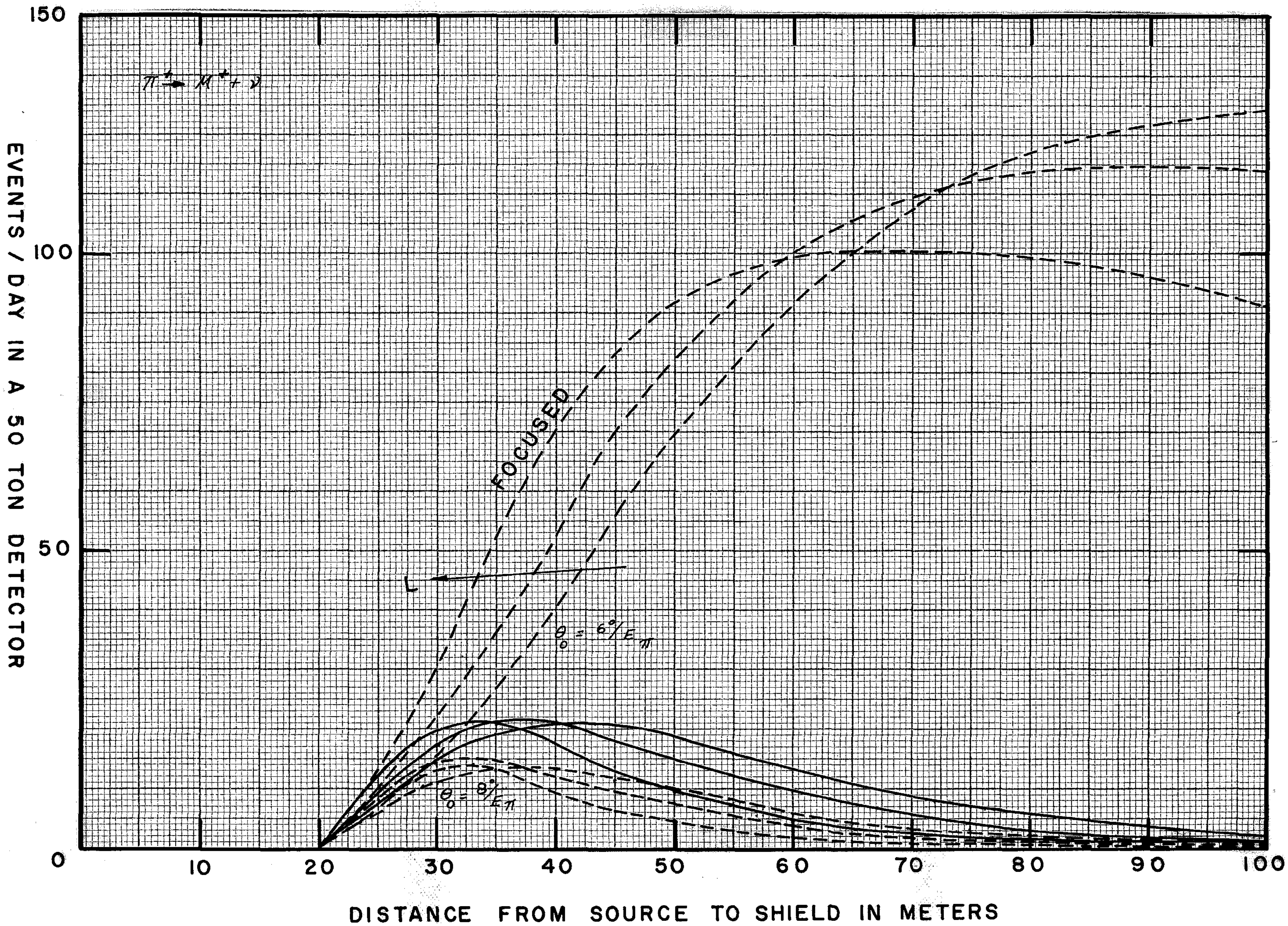
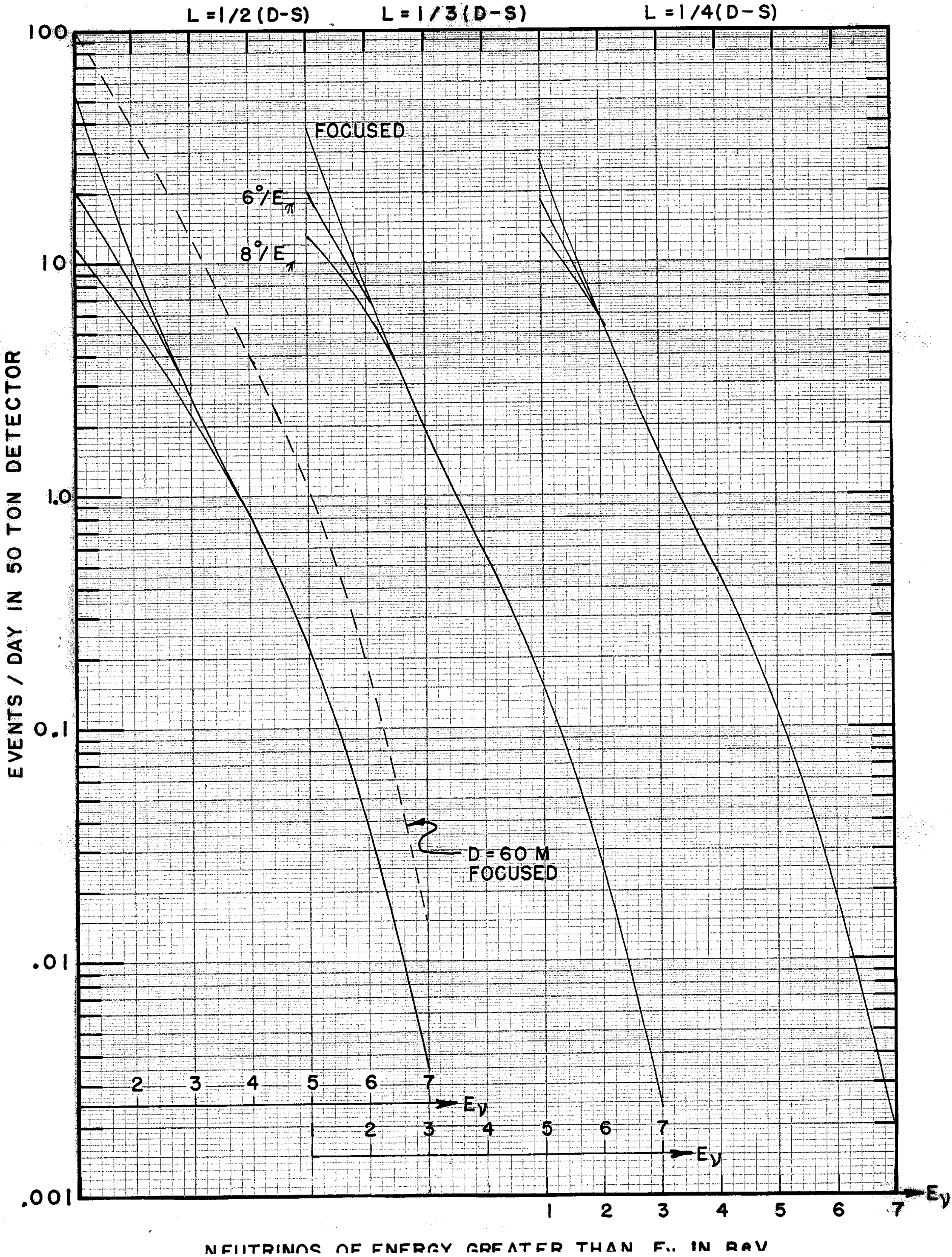
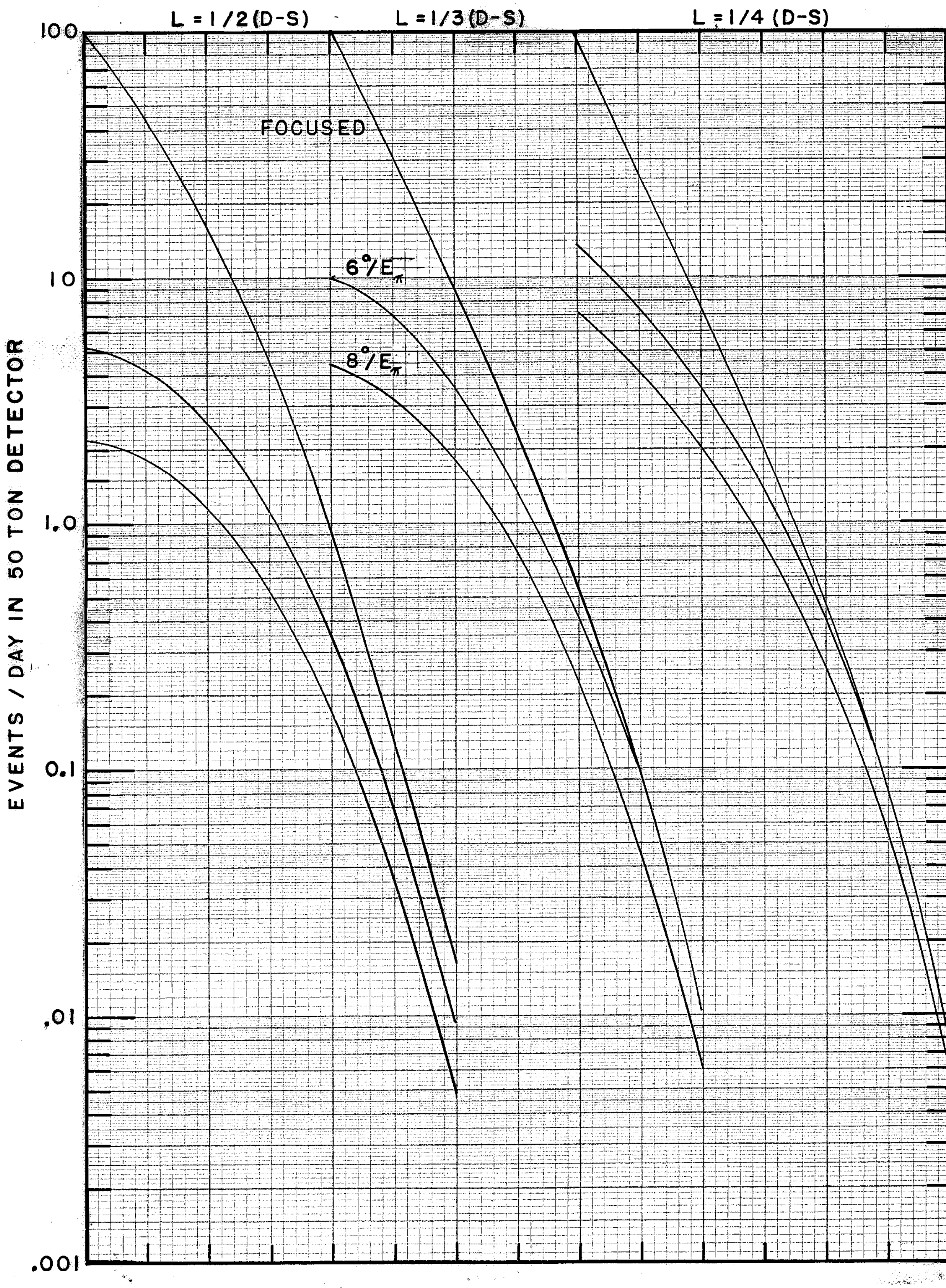


FIG. 8 D = 35 M



L = 1/2 (D-S)
L = 1/3 (D-S)
L = 1/4 (D-S)
FOCUSED
6°/Eπ
8°/Eπ
D = 60 M
FOCUSED
EVENTS / DAY IN 50 TON DETECTOR
100
10
1.0
0.1
.01
.001
2 3 4 5 6 7
Eν
2 3 4 5 6 7
Eν
1 2 3 4 5 6 7
Eν
NEUTRINOS OF ENERGY GREATER THAN Eν IN BeV

FIG. 9 D=60M



L = 1/2 (D-S)
L = 1/3 (D-S)
L = 1/4 (D-S)
100
10
1.0
0.1
.01
.001
FOCUSED
6°/E$_\pi$
8°/E$_\pi$
EVENTS / DAY IN 50 TON DETECTOR

FIG. 10 $\bar{\nu}_e$ EVENT RATES FOR A 10 RADIATION LENGTH SOURCE

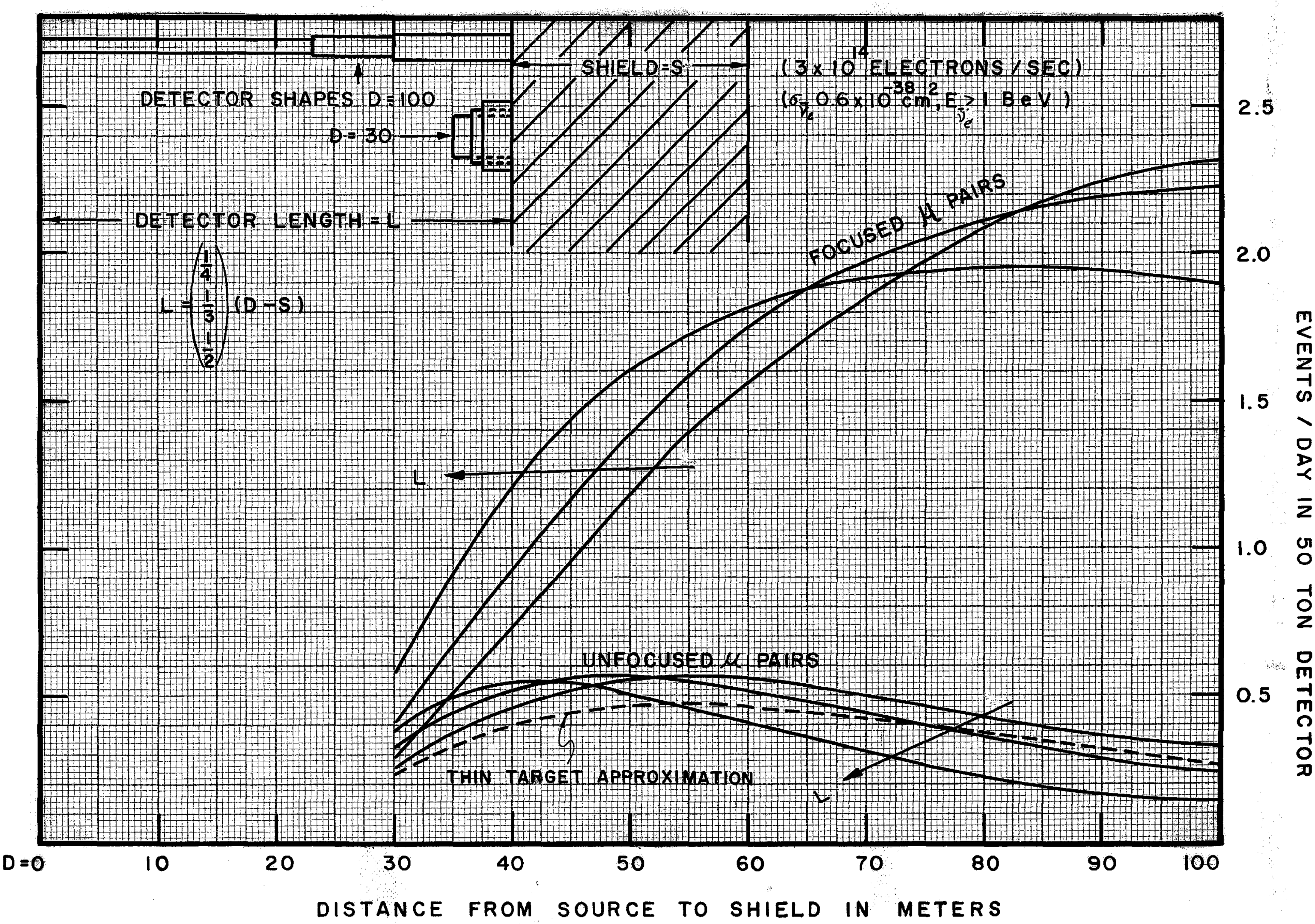


FIG. 11 ν_μ EVENT RATES FROM μ PAIRS

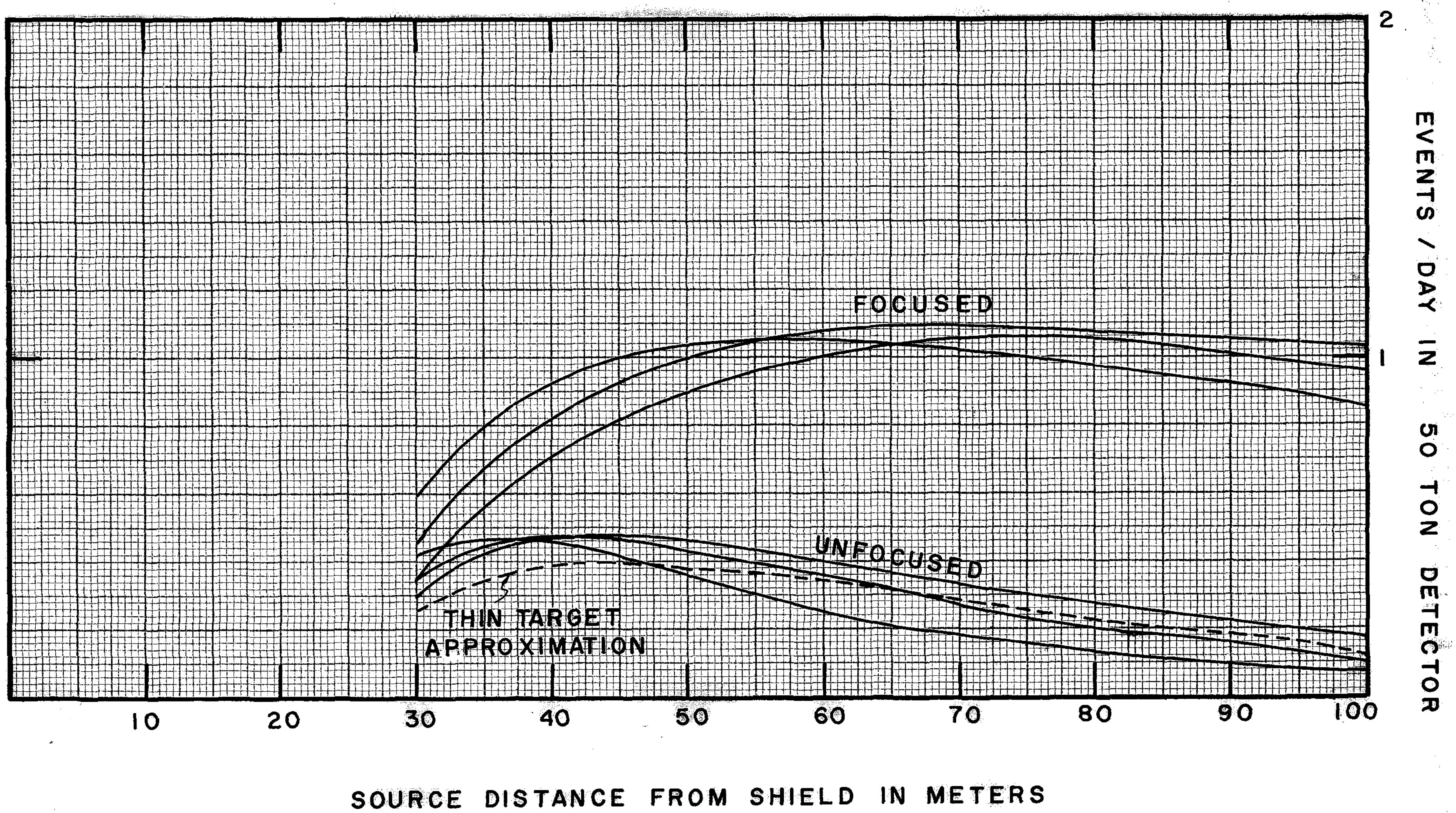
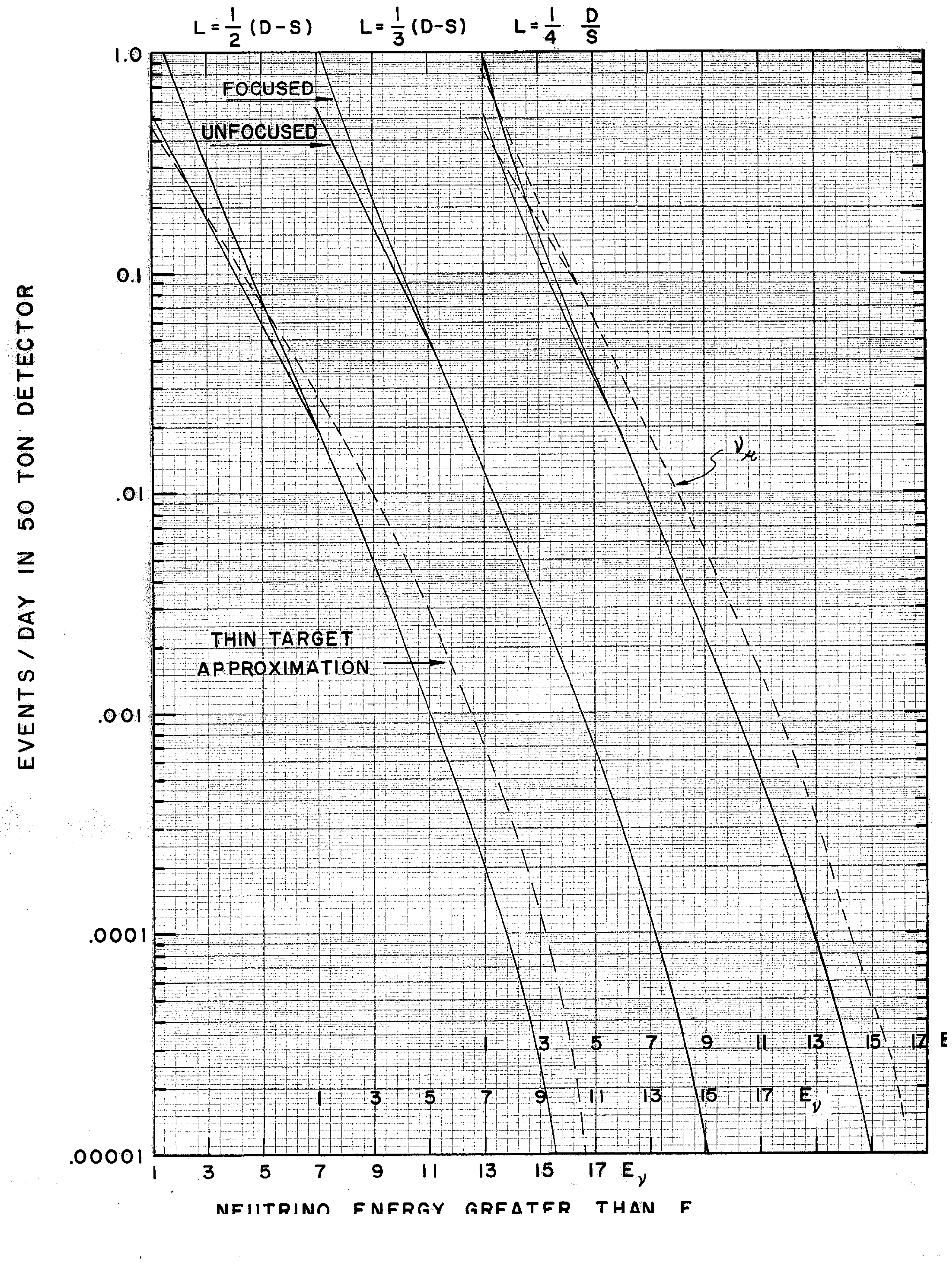


FIG. 12, D = 45 M



L = 1/2 (D-S)
L = 1/3 (D-S)
L = 1/4 D/S
FOCUSED
UNFOCUSED
THIN TARGET APPROXIMATION
ν_μ
1.0
0.1
.01
.001
.0001
.00001
EVENTS / DAY IN 50 TON DETECTOR
1 3 5 7 9 11 13 15 17 E_γ
1 3 5 7 9 11 13 15 17 E_ν
1 3 5 7 9 11 13 15 17
NEUTRINO ENERGY GREATER THAN E

FIG. 13, D = 100 M

$L = \frac{1}{2}(D-S)$ $L = \frac{1}{3}(D-S)$ $L = \frac{1}{4}(D-S)$