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Electromagnetic Properties of the $I^\pi = 11/2^-$ isomers in the cadmium isotopes: What can we learn from them?

Y. Lei[1], S. Pittel[2] and H. Jiang[3,4]

[1] Key laboratory of neutron physics, Institute of Nuclear Physics and Chemistry, China Academy of Engineering Physics, Mianyang 621900, China
[2] Bartol Research Institute and Department of Physics and Astronomy, University of Delaware, Newark, Delaware 19716, USA
[3] School of Arts and Sciences, Shanghai Maritime University, Shanghai 201306, China
[4] Department of Physics, Shanghai Jiao Tong University, Shanghai 200240, China

E-mail: pittel@bartol.udel.edu

Abstract. The electromagnetic features of the $11/2^-$ isomers in the odd-mass Cd isotopes are shown to exhibit an an anomaly near $N = 70$. We report shell-model calculations of these isotopes aimed at describing these properties. We find a sudden phase change in the hexadecapole component of the wave functions precisely at $N = 70$, which gives rise to different linear relations between the Q and μ values before and after $N = 70$, as needed to reproduce the experimental data. The particle-hole transformation properties associated with a strong subshell closure at $N = 70$ followed by fairly close-lying neutron orbitals from $N = 70 - 82$ is suggested as a possible explanation for this phase change.

1. Introduction

A recent study of the $I^\pi = 11/2^-$ isomers in the Cd-isotope chain [1] indicates that the quadrupole and magnetic moments (denoted by Q and μ, respectively) both vary roughly as linear functions of neutron number. However, as was also noted, there is a noticeable difference in the linear relation for the μ values between $^{111-117}$Cd and $^{119-127}$Cd, separated by $N = 70$. Indeed, if one performs a linear fit to the quadrupole moments, as shown in fig. 1(a), a similar behavior is observed: the best-fit linear relation perfectly describes the quadrupol moments of $^{111-117}$Cd, while residues from the same fit obviously exceed experimental errors for $^{119-127}$Cd, as shown in fig. 1(b). Thus, the quadrupole moment data also exhibit a change in linear behavior before and after $N = 70$.

A density functional study [2] recently reported suggested that the Q linearity in the Cd isotopes has a different mechanism for $N < 70$ and $N > 70$. In this work, we study the mechanism for the behavior of both the quadrupole moments and magnetic moments at $N = 70$ in the framework of a simple shell-model treatment, to see whether we can explain the data and in doing so obtain new insight on the structural properties of these isotopes. Further details can be found in [3] where the results of this study were first presented.

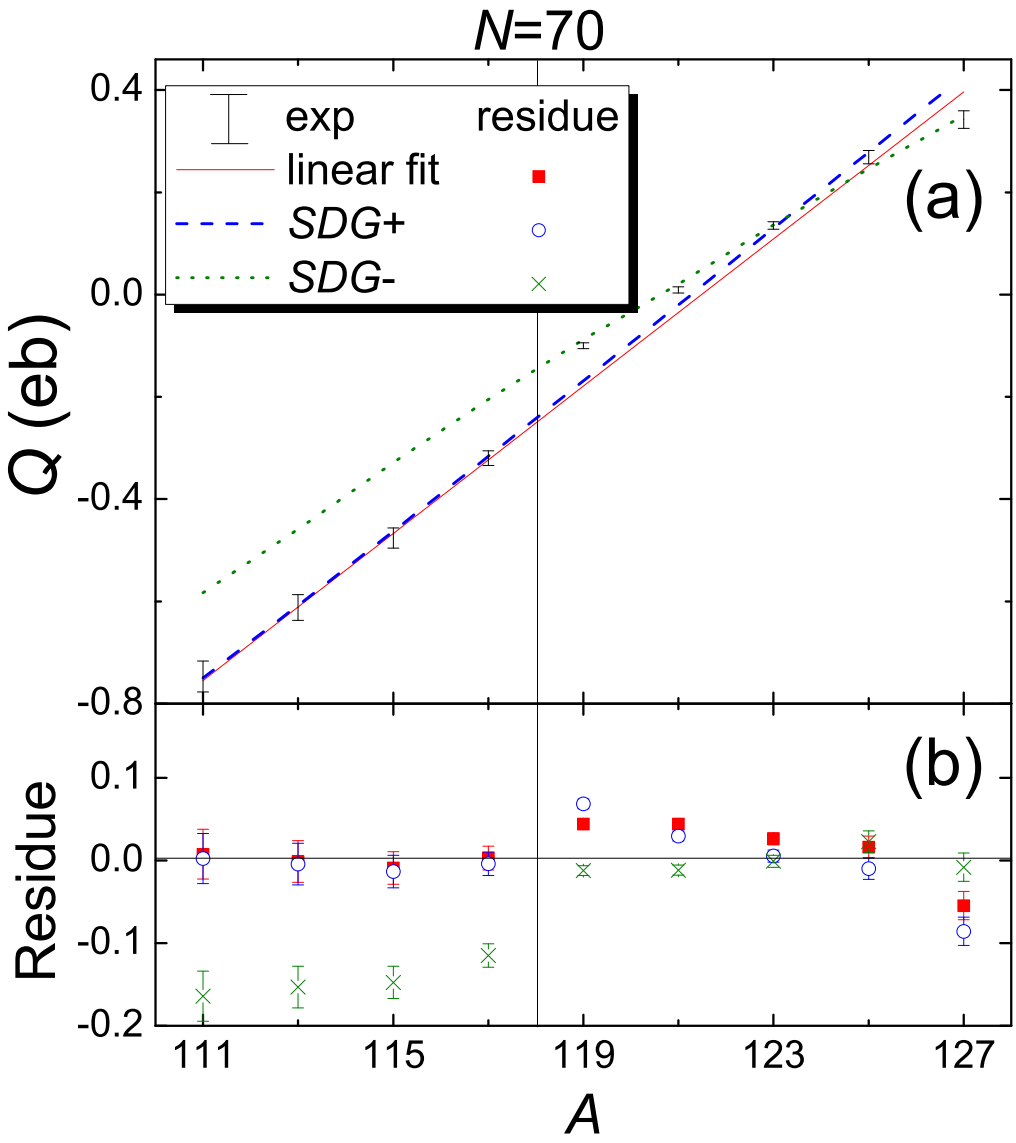

Figure 1. Quadrupole moments of the $11/2^-$ isomers in the Cd isotopes. In panel (a), we compare experimental values [1], linear fitting results, and results from the $SDG+$ and $SDG-$ calculation. Panel (b) presents residues of the linear fit. The $N = 70$ position is highlighted. For $N < 70$, the residues are smaller than the experimental errors, so that a linear relation follows. In contrast, the residues for $N > 70$ are larger than experimental errors, which means that the same linear behavior is not respected. The $SDG+$ and $SDG-$ results are consistent with the $N < 70$ and $N > 70$ experimental data, respectively, as discussed in the text.

2. Calculations and Analysis

We begin by outlining the basic ingredients of our calculations:

1 We first assume simple, but reasonable, neutron and proton configurations from which to build the $11/2^-$ isomers.

2 We then calculate the quadrupole and magnetic-dipole matrix elements for the above configurations.

3 We next consider several trial wave functions for the isomers and fit their respective amplitudes to optimally describe the quadrupole- and magnetic-moment data for the $11/2^-$

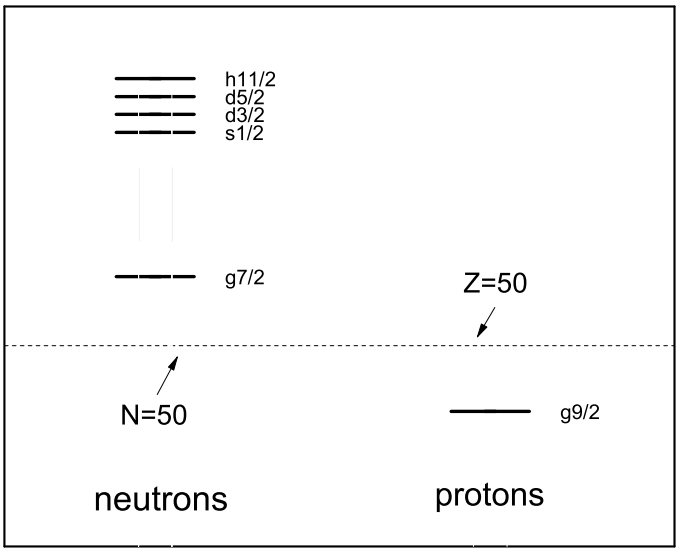

Figure 2. Schematic illustration of the single-particle levels of relevance to the Cd isotopes and their corresponding energies.

isomers.

As we will see, the procedure leads to a set of wave functions that nicely describe the evolution of the quadrupole and magnetic moments of the $11/2^-$ isomers, including the change in behavior at $N = 70$, and furthermore suggests a fairly simple picture as to why it occurs.

In fig. 2, we schematically show the relevant single-particle energy levels appropriate to our analysis of the $N = 48$ Cd isotopes. For protons, only the $g_{9/2}$ orbital (for holes) is relevant. For neutrons, the levels of the $N = 50 - 82$ shell are all in principle relevant. However, the $g_{7/2}$ orbital is fairly well isolated from the others, which are themselves fairly close in energy.

With that as background, we now describe the model space we use in our shell-model study. For the proton configuration, we assume that the two holes in the $Z = 50$ core both lie in the $g_{9/2}$ orbital. There are five such two-hole states, with total spins $J = 0, 2, 4, 6, 8$, which we denote $S_\pi = (g_{9/2} \otimes g_{9/2})^0$, $D_\pi = (g_{9/2} \otimes g_{9/2})^2$, $G_\pi = (g_{9/2} \otimes g_{9/2})^4$, $I_\pi = (g_{9/2} \otimes g_{9/2})^6$, and $K_\pi = (g_{9/2} \otimes g_{9/2})^8$, respectively.

Since the Cd isotopes are nearly spherical, we assume minimal proton-neutron configuration mixing. We thus assume, following [1], the simplest possible configuration for neutrons, a seniority-one configuration involving a single unpaired $h_{11/2}$ particle and the remaining particles paired to spin-zero and distributed over the close-lying levels shown in fig. 2. We denote this neutron configuration as ϕ_ν.

With the above assumptions, our basis space for treating the $11/2^-$ isomers consists of only five configurations: $\tilde{S} = \phi_\nu \otimes S_\pi$, $\tilde{D} = \phi_\nu \otimes D_\pi$, $\tilde{G} = \phi_\nu \otimes G_\pi$, $\tilde{I} = \phi_\nu \otimes I_\pi$, and $\tilde{K} = \phi_\nu \otimes K_\pi$.

The Q and μ matrix elements for the neutron configurations can be calculated straightforwardly based on the seniority-one nature of ϕ_ν, using the well-known seniority

reduction formulae [4] that apply for either a single orbit or a set of *degenerate* orbits, as assumed to approximately apply here. The proton Q and μ matrix elements of the $(g_{9/2})^{-2}$ configurations can likewise be calculated simply based on the same formalism.

We need to choose appropriate effective charges and Lande g factors for the analysis. The proton effective charge are taken from a fit to the experimental quadrupole moment of the 2_1^+ state of ^{112}Cd, which should have as predominantly D_π structure. Some minor modifications to this simple prescription are described in [3]. The usual Lande g factors are assumed. The neutron effective charge will be obtained from a fit to the quadrupole moments, to be described below.

Since S_π and D_π configurations dominate the low-lying states of the Cd isotopes, we first consider as a trial wave-function $C_S\tilde{S} + C_D\tilde{D}$, where C_S and C_D are wave function amplitudes. The corresponding probabilities C_S^2 and C_D^2 can be obtained from the magnetic moment data on the $11/2^-$ isomers. Using the values that emerge, we then determine the neutron effective charge e_ν so as to optimize the reproduction of the associated quadrupole moments. We consider both possibilities for the relative sign of C_D with respect to C_S, with the eventual choice determined by which yields results in closer accord to experiment. This is referred to as our "SD fit". We present the optimal e_ν and the root-mean-square deviation (RMSD) of the SD fit in the top row of Table 1. Both the resulting $e_\nu = 7.334$e and the RMSD=$0.456eb$ are much too large for this to be viewed as an acceptable fit. Thus we need to consider additional proton configuration.

Table 1. Fitting results with SD, SDG, SDI and SDK trial wave functions.

	e_ν (e)	RMSD (eb)
SD	7.334±2.110	0.456
SDG	1.496±0.221	0.009
SDI	3.079±6.479	0.018
SDK	3.073±10.44	0.018

We thus consider three other trial wave-functions: (a) $C_S\tilde{S}+C_D\tilde{D}+C_G\tilde{G}$, (b) $C_S\tilde{S}+C_D\tilde{D}+C_I\tilde{I}$ and (c) $C_S\tilde{S} + C_D\tilde{D} + C_K\tilde{K}$, which we denote "$SDG$, SDI and SDK", respectively. Here too C_G, C_I and C_K are wave function amplitudes to be determined analogously through optimization. The final best-fit values and the RMSD are also listed in Table 1. All three fits achieve reasonable levels of agreement, with RMSD values$\sim 0.01eb$. Therefore, in what follows we limit ourselves to these trial wave functions.

When comparing the various fits, we conclude that the SDG fit seems to be the best for two principal reasons:

- The SDG fit provides the smallest RMSD.
- The SDI and SDK fits both yield an unusually large $e_\nu \sim 3$e value, as did the SD fit discussed earlier.

Therefore, we only consider the SDG wave-function and their associated best-fit e_ν value in the analysis to follow.

For the SDG wave functions, the choice of the C_G phase relative to that of C_S can lead to different quadrupole moments. We classify the SDG wave functions according to the relative phase of C_G with respect to C_S (assumed positive), with $SDG+$ having $C_G > 0$ and and $SDG-$ having $C_G < 0$.

In fig. 1, the calculated quadrupole moments associated with the $SDG+$ and $SDG-$ wave functions are compared with the experimental quadrupole moments. The $SDG+$ results are

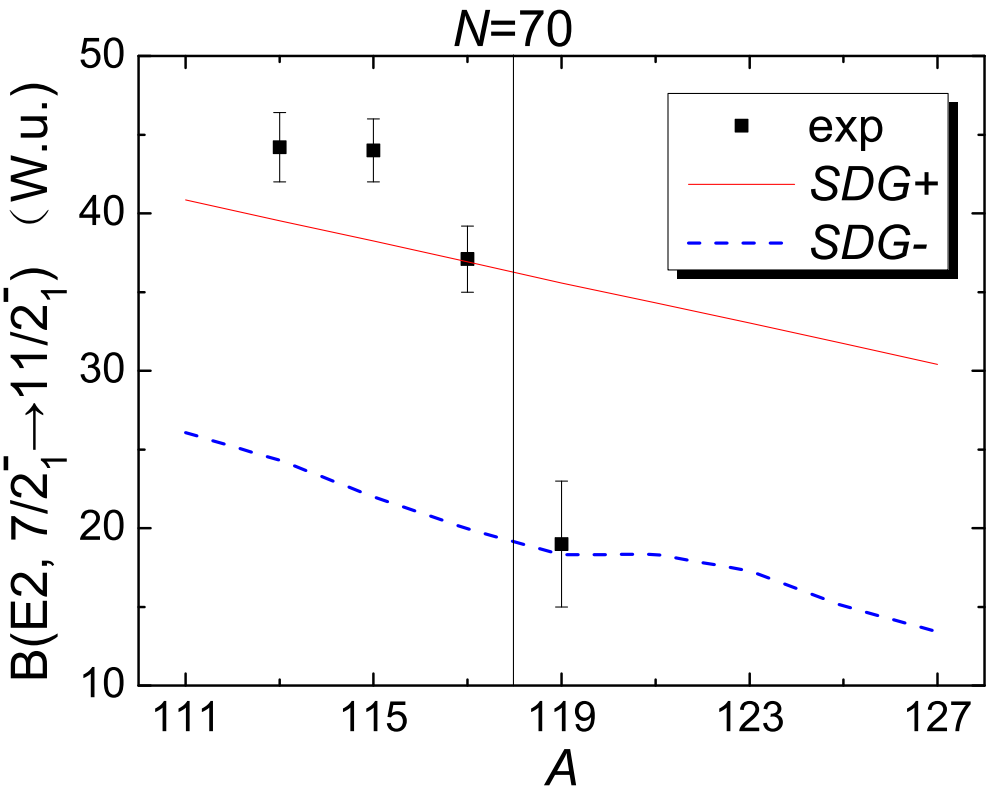

Figure 3. Calculated and experimental B(E2, $7/2_1^- \rightarrow 11/2_1^-$) values. Calculated results based on both the $SDG+$ and $SDG-$ wave functions for the $11/2^-$ isomers are presented. The experimental data (exp) is from ref. [5]. The sudden $N = 70$ phase change is highlighted.

compatible with the best-fit linear relation of the experimental quadrupole moments, and provide a better description for $N < 70$. In contrast, the $SDG-$ results exhibit some curvature beyond $N = 70$, as in the experimental data. From these results, we conclude that there is a C_G phase change across $N = 70$. This provides a plausible explanation for the change in Q linearity on the two sides of $N = 70$, as reflected in fig. 1.

The precise difference between the $SDG+$ and $SDG-$ quadrupole moments is observable but not very pronounced. It is desirable therefore to use other electromagnetic data to more clearly isolate the C_G phase in the various isotopes. In particular, we will consider the systematic strong B(E2, $7/2_1^- \rightarrow 11/2_1^-$) values in $^{113-119}$Cd [5], to see whether they can provide additional information. We will calculate these B(E2) values using both the $SDG+$ and $SDG-$ wave-functions obtained above, and carry out an isotope-by-isotope comparison between the experimental and calculated results.

As necessary input to the B(E2) calculation, we must first identify an appropriate wave function to use for the initial $7/2_1^-$ state. The strong E2 transition $7/2_1^- \rightarrow 11/2_1^-$ suggests a similarity between the $7/2_1^-$ state and the $11/2^-$ isomeric state We thus treat the $7/2_1^-$ state as a simple proton-neutron recoupling of the $11/2^-$ isomer, namely as $\phi_\nu \otimes D_\pi$ (this should be lower in energy than $\phi_\nu \otimes G_\pi$ and thus more important) *but coupled of course to* $J^\pi = 7/2^-$. Thus, in the spirit of the analysis to date, we assume a single configuration for the state. But of course we still need to see whether a good description of the data follows from such a simple choice.

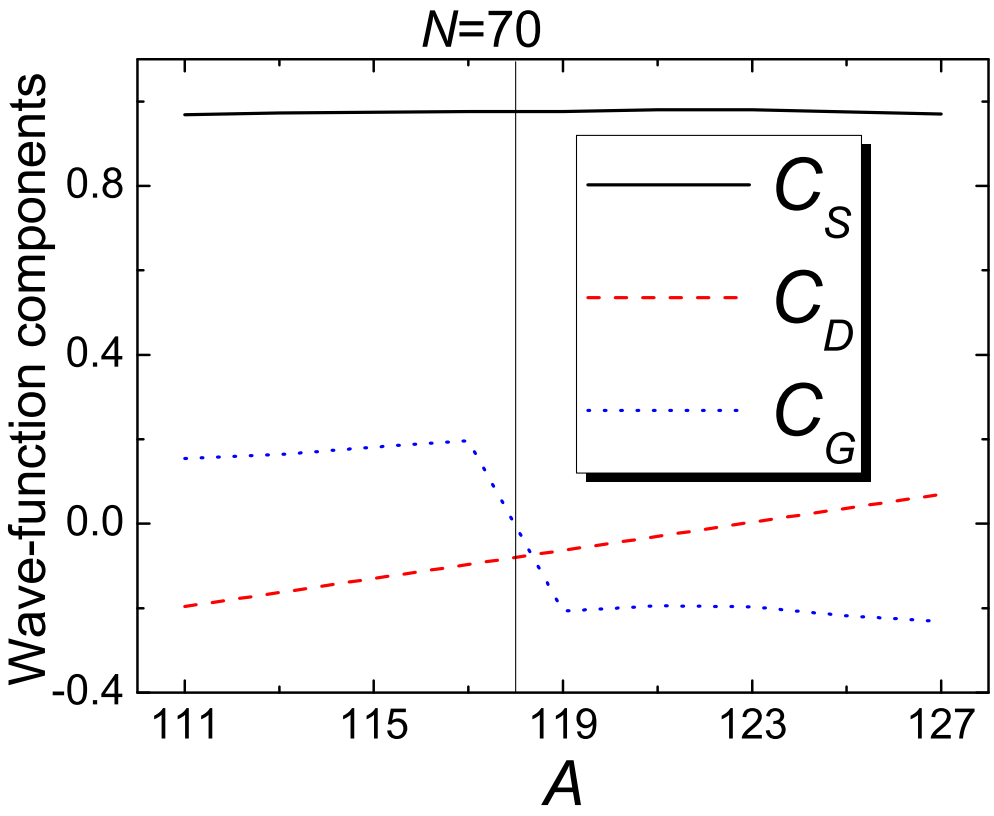

Figure 4. Wave functions from the final SDG fit to the quadrupole moments and B(E2) values. The $N = 70$ phase change in C_G is highlighted.

The calculated B(E2, $7/2_1^- \rightarrow 11/2_1^-$) values that emerge from both the $SDG+$ and $SDG-$ wave functions are compared with the available experimental data in fig. 3. This comparison confirms a sudden phase change of C_G at $N = 70$: the $N < 70$ B(E2) values clearly favor the $SDG+$ results ($C_G > 0$), whereas beyond $N = 70$, only the $SDG-$ calculation can fit the single experimental B(E2) data point. Based on these results, we summarize our final SDG wave functions from the Q-fitting procedure in fig. 4.

What we see from the figure is that the C_S value is close to 1 for all the Cd isotopes. Thus, the $11/2^-$ isomer is constructed with a dominant $h_{11/2}$ single-particle configuration, with perturbations arising from D_π and G_π excitations. The second feature to note is the rapid change of phase exhibited by C_G, precisely at $N = 70$. Lastly, note that that the D_π contributions also changes phase near $N = 70$, but it does so very smoothly.

How might we understand these results? In perturbation theory, changes in C_D and C_G relative to C_S reflect sign changes in the associated $p - n$ interaction coupling matrix elements. The relevant interactions correspond to scalar products of multipole operators in the $Q \cdot Q$ channel (responsible for C_D) and the $H \cdot H$ channel (responsible for C_G) . Whereas the proton reduced matrix elements are simple and do not evolve with neutron number, the neutron reduced matrix elements do evolve with neutron number, and must therefore be responsible for the sign change. As is well known [6], the matrix elements of an even multipole operator, in either a single orbit or a series of degenerate orbits, changes sign at mid-shell due to a change from particle-like to hole-like behavior.

As noted earlier, the neutron orbits from $N = 58-82$ (the $s_{1/2}$, $d_{3/2}$, $d_{5/2}$, and $h_{11/2}$) are fairly close in energy and well isolated from the lower $g_{7/2}$. Thus, they seem to provide the needed

well-isolated subshell. We would expect, therefore, that both the quadrupole and hexadecapole matrix elements change sign at mid-shell, which is precisely at $N = 70$. Since C_D evolves very smoothly with N, the effect of the sign change shows up most dramatically in C_G.

On the basis of these arguments, we conclude that the dramatic change in the sign of C_G seems to be a manifestation of a fairly isolated $N = 58 - 82$ subshell with relatively close-lying orbits and the particle-hole transformation properties that arise in such circumstances.

3. Summary

Summarizing, we have extracted shell-model wave functions of the $11/2^-$ isomers in the odd-mass $^{111-129}$Cd by fitting their electromagnetic features, including both electromagnetic moments and B(E2) values. A phase change in the hexadecapole component of the extracted wave functions explains the change in linear behavior of the quadrupole and magnetic moments for $N < 70$ and $N > 70$ as well as the BE2($7/2_1^- \rightarrow 11/2_1^-$) behavior. According to perturbation theory, this phase change at $N = 70$ is related to a change in sign of the hexadecapole proton-neutron matrix elements as we pass through $N = 70$, if we assume a strong subshell effect taking place for this neutron number.

Acknowledgments

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