Physics Vol. 1, No. 1, pp. 1–2, 1964. Pergamon Press, Inc. Printed in the United States.

ON THE EXISTENCE OF MAGNETIC MONOPOLES*

R. A. FERRELL† and J. J. HOPFIELD
University of California, Berkeley, California

(*Received* 18 *March* 1964)

Abstract

By the Bohm-Aharonov effect, the flux within a long slender solenoid is observable unless it is a multiple of hc/e. *In this case only the ends of the solenoid produce any physical effect and they behave as magnetic monopoles with strength quantized according to Dirac.*

IN view of the current interest and search for magnetic monopoles [1], and because of the widespread feeling that the possibility of monopoles constitutes a "slot" in nature which is completely vacant, it seems worthwhile to point out that there does already exist one familiar, prosaic physical system whose properties approach those of a pair of monopoles; namely, a long slender solenoid. If the magnetic flux contained in the solenoid is ϕ, then there is at the two ends the same isotropic radial magnetic field which is associated with a magnetic pole of source strength $g = \phi/4\pi$. In the absence of quantum effects, a long solenoid has no influence on the field-free region outside it and in the limit of vanishing solenoid radius we are left only with the two equal and opposite magnetic monopoles of arbitrary separation and strength at the two ends of the solenoid.

But, in fact, wave mechanical diffraction effects in the field-free region surrounding the solenoid render the enclosed flux detectable, as clearly discussed by Bohm and Aharonov [2]. This is the case unless the flux is an integral multiple of the basic quantum hc/e, where h, c, and e are Planck's constant, the velocity of light, and the electron charge, respectively. Choosing the lowest non-trivial value gives $g = \phi/4\pi = hc/4\pi e$, or $eg/hc = \frac{1}{2}$, Dirac's quantization condition [3]. When this condition is satisfied the solenoid becomes unobservable in the limit of vanishing radius and can be identified with Dirac's string singularity [4].

For the purpose of making this result completely rigorous, we suppose that all attempts to detect the infinitesimally thin solenoid are to be carried out by scattering particles which obey the Schrödinger wave equation. To prevent the particles from entering the interior of the solenoid, we surround it by an impenetrable cylinder of radius a. This corresponds to Dirac's condition [5], "a string must never pass through a charged particle." As the cross section for the scattering of free particles by a cylindrical hard core of radius a is proportional to $(\ln a)^{-2}$, the solenoid becomes undetectable in the limit $a \longrightarrow 0$.

In summary, the purpose of this note has been to exhibit a concrete example of a pair of Dirac magnetic monopoles. This is provided (at least in concept, if not in actuality) by a solenoid satisfying Dirac's quantization condition and vanishing in radius. If in this limit of a string singularity [6], the unobservable magnetic flux within the solenoid is omitted from the physically observable magnetic field $B(r)$, the latter is no longer divergence-free. Instead, it satisfies the point-source divergence condition $\text{div}\, B(r) = 4\pi g\,\delta^3(r - r_0)$ in the vicinity of a monopole of strength g situated at r_0. (δ^3 is the three-dimensional delta-function.) Although the monopoles discussed here can hardly be regarded as elementary particles [7], they do exhibit the magnetic properties required by Dirac's theory. It remains a legitimate question whether or not these are the only examples of Dirac monopoles occurring in nature.

*Research supported in part by the National Science Foundation.
† Permanent address: University of Maryland, College Park, Maryland.

References

1. K. W. FORD, *Scientific American* **209**, 122 (1963); E. GOTO, H. HOLM and K. FORD, *Phys. Rev.* **132**, 387 (1963); PURCELL, COLLINS, FUJII, HORNBOSTEL and TURKOT, *Phys. Rev.* **129**, 2326 (1963); N. A. PORTER, *Nuovo Cim.* **16**, 958 (1960).
2. Y. AHARONOV and D. BOHM, *Phys. Rev.* **115**, 485 (1959). See also W. EHRENBERG and R. E. SIDAY, *Proc. Phys. Soc. (London)* B **62**, 8 (1949).
3. P. A. M. DIRAC, *Proc. Roy. Soc.* **1334**, 60 (1931).
4. P. A. M. DIRAC, *Phys. Rev.* **74**, 817 (1948).
5. Reference 4, p. 822.
6. For a reformulation of quantum electrodynamics in which singular potentials can be avoided in the description of Dirac monopoles, see N. CABIBBO and E. FERRARI, *Nuovo Cim.* **23**, 1147 (1962) and S. MANDELSTAM, *Ann. Phys.* **19**, 1 (1962).
7. Because of the diverging magnetic field energy within the solenoid, the mass of the monopoles would have to be taken as infinite.