Application of the interacting boson model and the interacting boson-fermion model to β decays

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Abstract. The β decay is studied in the interacting boson model. The application to single-β decay is extended to two-neutrino double-β decay.

1. Introduction

The interacting boson model (IBM) [1, 2], including the interacting boson-fermion model (IBFM) [3] and the interaction boson-fermion-fermion model (IBFFM) [4], has been successful in describing the energy levels and the electromagnetic properties of various kinds of nuclei. One of the important applications is β decay [5, 6, 7, 8, 9, 10, 11, 12, 13, 14] because β decay is related to neutrino physics. In this school, I talk about the application of IBM to the β-decay. The decays include single-β decay from odd-A nuclei as well as double β-decay from even-even nuclei [15].

2. Description of β-decay in IBM

Single-β decay between odd-even nuclei has been studied in wide regions in IBM [5, 6, 8, 9, 10]. Double-β in IBM decay was studied by Scholten and Yu [7]. Recently, Barea, Kotila and Iachello studied extensively both neutrino-less double-β decay ($0\nu\beta\beta$) and two-neutrino double-β decay ($2\nu\beta\beta$) [12]. These works use the closure approximation in treating the intermediate states in odd-odd nuclei. I talk about the work on $2\nu\beta\beta$ with Iachello [15].

For $2\nu\beta\beta$, the Gamow-Teller (GT) $M_{2\nu}^{\mathrm{GT}}$ and the Fermi (F) matrix elements $M_{2\nu}^{\mathrm{F}}$ are calculated by [16]

$$M_{2\nu}^{\mathrm{GT}} = \sum_N \frac{\langle 0_F^+ || t^+\sigma || 1_N^+ \rangle \langle 1_N^+ || t^+\sigma || 0_1^+ \rangle}{\frac{1}{2}(Q_{\beta\beta} + 2m_e c^2) + E_N - E_I}, \qquad M_{2\nu}^{\mathrm{F}} = \sum_N \frac{\langle 0_F^+ || t^+ || 0_N^+ \rangle \langle 0_N^+ || t^+ || 0_1^+ \rangle}{\frac{1}{2}(Q_{\beta\beta} + 2m_e c^2) + E_N - E_I}, \tag{1}$$

where $t^{\pm}$ is the isospin increasing/decreasing operator, $\boldsymbol{\sigma} = 2\boldsymbol{s}$ is the Pauli spin matrix, while $Q_{\beta\beta}$ is the Q value of the double-β decay, and E_I and E_N are the energies of the initial and the intermediate states, respectively. The proton-neutron IBM is used in which the even-even core of the nucleus is treated as a system of proton bosons and neutron bosons of angular momentum zero (s-bosons) or angular momentum two (d-bosons), which represent proton pairs and neutron pairs outside the closed shell. The microscopic theory of IBM gives the images of the Fermi and

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Gamow-Teller transition operators as [5]

$$t^{\pm} \longrightarrow O^{\mathrm{F}} = \sum_{j} -\sqrt{2j+1}\big[P_{\pi}^{(j)} P_{\nu}^{(j)}\big]^{(0)}; \qquad t^{\pm}\sigma \longrightarrow O^{\mathrm{GT}} = \sum_{j'j} \eta_{j'j}\big[P_{\pi}^{(j')} P_{\nu}^{(j)}\big]^{(1)} \quad (2)$$

where $P_{\rho}^{(j)}$ represents the boson-fermion image of the particle-transfer operator expanded in terms of fermion $a_{jm}^{\dagger}$ and boson $s_{\pi,\nu}^{\dagger}, d_{\pi,\nu}^{\dagger}$ operators: $P_{\rho,m}^{\dagger(j)} = \zeta_j a_{jm}^{\dagger} + \sum_{j'} \zeta_{jj'} s_{\rho}^{\dagger}[\tilde{d}_{\rho} \times a_{j'}^{\dagger}]_m^{(j)}$ with proper coefficients [17]. From these the $\log ft$ values can be calculated for β^- and β^+/EC transitions. In many calculations, the closure approximation is adopted in which the summation in (1) over the intermediate states N is replaced by the average. In the work that I am presenting, no closure approximation is made.

In the $2\nu\beta\beta$ decay from ^{128}Te to ^{128}Xe, the states in the intermediate nucleus ^{128}I are accounted for in the proton-neutron IBFFM. Some of the low-lying states are shown in Fig. 1. In the summation for the matrix elements, the states 1^+ up to 3 MeV in excitation energy

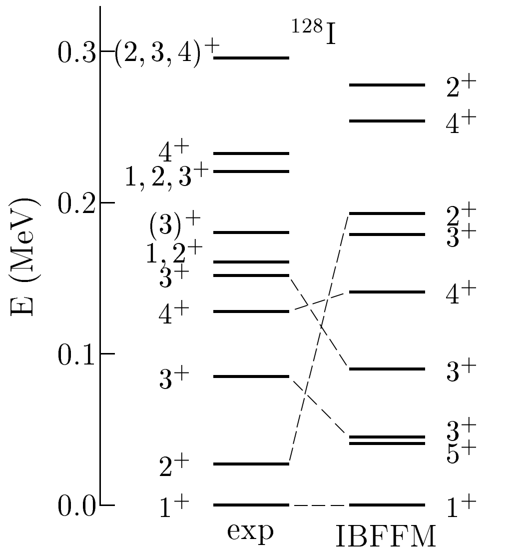

Figure 1. Energy levels in ^{128}I. The experimental data are from [18].

are included.

From ^{128}I, some of single-β decay (β^-, β^+/EC) are experimentally observed. Table 1 shows the $\log ft$ values of electron capture (EC) from ^{128}I, while those to ^{128}Xe are shown in Table

Table 1. The $\log_{10} ft$ values of EC from ^{128}I to ^{128}Te. The data are from [18].

transition	exp	cal	quenched
$1_1^+ \to 0_1^+$	5.049 (7)	3.836	5.15 (9)

2. Introducing a common hindrance factor: $h \approx 4.5$, which is equivalent to a quenched axial vector coupling constant: $g_{A,\mathrm{eff},\beta} = 1.269/h = 0.28$, we obtain a reasonable agreement, as shown as "quenched" in the Tables. The $B(\mathrm{GT})$ values from ^{128}I have been also extracted from (^{3}He, t) reaction [19]. The related values are: ^{128}I, $B(\mathrm{GT})_{\mathrm{g.s.}}^{(^3\mathrm{He},t)} = 0.079$ (8); $\sum = 0.829$ (50). The corresponding IBM values are: $^{128}\mathrm{I} = 1.676$, $\sum = 15.09$. if we use the same quenched value of

Table 2. The $\log_{10} ft$ values of β^- decay from ^{128}I to ^{128}Xe. The data are from [18].

transition	exp	cal	quenched
$1_1^+ \to 0_1^+$	6.061 (5)	4.665	5.98 (9)
$1_1^+ \to 0_2^+$	7.748 (24)	5.262	6.57 (9)
$1_1^+ \to 0_3^+$	7.84 (6)	5.712	7.02 (9)
$1_1^+ \to 2_1^+$	6.495 (7)	5.212	6.52 (9)
$1_1^+ \to 2_2^+$	6.754 (9)	6.446	7.76 (9)

$g_{A,\mathrm{eff},\beta} = 0.28$, then we have $B(\mathrm{GT})$[IBM-quenched] $= 0.082$, $\sum$[IBM-quenched] $= 0.735$. These values are in good agreement with the (^{3}He, t) values.

Figure 2 shows the contributions from the intermediate states in ^{128}I to the GT matrix element in Eq. (1). The single-state dominance (SSD) discussed in Refs. [13, 14], namely, the

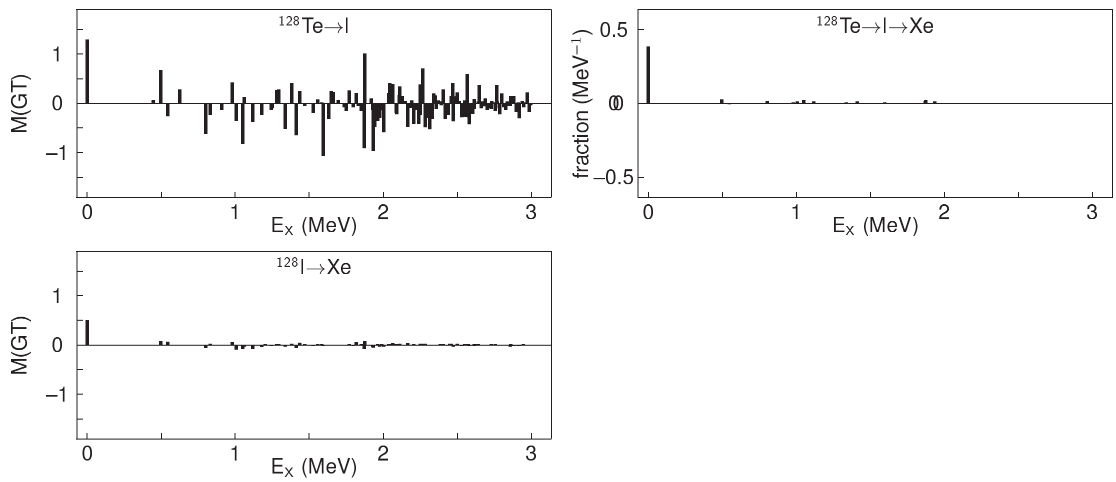

Figure 2. The values of $\langle 1_N^+ || t^+\sigma || 0_1^+ \rangle$ (top-left), $\langle 0_1^+ || t^+\sigma || 1_N^+ \rangle$ (bottom-left) and $\langle 0_1^+ || t^+\sigma || 1_N^+ \rangle \langle 1_N^+ || t^+\sigma || 0_1^+ \rangle / (\frac{1}{2}(Q_{\beta\beta} + 2m_e c^2) + E_N - E_I)$ (top-right), for the double-β decay from the lowest 0^+ in ^{128}Te to the lowest 0^+ in ^{128}Xe through the intermediate 1^+ in ^{128}I, plotted as a function of the excitation energy.

dominance of 1_1^+ in the summation, is seen in the figures. Similar analysis has been made for the Fermi decay, as well as those from ^{130}Te. Table 3 shows the nuclear matrix elements calculated by (1) and a similar formula for the decay to a state 2^+. The inverse half-life of $0_1^+ \to 0_F^+$ can be calculated from

$$|M_{2\nu}^{\mathrm{calc}}| = g_A^2 \left| M_{2\nu}^{\mathrm{GT}} - \left(\frac{g_V}{g_A}\right)^2 M_{2\nu}^{\mathrm{F}} \right| \tag{3}$$

by multiplying the lepton phase-space integral. Table 4 shows the thus obtained nuclear matrix element as "calu". By introducing the same quenched $g_{A,\mathrm{eff},\beta\beta} = g_{A,\mathrm{eff},\beta} = 0.28$ in

$$|M_{2\nu}^{\mathrm{quenched}}| = g_{A,\mathrm{eff},\beta\beta}^2 |M_{2\nu}^{\mathrm{calc}}|, \tag{4}$$

while the ratio $g_V/g_A = 1/1.269$ in (3) is fixed, we obtain the values shown as "quenched" in Table 4, which are consistent with the experimental values.

Table 3. Nuclear matrix elements $M_{2\nu}^{\mathrm{GT}}$, $M_{2\nu}^{\mathrm{F}}$ of transitions from the ground state of ^{128,130}Te to some states in ^{128,130}Xe. The sign of $M_{2\nu}^{\mathrm{GT}}$ is chosen to be positive.

transition	^{128}Te→^{128}Xe	^{130}Te→^{130}Xe
GT		
$0_1^+ \to 0_1^+$	0.297	0.273
$0_1^+ \to 2_1^+$	0.00718	0.00639
$0_1^+ \to 0_2^+$		0.668
F		
$0_1^+ \to 0_1^+$	−0.0353	−0.0309
$0_1^+ \to 0_2^+$		−0.112

Table 4. Two-neutrino double-β decay matrix elements, $|M_{2\nu}|$ in IBFM.

	exp	calc	quenched
^{128}Te	0.044 (6)	0.514	0.040 (8)
^{130}Te	0.031 (4)	0.470	0.037 (8)

3. Conclusion

Use of a single value of $g_{A,\mathrm{eff},\beta} = g_{A_{\mathrm{eff}},\beta\beta}$ appears to describe well both single-β and double-β decay in a consistent way. The question of the small value of $g_{A,\mathrm{eff}}$ is a subject of further study.

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