# PHYSICS WITH PHOTONS OF NON-ZERO REST MASS

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This study is done under the following assumptions: A) The results presented in the two relativistic works of Henri Poincaré "Sur la dynamique de l'électron" (ref.1 and 2) are true. B) The "second principle" of Einstein, the constancy of the velocity of light, in "Zür Elektrodynamik der bewegten Körper" (ref. 4), is not necessarily true and the photons can have an extremely small but non-zero rest mass $m_o$.

Of course, all photons will be considered as identical, an already common assumption for electrons, for protons and for all particles.

The results are simple and lead to a small modification of Maxwell equations for the propagation of electromagnetic fields.

The solutions can be developed and lead to possibilities of verifications and of measures of the rest mass $m_o$ either by astronomical observations or by astronautical experiments. They lead also to new cosmological perspectives.

### Introduction

Since it is now clear that among the founding texts of Special Relativity are the two works of Henri Poincaré "Sur la dynamique de l'électron" (ref. 1 and 2), it is interesting to look for their differences with the work of Einstein (ref. 4). The ref. 2, the "Palermo Memoir" contains a mine of relativistic results ignored by the Einstein text, but this Einstein text has also some additions: three small applications of the Lorentz transformation (the aberration of stars, the Doppler-Fizeau effect and the radiation pressure on moving mirrors) and a main difference: the "second principle": the constancy of the velocity of light. As a consequence, for Einstein, the photons must have a zero rest mass.

For Poincaré the constancy of the velocity of light is only a physical ascertaining and his "Lorentz transformation" is only a direct consequence of his Relativity Principle without the necessity of the constancy of the velocity of light. Hence, the possibility remains that the photons have a very small, but non-zero, rest mass. Their velocity, the velocity of light, would then be extremely near to the limit velocity c and would be an increasing function of their energy.

Several comparisons allows to think that this idea is possible: A) At first the neutrinos were considered as massless, but today their rest mass is supposed to be very small but non zero. B) The same evolution of ideas occurs for the hypothetical "gravitons". C) In transparent mediums the velocity of light is much smaller than the limit velocity c and some fast particles can go faster than the photons which gives the famous Cerenkov effect.

Because of the prestige of Einstein, only a few scientists have investigated the possibility of a non-zero rest mass for the photon (for instance ref. 5–17) with few results and few possibilities of verifications. However, the huge progress of astronomy, astronautics and astrophysics allows to consider, that question again and some anomalies of pulsars as well as the Kotov paradox can perhaps be explained by a non-zero rest mass of the photons.

## Nomenclature of main elements

c : limit or maximal velocity for material bodies = 299 792 458 m/s
$m_o$ : rest mass of photons
$\varphi$, $\varphi_1$ : phase velocity
g, $g_1$ : group velocity (or "velocity of light") ; $g\varphi = g_1\varphi_1 = c^2$
$\nu$ : frequency
$\lambda$ : wavelength ; $\lambda\nu = \varphi$
h = Planck's constant = $6.626\,196 \times 10^{-34}$ kg.m$^2$.s$^{-1}$
$\hbar = h / 2\pi$ = reduced Planck's constant = $1.054\,592 \times 10^{-34}$ kg.m$^2$.s$^{-1}$
$P_o$ = proper period of photons = $h / m_o c^2$
$\nu_o$ = proper frequency of photons = $1 / P_o = m_o c^2 / h$
$\omega_o$ = proper pulsation of photons = $2\pi\nu_o = m_o c^2 / \hbar$
$\mu_o$ = magnetic permeability of vacuum = $4\pi \times 10^{-7}$ henry per meter
$\varepsilon_o$ = permittivity of vacuum = $8.854\,187\,818 \times 10^{-12}$ farad per meter; $\mu_o\varepsilon_o c^2 = 1$
**E**, **E₁** : electric field
**B**, **B₁** : magnetic induction
V : scalar potential
**A** : vector potential
$\Delta$ : Laplacian. $\Delta U = \partial^2 U/\partial x^2 + \partial^2 U/\partial y^2 + \partial^2 U/\partial z^2$
**E**, $\varepsilon$ : arbitrary fixed vectors
**u** : arbitrary fixed unit vector
**r** : radius-vector (x, y, z)

## 1. About the photon

The photon is a very mysterious quantic phenomenon, moving as a wave, but appearing as a particle. It is a boson with spin one and rest mass either zero, or extremely small, it carries energy, it has a frequency (in a given referential) and presents phenomena of polarization.

If we assume a very small but non-zero rest mass, the photon goes slower than the limit velocity c and we can study it **in the referential in which it is at rest.** In such referentials and in vacuum the basic hypothesis is that all photons are identical. In the other referentials, according to their velocity, they present frequency and energy that can be studied by the usual Lorentz transformations, that will be our main tool.

We will see that if photons have a non-zero rest mass they have also a well defined proper period and thus the phenomenon of photon can perhaps be associated to vibrations or rotations, which fit particularly well with the property of polarization and the different possible presentations of a rotating phenomenon.

## 2. Phase velocity and group velocity

The dual nature of particles, wave and corpuscle, appears very clearly for photons. We will see that the light waves (phase velocity) goes faster than the limit velocity c but the photons (group velocity) goes slower than c.

The group velocity is defined as the velocity at which we must move in order that some close waves remain with the same phase difference. In the case when that difference is zero, we obtain a concentration of energy, and matter, and thus the group velocity is the natural velocity of matter.

Let us consider an ordinary sinusoidal wave moving along the x-axis:

$$F(x, t) = A \cos\{2\pi [\nu t - (x/\lambda) + f_o]\}. \quad (1)$$

As usual, $\boldsymbol{A}$ is the amplitude, $\nu$ is the frequency, t is the time, $\lambda$ is the wavelength and $2\pi f_o$ is the phase at origin. The sum $2\pi[\nu t - (x/\lambda) + f_o]$ is the phase, and the phase velocity $\varphi$ is given by:

$$\varphi = \lambda\nu = \text{phase velocity}. \quad (2)$$

It is the velocity at which the sinusoid moves along the x-axis.

Let us consider now two or several neighboring waves with phase $2\pi[\nu_j t - (x/\lambda_j) + f_{oj}]$, with j = 1, 2, 3... and with all $\nu_j \approx \nu$ and all $\lambda_j \approx \lambda$. The phase differences of these waves remain constant for the group velocity g if and only if :

$$\delta\nu\, t - gt\, \delta(1/\lambda) = 0\,, \quad (3)$$

that is: $$g = \delta\nu / \delta(1/\lambda) = -\lambda^2\, d\nu/d\lambda = \text{group velocity}\,, \quad (4)$$

if with equation (2), we consider the function $\varphi(\lambda)$ instead of the function $\nu(\lambda)$, the relation (4) becomes:

$$g = \text{group velocity} = \varphi - \lambda(d\varphi/d\lambda)\,. \quad (5)$$

The phase velocity $\varphi$ and the group velocity g are equal only when $(d\varphi/d\lambda) = 0$ .

## 3. The aberration of stars

Let us consider successively the two points of view.

A) The photon as a corpuscle moving with the group velocity g.

The astronomer at the origin receive at the time t = 0 a photon coming from the direction given by the angle a (fig. 1).

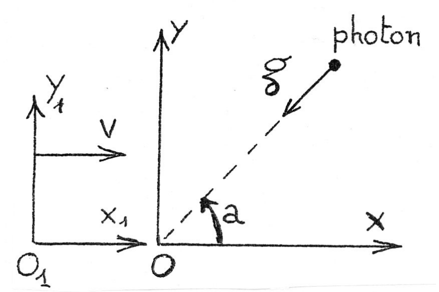

*Fig. 1. Arrival of a photon from the direction given by the angle a.*

At the time t = – 1 the photon was at the following point of space time :

$$\left.\begin{aligned} x &= g \cos a \\ y &= g \sin a \\ z &= 0 \\ t &= -1 \end{aligned}\right| \quad (6)$$

However, for an astronomer moving along the x-axis with the velocity v that same point of space-time is given by the corresponding "Lorentz transformation" (remember that the Lorentz transformation is independent of the property of light, it only depends of the velocity v and the limit velocity c, that is no more "the velocity of light").

The space-time coordinate (6) becomes:

$$\left.\begin{aligned} x_1 &= \gamma\,(x - vt) = \gamma\,(g \cos a + v)\ ; \quad \text{with } \gamma = [1 - (v^2/c^2)]^{-0.5} \\ y_1 &= y = g \sin a \\ z_1 &= z = 0 \\ t_1 &= \gamma\,(t - vx/c^2) = -\gamma\,[1 + (v\, g \cos a / c^2)] \end{aligned}\right| \quad (7)$$

Hence for the second astronomer the angle $a_1$ of the direction of the photon is given by:

$$\tan a_1 = y_1 / x_1 = g \sin a / \gamma\,(g \cos a + v)\,. \quad (8)$$

Notice that in the classical case when g = c this equation (8) gives the classical Einsteinian relation that can then be simplified into: $(c - v)^{0.5} \tan(a/2) = (c + v)^{0.5} \tan(a_1/2)$.

Also notice the velocity $g_1$ of the photon for the moving astronomer:

$$g_1^2 = (x_1^2 + y_1^2) / t_1^2 = \{c^4 (g + v \cos a)^2 + c^2 v^2 \sin^2 a (c^2 - g^2)\} / (c^2 + vg \cos a)^2 , \quad (9)$$

that gives:

$$c^2 - g_1^2 = c^2 (c^2 - g^2)(c^2 - v^2) / (c^2 + vg \cos a)^2 , \quad (10)$$

and also the beautiful symmetrical relation:

$$c^2 (v + g \cos a - g_1 \cos a_1) = g\, g_1\, v \cos a \cos a_1 . \quad (11)$$

B) Let us consider now the wave point of view.

The plane wave, moving with the phase velocity $\varphi$, arrives from the direction given by the angle a (fig.2) at the origin at the time t = 0.

The space-time equation of the wave is:

$$x \cos a + y \sin a + \varphi t = 0. \quad (12)$$

For the astronomer moving with the velocity v along the x-axis we can do again the Lorentz transformation (7), that lead to:

$$\gamma x_1 [\cos a + (v\varphi / c^2)] + y_1 \sin a + \gamma t_1 [\varphi + v \cos a] = 0. \quad (13)$$

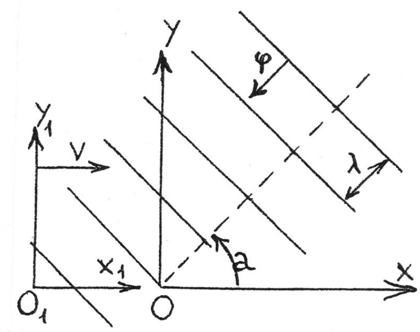

*Fig. 2. Arrival of a wave from the direction given by the angle a.*

The ratio of the coefficients of $y_1$ and $x_1$ gives the tangent of the angle $a_1$

$$\tan a_1 = \sin a / \gamma [\cos a + (v\varphi / c^2)] . \quad (14)$$

The identity between (8) and (14) requires:

$$g\, \varphi = c^2 \quad (15)$$

and we have to verify the compatibility of this equation and the group velocity relation (5).

The analysis of (13) gives also the phase velocity $\varphi_1$ for the moving astronomer:

$$\varphi_1^2 = \gamma^2 [\varphi + v \cos a]^2 / \{ \gamma^2 [\cos a + (v\varphi / c^2)]^2 + \sin^2 a\} =$$
$$= c^4 [\varphi + v \cos a]^2 / \{ (c^2 + v\varphi \cos a)^2 + (\varphi^2 - c^2) v^2 \sin^2 a \} \quad (16)$$

and, if (15) is verified, the equations (9) and (16) lead to the following extension of (15):

$$g_1\, \varphi_1 = c^2 . \quad (17)$$

## 4. The Doppler-Fizeau effect

Let us consider the fig. 2. The astronomer at rest at the origin receives in one second all the waves of an interval of length $\varphi \times$ (one second), meanwhile the astronomer moving from $O_1$ to O receives the waves of the interval of length $(\varphi + v \cos a) \times$ (one second), but he receives them in an interval of time that is for him only $[1 - (v^2/c^2)]^{0.5}$ second. Hence the ratio $\nu_1 / \nu$ of the two frequencies is:

$$\nu_1 / \nu = \gamma (\varphi + v \cos a) / \varphi . \quad (18)$$

Notice that, with the equations (14) and (16), this relation is perfectly symmetrical:

$$\nu / \nu_1 = \gamma (\varphi_1 - v \cos a_1) / \varphi_1 . \quad (19)$$

## 5. The energy of photons

The energy of a particle of rest-mass $m_o$ and velocity g is given by the usual relativistic expression:

$$E = m c^2 = m_o c^2 [1 - (g^2/c^2)]^{-0.5} . \quad (20)$$

Hence the relation (10) leads, for the energy $E_1$ measured by the moving astronomer, to:

$$E_1 / E = [ (c^2 - g^2 ) / (c^2 - g_1{}^2 ) ]^{0.5} = \gamma (c^2 + vg \cos a ) / c^2 . \qquad (21)$$

Thus, if again we consider as true the relation (15), the comparison of (18) and (21) leads to

$$E_1 / E = \nu_1 / \nu . \qquad (22)$$

The ratio $E / \nu$ is the same for all astronomers: it is the famous Planck constant, that is valid even for photons of non-zero rest mass.

$$E = h \nu ; \qquad h = 6.\ 626\ 196 \times 10^{-34}\ \text{kg.m}^2.\text{s}^{-1} . \qquad (23)$$

## 6. Compatibility of the relations g = group velocity = $\varphi - \lambda(d\varphi/d\lambda)$ and $g \varphi = c^2$

It remains to verify the compatibility of the relations (5) and (15), that is the relations g = group velocity = $\varphi - \lambda(d\varphi/d\lambda)$ and $g \varphi = c^2$ .

We will use systematically the relation $g \varphi = c^2$ , and we will verify that it leads to the other relation.

However, from (20) and (23):

$$h \nu = m_o c^2 [1 - (g^2 /c^2 )]^{- 0.5} ; \quad \text{that is : } g^2 = c^2 - (m_o{}^2 c^6 / h^2 \nu^2). \qquad (24)$$

The wavelength $\lambda$ is the ratio $\varphi/\nu$ , which leads to:

$$c^4 / \varphi^2 = g^2 = c^2 - (m_o{}^2 c^6 \lambda^2 / h^2 \varphi^2). \qquad (25)$$

Let us multiply the left and right members by $\varphi^2/c^2$ , the relation between $\varphi$ and $\lambda$ is then:

$$c^2 = \varphi^2 - (m_o{}^2 c^4 \lambda^2 / h^2), \qquad (26)$$

that leads to:

$$\varphi\, d\varphi = m_o{}^2 c^4 \lambda\, d\lambda / h^2 \qquad (27)$$

and thus:

$$g = \text{group velocity} = \varphi - \lambda(d\varphi/d\lambda) = \varphi - (m_o{}^2 c^4 \lambda^2 / h^2 \varphi ). \qquad (28)$$

Let us now multiply the left and right members by $\varphi$ and, with $g \varphi = c^2$, we will obtain again the relation (26). The compatibility we were looking for is verified.

## 7. The proper period and the proper frequency of the photon

Let us consider the figures 1 and 2 or the below fig. 3. The photon with velocity g moves straight into an electromagnetic field moving with a faster phase velocity $\varphi$ and a wavelength $\lambda$ .

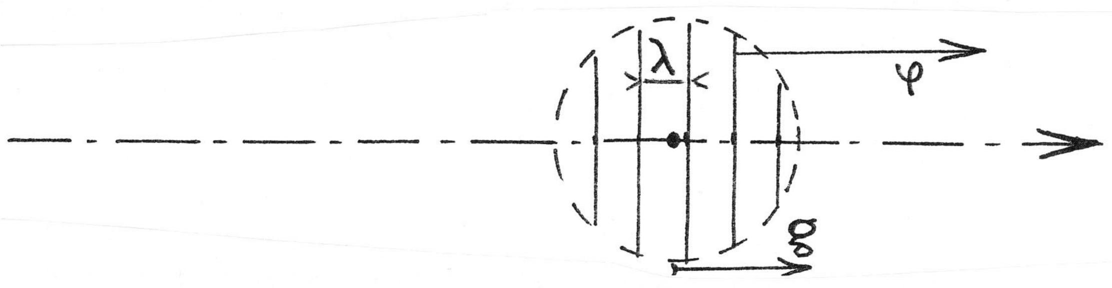


*Fig. 3. The photon (velocity g) moves in a faster electromagnetic field (velocity $\varphi$ ).*

For the astronomer at rest the photon moves periodically in the electromagnetic field with the period $\lambda / (\varphi - g)$. However for the photon itself, that has a slower proper time in this referential, that period is the proper period $P_o = \lambda [1 - (g^2 /c^2 )]^{0.5} / (\varphi - g)$ .

It is essential to notice that this proper period $P_o$ is **fixed and independent of the velocity g**. Indeed, with (24):

$$[1 - (g^2/c^2)]^{0.5} = m_o c^2 / h\nu \qquad (29)$$

and with (26):

$$m_o^2 c^4 \lambda^2 / h^2 = \varphi^2 - c^2 = \varphi^2 - g\varphi = \varphi(\varphi - g) \qquad (30)$$

hence:

$$P_o = \lambda\, [1 - (g^2/c^2)]^{0.5} / (\varphi - g) = \lambda\, (m_o c^2 / h\nu) / (m_o^2 c^4 \lambda^2 / h^2 \varphi) = h / m_o c^2 . \qquad (31)$$

The photon can thus be considered as a periodic phenomenon, with for instance a vibration or a rotation, with always the proper period $P_o = 1/\nu_o = h/m_o c^2$, and with thus the proper frequency $\nu_o = m_o c^2 / h$.

Notice that this unification of the photons appears only for non-zero rest mass $m_o$.

## 8. Table of the relations of photons of non-zero rest mass $m_o$

With the "proper frequency of photons" $\nu_o = m_o c^2/h$ presented in the previous section the basic relations of photon motions, as deduced from (23), (2), (15), (24), (25), become simple.

A) For the constants c, h, $m_o$, $P_o$, $\nu_o$.

$$P_o = \text{"proper period of photons"} = 1/\nu_o = h/m_o c^2 . \qquad (32)$$

B) For the variables E (energy), $\nu$ (frequency), $\lambda$ (wavelength), $\varphi$ (phase velocity) and g (group velocity).

$$E = h\nu \quad ; \quad \varphi = \lambda\nu \quad ; \quad g\,\varphi = c^2 = \lambda^2(\nu^2 - \nu_o^2)$$
$$\varphi^2 / c^2 = c^2 / g^2 = \nu^2/(\nu^2 - \nu_o^2) = 1 + (\nu_o^2 \lambda^2 / c^2) . \qquad (33)$$

## 9. The modification of Maxwell equations of electromagnetic fields

These very famous Maxwell's equations are:

$$\text{div}\, \mathbf{E} = \rho/\varepsilon_o \; ; \quad \text{div}\, \mathbf{B} = 0 \; ; \quad \text{curl}\, \mathbf{E} = -\,\partial\mathbf{B}/\partial t \; ; \quad \text{curl}\, \mathbf{B} = \mu_o\, [\, \mathbf{j} + \varepsilon_o\, \partial\mathbf{E}/\partial t\, ]$$
$$\text{div}\, \mathbf{j} = -\,\partial\rho/\partial t \; ; \quad \mathbf{E} = -\text{grad}\, V - \partial\mathbf{A}/\partial t ; \quad \mathbf{B} = \text{curl}\, \mathbf{A} \qquad (34)$$

with as usual the vectors in bold character and with :

**E** = electric field (in volts per meter)
**B** = magnetic induction (in teslas)
t = time (in seconds)
$\rho$ = density of charges (in coulombs per cubic meter)
**j** = density of electric current (in amperes per square meter)
V = scalar potential ( in volts)
**A** = vector potential (in volt-second per meter)

and for the constants :

$\mu_o$ = magnetic permeability of vacuum = $4\pi \times 10^{-7}$ henry per meter
$\varepsilon_o$ = permittivity of vacuum = $8.854\,187\,818 \times 10^{-12}$ farad per meter
$\mu_o\, \varepsilon_o\, c^2 = 1$

The simplest solutions are the plane waves in vacuum ($\rho = 0$; **j** = 0 ), for instance those propagating into the Ox direction:

$$u = x - ct\ ; \quad \text{with} \quad c = [\mu_o\, \varepsilon_o\,]^{-0.5} = 299\,792\,458 \text{ meter per second}$$
$$\mathbf{E} = [\,0\ ;\ cF(u)\ ;\ cG(u)\,]\ ; \qquad \mathbf{B} = [\,0\ ;\ -G(u)\ ;\ F(u)\,] \qquad (35)$$
F(u) and G(u) are two arbitrary continuously differentiable functions

The Maxwell's waves propagate with the limit velocity c, while the photons of above sections have the smaller group velocity g and their waves have the larger phase velocity $\varphi$ that is related to g , to the wavelength $\lambda$ and to the frequency $\nu$ by the relations (33); hence we have to look for a small modification.

Let us consider such a sinusoidal wave as in (1):

$$F(x, t) = A\ \cos\{2\pi\,[\nu t - (x/\lambda) + f_o]\} \qquad (36)$$

and notice that, for any $\nu$ and $\lambda$ satisfying (33), F(x, t) is a solution of the following differential equation :

$$(\,\partial^2 F/\,\partial t^2\,) + \omega_o^2\, F = c^2\, \partial^2 F/\,\partial x^2\ , \qquad (37)$$

where $\omega_o$ is the "proper pulsation of photons" :

$$\omega_o = 2\pi\nu_o = 2\pi\, m_o c^2 / h = m_o c^2 / \hbar \qquad (38)$$

with, as usual:

$$\hbar = h / 2\pi = \text{reduced Planck's constant} = 1.054\,592 \times 10^{-34}\ \text{kg.m}^2.\text{s}^{-1}\ . \qquad (39)$$

On the other hand, in vacuum, the Maxwell equations (34) imply the well known waves equations:

$$\partial^2\mathbf{E}/\,\partial t^2 = c^2\,\Delta\mathbf{E}\ ; \qquad \partial^2\mathbf{B}/\,\partial t^2 = c^2\,\Delta\mathbf{B}\ , \qquad (40)$$

where the notation $\Delta$ is the usual Laplacian notation:

$$\Delta U = \partial^2 U/\,\partial x^2 + \partial^2 U/\,\partial y^2 + \partial^2 U/\,\partial z^2\ . \qquad (41)$$

Then, with (37)-(41), we are inclined to think that in vacuum, in our hypotheses, the electric field **E** and the magnetic induction **B** must obey the following generalization of Maxwell equations:

$$(\,\partial^2\mathbf{E}/\,\partial t^2\,) + \omega_o^2\,\mathbf{E} = c^2\,\Delta\mathbf{E}\ ; \qquad (\,\partial^2\mathbf{B}/\,\partial t^2\,) + \omega_o^2\,\mathbf{B} = c^2\,\Delta\mathbf{B}. \qquad (42)$$

Let us consider now a force **F** applied on a point of rest mass $M_o$ and velocity **W**, the usual Poincaré generalization of the law of inertia is:

$$\mathbf{F} = M_o\,.\,d\,(\Gamma\,\mathbf{W})/dt\ ; \qquad \Gamma = [\,1 - (\,W^2 / c^2\,)]^{-0.5}\ , \qquad (43)$$

which leads to usual non-uniform motions. However, if we consider the motion of that same point as seen in another inertial referential, for instance the referential of velocity v along the x-axis, as used in figures 1 and 2, we find that the rest mass $M_o$ has a corresponding non uniform motion of variable velocity $\mathbf{W}_1$ and is now moved by the force $\mathbf{F}_1$ that is related to the force **F** by:

$$\mathbf{F}_1 = \mathbf{F}_{1E} + \mathbf{W}_1 \times \mathbf{F}_{1B} \qquad (44)$$

with:

$$F_{1Ex} = F_x\ ; \qquad F_{1Bx} = 0\ ; \qquad \gamma = [\,1 - (\,v^2 / c^2\,)]^{-0.5}$$
$$F_{1Ey} = \gamma\, F_y\ ; \qquad F_{1By} = \gamma v\, F_z / c^2$$
$$F_{1Ez} = \gamma\, F_z\ ; \qquad F_{1Bz} = -\gamma v\, F_y / c^2 \qquad (45)$$

This famous Poincaré result is of course related to the expression of the Lorentz electromagnetic force on a charge q and velocity **V** :

$$\mathbf{F} = q\,(\,\mathbf{E} + \mathbf{V} \times \mathbf{B}\,)\ ; \qquad \mathbf{F}_1 = q\,(\,\mathbf{E}_1 + \mathbf{V}_1 \times \mathbf{B}_1\,) \qquad (46)$$

with the following Poincaré generalization of (45) :

$$E_{1x} = E_x\ ; \qquad B_{1x} = B_x$$
$$E_{1y} = \gamma E_y - \gamma v B_z\ ; \qquad B_{1y} = \gamma B_y + \gamma v\, E_z / c^2$$
$$E_{1z} = \gamma E_z + \gamma v B_z\ ; \qquad B_{1z} = \gamma B_z - \gamma v\, E_y / c^2 \qquad (47)$$

As usual we will adopt the Lorentz expressions (46) as definition of the electric field **E** and the magnetic induction **B** and the relations (46)–(47) will remain valid for photons of non-zero rest mass, since they are a direct consequence of the expression (43) and the Lorentz transformation.

Let us consider now a photon and its corresponding wave **in the referential in which the photon is at rest.** Our main hypothesis is that in these conditions, in vacuum, all photons are identical. A simple hypothesis, corresponding to the assumption of Ritz (ref. 5), is that then the magnetic induction **B** vanish and thus, with suitable fixed vectors **E** our sinusoidal stationary waves are:

$$\mathbf{E} = \mathsf{E} \exp(2\pi i \nu_o t) ; \qquad \mathbf{B} = 0. \qquad (48)$$

The other sinusoidal waves can be obtained by the transformation (47), with its corresponding Lorentz transformation, and by the similar transformations leading to arbitrary inertial referentials.

Let us consider a stationary wave as (48) in the referential $Ox_1y_1z_1$ of figures 1 and 2:

$$\mathbf{E_1} = \mathsf{E} \exp(2\pi i \nu_o t_1) ; \qquad \mathbf{B_1} = 0 . \qquad (49)$$

Since in the Lorentz transformation :

$$t_1 = \gamma [t - (vx / c^2)] \qquad (50)$$

we will obtain:

$$\exp(2\pi i \nu_o t_1) = \exp\{2\pi i \gamma \nu_o [t - (vx/c^2)]\} = \exp\{2\pi i [\nu t - (x / \lambda)]\}, \qquad (51)$$

hence in the Oxyz referential the frequency of the wave will be $\nu = \gamma \nu_o$ and its wavelength will be $\lambda = c^2 / \gamma \nu_o v$ . Notice that $\nu$ and $\lambda$ agree with the relation (33): $c^2 = \lambda^2 ( \nu^2 - \nu_o^2 )$ .

Starting from (49), the reverse of the transformation (47) gives then:

$$\left.\begin{aligned} \mathbf{E} &= [ \mathsf{E}_x , \gamma \mathsf{E}_y , \gamma \mathsf{E}_z ] . \exp\{2\pi i [\nu t - (x / \lambda)]\} \\ \mathbf{B} &= (v / c^2) [ 0, -\gamma \mathsf{E}_z , \gamma \mathsf{E}_y ] . \exp\{2\pi i [\nu t - (x / \lambda)]\} . \end{aligned}\right| \qquad (52)$$

We can generalize this to all directions of space and obtain the arbitrary sinusoidal electromagnetic wave:

$\mathbf{E} = \varepsilon \exp\{2\pi i [\nu t - (\mathbf{r.u} / \lambda)]\}$ ; $\mathbf{B} = (\mathbf{u} \times \varepsilon / \lambda\nu ) \exp\{2\pi i [\nu t - (\mathbf{r.u} / \lambda)]\}$

**u** is an arbitrary fixed unit vector ; **r** = (x, y, z) is the usual radius vector

$\varepsilon = [ \mathsf{E}_x , \gamma \mathsf{E}_y , \gamma \mathsf{E}_z ]$ is an arbitrary fixed vector

$\lambda$ and $\nu$ are related by (33) that is by: $c^2 = \lambda^2 ( \nu^2 - \nu_o^2 )$ . (53)

We can notice that these waves satisfy the conditions (42) and we can now consider that the general electromagnetic field in vacuum is a linear composition of these basic sinusoidal waves.

Two basic linear equations can be immediately verified for all solutions of (53), and thus for their linear compositions and for the general electromagnetic field:

$$\text{div}\, \mathbf{B} = 0 ; \qquad \text{curl}\, \mathbf{E} = - \partial \mathbf{B} / \partial t . \qquad (54)$$

These two equations already appear among the Maxwell's equations (34), but of course they are insufficient.

In order to go further, we will need the usual "scalar potential" V and "vector potential" **A** related to **E** and **B** by:

$$\mathbf{E} = - \text{grad}\, V - \partial \mathbf{A} / \partial t ; \qquad \mathbf{B} = \text{curl}\, \mathbf{A} . \qquad (55)$$

The two equations (54) are obvious consequences of (55), but notice that (55) is insufficient for the definition of V and **A** : we have there a small choice .

<u>We will choose V and **A** such that in (48) for stationary waves:</u>

$$\left.\begin{aligned} &\mathbf{E} = \mathsf{E} \exp(2\pi i \nu_o t) = - \partial \mathbf{A} / \partial t ; \qquad \mathbf{B} = 0 = \text{curl}\, \mathbf{A} \\ &V = 0 ; \qquad \mathbf{A} = ( \mathsf{E}\, i / 2\pi\nu_o ) \exp(2\pi i \nu_o t) \end{aligned}\right| \qquad (56)$$

For the other waves, Henri Poincaré noticed in his Palermo memoir (ref. 2) that the set (**A**, V/c) is a "quadrivector", the transformation of which, in a Lorentz transformation, is identical to that of the quadrivector (x, y, z, ct). He also gave a long and impressive list of quadrivectors with all the same type of transformation (for the choice c = 1): the force by unit of volume and the work by unit of time, the current and the charge by unit of volume, the momentum and the energy, etc. These impressive results don't appear in Einstein paper and it is obvious that Henri Poincaré is the real founder of Relativity and that he perfectly understood the subject (for an updated presentation of this Palermo Memoir see the three references 19 in Russian, English and French).

For the general sinusoidal wave (53) the scalar potential V and the vector potential **A** are then the following:

$$\mathbf{E} = \varepsilon \ \exp\{2\pi i\,[\nu t - (\mathbf{r.u} / \lambda)]\}\ ; \quad \mathbf{B} = (\mathbf{u}\times\varepsilon / \lambda\nu)\exp\{2\pi i\,[\nu t - (\mathbf{r.u} / \lambda)]\}$$
$$V = \mathbf{E.u}\ i\, c^2 / 2\pi\,\lambda\,\nu_o^2 ; \quad \mathbf{A} = (i / 2\pi\nu)\,\{\,\mathbf{E} + [(\nu^2 - \nu_o^2)\,\mathbf{u}\,(\mathbf{E.u}) / \nu_o^2\,]\}. \qquad (57)$$

We can now express the modified Maxwell's equations in vacuum that are verified by all waves (57) and, under our assumptions, by all electromagnetic fields.

Let us write first the four equations (54), (55) that were already given by Walter Ritz (ref. 5), they can be called "Ritz equations" and are identical to those of Maxwell:

$$\mathrm{div}\,\mathbf{B} = 0\ ; \quad \mathrm{curl}\,\mathbf{E} = -\,\partial\mathbf{B} / \partial t\ ; \quad \mathbf{E} = -\,\mathrm{grad}\,V - \partial\mathbf{A} / \partial t\ ; \quad \mathbf{B} = \mathrm{curl}\,\mathbf{A}\,. \qquad (58)$$

The two remaining equations are the following:

$$\partial\mathbf{E} / \partial t = c^2\,\mathrm{curl}\,\mathbf{B} + \omega_o^2\,A\ ; \qquad c^2\,\mathrm{div}\,\mathbf{E} + \omega_o^2\,V = 0\,. \qquad (59)$$

Notice that these equations become identical to those of Maxwell in the case when the photon rest mass $m_o$ is zero (that is when the proper pulsation $\omega_o$ is zero). Also notice the three following properties:

I ) In the case when the photon rest mass $m_o$ is not zero, the scalar potential V and the vector potential **A** are no more mathematical commodities ; with the definition given in (56) and the equations (59) they become physical quantitities.

II ) With the two equations of (59), the elimination of **E** leads to**:**

$$c^2\,\mathrm{div}\,\mathbf{A} + \partial V / \partial t = 0\,. \qquad (60)$$

This equation is known as "Lorentz gauge", it can also be used for suitable **A** and V, if the photon rest mass is zero.

III ) A consequence of the modified Maxwell's equations in vacuum (58)–(60) is a general property similar to (42):

$$(\,\partial^2 f / \partial t^2\,) + \omega_o^2\,f = c^2\,\Delta f\,, \qquad (61)$$

the scalar f being either the scalar potential V or any component of **E**, **B** or **A**.

It would be useful to extend these results out of the vacuum, i.e. when ρ and **j** are not zero, but certainly the "Ritz equations" given in (58) remain the same, as well as the equation div **j** = $-\,\partial\rho / \partial t$ , the classical definition equation relating ρ and **j**. The two remaining equations will become certainly be very near to a mixture of (34) and (59), that is:

$$(\mathbf{j}/\varepsilon_o) + \partial\mathbf{E} / \partial t = c^2\,\mathrm{curl}\,\mathbf{B} + \omega_o^2\,A; \qquad c^2\,\mathrm{div}\,\mathbf{E} + \omega_o^2\,V = c^2\,\rho/\varepsilon_o\,. \qquad (61.5)$$

## 10. The plane waves

When f is only function of x and t, the general solution of (61) is simple; let us put:

$$u = 0.5\,[\,t - (x/c)\,]\ ; \qquad v = 0.5\,[\,t + (x/c)\,]. \qquad (62)$$

The linear second order differential equation (59) becomes:

$$f(x,t) = f(u,v) \, ; \qquad (\partial^2 f / \partial u \, \partial v) + \omega_o^2 f = 0 \, . \tag{63}$$

In the Maxwell case, i.e. when $\omega_o = 0$, the solution is of course $f = \phi(u) + \psi(v)$, with $\phi(u)$ and $\psi(v)$ two arbitrary functions of class $C_2$ , as already used in (32). In the general case we obtain:

$$f = \phi(u) + \psi(v) + \int_0^u \phi(\alpha)\, d\alpha \sum_{k=1}^{\infty} [(-v\,\omega_o^2)^k (u-\alpha)^{k-1} / k!\,(k-1)!] +$$
$$+ \int_0^v \psi(\beta)\, d\beta \sum_{k=1}^{\infty} [(-u\,\omega_o^2)^k (v-\beta)^{k-1} / k!\,(k-1)!] \, . \tag{64}$$

We will use 0! = 1, as usual, and for all functions $\phi(u)$ and $\psi(v)$ of class $C_2$, the expression (64) is always converging.

Notice that if $\phi(u)$ is zero for all negative or zero u and $\psi(v)$ is zero for all positive or zero v, then f will remain zero for all negative or zero u and positive or zero v , that is for $x \geq c|t$: the electromagnetic field don't expand faster than the limit velocity c.

## 11. Expression of the electromagnetic energy

The usual Maxwellian expression of the energy W of an electromagnetic field at a given time in a given volume $\Omega$ is the following:

$$W = (\varepsilon_o / 2).\int_\Omega (E^2 + c^2 B^2)\, d\Omega \, . \tag{65}$$

For photons with a non-zero rest mass we will of course obtain a neighboring expression.

Let us consider in the Oxyz space a bounded domain B in which, at some given instant t, the electromagnetic parameters **E**, **B**, V, **A** are arbitrary and out of which they are zero, and let us consider a wider domain $\Omega$ out of which the electromagnetic field will remain zero for some time; hence in $\Omega$ the energy W will remain constant during that duration.

We can now study the time variation of the expression given in (63) and of some similar expressions.

$$d\left[\int_\Omega E^2 \, d\Omega\right]/dt = 2\int_\Omega [\mathbf{E}.\,\partial\mathbf{E}/\partial t]\, d\Omega = 2\int_\Omega [\mathbf{E}.\,(c^2 \,\text{curl}\,\mathbf{B} + \omega_o^2 A)]\, d\Omega$$
$$d\left[\int_\Omega B^2 \, d\Omega\right]/dt = 2\int_\Omega [\mathbf{B}.\,\partial\mathbf{B}/\partial t]\, d\Omega = -\,2\int_\Omega [\mathbf{B}.\,\text{curl}\,\mathbf{E}]\, d\Omega$$
$$d\left[\int_\Omega V^2 \, d\Omega\right]/dt = 2\int_\Omega [V.\,\partial V/\partial t]\, d\Omega = -\,2\int_\Omega [V.\,c^2\,\text{div}\,\mathbf{A}]\, d\Omega$$
$$d\left[\int_\Omega A^2 \, d\Omega\right]/dt = 2\int_\Omega [\mathbf{A}.\,\partial\mathbf{A}/\partial t]\, d\Omega = -\,2\int_\Omega [\mathbf{A}.\,(\mathbf{E} + \text{grad}\,V]\, d\Omega \tag{66}$$

and hence:

$$d\left\{\int_\Omega [E^2 + c^2 B^2 + \omega_o^2 A^2 + (\omega_o^2 V^2 / c^2)]\, d\Omega\right\}/dt =$$
$$= 2\int_\Omega [c^2 (\mathbf{E}.\text{curl}\,\mathbf{B} - \mathbf{B}.\,\text{curl}\,\mathbf{E}) - \omega_o^2 (\mathbf{A}.\text{grad}V + V.\text{div}\,\mathbf{A})]\, d\Omega =$$
$$= 2\int_\Omega \{\partial[c^2(E_z B_y - E_y B_z) - \omega_o^2 A_x V]/\partial x + \text{similar } \partial/\partial y \text{ and } \partial/\partial z \text{ terms}\}\, d\Omega = \mathbf{0}. \tag{67}$$

Then, the expression of the electromagnetic energy W at some given time t in the volume $\Omega$ is :

$$W = (\varepsilon_o / 2).\int_\Omega [E^2 + c^2 B^2 + \omega_o^2 A^2 + (\omega_o^2 V^2 / c^2)]\, d\Omega \, , \tag{68}$$

that indeed gives (65) when $\omega_o = \mathbf{0}$.

## 12. Expression of the electromagnetic momentum

The usual Maxwellian expression of the electromagnetic momentum **Q** at some given time t and in a given volume Ω is:

$$\mathbf{Q} = \varepsilon_o \int_\Omega \mathbf{E} \times \mathbf{B} \; d\Omega \qquad (69)$$

and, if the photon has a non-zero rest mass, a study similar to that of the previous section leads to the expression generalizing the Maxwellian one:

$$\mathbf{Q} = \varepsilon_o \int_\Omega [\mathbf{E} \times \mathbf{B} + (\omega_o^2 \, V \, \mathbf{A} / c^2 )] \, d\Omega \, . \qquad (70)$$

It is then possible to verify that for a bounded and isolated set of electromagnetic waves, with then a bounded total energy and a bounded total momentum, the energy and the momentum are a "quadrivector", the transformation of which are identical the Lorentz transformations of the space-time quadrivector x, y, z, t when the referential is modified.

## 13. Test observations and experiments

The first idea is of course to measure the velocity of light for some photon with a large wavelength. However, even for very large radio wavelengths, that velocity remains extremely close to the limit velocity c because in any case the rest mass of photons is very small.

Hence, we have to go to the most extreme limit, either with the possibilities and the accuracy of space experiments or with the exceptional opportunities given by the astronomical observations.

A possible space experiment would be the following: Consider two space probes A and B in the interplanetary space, separated by a very large distance D. A kilometric waves and its harmonic of rank two or three are send from A to B. The harmonic has a shorter wavelength and goes faster than the main wave. The velocity difference can be appreciated by the figure of the signal at reception and its comparison with its figure at emission. The effect is proportional to the product $Dm_o^2\lambda^2$, which emphasize the interest of long waves.

The accuracy of the measure of $m_o$ can thus reach $10^{-49}$ or even $10^{-50}$ kg. This accuracy can perhaps be greatly improved if very accurate transponders allow to use the signal back and forth many times.

Notice however that this experiment requires an accurate analysis of the interplanetary medium, since the presence of electronic plasmas modify the transmission, as we are going to see for the observation of pulsars.

The pulsars are certainly very promising astronomical objects for our analysis: they use radio waves and have well identified "pulses" that can be analyzed at very various and very large wavelengths.

The Crab Nebula pulsar was discovered almost thirty years ago and it has been observed regularly in many wave bands. The observations (ref. 22) show successive more or less regular "pulses" and the observers noticed a delay between the energetic waves (gamma, X, ultraviolet, visible, infrared) and the radio waves. They give two possible explanations: either a phase difference or the possibility that radio waves originate from a place about 100 km closer to the center of the neutron star... which seems difficult in a so small star.

However, we must understand the difficulty of the analysis. The presence of electronic plasmas in interstellar space gives effects very similar to those of a non-zero rest mass of photons (ref. 20) and we cannot go there for an in situ analysis of these plasmas.

For direct propagation of electromagnetic waves a rest mass $m_o$ for photons has effects similar to an electronic density N of interstellar plasma if :

$$m_o^2 = N \hbar^2 e^2 / c^4 \varepsilon_o m_e \qquad (71)$$

with: e = charge of the electron = $-1.602\ 19 \times 10^{-19}$ coulomb,
$m_e$ = mass of the electron = $9.109\ 56 \times 10^{-31}$ kg ,

that gives: N (number of electrons per cubic meter) = $2.282 \times 10^{98} (m_o/\text{kg})^2$ . (72)

The ANTF pulsar catalogue (ref. 18), gives informations about the 1510 known pulsars (directions, distances, dispersion coefficient related to the presence of electronic plasmas, etc). These pulsars appear into all right ascensions and from 83° North to 83° South (but with a greater density in the vicinity of the galactic center). Their distances goes from 130 parsecs to 57 000 parsecs.

Almost all these pulsars belong to our galaxy, but five of them are in the large Magellanic cloud and the most distant is in the small Magellanic cloud (the pulsar J0045−7319).

The huge majority of the pulsars of the catalogue show that, in their directions, the Galaxy has a rather uniform electronic plasma, with an electronic density between 20 000 and 36 000 electrons per cubic meter – and then also the same density of positive ions. But the six pulsars in the Magellanic clouds indicate a much smaller density in the intergalactic space (which is natural : if the electronic plasma had everywhere the density it has in the galaxy it would represent 98% of the mass of Universe... ). If $m_o = 0$, this intergalactic density is already below 2000 electrons per cubic meter and, with (70), this forbid $m_o$ to be larger than $[\ 2000 / 2.282 \times 10^{98}\ ]^{1/2}$ kg , that is $3 \times 10^{-48}$ kg.

The upper limit given by Ryutov (ref. 13) through the analysis of the solar wind is even better: $m_o \leq 10^{-52}$ kg.

When it will become possible to observe pulsars in the outer galaxies it will become more easy to separate the effects of the electronic plasmas and those of a non-zero rest mass for photons, especially if pulsars at very different distances are almost in the same direction.

Another possibility is the analysis of the radio emission of a very rapidly rotating double system: the radio waves have not only a delay with respect to the energetic waves, but they have also a slight distortion : they go faster when their emitter has a negative radial velocity than when it has a positive radial velocity. This difference is related to the Doppler-Fizeau effect and to the difference of frequencies. However only very fast double systems, with an orbital velocity above 1000 km/s , will show measurable effects.

With all this information, if we consider the proper period $P_o$ of photons presented in section 8, the best candidate for that period is probably the famous Kotov period of 2h 40mn 0.6s that appears in so many astrophysical phenomena (ref. 21). The rest mass $m_o$ of photons would then be: $7.68 \times 10^{-55}$ kg.

## 14. Interaction photon-photon

The famous Compton effect is the result of the interaction (or the collision) electron-photon. It has been considered as the proof of the existence of photons.

Similarly we can consider the collision of two photons in the two following referentials:

A) Collision at $t_1 = 0$, at the origin in the referential of the center of mass.

The two photons have opposite velocities before and also after the collision, and (if we choose the plane of motion as $O_1x_1y_1$ plane), we will obtain:

I) For the first photon $x_1 = g_1 t_1 \cos a_1$ ; $y_1 = g_1 t_1 \sin a_1$ ; $z_1 = 0$.

II) For the second photon: at any time $x'_1 = -x_1$ ; $y'_1 = -y_1$ ; $z'_1 = -z_1$ (73)

The angle $a_1$ will have a given value for negative times and another value for positive times, but the velocity $g_1$ will be the same in both cases (conservation of the energy).

B) Collision at t = 0 , at the origin in the referential with velocity v with respect to the first referential.

We will of course use the Lorentz transformation (7), which leads to:

$$\gamma = [1 - (v^2/c^2)]^{-0.5}$$
$$x = \gamma t_1 (g_1 \cos a_1 + v) ; \quad y = g_1 t_1 \sin a_1 ; \quad z = 0; \quad t = \gamma t_1 [1 + (v g_1 \cos a_1 /c^2)]$$
$$x' = \gamma t_1 (v - g_1 \cos a_1) ; \; y' = - g_1 t_1 \sin a_1 ; \; z' = 0 ; \; t' = \gamma t_1 [1 - (v g_1 \cos a_1 /c^2)] . \quad (74)$$

From the results of the above sections 5 and 8 we deduce the energy E and E' in this second referential :

$$E = h\nu = m_o c^2 (c^2 + vg_1 \cos a_1) [(c^2 - g_1^2)(c^2 - v^2)]^{-1/2}$$
$$E' = h\nu' = m_o c^2 (c^2 - vg_1 \cos a_1) [(c^2 - g_1^2)(c^2 - v^2)]^{-1/2} \quad (75)$$

Notice that the sum E + E' is independent of the angle $a_1$: the total energy is conserved after the collision.

Let us assume now that the first photon is a usual energetic photon, while the second is a "slow photon" with a velocity less than 280 000 km/s. Even for metric radio waves, if we use the photon rest mass $m_o$ presented at the end of section 13, the ratio E / E' will be above $10^{12}$ , which implies:

$$1 - 2 . 10^{-12} < vg_1 \cos a_1 /c^2 < 1 \quad (76)$$

and then:

$$1 - 2 . 10^{-12} < v/c < 1 ; \quad 1 - 2 . 10^{-12} < g_1 /c < 1; \quad 1 - 2 . 10^{-12} < \cos a_1 < 1 \quad (77)$$
$$\gamma = [1 - (v^2/c^2)]^{-0.5} > 500\,000 . \quad (78)$$

Let us now consider a very rough model of photon-photon collision: nothing happens if the distance photon-photon remain larger than some ε and, for instance for some quantic reason, the deviation of the angle $a_1$ is always the fixed small quantity $\delta a_1$ (in an arbitrary direction) if the distance photon-photon has a minimum smaller than ε .

With (74), (77) and (78) a deviation $\delta a_1$ of the angle $a_1$ gives only a deviation $\delta a_1/2\gamma$ , that is less than $\delta a_1 /10^6$ of the direction of the first photon in the second referential.

In these conditions, if we assume that a usual fast photon meet in the average N slow photons on its way along a Megaparsec, we will obtain:

A) A decrease of its energy in the following ratio:

$$E\,(\text{final}) / E\,(\text{initial}) = [1 - (\delta a_1^2 / 4)]^N . \quad (79)$$

B) A deviation of the direction of its motion smaller than the angle β given by:

$$\beta = \sqrt{N}\, [\delta a_1 /10^6] . \quad (80)$$

For instance if we assume that the decrease of energy is precisely that given by the Hubble's law, we will get :

$$N (\delta a_1^2 / 4) = (70 \text{ km/s}) / (300\,000 \text{ km/s}) = 1/4300 \quad ; \quad \beta = 3 . 10^{-8} = 0.007'' . \quad (81)$$

The deviation β remains negligible. Let us recall that this deviation has been obtained for metric waves and it is much less for more energetic photons.

Hence, surprisingly, the study of photons with non-zero rest mass leads to an alternative to the usual model of expansion of galaxies. Of course there is a constraint: a massive majority of not yet discovered slow photons – their energy is so small – but let us recall that we already know that for each usual particle (electron, proton, neutron) there is more than one hundred million photons, almost all of them being "cold photons" of the "cosmic background radiation" at 2.7 K.

This new model has also two advantages: it allows to obtain the famous "missing mass" and it agrees with the anisotropy of the Hubble "constant" that is larger in the direction of concentration of matter.

## Conclusions

The analysis of the possibility of a non-zero rest mass for photons leads to coherent results and a small modification of Maxwell's equations of electromagnetic fields. However the likely smallness of that rest mass restrict its physical effects to radio waves, these effects remain negligible for energetic waves, and even for infrared waves.

Several possibilities of measure of that rest mass exist either by astronomical observations or by astronautical experimentation.

Notice that photons with non-zero rest mass have been considered for the possibilities of non cosmological redshifts (see a general synthesis in ref. 23). These redshifts, competing with that given by the classical Doppler-Fizeau effect of the expansion of intergalactic space, require both a non-zero rest mass of photons and the presence of a very large quantity of "slow photons" with a velocity smaller than 280 000 km/s.

These possibilities of considerable modifications of the cosmological perspectives worth a careful analysis. Some of them give an explanation of the "missing mass", they are in agreement with the invariability of the Kotov period (ref. 21), and with the anisotropy of Hubble constant that is larger in the directions of heavy concentrations of matter.

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