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UNIQUE DETERMINATION OF THREE PROTON-PROTON SCATTERING PARAMETERS AT 9.69 MEV*

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ABSTRACT

The well established fact that the spin-orbit interaction is of short range compared to one-pion-exchange (OPE) causes the phase shift combination $\Delta_{LS} = \left(-2\delta_{1,0} - 3\delta_{1,1} + 5\delta_{1,2}\right)/12$ to be small compared to $\Delta_T = 5\big(2\delta_{1,0} - 3\delta_{1,1}$ $+$ [redacted] $\delta_{1,2}\big)/72$ at low energy. We find that this makes it impossible to carry through an unambiguous phase shift analysis of existing data near 10 MeV for δ_0 and $\delta_{1,J}$, but also makes it possible to show that the ratio Δ_{LS}/Δ_T must lie between 0.07 and 0.15 at 9.69 MeV. Using this result, we can then uniquely determine δ_0^E, Δ_T, and $\Delta_C = \left(\delta_{1,0} + 3\delta_{1,1} + 5\delta_{1,2}\right)/9$ and obtain $\delta_0^E = 55.69° \pm 0.28°$, $\Delta_T = 0.91° \pm 0.28°$, $\Delta_C = -0.020° \pm 0.029°$ with relative correlations (OT) = -.96, (OC) = .47, (TC) = -.30. The result is stable against variation of Δ_{LS}/Δ_T over the full physically allowable range, against the extrapolation needed to include A_{yy}/A_{xx} (90°) at 11.4 MeV in the data set, against vacuum polarization corrections, against whether or not phases with $J > 2$, ${}^3F_2$, and (marginally) ϵ_2 are dropped or given their OPE values. The data require ${}^1D_2$ to be within 30% of the OPE prediction, and the value of Δ_T given above also agrees with this prediction to the same accuracy. This is important as it shows conclusively that p-p differential cross sections below 10 MeV can be analyzed uniquely and stably for only two nuclear parameters $\left(\delta_0^E \text{ and } \Delta_C\right)$ by using the (coulomb corrected) OPE prediction for all other nuclear scattering parameters. The small value of Δ_C is presumably due to the delicate cancellation between the weakly repulsive OPE central P-wave interaction and the strongly attractive intermediate range attraction arising from the exchange of two interacting pions in the $I = 0$ $\pi - \pi$ S state. A single (model dependent) parameter measuring the strength of this attraction relative to OPE will account both for this value of Δ_C and for the four values previously determined at 3 MeV and below. The value of δ_0^E again confirms the predicted OPE shape correction in the ${}^1S_0$ state to modest accuracy and, taken together with results below 3 MeV and near 27 MeV, establishes the existence of the effect beyond reasonable doubt.

This paper has two objectives. The first is simply to determine what quantitative nuclear information can be extracted from the relative[1] and absolute[2] proton-proton differential cross section measurements at 9.69 MeV by adding to the analysis the recent polarized-target polarized-beam measurement[3] of $A_{yy}/A_{xx}(90°)$ at 11.4 MeV. The second is to determine to what extent this data supports the assumption that the longest range strong interaction between two pions is due to one-pion-exchange (OPE), and hence to provide additional justification for the simplifications which this assumption makes possible in the analysis of proton-proton scattering measurements at lower energy.

At first sight, the assumption that the longest range contribution to the strong interaction between two protons is OPE is all that is needed for a unique phase shift analysis of this data. Quantitatively this assumption tell us immediately that there will be no appreciable contribution, at the level of accuracy of the data, to states with $J > 2$, or to $^3F_2$, and that, to the same accuracy, shorter range interactions will not significantly shift $^1D_2$ or ϵ_2 away from the OPE prediction. This leaves only δ_0 and $\delta_{1,J}$ with $J = 0, 1, 2$ to be determined (we use the Stapp "nuclear-bar" parameterization[4] throughout). Clementel and Villi[5] have shown that, under these conditions, the differential cross section allows a fourfold continuum of solutions; however, if δ_0 is fixed anywhere below the maximum set by the value of $\sigma(90°)$, the solution for $\delta_{1,J}$ is unique, up to a fourfold algebraic ambiguity. Both theory and experiment agree that the correct solution has the $+-+$ tensor signature for $^3P_{0,1,2}$ from 27 to 210 MeV, so we are on firm ground if we pick only that solution. Further, Iwadare[6] has shown that a single spin-dependent experiment, for example C_{nn}, added to the differential cross section data, will determine δ_0 and hence a unique solution; in principle, a second experiment is needed to resolve the algebraic ambiguity empirically, but we have already noted that this is no longer necessary. However, our attempts to obtain a solution on this basis led to ambiguous results, so further analysis is needed.

It is easy to show[7] that, in Born approximation, the central, tensor, and spin-orbit interactions in the triplet-odd P states determine three linear combinations of the $\delta_{1,J}$ which are, respectively,

$$
\begin{aligned}
\Delta_C &= \left(\delta_{1,0} + 3\,\delta_{1,1} + 5\,\delta_{1,2}\right)/9 \\
\Delta_T &= 5\left(2\,\delta_{1,0} - 3\,\delta_{1,1} + \text{[illegible]}\,\delta_{1,2}\right)/72 \qquad (1) \\
\Delta_{LS} &= \left(-2\,\delta_{1,0} - 3\,\delta_{1,1} + 5\,\delta_{1,2}\right)/12
\end{aligned}
$$

The Clementel-Villi ambiguity occurs because, although Δ_C appears linearly in the differential cross section (in the coulomb-nuclear P-wave interference term) and hence has a known sign, the other two combinations occur quadratically, allowing four choices of sign for the same fit to the cross section data. If one of these two combinations is close to zero, the four solutions are approximately degenerate leading to a broad region in the parameter space having no unique solution and no well-defined minimum in χ^2. In our case, the short range of the spin-orbit term (the evidence for which we will discuss in a moment) relative to the long-range OPE tensor interaction makes Δ_{LS} small compared to Δ_T at this energy, and hence explains why our attempt to obtain a direct solution was frustrated. This analysis also makes it clear that if we can use additional information to fix the range over which Δ_{LS}/Δ_T can vary, and show that within those limits the other three parameters (δ_0, Δ_C, and Δ_T) are uniquely determined, we can still achieve an acceptable analysis. We believe that this can be done in an essentially model-independent way as follows.

We have identified the cause of the difficulty as due to the short range of the spin-orbit interaction. The evidence for this fact is quite strong. As already noted, the $^3P_{0,1,2}$ phases retain the $+-+$ OPE tensor signature up to 210 MeV, but change to the $--+$ LS signature above that energy. Taking account of centrifugal shielding, this is already strong evidence that the spin-orbit force is of short range. This interpretation then requires the $^3F_{2,3,4}$ and higher angular momentum triplet-odd partial waves to retain the $+-+$ OPE tensor signature up to much higher energy, and this is in fact the case over the entire elastic scattering range. Further, we know from a number of lines of evidence that massive vector mesons, in particular the ω, are strongly coupled to nucleons and that the Thomas term arising from the exchange of these boson resonances corresponds to a spin-orbit interaction of the right sign and approximately the right range and strength to account for the effect. These massive vector mesons will also give rise to a strong short-range repulsion between two nucleons, and a strong short-range attraction in the nucleon-antinucleon system, for which again there is ample evidence. Since there is no reasonable doubt that we have both demonstrated the existence of a short-range spin-orbit interaction and its physical origin, it remains simply to show that this fact alone allows us to set sufficiently accurate limits on the ratio of Δ_{LS}/Δ_T to be expected at 10 MeV.

The qualitative fact that, granted the long-range OPE tensor interaction and a short-range spin-orbit interaction, centrifugal shielding will make the Δ_{LS}/Δ_T ratio small, is obvious. To make this quantitative, we need models with these two features which are in reasonable agreement with the ^3P phases which have been measured at several energies between 27 and 330 MeV. To demonstrate model-independence, we need to show that models sharing these characteristics (which we argue are physically demonstrated), but otherwise use divergent physical assumptions, still lead to the same prediction. Fortunately, a number of such models are ready to hand. The explanation of the effect as due to the vector mesons has been incorporated in a number of one-boson-exchange models,[8] of which we select that of Scotti and Wong[9] as typical. The phenomenological potential models used are the Yale potential,[10] which has a hard core, OPE, and adjusted static potentials of the form $\sum_{n=1} a_n e^{-2x}/x^n$, with $x = m_\pi c\, r/\hbar$ and the Hamada-Johnston potential[11] which again has a hard core but used the form $\sum_{n=2} a_n \left(e^{-x}/x\right)^n$. A representative non-local model is that of Feshbach, Lomon, and Tubis[12] which produces the spin-orbit effect via an energy-independent boundary condition at finite radius inside the one plus two pion exchange static potentials suggested by field theory. In all cases, the predictions quoted in Table 1 for Δ_{LS}/Δ_T lie in the range between 0.07 and 0.15, which we believe amply demonstrates our contention. Further evidence, if anyone still thinks this required, is supplied by energy-dependent phase shift analyses[13,14,15] which also give predictions lying within this range.

Before we can proceed to analysis for the parameters at 9.69 MeV, we must either extrapolate the measurement of $A_{yy}/A_{xx}(90°)$ at 11.4 MeV to that energy, or extrapolate the phase shifts to make a prediction at 11.4 MeV. We have chosen to do the latter by simply multiplying the 9.69 MeV parameters by an assumed ratio of $\delta(11.4)/\delta(9.69)$ before computing the single point at 11.4 to include in the χ^2 sum. For this ratio we again used the same models,[16] but found the effect of these different assumptions on the result so negligible as not to be worth quoting. As anticipated, the analysis for δ_0^E, Δ_C, and Δ_T is now unique, and the results are given in Table 2a. This table also gives results at the ends of the physically allowable range for Δ_{LS}/Δ_T. As was hoped, the parameter values are stable within this range, and therefore, we claim, firmly established. It remains to explore the sensitivity of the analysis to the other assumptions which have been made, and to interpret the significance of the values obtained.

In making this analysis, we have included the scattering amplitude due to vacuum polarization as given by Gursky and Heller,[17] and those vacuum polarization phase shifts which are needed ($\tau_0 \ldots \tau_3$) as computed by Heller.[18] Omitting these corrections entirely leads to an increase in χ^2 of about one, and little shift in the parameter values, so they are barely significant. However, their inclusion means that we obtain the physical phase shift δ_0^E which includes the effect of vacuum polarization,[18] rather than the nuclear phase shift δ_0^c which the same strong interaction would produce in the absence of vacuum polarization; we defer discussion of this additional correction to a later paragraph. As can be seen from Table 2b, the omission of $^3F_2$ from the analysis has no effect, as anticipated, and the omission of ϵ_2 produces a barely significant change. However, if we omit $^1D_2$, ϵ_2 and $^3F_2$, or $^1D_2$ alone, χ^2 rises by an unacceptable amount. Turning again to model predictions, we find that $^1D_2$ is predicted to be within 20% of the OPE prediction at this energy, and as is shown in Table 2b, the analysis is stable within those limits. Thus we have not only demonstrated insensitivity of the parameters within the physically acceptable limits, but also shown that the data is sufficiently accurate to <u>require</u> $^1D_2$ to lie approximately within those limits, giving modest additional support to the long-range OPE assumption for this state.

Although we have shown that cross section and C_{nn} data of currently available precision cannot possibly give any information about the ratio Δ_{LS}/Δ_T within the limits predicted from the behavior of the P-waves at higher energy, and that these limits can be firmly established, it would obviously still be desirable to have a direct measurement of this ratio at low energy. We have, therefore, computed predictions for a number of spin-dependent measurements at 9.69 MeV and give these in Table 3. ~~Then~~ The error quoted for the column $\Delta_{LS}/\Delta_T = 0.11$ is the uncertainty predicted by our analysis, using the error correlations quoted in the abstract. Measurement to better than this accuracy would either allow a reduction of the uncertainty in the three parameters we determine, or reveal an experimental inconsistency between the two sets of measurements. The spread between the columns labeled .07 and .15 gives the minimum precision required to obtain any information about the desired ratio, provided we are correct in our arguments which led to these limits. As can be seen at a glance, the situation looks pretty hopeless. Although results are given only at one angle, we have examined the full angular range, and find no particularly significant increase in sensitivity as the angle is varied. The most promising experiments appear to be A and A', and full angular predictions for these are given in Figs. 1 and 2. We also note that our analysis allows us to predict the absolute value of $A_{xx}(90°)$ at 11.4 MeV, for which we find -0.984 ± 0.008, and hence provide an absolute normalization for the Saclay experiments.[3] However, as was first pointed out by Catillon,[19] and has been demonstrated in detail elsewhere,[16] this is a direct consequence of our assumption about Δ_{LS}/Δ_T and hence gives no additional support to this normalization.

At first sight the value of Δ_C we obtain is anomalous compared to the OPE prediction, since it is an order of magnitude smaller and ten standard deviations away from the prediction. This same anomaly was encountered and interpreted in the previously reported analysis[20] of data below 3 MeV. As will be discussed below, the OPE interaction is relatively weak compared to the intermediate range attraction needed to account for the observed $^1S_0$ scattering length and effective range, and can only be detected in the $^1S_0$ state by measuring a small deviation from the shape-independent effective range approximation. If this intermediate range attractive interaction is approximately spin independent, it should also occur in the triplet-odd states. Since the effective central interaction due to OPE is repulsive and three times weaker in the triplet-odd states than in the singlet-even states, where we have already seen it is barely detectable, we would then predict that it will be completely swamped by the attractive intermediate range interaction, and anticipate a large positive value for Δ_C. This expectation is confirmed by phase shift analyses at 27 MeV and above. However, at low enough energy, centrifugal shielding reduces the importance of the intermediate range attracti~~ve~~on relative to OPE, and produces a delicate cancellation between attraction and repulsion leading to the small value of Δ_C obtained here, and at 3 MeV. All that is needed to make this interpretation plausible is a physical explanation for the spin-independent intermediate range attraction. But most theories of the pion-pion interaction agree that the interaction is attractive in the $I = 0$ S state. Whether or not this interaction is strong enough to produce an S-wave resonance, or σ meson, the resulting correlation between two pions will, in the nuclear force, produce an intermediate range spin-independent attraction, and hence account for the effect we observe.

At first sight, the determination of a very precise value of Δ_C at four energies below 3 MeV and again at 10 MeV should allow one to compute an accurate value for the central P-wave scattering length and effective range. However, if one is willing to accept the physical interpretation we have just given, this is no longer true. We argue that any acceptable model for the effective central interaction in the triplet-odd P waves must include (1) OPE, (2) an attractive intermediate range attraction due to the $I = 0$ $\pi - \pi$ S-wave or the σ meson, and (3) short range repulsion due to the vector mesons. As was noted previously[20] a single parameter in such a model, specifically the ratio of the strength of the intermediate range attraction to the strength of OPE, will produce agreement with the measured values of Δ_C at 3 MeV and below, independent (over broad limits) of the range assumed for the intermediate range attraction, or the ratio of the strength of the intermediate range attraction to the strength of the short range repulsion. We find that this is still true when we include the value of Δ_C determined by this analysis at 10 MeV, and conclude that, within this framework, p-p scattering data below 10 MeV determines only a single P-wave parameter which measures the average strength of the intermediate range attraction relative to OPE. The actual value of this parameter will depend critically on the specific P-wave model used, but within a given context, it is known precisely, and we urge that any models aimed at a quantitative description of p-p scattering in the neighborhood of 10 MeV and below include the measured values of Δ_C as part of the data they are required to fit.

The implications of these results for the analysis of p-p scattering experiments below 10 MeV are very important. It has already been shown[20,21] that the best available differential cross sections at 3 MeV and below determine, at a single energy, only δ_0^E and Δ_C, and that the values so determined are stable, if one assumes (1) either OPE or no nuclear contribution from $J > 2$, $^3F_2$, or ϵ_2, (2) $^1D_2$ within a factor of 2 of the OPE prediction, (3) Δ_T no more than 50% different from the OPE prediction, and (4) Δ_{LS} no larger than 50% of the OPE prediction for Δ_T. But we have just shown that even at 10 MeV all these requirements are already empirically satisfied to much higher accuracy than this; a fotiriori we conclude that they are also satisfied to much higher precision than is required for unique and stable single-energy analyses at 3 MeV and below. We therefore insist that, so far as nuclear effects go, everything except δ_0^E and

and Δ_C can be reliably predicted to much more than the requisite accuracy, and that these two parameters can be determined directly from the data without ambiguity. Of course, if some way is found to increase the precision of the data, this question should be reopened, but this much improvement in experimental technique does not appear likely in the near future. The interpretation of the values of the nuclear parameters so determined is not simple, since it involves subtle questions about electromagnetic corrections that we are not discussing here. All we are saying is that, so far as data analysis goes, the nuclear physics should be confined to the determination of two empirical parameters, and can be cleanly separated from the question of how to calculate or interpret electromagnetic corrections.

The value of δ_0^E we give above still includes the effect of vacuum polarization,[18] so cannot be directly compared to model calculations which do not include vacuum polarization. Since no calculations including vacuum polarization exist at this energy, we use a roundabout, but we believe sufficiently accurate, method to obtain a phase shift that can be compared with existing calculations. Foldy and Eriksen[22] have computed the correction to be applied to δ_0^E in order to give the phase shift δ_0^c which the same nuclear interaction would give in the absence of vacuum polarization at the energies of our previous analysis.[20] We apply these corrections, fit two parameters in the Hamada-Johnston potential to this corrected data (which takes only a slight adjustment), and use this potential to compute the scattering length and effective range, a_c and r_c, from the zero energy wave function. Alternatively, we fit the empirical values of δ_0^E directly to the effective range expansion given by Heller[18] and thus determine a_E and r_E. Ideally, we should also include a shape correction to this fit computed from the same potential but including vacuum polarization, but since the shape correction is only determined to 30% by the data, and the effect of vacuum polarization on the shape correction is of order 1/137 of the correction, we are satisfied that we can use the shape correction computed in the absence of vacuum polarization instead. We then compare the phase shift at 9.69 MeV predicted by the Heller expansion to that predicted by the conventional expansion, and find it larger by 0.14°. Again assuming that we can ignore the vacuum polarization correction to the shape correction, we therefore conclude that $\delta_0^c = 55.55° \pm 0.29°$ at 9.69 MeV. Since the correction we find this way is only half the experimental error in the phase shift, our estimate could be out by a factor of two without affecting anything we say below; actually we believe it to be much better than that.

Since we have already determined a value for a_c and r_c, we can also predict what the shape-independent effective range approximation would give at this energy and find 54.40°. The statistical error in this prediction is only ± 0.04°, which is much too optimistic if one starts thinking about sources of systematic error and model-dependent corrections, but we believe that it is safe to say that the empirical value of δ_0^c is larger than that given by the shape-independent approximation by two to three standard deviations. Theoretically, this effect is not only expected, but we believe its absence would lead to serious theoretical difficulties. It was shown long ago[23,24] that the long-range OPE interaction should produce a deviation from the shape-independent approximation in this energy range in this direction of approximately this magnitude, and this prediction has been confirmed to an accuracy of about 30% (which is consistent with experimental error) both below 3 MeV,[20] and more recently in the neighborhood of 27 MeV.[25] Quantitatively, this new confirmation of the prediction at 10 MeV is not quite so good, since the potential model used predicts 55.17° rather than the observed 55.55 ± 0.29°, but the discrepancy is hardly significant. We note without comment that if the absolute value of the cross section quoted by the Minnesota group[1,2] should turn out to be 0.5 to 1% too high, there would be complete agreement between theory and experiment.

We summarize our conclusions as follows: (1) We find that, because of the small value of the Δ_{LS}/Δ_T ratio, it is impossible to obtain a unique 4-parameter phase shift analysis near 10 MeV using data of currently available precision, and unlikely that new experiments will improve this situation in the near future. (2) We believe that this situation is completely understood in terms of the firmly-established short-range character of the spin-orbit interaction, and that available information at higher energy allows one to predict the value of Δ_{LS}/Δ_T to high precision at 10 MeV, independent of any specific model for the p-p interaction. (3) Under this assumption, we have uniquely determined δ_0^E, Δ_C, and Δ_T from existing data at 9.69 and 11.4 MeV. (4) ${}^1D_2$ and Δ_T have to be within 30% of their OPE values at this energy, and all other nuclear phase parameters except δ_0^E and Δ_C can be safely ignored at this and lower energies. (5)The limits just established are more than sufficient to justify the use of (coulomb corrected) OPE values for <u>all</u> nuclear parameters except δ_0^E and Δ_C at any energy below 10 MeV, and hence to allow unique and stable determinations of these two parameters at a single energy using only p-p differential cross section data. (6) The anomalously small value of Δ_C compared to the OPE prediction is again explicable as due to a delicate cancellation between the weakly repulsive long range OPE interaction and the strong intermediate range attraction to be expected from the exchange of two pions interacting attractively in the $I = 0$ $\pi - \pi$ S state. A single (model-dependent) parameter which measures the average strength of this attraction compared to OPE suffices to explain both this value and four other values of Δ_C previously determined below 3 MeV. This single parameter is precisely determined by the data within the framework of models of this general character. (7) The Foldy correction at 9.69 MeV is estimated to be only about 0.14° , and if this is even approximately correct, the $^1S_0$ phase shift determined by this analysis again confirms the OPE prediction that it must lie above the value predicted by the shape-independent effective range approximation at this energy. Quantitatively, the deviation is too large, but by only somewhat more than one standard deviation. Taken together with corresponding results below 3 MeV[20] and near 27 MeV,[27] we contend that the predicted OPE shape correction to the $^1S_0$ effective range expansion is now firmly established, and can be used with confidence over the entire energy range (0 - 27 MeV) where it is significant.

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TABLE 1

Predictions of various p-p models for the ^3P scattering parameters at 9.69 MeV. The OPE prediction has been multiplied by $e^2 = 2\pi n/(e^{2\pi n} - 1)$, $n = e^2/\hbar v_{lab}$ in $= .84259$ order to roughly correct for the static coulomb effect which was included in the model calculations.

	Δ_C	Δ_T	Δ_{LS}	Δ_{LS}/Δ_T
OPE	-.3203°	1.030°	0	0
Scotti-Wong (SW)	.2507°	1.074°	.155°	.145
Yale (Y)	.0502°	1.089°	.160°	.147
Hamada-Johnston (HJ)	.0176°	.911°	.109°	.119
Feshbach-Lomon-Tubis (FLT)	.0769°	1.045°	.073°	.070

TABLE 2

Sensitivity of the analysis to the theoretical assumptions used

(a) Sensitivity to the ratio of Δ_{LS}/Δ_T assumed

Δ_{LS}/Δ_T	$^1S_0$	Δ_C	Δ_T	χ^2	A_{xx}
.07	55.66 ± .29°	-.021 ± .029°	.94 ± .29°	11.2702	-.982 ± .009
.11	55.69 ± .28°	-.020 ± .029°	.91 ± .28°	11.2859	-.984 ± .008
.15	55.72 ± .27°	-.020 ± .029°	.88 ± .27°	11.3048	-.986 ± .007

(b) Effect of higher partial waves at $\Delta_{LS}/\Delta_T = .11$

Case	$^1S_0$	Δ_C	Δ_T	χ^2	A_{xx}
$^1D_2 = {}^3F_2 = \epsilon_2 = 0$	55.33 ± .30°	-.037 ± .028°	1.22 ± .22°	18.4491	-.979 ± .008
$^1D_2 = 0$	55.72 ± .28°	-.074 ± .029°	.87 ± .28°	25.7215	-.985 ± .088
$^1D_2$=1.2 OPEC	55.69 ± .28°	-.009 ± .029°	.92 ± .28°	11.4357	-.984 ± .008
$^1D_2$=.8 OPEC	55.70 ± .28°	-.031 ± .029°	.91 ± .28°	12.1565	-.984 ± .008
$\epsilon_2 = 0$	55.64 ± .30°	+.013 ± .028°	.97 ± .28°	11.7125	-.987 ± .008
ϵ_2=0.8 OPEC	55.67 ± .28°	-.015 ± .029°	.94 ± .28°	11.1941	-.984 ± .008
ϵ_2=1.2 OPEC	55.72 ± .27°	-.026 ± .030°	.88 ± .28°	11.4450	-.984 ± .008
$^3F_2 = 0$	55.69 ± .28°	-.016 ± .029°	.91 ± .28°	11.3069	-.984 ± .008

TABLE 3

Sensitivity of p-p observables at 9.69 MeV to the ratio Δ_{LS}/Δ_T Measurement of the observable to better than the error given in the last column would give new information on δ_0^E, Δ_C, and Δ_T. Measurement to better than the spread between the first two columns is required to get any information on Δ_{LS}/Δ_T.

Observable	θ_{cm}	$\Delta_{LS}/\Delta_T = .07$	$\Delta_{LS}/\Delta_T = .15$	$\Delta_{LS}/\Delta_T = .11$
$-R, R^1$	90°	.0725	.0725	.0725 ± .0188
A, A^1	90°	.0723	.0638	.0679 ± .0197
D	90°	-.0138	-.0123	-.0130 ± .0070
C_{NKP}	46°	.1913	.1878	.1895 ± .0179
C_{PNP}	90°	-.0116	-.0179	-.0148 ± .0055
C_{KNP}	90°	.1300	.1237	.1268 ± .0327
A_{zz}	90°	-.9845	-.9845	-.9845 ± .0084
A_{yy}	90°	-.9720	-.9747	-.9735 ± .0142
A_{xx}	90°	-.9876	-.9902	-.9890 ± .0059
A_{zx}	46°	-.0154	-.0140	-.0146 ± .0033
P	46°	-.00077	-.00065	-.00071 ± .00036
C_{KP}	90°	.00154	.00284	.00223 ± .00124
C_{KK}, C_{PP}	90°	-.9860	-.9873	-.9867 ± .0071

Figure Captions

Figure 1. Prediction of A' vs scattering angle at 9.69 MeV.

(1) Upper bound for A' with $\Delta_{LS}/\Delta_T = .11$

(2) A' with $\Delta_{LS}/\Delta_T = .07$

(3) A' with $\Delta_{LS}/\Delta_T = .11$

(4) A' with $\Delta_{LS}/\Delta_T = .15$

(5) Lower bound for A' with $\Delta_{LS}/\Delta_T = .11$

Figure 2. Prediction of A vs scattering angle at 9.69 MeV.

(1) Upper bound for A with $\Delta_{LS}/\Delta_T = .11$

(2) A with $\Delta_{LS}/\Delta_T = .07$

(3) A with $\Delta_{LS}/\Delta_T = .11$

(4) A with $\Delta_{LS}/\Delta_T = .15$

(5) Lower bound for A with $\Delta_{LS}/\Delta_T = .11$

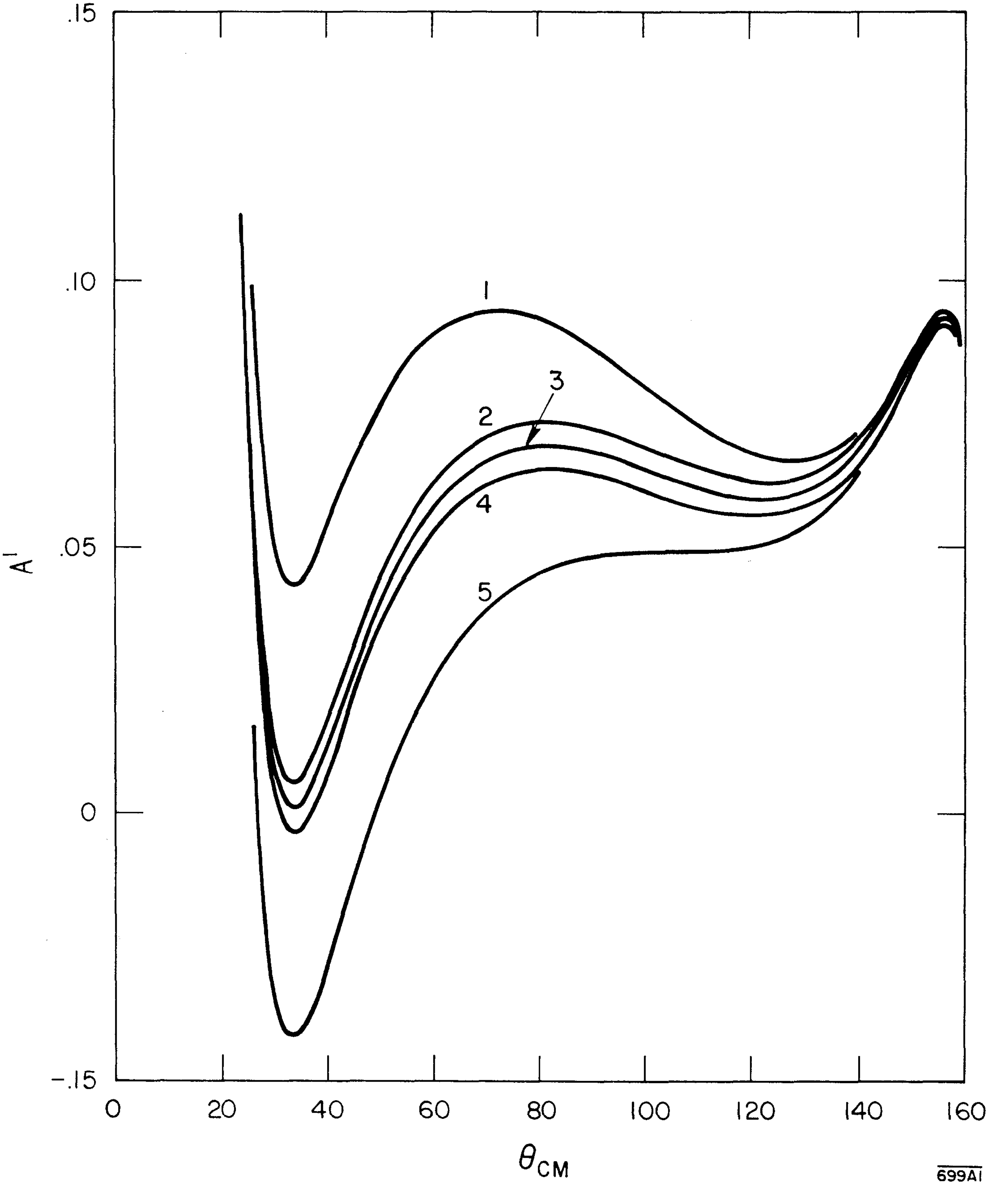

Fig. 1

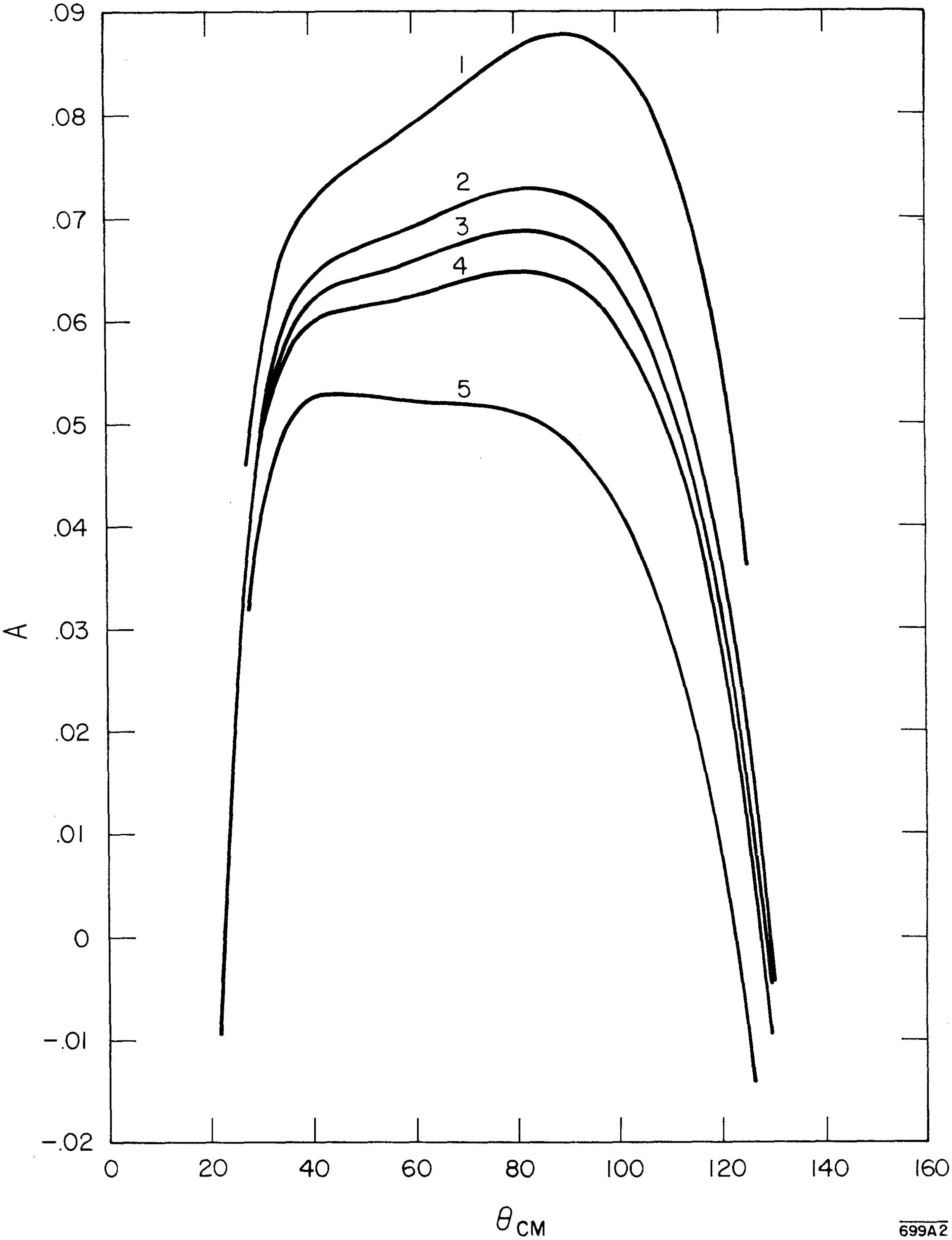
.09
.08
.07
.06
.05
.04
.03
.02
.01
0
-.01
-.02
A
0
20
40
60
80
100
120
140
160
θ_{CM}
1
2
3
4
5
699A2

Fig. 2