A method to calibrate the energy of air showers with ultra-high energy photons

P. HOMOLA[1,2], M. RISSE[1]

[1] *University of Siegen, Siegen, Germany*
[2] *H. Niewodniczański Institute of Nuclear Physics PAN, Kraków, Poland*

homola@hep.physik.uni-siegen.de

Abstract: Calibrating the absolute energy scale of air showers initiated by ultra-high energy cosmic rays is an important experimental issue. Currently, the corresponding systematic uncertainty amounts to 15-20% using the fluorescence technique. Here we describe a new, independent method which can be applied if ultra-high energy photons are observed. While such photon-initiated showers have not yet been identified, the capabilities of present and future cosmic-ray detectors may allow their discovery. The method makes use of the geomagnetic conversion of an UHE photon (preshower effect), which significantly affects the subsequent longitudinal shower development. The conversion probability depends on the energy of the UHE photon and can be calculated accurately by QED. The comparison of the observed fraction of converted photon events to the expected one allows the determination of the absolute energy scale of the observed photon air showers and, thus, an energy calibration of the air shower experiment. We provide details of the method and estimate the accuracy that can be reached as a function of the number of observed photon showers.

Keywords: extensive air showers, geomagnetic cascading, gamma conversion, PRESHOWER, energy calibration.

1 Introduction

The primary energies of UHECR could be calibrated with well identified photon air showers taking into account the so-called preshower effect [1]. The method for the energy calibration proposed here requires, firstly, the detection of at least a few photon air showers at UHE where the conversion of a primary γ into an $e^{\pm}$ pair becomes important. And secondly, the experiment should provide shower observables sensitive to the primary type. The total probability of the UHE photon conversion, hereafter P_{conv}, is closely related to the primary photon energy E. In the following it is shown how this relation can be used to determine the energy scale for the observed showers.

2 P_{conv} vs. E

The number of pairs created by a high-energy photon in the presence of a magnetic field per path length dr can be expressed in terms of the attenuation coefficient $\alpha(\chi)$ [2]:

$$n_{pairs} = n_{photons}\{1-\exp[-\alpha(\chi)dr]\}, \qquad (1)$$

where

$$\alpha(\chi) = 0.5(\alpha_{em} m_e c/\hbar)(B_\perp/B_{cr})T(\chi) \qquad (2)$$

with α_{em} being the fine structure constant, $\chi \equiv 0.5(E/m_e c^2)(B_\perp/B_{cr})$, $B_\perp$ is the magnetic field component transverse to the direction of the photon motion, $B_{cr} \equiv m_e^2 c^3/e\hbar = 4.414 \times 10^{13}$ G and $T(\chi)$ is the magnetic pair production function. The approximations behind Eq. (2) are discussed in Ref. [2]. $T(\chi)$ can be well approximated by:

$$T(\chi) \cong 0.16\chi^{-1}K^2{}_{1/3}(\frac{2}{3\chi}), \qquad (3)$$

where $K_{1/3}$ is the modified Bessel function of order 1/3. For small or large arguments $T(\chi)$ can be approximated by

$$T(\chi) \cong \begin{cases} 0.46\exp(-\frac{4}{3\chi}), & \chi \ll 1; \\ 0.60\chi^{-1/3}, & \chi \gg 1. \end{cases} \qquad (4)$$

We use Eq. (1) to calculate the probability of γ conversion over a small path length dr:

$$p_{conv}(r) = 1-\exp[-\alpha(\chi(r))dr] \simeq \alpha(\chi(r))dr. \qquad (5)$$

The total probability of gamma conversion for a given primary energy and arrival direction can be found by calculating

$$P_{conv}(E,\theta_0,\phi_0) = 1-\prod_i(1-p_{conv}(r_i)) \qquad (6)$$

where i numbers the steps along the trajectory of the simulated primary γ. It can be checked that $T(\chi)$ is a strictly increasing function until its maximum around $\chi \simeq 6.5$ which corresponds to gamma energies up to 5×10^{20} eV and the magnetic field strengths up to 0.6 G. It implies that for a constant value of $B_\perp$ also $\alpha(\chi)$ is a strictly increasing function of the primary photon energy (see Eq. 2) for any terrestrial geomagnetic conditions and any realistic photon energies. A further conclusion is that also p_{conv}, through its dependence on $\alpha(\chi)$ (Eq. 5), and consequently P_{conv} (Eq. 6) are strictly increasing functions of E for any energies and geomagnetic field strengths of interest. The key idea here is that one can determine E provided P_{conv} can be measured. The energy calibration method presented in this paper is based on this idea.

Our calculations of gamma conversion probability were performed with program PRESHOWER [3] where a propagation step $dr = 10$ km is used. Such a length of the step was checked to be optimal for the numerical procedures

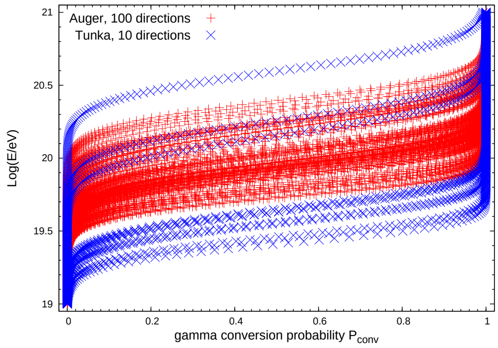

Fig. 1: Examples of E vs. P_{conv} relations for different locations: Auger - weak local B and Tunka - extremely strong local B. For each location the E vs. P_{conv} relations are plotted for random arrival directions (100 directions for Auger South, 10 directions for Tunka). The energies of the presented photon events were taken at random from the range $19.8 < \log(\mathrm{E/eV}) < 20.2$ from a power law with index 2.84.

used. In PRESHOWER the γ propagation starts at around 30000 km a.s.l. and ends at 112 km a.s.l.

Fig. 1 shows examples of the relations $E(P_{conv})$ for two different observatory locations and different arrival directions. The energies of the presented photon events were taken at random from the range $19.8 < \log(\mathrm{E/eV}) < 20.2$ and according to the spectrum with index 2.84. The strength of the local geomagnetic field is significantly different at the two sites: around 0.23 G at the location of the Pierre Auger Observatory [4] in Malargüe, Argentina, and around 0.58 G at the location of the Tunka experiment [5] in Russia, near the Baikal Lake. Apart from a shift in energy, the relations follow a very similar behaviour. We also note that the slope steepness $s \equiv \Delta \log(\mathrm{E/eV}) / \Delta P_{conv}$ determines the precision of finding E: decreasing s results in a decreasing uncertainty of E, provided the uncertainty of the P_{conv} measurement (ΔP_{conv}) is constant.

3 Energy calibration method

In an idealized case, let us first consider the observation of $n \gg 1$ photon events of same primary energy and arrival direction. Out of these, k are observed to be initiated by preshowers (primary γ converted), $0 \ll k \ll n$. We are interested in estimating P_{conv}. The probability of observing k preshower events out of n photon-induced air showers is given by a binomial probability distribution:

$$P(P_{conv}, n, k) = \binom{n}{k} P_{conv}^{k} (1 - P_{conv})^{n-k} \qquad (7)$$

P has a maximum for $P_{conv} = k/n$. In other words, the observed ratio k/n is the best estimate for P_{conv} and as such can be used to determine E based on the relation (6). The uncertainty of the P_{conv} estimate can be found by checking the cumulative binomial distributions for different values of P_{conv} and finding the distributions for which the observed k can be excluded at a specified confidence level, e.g. 95%. Applying the relation (6) to the relevant values of P_{conv} gives the range of energies containing the best-estimate-energy E with a desired confidence level. For an example let's consider $n = 100$ photon events out of which $k = 30$ converted and all the events arrived at the Pierre Auger Observatory from the geographical South at zenith angle of 13°. The observed fraction of converted photons, 30/100, is the best estimate for P_{conv}. Having this estimate, i.e. $P_{conv} = 0.3$, one can use the relation (6) for the considered arrival direction to find the best estimate of the energy of the observed photons: $E = 8.36 \times 10^{19}$ eV. $k = 30$ can be excluded at the 84.1% confidence level (corresponding to 1σ) for $P_{conv} \leq 0.261$ and $P_{conv} \geq 0.353$ ($\Delta P_{conv} = 0.092$) which corresponds to $E \leq 8.02 \times 10^{19}$ eV and $E \geq 8.79 \times 10^{19}$ eV, respectively (Δ logE $= 0.040$). In this way we find the primary gamma energy and its uncertainty: $E = 8.36^{+5\%}_{-4\%} \times 10^{19}$ eV. For a fixed k/n the uncertainty of P_{conv} increases with decreasing n. This transfers directly into the increase of the energy uncertainty. E.g. for $k = 6$ and $n = 20$, $\Delta P_{conv} = 0.16$, Δ logE $= 0.08$ and the energy determination is more uncertain: $E = 8.36^{+9\%}_{-8\%} \times 10^{19}$ eV. If k is close to 0 or to n the slope s is larger than in the case of $0 \ll k \ll n$ (see Fig. 1) and the precision of the energy determination is reduced. E.g. for $k = 10$ and $n = 100$, $s \simeq 1$ (comparing to $s \simeq 0.5$ for $k = 30$ and $n = 100$) which transfers into $E = 8.36^{+7\%}_{-6\%} \times 10^{19}$ eV.

3.1 A more realistic scenario

In a more realistic scenario, each of the photon events (in total perhaps just a few) have different energies and arrive from different directions. In this case a measurement of a single value of P_{conv} for a certain direction is not possible anymore. Nevertheless, the energy calibration based on an alternative measure of primary gamma conversion is still possible, as the shower experiment provides us with a relative measurement of shower energies, only the absolute energy scale needs to be determined. Consider n photon events numbered by index i=1,...,n, out of which k were identified as initiated by preshowers. Let's assume the events arrive at an arbitrary location $Loc_0 \equiv (lat_0, long_0)$, where lat_0 and $long_0$ are the geographic coordinates of the observation site, and from randomly distributed arrival directions $Dir_0(i) \equiv (\theta_0(i), \phi_0(i))$, where $\theta_0(i)$ are the zenith angles and $\phi_0(i)$ are the azimuth angles of arrival directions. The reconstruction of the events based on the properties of the detector and other experimental conditions gives us the initial energies $E_{ini}(i)$ and we assume that the relations between these energies are determined with a good precision. In other words, $E_{ini}(i)$ might differ from the true primary energies but all of them differ in the same way, i.e. by a certain factor. The purpose of the following method is to determine this factor. For the following considerations we define also the "observed" combination C of preshower events: $C(i) = 0$ if there was no preshower and $C(i) = 1$ if an event was initiated by a preshower. We have $\sum_i C(i) = k$.

We look for the true values of primary energies. For the clarity of the initial considerations let's assume no uncertainties in primary photon identification, identification of "preshower" events, relations between the primary energies, and the observed arrival direction. We will discuss the influence of these uncertainties later.

To determine the true values of primary photon energies (i.e. the absolute energy scale) the following procedure is proposed (a so-called "bootstrapping" approach):

1. Probability of occurring of combination C is :

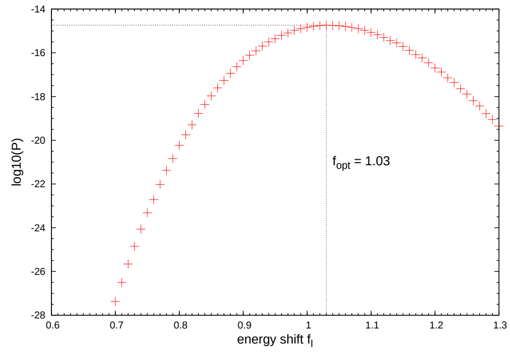

Fig. 2: The probability values (see Eq. 8) for energy shifts f_l ranging from 0.7 to 1.3. The shift $f_l = 1.0$ corresponds to the true energies. The plot was obtained for the artificial data set shown in Fig. 1 for the Auger location. An optimum shift of $f_{opt} = 1.03$ is found.

$$P(C, Loc_0) = \prod_{i_{conv}=1}^{k} P_{conv}(i_{conv}) \prod_{i_{noconv}=k+1}^{n} (1 - P_{conv}(i_{noconv})) \tag{8}$$

where i_{conv} numbers the converted events, i_{noconv} numbers the unconverted events and $P_{conv}(i)$ is the probability that the i-th photon arriving at a site Loc_0 from a direction $Dir_0(i)$ converts into an $e^{\pm}$ pair. We generate a set of scanning energies $\{E_{scan}(i)\}$ by shifting the measured photon energies $\{E_{ini}(i)\}$, i.e. multiplying all of them by the same factor f: $E_{scan}(i) = f \cdot E_{ini}(i)$. Having the set of scanning energies we use Eq. (6) to compute the relevant values of scanning conversion probabilities $\{P_{conv-scan}(i)\}$.

2. We repeat step 1. with different factors f_l and find the optimum shift f_{opt}, for which $P(C, Loc_0, \{P_{conv-scan}(i)\})$ reaches the maximum. Then f_{opt} is the energy shift that fits best the observation (i.e. combination C). See Fig. 2 for an example, where the true energy scale is reproduced within 3%.

3. The optimum energy shift f_{opt} determines a set of conversion probabilities: $\{P_{opt}(i)\}$. Each of the probabilities $P_{opt}(i)$ determines the expectation of the status of the observed photon event: initiated by a preshower (probability of occurrence $P_{conv}(i)$) or by a photon which did not undergo a pair production process above the atmosphere (probability of occurrence: $1 - P_{conv}(i)$). For further use let's name this event status the "conversion flag". In this step we generate n_c random combinations $\{C(i_c)\}$ ($i_c = 1, ..., n_c$) of "conversion flags".

4. We repeat steps 1. and 2. for each combination $C(i_c)$ and get the distribution of the optimum energy shifts $\{f_{opt}(i_c)\}$. The RMS of this distribution is the uncertainty of f_{opt}. See Fig. 3 for an example where a 5% uncertainty is obtained.

We note that this method gives a two-sided confidence interval only if both $i_{conv} > 0$ and $i_{noconv} > 0$ which is the case we focus on in this paper. For $i_{conv} = 0$ or $i_{noconv} = 0$ (i.e. all photons either converted or unconverted), a one-sided confidence interval will result which still can serve to place limits to the energy scale.

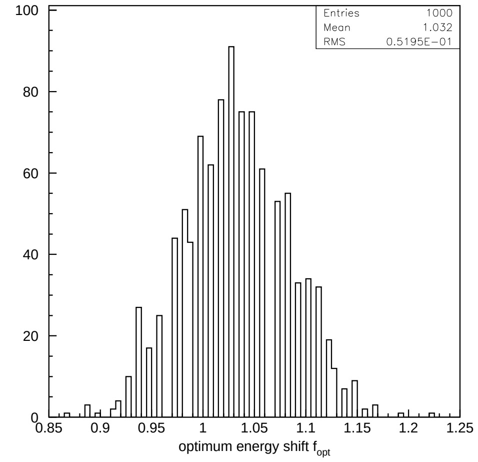

Fig. 3: Distribution of the optimum energy shifts for the artificial data sample as in Figs. 1 (the Auger location) and 2. For the set of $P_{conv}(i)$ relevant to the best energy shift 1000 combinations of "conversion flags" (see the text for an explanation) was generated. For each combination an optimum energy shift was computed and added to the distribution.

4 Accuracy of the method

To get an idea on the accuracy of the method for different sizes of the photon data samples, we repeated the steps 1-4 of the method described in the previous Section for different subsamples of the previously used 100 photon events arriving at the Auger site (the same as e.g. in Fig. 2). The size of the subsamples ranged from n =3 to 100 events. For each subsample 100 random combinations of "conversion flags" were generated for the true photon energies. We only use combinations that contain "conversion flags" of both types, where the method provides a two-sided confidence interval. (Even for $n = 2$ interesting results might be obtained, but the case of just one type of conversion flag happens more often, and we leave this for a further, more detailed study.) The resultant mean values with their uncertainties and the RMS values for different number of events in a subsample are plotted in Figs. 4 and 5.

As can be seen in Fig. 4, the systematic uncertainty is always below 5% and becomes less important with increasing n (below 2% for $n \geq 10$). Looking at Fig. 5 it can be seen that the statistical uncertainty of the presented method decreases as expected with increasing n, with values below 20% even for a few photon events only, and below 10% for $n \geq 25$.

5 Experimental uncertainties

To complete the analysis one has to consider the experimental uncertainties of the reconstructed energies, the measured arrival directions and the uncertainties of primary identification. There are three types of possible primary misidentification, each with different impact on the total probability (see Eq. 8):

a) converted photon $\leftrightarrow$ unconverted photon:
difference in the total probability: $P_{conv} \leftrightarrow 1 - P_{conv}$

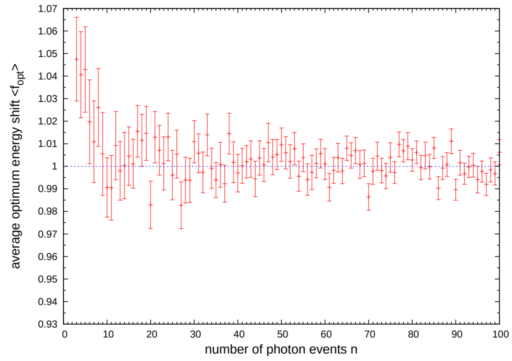

Fig. 4: Average optimum energy shifts for different subsamples (varying n) of the arbitrarily chosen artificial photon data sample as in Fig.1 for the Auger site. The average values and their uncertainties were obtained for 100 random combinations of "conversion flags". The combinations were generated for the energy shift $f_l = 1.0$ (blue dotted line).

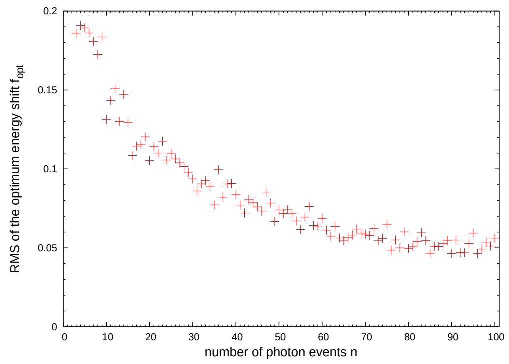

Fig. 5: RMS values of the energy shifts for different subsamples (varying n) of the arbitrarily chosen artificial photon data sample as in Fig.4.

b) proton $\leftrightarrow$ unconverted photon:
difference in the total probability: $1 \leftrightarrow 1 - P_{conv}$

c) proton $\leftrightarrow$ converted photon:
difference in the total probability: $1 \leftrightarrow P_{conv}$.

As discussed earlier, for a given set of true values $\{P_{conv}(i)\}$ many combinations of events with different "conversion flags" could be observed. One has to take into account that each of these combinations can be interpreted in many ways – according to individual identification uncertainties a), b) and c). These uncertainties could be obtained with dedicated studies like in Ref. [6].

While a detailed, quantitative discussion of the impact of these uncertainties is beyond the scope of this paper, we note that all the uncertainties listed above could be introduced in the method presented here in a fairly straightforward way. One has to generate many artificial data sets taking the energy, arrival direction and interpretation of each event at random, according to the measured values and experimental uncertainties. Then, for each of these sets, the steps 1-4 of Sec. 3 can be performed to get the optimum energy shift f_{opt} and its uncertainty.

6 Conclusions

We presented a new method to determine the energy scale of the highest energy cosmic rays. The method is based on using a one-to-one relation between the probability of pair production of a high energy photon induced by the geomagnetic field and the primary energy of this photon. The method works already for a small data sample of converted and unconverted photon events, provided a sufficiently accurate ability of the experiment to identify photons and to distinguish between the two different classes of photon events (converted and unconverted). If so, uncertainties of well below 20% could be reached with just a few events only, which would already allow an independent cross-check of the energy calibration of the shower experiment used so far. A closer analysis, particularly of the systematic uncertainties related to special properties of a data set, is in progress.

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