



Theory Predictions for Higgs Physics at the LHC

*Predicciones Teóricas para la Física de Higgs
en el LHC*

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PRÓLOGO

La partícula descubierta en la búsqueda del bosón de Higgs en el Gran Colisionador de Hadrones (LHC) del CERN, con una masa de aproximadamente 125 GeV, puede ser identificada como uno de los bosones de Higgs neutrales de varias extensiones supersimétricas del Modelo Estándar de la física de partículas. Evaluamos aquí algunas predicciones para las masas de los bosones de Higgs, mediante el método Feynman-diagramático, en el marco de la extensión supersimétrica mínima del Modelo Estándar, denominada MSSM (por sus siglas en inglés), así como para su generalización al modelo denominado Next-to-MSSM, o simplemente NMSSM. Las predicciones se basan en las contribuciones completas a un lazo, complementadas con las correcciones principales a dos lazos extraídas de nuestras publicaciones. Estos modelos representan descripciones elegantes y bien motivadas de la fenomenología observada en la física de altas energías, donde en el caso del NMSSM se agrega un campo singlete escalar (junto con su “pareja supersimétrica”) al sector de Higgs con respecto al del MSSM. A fin de comparar con los datos experimentales, es necesario alcanzar el mismo nivel de precisión en las predicciones para las masas de los bosones de Higgs.

El sector de Higgs del MSSM contiene cinco grados de libertad físicos, los cuales para el caso de parámetros reales se pueden clasificar en los neutrales CP -pares h y H , el CP -impar neutro A y los bosones de Higgs cargados H^+ y H^- . El bosón de Higgs más ligero, h , se puede identificar con la señal de Higgs neutral del LHC a ~ 125 GeV. La fenomenología de esta partícula depende crucialmente de las propiedades predichas para los otros estados del sector de Higgs del modelo, como por ejemplo la masa del bosón cargado, $M_{H^\pm}$. Analizamos aquí en detalle el cálculo diagramático para esta masa, incluyendo la evaluación de las contribuciones completas a un lazo con la adición de correcciones a dos lazos de orden $O(\alpha_t \alpha_s)$ tomadas de [1].¹ El resultado a un lazo conduce a modificaciones considerables en la predicción de $M_{H^\pm}$, alcanzando hasta ~ 8 GeV para valores relativamente pequeños de M_A . Incluso efectos mayores pueden ocurrir dependiendo del signo y el tamaño del parámetro μ , el cual interviene en las

¹ Estas contribuciones a dos lazos ascienden típicamente a cerca del 20% de las correcciones a un lazo para los ejemplos que hemos estudiado.

correcciones que afectan a la relación entre la masa y el acoplamiento de Yukawa del quark bottom.

Para la predicción de las masas de Higgs en el NMSSM, debido al singlete adicional se tienen para el sector de Higgs siete grados de libertad físicos en total, que en el caso de parámetros reales pueden clasificarse como tres bosones CP -pares $h_{1,2,3}$, dos bosones CP -impares $A_{1,2}$, y el par de bosones de Higgs cargados $H^\pm$. El cálculo para estas masas se realiza en este trabajo tomando en cuenta el NMSSM completo a orden de un lazo, complementado por las contribuciones de dos lazos a las autoenergías de los bosones de Higgs aproximadas en [2] por sus homólogas del MSSM.² Tener en cuenta estas correcciones en la predicción de la masa del bosón de Higgs del NMSSM identificable con la señal observada en el LHC es crucial para alcanzar una precisión a un nivel similar al del MSSM. Nuestros resultados se ejemplifican para un escenario genuino del NMSSM.

El procedimiento para obtener dicho nivel de precisión para las predicciones teóricas en el marco de ambos modelos incluyó la derivación de la renormalización para los correspondientes sectores de Higgs, la cual se describe en detalle en este trabajo. En el caso del MSSM, se realiza una comparación entre los esquemas de renormalización para los casos reales y complejos de parámetros, mientras que para el NMSSM se consideró el caso CP -conservativo (parámetros reales). Los esquemas de renormalización implementados fueron híbridos de tipo on-shell/ $\overline{DR}$ en todos los casos.

También se analizan varios aspectos teóricos adicionales del NMSSM. La condición para el “límite al MSSM”, en el cual los resultados MSSM se recuperan del (más general) NMSSM, se derivan analíticamente y se verifican numéricamente para dos escenarios. Además, proponemos un esquema alternativo de renormalización (no publicado aún) para el NMSSM, y se discuten sus características y posibles ventajas. Finalmente, se deriva en detalle una propiedad de simetría de la teoría con respecto a (los signos de) los parámetros genuinos del NMSSM λ y κ .

² Estas incluyen las contribuciones de orden $O(\alpha_t\alpha_s, \alpha_b\alpha_s, \alpha_t^2, \alpha_t\alpha_b)$, así como un compendio de los logaritmos dominantes y subdominantes del sector del quark top y del top escalar.

PREFACE

The particle discovered in the Higgs boson searches at the Large Hadron Collider (LHC) at CERN with a mass of about 125 GeV can be identified with one of the neutral Higgs bosons of several supersymmetric extensions of the Standard Model of particle physics. We calculate here some predictions for the Higgs-boson masses within the Minimal Supersymmetric Standard Model (MSSM) and the Next-to-Minimal Supersymmetric Standard Model (NMSSM), obtained by Feynman-diagrammatic methods. The predictions are based on the full one-loop contributions, supplemented with the leading two-loop corrections extracted from our publications. These models represent elegant and well-motivated descriptions for the observed phenomenology in high energy physics, where in the case of the NMSSM a scalar singlet (together with its “superpartner”) is added to the Higgs sector of the MSSM. In order to compare with experimental data, the same level of accuracy in these predictions for the Higgs-boson masses is a necessity.

The Higgs sector of the MSSM contains five physical degrees of freedom, which for the case of real parameters can be classified as the neutral CP -even h and H , the neutral CP -odd A , and the charged Higgs bosons H^+ and H^- . The lightest Higgs boson, h , can be identified with the LHC neutral Higgs signal at ~ 125 GeV. The phenomenology of this particle depends crucially on the predicted properties of the other Higgs states of the model, e.g. the mass of the charged Higgs boson, $M_{H^\pm}$. We analyse in detail the diagrammatic calculation of this mass, including the evaluation of the complete one-loop contributions with the addition of two-loop corrections of $O(\alpha_t \alpha_s)$ taken from [1].³ The one-loop result leads to sizable shifts in the $M_{H^\pm}$ prediction, reaching up to ~ 8 GeV for relatively small values of M_A . Even larger effects can occur depending on the sign and size of the μ parameter that enters the corrections affecting the relation between the bottom-quark mass and the bottom Yukawa coupling.

For the prediction of the Higgs masses in the NMSSM, due to the additional singlet we have seven physical Higgs d.o.f. in total, which in the real parameter case can be classified as three CP -even bosons $h_{1,2,3}$, two CP -odd bosons, $A_{1,2}$,

³ These two-loop contributions amount to typically about 30% of the one-loop corrections for the examples that we have studied.

and the charged Higgs boson pair $H^\pm$. The calculation for these masses is performed in this work taking into account the full NMSSM at one-loop order, supplemented by the two-loop contributions to the Higgs-boson self-energies approximated in [2] by their MSSM counterparts.⁴ Taking these corrections into account in the prediction for the mass of the NMSSM Higgs boson that is identified with the observed signal at the LHC is crucial in order to reach a precision at a similar level as in the MSSM. Our results are exemplified for a scenario genuine to the NMSSM.

The procedure for obtaining such level of accuracy for theoretical predictions within both models included the derivation of the renormalization for the corresponding Higgs sectors, which is described in detail in this work. In the case of the MSSM, a comparison between renormalization schemes for the real and the complex parameter cases is performed, while for the NMSSM the CP -conserving (real parameters) case was considered. Hybrid on-shell/ $\overline{\text{DR}}$ renormalisation schemes were implemented in all cases.

Several additional theoretical aspects of the NMSSM are also analysed. The condition for the “MSSM-limit”, in which the MSSM results are recovered from the (more general) NMSSM, are derived analytically and are numerically verified for two scenarios. In addition, we propose an alternative renormalization scheme (not published yet) for the NMSSM, and its features and possible advantages are discussed. Finally, a symmetry property of the theory with respect to (the signs of) the genuine NMSSM-parameters λ, κ is derived in detail.

⁴ These include contributions at $O(\alpha_t \alpha_s, \alpha_b \alpha_s, \alpha_t^2, \alpha_t \alpha_b)$, as well as a resummation of leading and subleading logarithms from the top/scalar top sector.

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Introduction

An important goal of the particle physics strategy at the high energy frontier, currently being pursued by the Large Hadron Collider (LHC) at CERN, is to elucidate the nature of electroweak symmetry breaking. The existence of electroweak massive gauge bosons ($W^\pm, Z$), together with the successful description of their behavior by the non-abelian gauge theory, requires that some form of such a breaking be realized in nature. An attractive theoretical suggestion for such dynamics is the Higgs mechanism, which implies the existence of one or more Higgs bosons (depending on the specific model considered). Therefore, the search for these bosons is an important cornerstone in the physical programs of present and future high energy colliders.

The spectacular discovery of a new boson of this kind with a mass around 125 GeV by the ATLAS and CMS experiments [3,4] in 2012 constitutes a milestone in this quest for understanding the physics of electroweak symmetry breaking. Any model describing electroweak physics needs to provide a state that can be identified with the observed signal. Within the present experimental uncertainties, the properties of the observed state are compatible with the interpretations from the predictions of the Standard Model (SM) of particle physics, but also as a Higgs boson of extended models.

One of the prime candidates for physics beyond the SM is supersymmetry (SUSY), which predicts two scalar partners for all SM fermions, as well as fermionic partners to all bosons. The most widely studied SUSY framework is the Minimal Supersymmetric Standard Model (MSSM) [5-7], which keeps the number of new fields and couplings to a minimum. In addition, in contrast to the single Higgs doublet of the (minimal) SM, the Higgs sector of the MSSM contains two scalar doublets leading to a spectrum of five physical states, which in the CP -conserving case (i.e. real soft SUSY-breaking parameters) consist of the light and heavy CP -even h and H , the CP -odd A , and the charged Higgs bosons $H^\pm$. It was shown that the signal discovered at about 125 GeV can be interpreted in principle as the light but also as the heavy CP -even Higgs boson of the MSSM [8-12]. In both cases, searching for instance for the charged Higgs bosons as well could be crucial to investigate that scenarios, and the discovery of a charged Higgs particle would constitute an unambiguous sign of physics beyond the SM. We will investigate here the Higgs sector of this model both in the

case of real and complex parameters, and we discuss the improved prediction for the mass of the charged Higgs bosons, whose results we then incorporated into the public tool FEYNHIGGS [13-18] and have been published in [1].

The MSSM acquires its most immediate and well-motivated extension in the so-called Next-to-Minimal Supersymmetric Standard Model (NMSSM). For this model (see [19,20] for a review) in the CP -conserving case (which we assume throughout this work), the states in the Higgs sector can be classified as three CP -even bosons, h_i ($i = 1,2,3$), two CP -odd bosons, A_j ($j = 1,2$), and the charged Higgs boson pair $H^\pm$. In addition, the SUSY partner of the singlet Higgs (called “singlino”) extends the neutralinos to a total of five. In the NMSSM the lightest but also the second lightest CP -even Higgs boson can be interpreted as the neutral signal observed at ~ 125 GeV [21]. In order to improve the prediction for the Higgs masses from this model, we performed the calculation of the full NMSSM one-loop contributions, supplemented by the two-loop corrections from our publications, approximated by the one from the MSSM (within a mixed on-shell and $\overline{DR}$ renormalisation scheme) and including the resummation of logarithms involving large SUSY masses. It was shown that this approximation is valid for a wide range of parameters. The prediction and its quality will be discussed. In a future, the presented calculation will be included in the NMSSM version of FEYNHIGGS.

In summary, these are the methodology and tools implemented for the different models and levels of precision for the amplitudes in perturbative quantum field theory:

Amplitudes	MSSM neutral and charged Higgs	NMSSM neutral Higgs
Leading 1-loop	By-hand Own Mathematica code FEYNHIGGS (now implemented for the charged Higgs)	Own Mathematica code
Full ⁵ 1-loop	Own Mathematica code + FEYNARTS, with preexisting model file + FORMCALC	Own Mathematica code + FEYNARTS, with a model file created by us using SARAH + FORMCALC
Leading 2-loop ⁶	FEYNHIGGS (already implemented for neutral Higgs, now implemented for the charged Higgs)	MSSM FeynHiggs (already implemented)

The author also wishes to emphasize the meticulous and laborious work done to express and present the equations and results as clear, simple and compact as possible.

⁵ “Full” means “with the virtual contributions from all sectors of the theory”.

⁶ The inclusion of these two-loop MSSM-like corrections was analysed by collaborators, in order to add them in our publications.

This dissertation is organized as follows:

- In this Section we review the SM and introduce all relevant information about the motivation and features of its supersymmetric extensions. It is discussed the basics of the Higgs theory and how the Higgs mechanism works in general. We also emphasize the need to improve the theoretical predictions in order to test the possibility that the SM-like Higgs boson discovered at the LHC could fit within these beyond-SM models, and accordingly the general spirit and main features of the renormalisation method (which is the main procedure implemented in this work in order to perform such higher-order predictions) are described.
- An overview of supersymmetry is given in Section 2, discussing its motivations and application, and introducing the main features of the MSSM and the NMSSM.
- In Section 3, masses and mixing relevant to the Higgs sector within the MSSM are evaluated and numerically analysed, performing the renormalisation involved in the computation of the full one-loop result, and briefly mentioning the main two-loop corrections.
- Our corresponding analytical and numerical analysis within the NMSSM is addressed in Section 4, including a discussion of the renormalisation and the incorporation of the radiative corrections, and performing a phenomenological comparison between several well-motivated scenarios.
- We proposed and discussed an alternative renormalisation scheme (not published yet) for the NMSSM in Section 5, exploring the possible advantages that this could yield, comparing it with the scheme that we implemented in sect. 4.
- The summary and conclusions are presented in Section 6.

1.1 This work

This doctoral work focused on the renormalisation of the Higgs sector (and the subsequent calculation with radiative corrections of the boson masses) at full one-loop level (i.e., at next-to-leading order in perturbation theory) within the MSSM and the NMSSM. The results are supplemented with the addition of the leading two-loop contributions developed and analysed in our publications. The author made use of existing public computer algebraic and numerical codes (such as FEYNARTS [22-24], FORMCALC [25] and

FEYNHIGGS) together with many others that he had developed on his own, mostly in Mathematica language, both to perform the analytic calculation and the numerical analysis.

It was performed in detailed the derivation and analysis of all analytic calculations for the radiative contributions to the Higgs masses for the supersymmetric models studied here, finding different ways to cross check the results and providing numerical fits needed to compare with phenomenology. He also collaborated in the calculation of some MSSM higher-order corrections for their implementation in FEYNHIGGS (see www.feynhiggs.de).

Once reproduced and improved the main radiative contribution for the MSSM case, the author performed the renormalisation of the NMSSM Higgs sector and implemented the Feynman rules that describe the interaction of the various particles of the model, with the help of the computer tool FEYNARTS 3.9, where the implementation of the NMSSM with real parameters was based on a model-file we generated using SARAH [26]. The diagrams and corresponding amplitudes obtained from this code, together with further evaluation by his brother FORMCALC 7.4, was used to generate and perform the calculation of the full one-loop self-energies and radiative contributions.

Checks with partial results available in the literature were performed to confirm the correctness of the outputs. This comparison also showed the effects of the newly evaluated higher-order corrections with respect to the known results, and it allowed to perform the corresponding phenomenological analyses both within the MSSM and focused on the deviations from it for the NMSSM.

In this way, the various calculations for the masses of the (both neutral and charged) Higgs bosons and their decay probabilities (two crucial ingredients for the Higgs boson searches) were performed. The next objective of the project will be the implementation of the NMSSM results into the public computer numerical FEYNHIGGS structure (of which the Director of this thesis is the main author), in order to improve the phenomenological analysis and to facilitate to the experimental particle physics community the accurate theory predictions essential to fully exploit and correctly interpret the experimental data within the context of physics beyond the SM and Higgs sectors of non-standard nature.

1.2 The Standard Model and beyond

The Standard Model (SM) of particle physics is a theory concerning the electromagnetic, weak and strong interactions, as well as classifying all the subatomic elementary particles

known. Although it is believed to be theoretically self-consistent [27] and has demonstrated huge and continued successes in providing experimental predictions, it does leave some phenomena unexplained and it falls short of being a complete theory of fundamental interactions. The SM is a paradigm of a quantum field theory, which exhibits a wide range of physics including spontaneous symmetry breaking, anomalies and non-perturbative behavior. It is used as a basis for building more exotic models that incorporate hypothetical particles, extra dimensions or elaborated symmetries (such as supersymmetry) in an attempt to explain experimental results at variance with the SM, such as the existence of dark matter and neutrino oscillations.

In 1967 Steven Weinberg and Abdus Salam incorporated the Higgs mechanism [28] into Glashow's electroweak model, giving it its modern form. The Higgs boson existence was predicted by this mechanism, which describes how SM's fundamental particles obtain mass, and it was the last particle of the model to be observed: a Higgs boson was confirmed to exist on July 4, 2012, although efforts to confirm that it has all of the properties predicted by the SM are ongoing. The other main component of the SM is the theory of the strong interaction (QCD), to which many contributed, acquiring its modern form around 1973–74 when experiments confirmed that the hadrons were composed of fractionally charged quarks.

1.2.1 Particle content

The SM includes 12 elementary fermions (spin 1/2 particles), and each fermion has a corresponding antiparticle. These are the six types (“flavors”) of leptons and quarks, placed in three families/generations, see tab. 1.1. They are influenced by the weak nuclear force. The defining property of the quarks is that they also carry “color” charge, and hence interact via the strong interaction; they interact with other fermions also electromagnetically. The remaining six fermions do not carry color charge, but by virtue of carrying an electric charge, the electron, muon and tau all do interact electromagnetically too. The three neutrinos do not carry electric charge, so their interaction is via the weak force only.

The SM explains forces as resulting from matter particles exchanging other particles, generally referred to as “force-mediating” particles or “gauge bosons”. The Feynman diagram calculations, which are a graphical representation of the perturbation theory approximation, invoke force-mediating particles, and when applied to analyse high-energy scattering experiments are in reasonable agreement with the data. The different types of gauge bosons are:

- Massless photons mediate the electromagnetic force between electrically charged particles.
- The W^+ , W^- , and Z gauge bosons mediate the weak interactions between particles of different “flavors” (quarks and leptons). They are massive, with the Z being more massive than the $W^\pm$.
- The eight massless gluons mediate the strong interactions between quarks. Because they have an effective color charge, they can also interact among themselves.

charge : spin :	QUARKS		LEPTONS		GAUGE BOSONS		
	u up	d down	ν_e electron neutrino	e electron	g gluon	$W^\pm$ W boson	H Higgs boson
	c charm	s strange	ν_μ muon neutrino	μ muon	γ photon		
	t top	b bottom	ν_τ tau neutrino	τ tau	Z Z boson		
	+2/3	−1/3	0	−1	0	±1	0
	1/2				1		0

Table 1.1: Zoo of the Standard Model of elementary particles, with the three generations of matter, the gauge bosons and the Higgs boson.

The Higgs particle is a massive scalar elementary particle theorized by Peter Higgs in 1964 and is a key building block in the SM [28], playing a unique role by explaining why the elementary particles, except the photon and gluon, are massive. Experiments to confirm and determine the nature of the Higgs boson using the Large Hadron Collider (LHC) at CERN began in early 2010, and were performed at Fermilab's Tevatron until its closure in late 2011. On 4 July 2012, the two main experiments at the LHC (ATLAS and CMS) both reported independently that they found a new particle with a mass of ~ 125 GeV, which is consistent with the Higgs boson [3,4]. Although it has several properties similar to the predicted “simplest” Higgs, they acknowledged that further work would be needed to conclude that it is indeed the Higgs boson, and exactly which version (model) of the SM-like Higgs is best supported if confirmed.

1.2.2 Theoretical aspects⁷

Particle physicists, using the concept of gauge symmetries, are able to build the SM to explain the fundamental particles and their interactions. The construction proceeds following the modern method of constructing most quantum field theories: the theory is formulated by first postulating a set of symmetries of the system, and then by writing down the most general renormalizable Lagrangian from its particle/field content that observes these symmetries. The local $SU(3)_{\text{color}} \times SU(2)_{\text{weak}} \times U(1)_{\text{em}}$ gauge group is the internal symmetry that essentially defines the SM, where the three factors roughly give rise to the three fundamental interactions. All particles appear to have these three kinds of gauge invariances.⁸

Fermions are chiral particles, which means that left and right handed fields transform differently. The left handed components are placed in electroweak $SU(2)$ doublets, while right handed components are placed in singlets:

$$\psi_\ell = (L_{1,2,3})_L, (e_{1,2,3})_R \quad , \quad \psi_q^c = (Q_{1,2,3}^c)_L, (u_{1,2,3}^c)_R, (d_{1,2,3}^c)_R \quad (1.1)$$

$$L_g = \begin{pmatrix} \nu_{e_g} \\ e_g \end{pmatrix} \quad , \quad Q_g = \begin{pmatrix} u_g \\ d_g \end{pmatrix}$$

$$f_L \equiv \frac{1}{2}(1 - \gamma^5)\psi_f \quad , \quad f_R \equiv \frac{1}{2}(1 + \gamma^5)\psi_f$$

$$e_g = e, \mu, \tau \quad , \quad u_g = u, c, t \quad , \quad d_g = d, s, b$$

For the quarks, the index $c=1,2,3$ is required to describe how they transform under the $SU(3)$ transformations.

The complete SM Lagrangian can be written as

$$L = L_{\text{fermion}} + L_{\text{gauge}} + L_{\text{Higgs}} + L_{\text{Yukawa}} \quad (1.2)$$

where L_{fermion} is given by

$$L_{\text{fermion}} = \sum_{f=\ell,q} \bar{\psi}_f i \gamma^\mu D_\mu \psi_f \quad . \quad (1.3)$$

⁷ This section is based on chapter 2 from [42].

⁸ In 1961 Glashow [43] proposed the $SU(2)_L \times U(1)_Y$ structure for the SM. Weinberg and Salam [36,37] extended his idea and employed the hypothesis of spontaneous symmetry breaking to generate masses of gauge boson and fermions.

D_μ is a covariant derivative given by

$$D_\mu \equiv \partial_\mu + ig_1 \frac{1}{2} Y B_\mu + ig_2 \frac{1}{2} \sigma_i^L W_\mu^i + ig_s T_a G_\mu^a \quad (1.4)$$

where B_μ is the $U(1)$ gauge field, Y is the weak hypercharge (the generator of the $U(1)$ group) and W_μ^i is the three-component $SU(2)$ gauge field. There are eight G_μ^a gauge field containing the gluons (the mediators of strong force), one for each generator T_a of $SU(3)$ transformations ($T_a \psi_\ell = 0$ for leptons). g_1 is the $U(1)$ gauge coupling, whereas g_2 and g_s are the $SU(2)$ and $SU(3)$ gauge couplings respectively. γ^μ are Dirac matrices, while σ_i^L are the Pauli matrices (infinitesimal generators of the $SU(2)$ group; the superscript “L” indicates that they only act on left fermions):

$$\sigma_i W_\mu^i = \begin{pmatrix} W_\mu^3 & \sqrt{2} W_\mu^- \\ \sqrt{2} W_\mu^+ & -W_\mu^3 \end{pmatrix}, \quad W_\mu^\pm \equiv \frac{1}{\sqrt{2}} (W_\mu^1 \pm i W_\mu^2) \quad (1.5)$$

The Lagrangian for the $SU(2)_L \times U(1)_Y$ gauge sector of the theory is

$$L_{\text{gauge}} = -\frac{1}{4} W_{\mu\nu}^i W_i^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \quad (1.6)$$

being $B_{\mu\nu}$ the field strength tensor for the $U(1)$ gauge boson B_μ , and $W_{\mu\nu}^i$ the ones for the $SU(2)$ gauge bosons:

$$\begin{aligned} B_{\mu\nu} &\equiv \partial_\mu B_\nu - \partial_\nu B_\mu \\ W_{\mu\nu}^i &\equiv \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g_2 \varepsilon_{ijk} W_\mu^j W_\nu^k \end{aligned} \quad (1.7)$$

ε_{ijk} is structure constant that appears due to non-abelian nature of the $SU(2)$ group.

In order to generate masses for fermions and bosons, Higgs mechanism is introduced, which will be discussed in sect. 1.3.2.

Electroweak theory

The electroweak (EW) force is transmitted by the exchange of the physical gauge bosons γ , $W^\pm$ and Z . By using eq. (1.3), the $U(1)$ and $SU(2)$ interaction terms for the first generation of leptons can be written as

$$\begin{aligned} L_{\text{lepton}} &= \frac{1}{2} g_1 (Y_L (\bar{\nu}_L \gamma^\mu \nu_L + \bar{e}_L \gamma^\mu e_L) + Y_R \bar{e}_R \gamma^\mu e_R) B_\mu \\ &\quad - \frac{1}{2} g_2 ((\bar{\nu}_L \gamma^\mu \nu_L - \bar{e}_L \gamma^\mu e_L) W_\mu^3 + \sqrt{2} (\bar{e}_L \gamma^\mu \nu_L W_\mu^- + \sqrt{2} \bar{\nu}_L \gamma^\mu e_L W_\mu^+)) \end{aligned} \quad (1.8)$$

Neutrinos do not have electromagnetic interactions, so terms of the form $\propto \bar{\nu}_L \gamma^\mu \nu_L A_\mu$ must be avoided. Diagonalizing,

$$A_\mu \equiv \frac{g_2 B_\mu - g_1 Y_L W_\mu^3}{\sqrt{g_1^2 Y_L^2 + g_2^2}} \quad , \quad Z_\mu \equiv \frac{g_1 Y_L B_\mu + g_2 W_\mu^3}{\sqrt{g_1^2 Y_L^2 + g_2^2}} \quad (1.9)$$

and the neutral current interactions in eq. (1.8) are modified as

$$\begin{aligned} L_{\text{lepton}} = & \frac{g_1^2 Y_L^2 + g_2^2}{2\sqrt{g_1^2 Y_L^2 + g_2^2}} (\bar{\nu}_L \gamma^\mu \nu_L) Z_\mu + \left(\frac{g_1^2 Y_L^2 - g_2^2}{2\sqrt{g_1^2 Y_L^2 + g_2^2}} \bar{e}_L \gamma^\mu e_L + \frac{g_1^2 Y_L Y_R}{2\sqrt{g_1^2 Y_L^2 + g_2^2}} \bar{e}_R \gamma^\mu e_R \right) Z_\mu \\ & + \left(\frac{g_1 g_2 Y_L}{\sqrt{g_1^2 Y_L^2 + g_2^2}} \bar{e}_L \gamma^\mu e_L + \frac{g_1 g_2 Y_R}{2\sqrt{g_1^2 Y_L^2 + g_2^2}} \bar{e}_R \gamma^\mu e_R \right) A_\mu \quad \supset \quad -e (\bar{e}_L \gamma^\mu e_L + \bar{e}_R \gamma^\mu e_R) A_\mu \quad . \end{aligned} \quad (1.10)$$

This gives

$$e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \equiv g_2 \sin \theta_W \quad , \quad Y_L = \frac{1}{2} Y_R \equiv -1 \quad . \quad (1.11)$$

1.2.3 Problems with the Standard Model

Physics beyond the Standard Model (BSM) refers to the theoretical developments needed to explain the deficiencies of the SM, such as the origin of mass, the strong CP problem, neutrino oscillations, matter-antimatter asymmetry, and the nature of dark matter and dark energy. Theories that lie beyond the SM include various extensions through supersymmetry, such as the Minimal Supersymmetric Standard Model (MSSM) [5-7] and Next-to-Minimal Supersymmetric Standard Model (NMSSM) [19,20], or entirely novel explanations, such as string theory, M-theory and extra dimensions.

Despite being the most successful theory of particle physics to date, the SM is not perfect. There are fundamental physical phenomena in nature that it does not adequately explain:

- ❖ SM does not explain gravity. Moreover, it is widely considered to be incompatible with the most successful theory of gravity to date, general relativity.
- ❖ Cosmological observations tell us that the SM explains about 5% of the energy present in the universe. It does not supply any fundamental particle that is good dark matter candidate.

- ❖ According to the SM, neutrinos are massless particles. Mass terms for the neutrinos can be added to it by hand, but these lead to new theoretical problems.
- ❖ The universe is made out of mostly matter. No mechanism sufficient to explain this matter-antimatter asymmetry exists in the SM.

Some features of the SM are added in an ad hoc way. These are not problems per se (the theory works fine with these features), but they imply a lack of understanding:

- *Hierarchy problem*: Within the SM, the mass of the Higgs gets some very large quantum corrections due to the presence of virtual particles.
- *Number of parameters*: The SM depends on 19 numerical parameters. Their values are known from experiment, but the origin is unknown.

1.2.4 SUSY can help

Supersymmetric theories such as the MSSM outweigh the SM in that they solve the “naturalness” or “hierarchy” problem of the latter, concerning in the occurrence of quadratically divergent quantum corrections for the mass of the Higgs particle, which implies that the couplings to the Higgs become too large at the TeV scale. Supersymmetry explains how a small Higgs mass can be protected from such corrections. Going beyond the MSSM, this model finds a well-motivated extension in the Next-to-Minimal Supersymmetric Standard Model (NMSSM) [19,20] which provides also a solution for naturally associating an adequate scale to the μ parameter appearing in the MSSM superpotential [29,30]: the introduction of a new singlet superfield, which only couples to the Higgs- and sfermion-sectors, gives rise to an effective μ -term, generated in a similar way as the Yukawa mass terms of fermions through its vacuum expectation value. We already pointed out that, both in the MSSM and the NMSSM, the lightest but also the second lightest neutral Higgs boson can be interpreted as the signal observed at about 125 GeV at CERN.

SUSY also provides a good candidate for dark matter. The determination of the cold dark matter (CDM) density imposes a corresponding constraint on the parameters of any model that provides the dominant fraction of the CDM density. This is, in particular, true for supersymmetric models with conserved R-parity, in which the CDM is provided by the stable lightest SUSY particle (normally a neutralino) [31].

1.3 Higgs physics

The Higgs boson is an elementary particle initially theorized in 1964, whose discovery was announced at CERN on 4 July 2012. This discovery should allow physicists to finally validate the last untested area of the SM's approach to fundamental particles and forces, guide other theories and discoveries in particle physics, and potentially lead to developments in “new” physics.

By March 2013, the newly discovered particle had been shown to behave, interact and decay in many of the ways predicted by the SM, and was also tentatively confirmed to have positive parity and zero spin, two fundamental attributes of a Higgs boson [32]. More data is needed to know if it exactly matches the predictions of the SM, or whether, as predicted by some theories, multiple Higgs bosons exist.

In the SM, the Higgs particle is a boson with zero spin and no electric charge neither color charge. It is a quantum excitation of one of the four components of the Higgs field. This field has a “Mexican hat” shaped potential with nonzero strength everywhere (including otherwise empty space) which in its vacuum state breaks the weak isospin symmetry of the electroweak interaction.

1.3.1 Spontaneous symmetry breaking

Spontaneous symmetry breaking describes systems where the equations of motion or the Lagrangian obey some symmetries, but the lowest-energy vacuum solutions do not exhibit that same symmetry. The underlying laws are invariant under a symmetry transformation. But the potential has an infinite number of possible minima (*vacuum* states), and once the system falls into a specific stable vacuum state, the symmetry will appear to be lost or “spontaneously broken”. Thus the system as a whole is symmetric, but it is never encountered with this symmetry, but only in one specific asymmetric state. Nevertheless, the fact that each outcome is equally likely (it corresponds to the same energy) is a reflection of the underlying symmetry [33], which is thus often dubbed “hidden symmetry”. Features of the symmetry are still there: it would take no “force” to freely displace the ground state in time, amounting to an oscillation of vanishing frequency, unlike the radial ones. This spinning mode is called Nambu-Goldstone boson [34,35]. While the symmetry of the equations is not reflected by the *individual* solutions, it does by the *range* of solutions.

Consider the following Lagrangian for a complex scalar field ϕ :⁹

$$L = \frac{1}{2} (\partial_\mu \phi)^* (\partial^\mu \phi) - V(\phi) \quad , \quad V(\phi) = \frac{1}{4} \lambda (|\phi|^2 - v^2)^2 \quad , \quad \lambda > 0 \quad (1.12)$$

If $v^2 > 0$, the minimum of the potential is at $|\phi| = v$: v is the vacuum expectation value (“vev”), $\langle \phi \rangle$, of field ϕ . To determine the particle spectrum, one must study the theory in the region of the minimum by putting $\phi = v + \eta(x)$ and expanding around $\eta = 0$:

$$L = \frac{1}{2} (\partial_\mu \eta)^* (\partial^\mu \eta) - \lambda v^2 \eta^2 + \lambda v \eta^3 + \frac{1}{4} \lambda \eta^4 + \text{const.} \quad (1.13)$$

The term in η^2 has the correct sign so it can be interpreted as the mass square, and the vacuum does not have the reflection symmetry of the original Lagrangian.

The Higgs mechanism [28], the spontaneous symmetry breaking of gauge symmetries, is an important component in understanding the origin of particle masses in the SM of particle physics. Spontaneous symmetry breaking of the $SU(2) \times U(1)$ gauge symmetry associated with the electroweak force generates masses for several particles, and separates the electromagnetic and weak forces. While at energies much greater than 100 GeV all electroweak particles behave in a similar manner, the Weinberg-Salam theory [36,37] predicts that, at lower energies, this symmetry is broken so that the photon and the massive W and Z bosons emerge.

1.3.2 Higgs mechanism¹⁰

The Higgs mechanism is essential to explain the generation mechanism of the mass in the SM. According to this theory, particles gain mass by interacting with the Higgs field that permeates all space. Particles such as the electron or the quarks constituents of protons are endowed with mass by this mechanism, technically through spontaneous symmetry breaking, where, however, due to the specific form of this symmetry breaking, instead of the usual transverse Nambu–Goldstone bosons a “longitudinal” massive boson, the Higgs boson, appears. The specific spontaneous symmetry breaking of the underlying local symmetry triggers conversion of the longitudinal field component to the Higgs boson, which interacts with itself and (at least a part of) the other fields in the theory, so as to produce mass terms for the three gauge bosons, and also to fermions.

⁹ Do not confuse this “ λ ” parameter with the one we will introduce for the NMSSM Higgs sector in the next chapters.

¹⁰ This section follows the procedure outlined in chapter 2 from [42] and sect. II from [44].

Spontaneous symmetry breaking offered a framework to introduce massive bosons into relativistic quantum field theories. The problem arises with Yang–Mills theory (also known as non-abelian gauge theory), which predicts massless spin-1 gauge bosons. Higgs mechanism implement spontaneous symmetry breaking in order to provide a way to give mass to these bosons without losing the gauge invariance of the theory. In practice, the introduction of some scalar fields with a non-zero vacuum expectation value (“vev” for short), and the subsequent expansion of those fields around a minimum of the potential, is just a way to introduced the desired mass terms for the gauge bosons but getting also additional terms (interaction terms) that ensure the Lagrangian remains gauge invariant.¹¹

The relativistic model was developed in 1964 by three independent groups –Robert Brout and François Englert [38], Peter Higgs [28], and Gerald Guralnik, Carl Richard Hagen and Tom Kibble [39]. These physicists discovered that when a gauge theory is combined with an additional field that spontaneously breaks the symmetry group, the gauge bosons can consistently acquire a nonzero mass. In spite of LHC announced results consistent with the Higgs particle, it should be stressed that further testing is needed to confirm the Higgs mechanism.

Abelian Higgs mechanism

Gauge invariance means that certain transformations of the gauge field do not change the energy at all. In order to give mass to a gauge theory, the gauge invariance must be broken by a “condensate”, which will then define a preferred phase, and the phase of this condensate will define the zero value of the field in a gauge invariant way. In order for the phase of the vacuum to define a gauge, the field must have a phase (also referred to as “to be charged”). And in order for a scalar field ϕ to have a phase, it must be complex, or (equivalently) it should contain two fields with a symmetry which rotates them into each other.

The only renormalizable model where a complex scalar field acquires a nonzero vev is the Mexican-hat model (1.12), where the field energy has the minimum energy value on the circle of points where the magnitude of ϕ is v . The abelian Higgs model is the Mexican-hat model coupled to electromagnetism:

$$L = |(\partial_\mu - i q A_\mu) \phi|^2 - V(\phi) \quad (1.14)$$

¹¹ In some sense, Higgs mechanism is nothing more that this way of finding which terms have to be added to the Lagrangian in order to have mass terms without losing gauge invariance. Again, we see that saying “hidden” is better than “broken” symmetry: the gauge symmetry is still there (that’s what we want!), but that is no longer evident with the Lagrangian written in term of the physical fields.

Now the phase of the vacuum is arbitrary, because gauge transformations change it. The potential energy density in the condensate is $E = \frac{1}{2} |\partial_\mu (\nu e^{iqA_\mu x})|^2 = \frac{1}{2} q^2 \nu^2 A_\mu A^\mu$; this energy is the same as a mass term $\frac{1}{2} m^2 A_\mu A^\mu$ where $m = q\nu$.

Standard Model Higgs

Mass terms are included in SM Lagrangian by the Higgs mechanism, using the idea of spontaneous symmetry breaking. With the Higgs field, the electroweak symmetry is spontaneously broken and the W and Z bosons acquire masses. Fermions, such as the leptons and quarks in the SM, can also acquire mass as a result of their interaction with the Higgs field, but not in the same way as the gauge bosons.

The Higgs field Φ is a scalar doublet of the group $SU(2)_L$ with hypercharge $Y=1$, a complex spinor with four real components (or equivalently with two complex components):

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} \quad (1.15)$$

which is coupled to the gauge fields through; before manifestly symmetry breaking, the non-abelian Higgs Lagrangian is

$$L_{\text{Higgs}} = |D_\mu \Phi|^2 - V(\Phi) \quad , \quad D_\mu = \partial_\mu + i\frac{1}{2}g_1 B_\mu + i\frac{1}{2}g_2 \sigma_i W_\mu^i \quad (1.16)$$

where the Higgs field self-interaction enters through the Higgs potential:

$$V(\Phi) = \frac{1}{4} \lambda^2 (\Phi^\dagger \Phi - \frac{1}{2} \nu^2)^2 \quad (1.17)$$

Again, the vev defines a preferred gauge where the vacuum is constant, and fixing this gauge fluctuations in (some) fields come with a non-zero energy cost. For $\nu^2, \lambda > 0$, V is minimized by all non-vanishing field configurations with $\Phi^\dagger \Phi = \frac{1}{2} \nu^2$. Selecting the minimum which is real and electrically neutral, one gets the vev

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu \end{pmatrix} \quad (1.18)$$

The scalar field ϕ^0 in eq. (1.15) can be written as

$$\phi^0(x) = \frac{1}{\sqrt{2}} (\nu + H(x) + i\chi(x)) \quad (1.19)$$

Choosing the particular gauge where $\phi^+ = \chi = 0$, denoted as the “unitary gauge”, the Higgs doublet field has the simple form

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + H(x) \end{pmatrix} \quad (1.20)$$

Three out of the four components $\{\text{Re}\phi^+, \text{Im}\phi^+, H, \chi\}$ would ordinarily amount to Goldstone bosons, if they were not coupled to gauge fields. However, after symmetry breaking, these three of the four degrees of freedom in the Higgs field mix with the three $W^\pm$ and Z bosons, and are only observable as spin components of these weak bosons, which are now massive, while the one remaining degree of freedom H becomes a new particle. The real field $H(x)$ thus describes physical neutral scalar particles, the Higgs bosons, with mass $M_H = \sqrt{\lambda}\nu$, see eq. (1.13).

The Higgs field, through the interactions specified by its potential, hence induces spontaneous breaking of three out of the four generators (“directions”) of the gauge group $SU(2) \times U(1)$. The gauge invariant Higgs-gauge field interaction in the kinetic part of eq. (1.16) gives rise to mass terms for the vector bosons in the non-diagonal form

$$L_{\text{Higgs}} \supset \frac{1}{2} M_W^2 W_\mu^+ W^{-,\mu} + \frac{1}{2} \frac{\nu^2}{2} (W_\mu^3, B_\mu) \begin{pmatrix} g_2^2 & g_1 g_2 \\ g_1 g_2 & g_1^2 \end{pmatrix} \begin{pmatrix} W^{3,\mu} \\ B^\mu \end{pmatrix} \quad (1.21)$$

$$M_W^2 = \frac{1}{2} g_2^2 \nu^2$$

The second term in the eq. (1.21) is not diagonal, and we already have defined the eigenvectors with definite mass, c.f. eq. (1.9). By inverting definitions of Z, A from eq. (1.9), we conclude that also the neutral gauge boson Z acquires mass:

$$\frac{1}{2} \frac{\nu^2}{2} (W_\mu^3, B_\mu) \begin{pmatrix} g_2^2 & g_1 g_2 \\ g_1 g_2 & g_1^2 \end{pmatrix} \begin{pmatrix} W^{3,\mu} \\ B^\mu \end{pmatrix} = \frac{1}{2} M_Z^2 Z_\mu Z^\mu \quad (1.22)$$

$$M_Z^2 = \frac{1}{2} (g_1^2 + g_2^2) \nu^2 = \frac{M_W^2}{\cos^2 \theta_W}$$

while the photon remains massless.

In spite of the introduction of spontaneous symmetry breaking, the mass terms oppose the chiral gauge invariance. One possibility is some kind of “Yukawa coupling” between the fermion field ψ and the Higgs field ϕ , with unknown couplings Y_ψ , which after symmetry breaking (more precisely: after expansion of the Lagrange density around a suitable ground state) again results in the original mass terms, which are now, however (i.e. by introduction of the Higgs field) written in a gauge-invariant way:

$$L_{\text{Yukawa}} = \sum_{g,c=1,2,3} \left(\frac{1}{3} Y_{u_g} (\bar{Q}_g^c)_L \Phi_c (u_g^c)_R + \frac{1}{3} Y_{d_g} (\bar{Q}_g^c)_L \Phi (d_g^c)_R + Y_{e_g} (\bar{L}_g)_L \Phi (e_g)_R \right) \quad (1.23)$$

where Y_{u_g} , Y_{d_g} and Y_{e_g} are up-quark down-quark and charged leptons Yukawa coupling, $\Phi_c \equiv i\sigma_2 \Phi^*$ and $g=1,2,3$ are the generation indices. After the EW symmetry is broken by the nonzero vev ν of the Higgs field, the Yukawa terms in eq. (1.23) yield the mass matrices of quarks and charged leptons:

$$m_{u_g} = Y_{u_g} \nu \quad , \quad m_{d_g} = Y_{d_g} \nu \quad , \quad m_{e_g} = Y_{e_g} \nu \quad (1.24)$$

Neutrinos are massless in the SM.

1.4 Experiment ask for higher-order calculations

The experimental value of the mass of the discovered Higgs boson [40],

$$M_{\text{Higgs}} = 125.09 \pm 0.21 (\text{stat.}) \pm 0.11 (\text{syst.}) \text{ GeV} \quad (1.25)$$

has an uncertainty of just a few permille so that only four years after the discovery of the Higgs boson its mass has become an electroweak precision observable. In order to make full use of this high-accuracy measurement to fully exploit and correctly interpret all the experimental data from colliders, theory predictions for this and other related quantities should be provided at the same level of accuracy as the experimental precision. To be able to do this match with experiment, it is necessary to complement the theoretical predictions at leading order (LO) by the evaluation of quantum effects (radiative corrections). These corrections can be interpreted as a perturbative expansion in the coupling constant. The first non-trivial term is to “one-loop”, the following corrections are of “two-loop” type, and so on. Each order represents an important step in the complexity of the calculation: the one-loop corrections are quite involved but (depending on the model) viable, whereas the higher orders are at the frontier of the current computational capacities. Current experimental accuracy requires the calculation of one- and (for some observables such as Higgs boson masses) also at least some two- loop corrections.

Turning to the specific case of the MSSM (which will be defined in detail in chapters 2 and 3), the Higgs sector can be expressed at lowest order in terms of the gauge couplings, the mass of the *CP*-odd Higgs boson, M_A , and $\tan\beta \equiv \nu_2 / \nu_1$ (the ratio of the two “vacuum

expectation values”).¹² All other masses and mixing angles can therefore be predicted, e.g. the charged Higgs boson mass, which results

$$M_{H^\pm}^2 = M_A^2 + M_W^2 \quad (1.26)$$

at tree-level. M_W denotes the mass of the W boson. However, higher-order contributions can give large corrections to this tree-level relation, as we will show in chapter 3. Furthermore, accurate predictions are required in order to be able of distinguishing experimentally between particles of similar properties, e.g. between the masses of the heaviest MSSM neutral Higgs boson H and A . The general features of the renormalisation procedure needed to evaluate such higher-order correction are described in the following section.

Theoretical predictions having an accuracy at the same level (or better) than the one of the experimental values are crucial to allow to constrain the available parameter spaces of the different considered models and so to discriminate between them. It is of particular interest for instance whether the NMSSM can be phenomenological distinguished from the MSSM by confronting Higgs sector measurements with the corresponding predictions of the two models. Keep in mind that this kind of comparisons, however, requires not only the appropriate theoretical and experimental accuracy, but also to treat the theoretical predictions within a coherent framework where, in some appropriate limit case of the parameter values, we recover the same results from both models, as we will see in the case of the MSSM-limit of the NMSSM in sects. 4.4 and 5.2.

1.5 Renormalisation

Renormalisation is a collection of techniques that are used to treat infinities arising in certain calculations, by altering values of building quantities to compensate for effects of the self-interactions that make the system behaves as if it had a different composition. When describing space and time as a continuum, certain statistical and quantum mechanical constructions are ill-defined. Renormalisation procedures are based on the requirement that certain physical quantities are equal to the observed values.

When developing quantum electrodynamics (QED) in the 1930’s, people discovered that in perturbative calculations many integrals were divergent. The divergences appear in calculations involving Feynman diagrams with closed loops of virtual particles in them.

¹² More details will be given in sect. 3.1.

While virtual particles obey conservation of energy and momentum, they can have any energy and momentum, even one that is not allowed by the relativistic energy-momentum relation $E^2 - p^2 = m^2$ for the observed mass m of that particle. When there is a loop, the momentum of the particles involved in the loop is not uniquely determined by the energies and momenta of incoming and outgoing particles, so to find the amplitude for the loop process one must integrate over *all* possible combinations of energy and momentum that could travel around the loop. These integrals are often *divergent*, that is, they give infinite answers.

1.5.1 Renormalised and bare quantities¹³

The solution was to realize that the quantities initially appearing in the theory's formulae (such as the Lagrangian), representing such things as the electron's electric charge and mass, as well as the normalizations of the quantum fields themselves, did not actually correspond to the physical constants measured in the laboratory. They were “bare” quantities that did not take into account the contribution of virtual-particle loop effects to the physical constants themselves. In order to make contact with reality, the formulae would have to be rewritten in terms of measurable, “renormalised” quantities. The charge of the electron, e.g., would be defined in terms of a quantity measured at a specific kinematic renormalisation point or subtraction point (which will generally have a characteristic energy, called the “renormalisation scale” or simply the “energy scale”).¹⁴ The parts of the Lagrangian left over, involving the remaining portions of the bare quantities, could then be reinterpreted as “counterterms”, involved in divergent diagrams exactly canceling out the problematic divergences for other diagrams.

For example, in the Lagrangian of QED

$$\begin{aligned} L &= \bar{\psi}_B (i\gamma_\mu (\partial^\mu + ie_B A_B^\mu) - m_B) \psi_B - \frac{1}{4} F_{B\mu\nu} F_B^{\mu\nu} \\ &\equiv Z_1 \bar{\psi} (i\gamma_\mu \partial^\mu + ie A^\mu) \psi - Z_0 \bar{\psi} m \psi - \frac{1}{4} Z_3 F_{\mu\nu} F^{\mu\nu} \end{aligned} \quad (1.27)$$

¹³ This section and sect. 1.5.2 are partially based on ideas outlined in chapters 3 and 4 from [42] and chapter 9 from [45].

¹⁴ The renormalisation point/scale is not itself a physical quantity: the physical predictions of the theory, calculated to all orders, should in principle be independent of the choice of this scale, as long as it is within the domain of application of the theory. The parameters in the Lagrangian depend on the renormalisation scale $\bar{\mu}$ in a way which is dictated by the independence of some physical quantities (such as the pole mass or scattering amplitudes) on this scale. One can exploit this fact to calculate the *effective* variation of physical constants with changes in scale. This variation is encoded by the *beta-functions*, and the general theory of this kind of scale-dependence is known as the “renormalisation group”. This *running* provide a convenient means of describing changes in the behavior of a field theory under changes in the energies involved in an interaction.

the original fields and coupling constant are really bare quantities, hence the subscript B above. A term in this Lagrangian, for instance, the electron-photon interaction, can then be written as $-e \bar{\psi} \gamma_\mu A^\mu \psi - (Z_1 - 1) e \bar{\psi} \gamma_\mu A^\mu \psi$. The *physical* constant e , the electron's charge, can then be defined in terms of some specific experiment; we set the renormalisation scale equal to the energy characteristic of this experiment, and the first term gives the interaction we see in the laboratory (up to small, finite corrections from loop diagrams). If the theory is renormalizable, the divergent parts of loop diagrams can all be decomposed into pieces with an algebraic form that can be canceled out by the counterterms.¹⁵ Here the term with the Z_1 counterterm insertion cancels out the divergence from the aforementioned loops.

If some physical quantity M is equal to some finite parameter (the finite part of some bare parameter) in the Lagrangian, m , at tree-level (i.e. without taking into account the loops contributions from perturbation theory), it could no longer be (and usually isn't) equal to m at higher-order: in that case we say that “ M acquires radiative corrections”. It has not to be with the physical properties of the quantity M , but with the definition of the m parameter. If from the same bare Lagrangian we performed two different set of renormalisation conditions, the finite input parameters of that Lagrangian will not be the same (they will be defined differently) in both cases. Let's see for instance the mass term in (1.27):

$$L_m = -m_B \bar{\psi}_B \psi_B \quad . \quad (1.28)$$

Renormalizing of L_m means to realize that actually in this original Lagrangian the quantities are bare quantities, which means that they are divergent, so we can write e.g. m_B as

$$m_B = m + \delta m \quad (1.29)$$

where m is some finite quantity, and the “counterterm” δm is the divergent part. By introducing these finites quantities explicitly so writing down the outputs from the theory in terms of them, it allows us to make predictions. The renormalizability feature of the theory means that all the divergences (from the counterterms contributions and loop calculations) cancel out in the expressions for physical quantities (observables), so we can express these quantities in terms of only the finite quantities m , etc.

¹⁵ Theories which lend themselves to renormalisation require a finite supply of counterterms, and all quantities becoming cutoff-independent at the end of the calculation. The theories gain a finite number of free parameters and therefore do not lose their predictive power.

1.5.2 Regularisation

Regularisation is a procedure through which one can deal with the infinities in the integrals in order to they cancel out with the divergences in the counterterms we introduce in the Lagrangian. An essentially arbitrary modification to the loop integrands, or *regulator*, can make the integrals converge. A regulator has a characteristic regularisation parameter known as the *cutoff*; taking this cutoff to the appropriate limit recovers the original integrals. With the regulator in place, divergent terms in the integrals then turn into finite but cutoff-dependent terms; after canceling out these terms with the contributions from cutoff-dependent counterterms, the cutoff is taken to the appropriate limit and finite physical results are recovered.

Through regularisation, each divergent quantity can be expanded in power of some quantity “ Δ ” that goes to infinity when we take the appropriate limit on the regularisation parameter. As a consequence, the expressions for the observables can be expressed as a Taylor expansion as well, so the cancelation of the divergences in perturbation theory must be satisfied order by order.

One of the most popular in modern use is *dimensional* regularisation, invented by Gerardus ’t Hooft and Martinus J. G. Veltman [41], which tames the integrals by carrying them into a space with a fictitious fractional number of dimensions. This will be the regularisation procedure implemented throughout this work, and it will be described in detail in sect. 4.5.

1.5.3 Renormalisation schemes

The finite+divergent decomposition (1.29) it’s not unique, however. We could write instead

$$m_B = \underbrace{m + \alpha}_{\equiv \tilde{m}} + \underbrace{\delta m - \alpha}_{\equiv \delta \tilde{m}} \quad (1.30)$$

If the shift α is finite, $\tilde{m}$ will be finite (like m) and $\delta \tilde{m}$ will be divergent (like δm). Thus, while the bare quantities of the Lagrangian are unique (they define the theory), their decomposition on finite and divergent parts (and as a consequence the results expressed in terms of that *finite* quantities derived from all the *bare* quantities of the theory) will be different in each case. One calls *scheme* to each specific choice for this decomposition. We find that in practice the notation is usually the same in every scheme, namely “ m ” and “ δm ”, but their definitions are not, and therefore the expressions for the observables in

terms of the “finite parameters” will not be the same (in general) either. It’s in this sense that we say that *the renormalisation scheme define the finite (“renormalised”) parameters* of the theory: the expressions for the observables will not be the same for every scheme; see sect. 6.2.3 for an example. Any relationship will be the same (it will have the same form) regardless of the scheme, however, if it involves *only* observable quantities.

The implementation of a specific renormalisation scheme means to characterize the decomposition (1.29) for each of the independent quantities (input parameter and fields) in the theory through some specific renormalisation conditions. Thus the arbitrariness in choosing the renormalisation scheme, and it is in this sense that we say that *the finite part of a counterterm it’s not uniquely defined*, while when we say that *the divergent part is scheme independent* we mean that the difference between the δm ’s from any two different schemes will be always finite.

Therefore, in actual calculations the counterterms introduced to cancel the divergences in Feynman diagram calculations beyond tree level must be fixed using a set of renormalisation conditions. Among the common renormalisation schemes, an “on-shell” renormalisation for some parameter m means that the decomposition in (1.29) is defined/implemented in such a way that the expression for some physical quantity M is equal to m not only at tree-level but also at higher-order, i.e., M does not receive radiative corrections. A “ $\overline{\text{DR}}$ ” renormalisation, on the other hand, entails that δm (expressed as an expansion in powers of the divergent quantity Δ) vanishes if Δ is set to zero; in this case we say that “ δm is purely divergent”.

Supersymmetry

In particle physics, supersymmetry (SUSY) is a proposed type of space-time symmetry that relates two basic classes of elementary particles: bosons, which have an integer-valued spin, and fermions, which have a half-integer spin. Each particle from one group is associated with a particle from the other, known as its “superpartner”, the spin of which differs by a half-integer. Since no superpartners have been observed yet, if supersymmetry exists it must be a spontaneously broken symmetry so that superpartners may differ in mass [46]. The simplest realization of spontaneously-broken supersymmetry, the so-called Minimal Supersymmetric Standard Model (“MSSM” for short) [5-7], is one of the best studied candidates for physics beyond the SM.

Experiments failed to discover supersymmetric partners so far. Supersymmetric models were and are constrained by a variety of experiments, including measurements of low-energy observables –for example, the anomalous magnetic moment of the muon at Brookhaven [47], the dark matter density measurement [48] and direct detection experiments [49,50]– and by particle collider experiments –including B-physics, Higgs phenomenology and direct searches for superpartners (sparticles), at the Large Electron-Positron Collider, and the Tevatron–. The first mass limits for squarks and gluinos were made at CERN by the UA1 and the UA2 experiments at the Super Proton Synchrotron. LEP later set very strong limits, which in 2006 were extended by the D0 experiment at the Tevatron [51]. The first run of the LHC found no evidence for SUSY either (all results were consistent with the SM), and as a result surpassed existing experimental limits from the LEP and Tevatron and set new limits on the expected parameter ranges [52]. The LHC resumed its search for supersymmetry and other new physics in its second run, and most physicists believe that supersymmetry must be discovered at the LHC if it is responsible for stabilizing the weak scale.

The search for Higgs bosons is a crucial test for SUSY that can be performed with present and future generation accelerators. Accurate predictions for the mass of these particle are therefore necessary to determine the potential for discovery and exclusion of the LHC and future linear colliders, where it would be possible eventually to measure the characteristics of these bosons with high precision [53]. During 2011 and 2012, the LHC discovered a

Higgs boson with a mass of about 125 GeV, and with couplings to fermions and bosons which are consistent with the SM [32]. This value seems to be relatively large for the MSSM, but it can be achieved through large radiative loop corrections (mainly from the top/stop sector): while MSSM predicts that the mass of the lightest Higgs boson should not be much higher than the mass of the Z boson, even in the absence of fine tuning (with the supersymmetry breaking scale on the order of 1 TeV), could reach 130 GeV. The lightest Higgs boson in the MSSM is also SM-like, consistent with measurements of the Higgs boson couplings by far [8,54].

2.1 Theoretical motivations for SUSY

In the SM, the electroweak scale receives enormous Planck-scale quantum corrections. The quantum mechanical interactions of the Higgs boson causes large (quadratically divergent in the renormalisation scale) radiative corrections to the Higgs mass and, unless there is an accidental cancellation, the natural size of this mass becomes so large that it undermines the internal consistency of the theory at the TeV scale (“hierarchy problem”). Supersymmetric models allow stabilizing the Higgs mass, reducing the size of those corrections by having automatic cancellations between partners and superpartners. In these models, scalars are related to fermions, and since fermion masses are only logarithmically divergent, scalar masses inherit the same radiative stability, owing to a minus sign associated with fermionic loops: the quantum contributions from the SM are (partially) canceled out by those of the corresponding supersymmetric partners. The hierarchy between the electroweak and Planck scales is thus achieved in a natural manner, without fine-tuning [46].

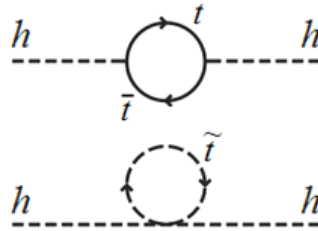


Figure 2.1: Cancellation of the Higgs boson quadratic mass renormalisation between fermionic top quark loop and scalar stop squark tadpole Feynman diagrams in a supersymmetric extension of the Standard Model.

Other attractive features of SUSY is that it allows the unification at high energies of weak, strong and electromagnetic couplings. In a supersymmetric theory, the renormalisation group evolution (“running”) of the three gauge coupling constants of the SM are modified,

and precise high-energy gauge couplings unification is achieved [55]. If superpartners are found in the near future, the apparent success of gauge coupling unification would suggest that a supersymmetric grand unified theory is a promising candidate for high energy scale physics.

SUSY provides candidate particles for dark matter. TeV-scale supersymmetric SM extensions, augmented with a discrete symmetry, typically provides a heavy stable particle which could serve as a weakly interacting massive particle (WIMP) dark matter candidate with a mass scale consistent with thermal relic abundance calculations [31]. In the MSSM, e.g., if R-parity is preserved,¹⁶ the lightest superparticle (LSP) is stable and is a neutralino, which does not have electromagnetic or strong interactions, so it is a WIMP. This makes the LSP a good dark matter candidate and falls into the category of cold dark matter (CDM) particle.

History

J.L. Gervais and B. Sakita (during 1971) [56], Yu. A. Golfand and E.P. Likhtman (also during 1971), and D.V. Volkov and V.P. Akulov (1972) [57], independently discovered supersymmetry in the context of quantum field theory (QFT), a radically new type of symmetry of space-time and fundamental fields, which establishes a relationship between elementary particles of different quantum nature, bosons and fermions,¹⁷ and unifies space-time and internal symmetries of microscopic phenomena. During 1971 Golfand and Likhtman were the first to show that the Poincarè algebra can be extended through introduction of four anticommuting spinor generators (in four dimensions), which later became known as “supercharges”.

Julius Wess and Bruno Zumino (during 1974) [58] identified the characteristic renormalisation features of four-dimensional supersymmetric field theories, which identified them as remarkable QFT’s, and they and Abdus Salam and their fellow researchers introduced early particle physics applications. During 1975 the Haag-Lopuszanski-Sohnius theorem [59] analysed all possible superalgebras in the general form, including those with an extended number of the supergenerators and central charges. The

¹⁶ R-parity is a proposed symmetry where “+1” is the value assigned for the SM-like particles and “-1” for the SUSY partners, see sect. 2.2.1. Any interaction term of the theory must have conserved R-parity, which means that the product of the R-parity values of all the incoming and of the outgoing particles must be equal, hence only processes that involve an even number of supersymmetric particles are allowed. This symmetry was assumed throughout all this work, both for calculations in the context of the MSSM as for its extension to the NMSSM.

¹⁷ According to the spin-statistics theorem, bosonic fields commute while fermionic fields anticommute; combining the two kinds of fields into a single algebra requires the introduction of a Z_2 -grading under which the bosons are the even elements and the fermions are the odd elements.

first realistic supersymmetric version of the SM was proposed during 1977 by Pierre Fayet and is the MSSM [60].

2.2 Minimal Supersymmetric Standard Model

The MSSM is the minimal supersymmetrical model as it considers only the minimum number of new particle states and new interactions consistent with phenomenology. It pairs bosons with fermions in such a way that each particle of the SM corresponds to another of opposite statistics; therefore every SM particle has a partner that has yet to be discovered. The MSSM imposes R-parity to explain the stability of the proton, and arbitrary/generic soft-breaking terms are added to the theory which break SUSY explicitly in order to parametrise the relevant features of supersymmetry breaking. This all means that there are 105 new parameters in the model.

2.2.1 Field content¹⁸

The MSSM incorporates bosonic supersymmetric partners for the SM fermions, called *sfermions*, as well as fermionic partners for the vector bosons and the Higgs bosons (*gauginos* and *higgsinos*):¹⁹

- There are four *neutralinos*, $\tilde{\chi}_{1,\dots,4}^0$, that are fermions and are electrically neutral. These four states are mixtures of the *Bino* and the neutral *Wino* (the neutral electroweak *Gauginos*) and the neutral *Higgsinos*. As the neutralinos are Majorana fermions, each of them is identical with its antiparticle. They primarily appear as particles in cascade decays of heavier particles, usually originating from colored supersymmetric particles such as squarks or gluinos. In R-parity conserving models, the lightest SUSY particle is stable and all supersymmetric cascades decays end up decaying into it; the parameters should be chosen such that this particle is the lightest neutralino, to have a viable dark matter candidate.

¹⁸ This section condensed chapters 6 and 8 from [46].

¹⁹ Incorporating supersymmetry into the SM requires doubling the number of particles: there is no way that any of the particles in the SM can be superpartners of each other.

- There are two *charginos*, which are fermions and are electrically charged. They are typically labeled as $\tilde{\chi}_{1,2}^{\pm}$ and are mixed states of the SUSY partners of $W^{\pm}$ and the charged Higgs bosons.
- The *squarks* are the scalar superpartners of the quarks and there is one version for each SM quark. There may be a remarkable left-right mixing of the *stops* $\tilde{t}$ (and of the *sbottoms* $\tilde{b}$) because of the high masses of the partner quarks top and bottom [61]. The number of squark particles are 2 for each quark, since the former are scalar particles (spin 0) and supersymmetry requires they match in spin degrees of freedom to the ones of the quarks (which are spin $\frac{1}{2}$ fermions). They decay to quarks and neutralinos or charginos, which further decay.
- *Gluinos* are the Majorana fermionic partners of the gluons, which means that they are their own antiparticles. They can only decay to a quark and a squark, and because gluinos are Majorana, they can decay to a quark+anti-squark or an anti-quark+squark with equal probability.
- *Sleptons* are the scalar partners of the leptons of the SM. As for the squarks, SUSY requires the existence of two slepton particles for each SM lepton. Sleptons will typically be searched for in direct production or in decays of charginos and neutralinos.

In the case of the Higgs boson, the supersymmetric extension of the SM is somewhat more involved. For most of the SM particles, doubling is very straightforward. But a single *higgsino* (the fermionic superpartner of the Higgs boson) would lead to inconsistencies due to gauge anomalies, and as a result it is not possible with a single Higgs doublet to get a theory supersymmetric and gauge invariant at the same time. The simplest theory turns out to be one with two higgsinos and therefore two scalar Higgs doublets. Another reason to use two scalar doublets instead of one is to have a supersymmetric theory with Yukawa couplings between Higgs and quarks (terms which generate the quarks masses) of both “up” and “down” types. In the SM, the down-type quarks couple to the Higgs field and the up-type quarks to its complex conjugate; but in a supersymmetric theory this is not allowed, so two types of Higgs fields are needed. The final MSSM particle spectrum is therefore as follows:

❖ Leptons/sleptons & quarks/squarks:

SM particle			Superpartner		
symbol	spin	R-parity	symbol	spin	R-parity
ν_g, e_g	1/2	+1	$(\tilde{\nu}_g)_{1,2}, (\tilde{e}_g)_{1,2}$	0	-1
u_g^c, d_g^c			$(\tilde{u}_g^c)_{1,2}, (\tilde{d}_g^c)_{1,2}$		

❖ EW gauge bosons/bosinos & gluons/gluinos:

SM particle			Superpartner		
symbol	spin	R-parity	symbol	spin	R-parity
B	1	+1	$\tilde{B}$	1/2	-1
$W_{1,2,3}$			$\tilde{W}_{1,2,3}$		
$g_{1,\dots,8}^c$			$\tilde{g}_{1,\dots,8}^c$		

❖ Higgs bosons & higgsinos:

SM particle			Superpartner		
symbol	spin	R-parity	symbol	spin	R-parity
$H_{1,2}$	0	+1	$\tilde{H}_{1,2}$	1/2	-1

with the subscripts $g = 1, 2, 3$ for the generations and $c = 1, 2, 3$ for the strong color charge. The mixing between the fields B and W_i gives rise to the photon γ and the Z and $W^\pm$ bosons (as in the SM), while the mixing between the $\tilde{B}$ and $\tilde{W}_i$ and the higgsinos $\tilde{H}_{1,2}$ gives rise to the charginos $\tilde{\chi}_{1,2}^\pm$ and the neutralinos $\tilde{\chi}_{1,\dots,4}^0$. From the eight d.o.f. in the two Higgs doublets $H_{1,2}$, three of them constitutes the longitudinal polarizations of Z and $W^\pm$ (which are therefore massive particles), while the remaining ones mix in three neutral physical fields $h_{1,2,3}$ (also labeled as h , H and A in the real case) and two charged Higgs fields $H^\pm$.

The MSSM Lagrangian

The model contains vector superfields associated with the SM gauge groups, which contain the vector bosons and the associated gauginos. It has also chiral superfields for the SM fermions and Higgs bosons (and their respective superpartners). The structure of the MSSM Lagrangian is:

- The first piece is the Kähler potential for the matter and Higgs fields, which produces the kinetic terms for those fields.
- The second piece is the gauge field “superpotential” that produces the kinetic terms for the gauge bosons and gauginos [62].
- It has in turn a superpotential for matter and Higgs fields, which generates the Yukawa couplings Y for the SM fermions and the mass term for the Higgsinos. After imposing R-parity, the renormalizable, gauge invariant operators in this superpotential are [63]

$$W = \sum_{g=1,2,3} (Y_{d_g} (\hat{H}_1 \cdot \hat{Q}_g) \hat{d}_g + Y_{e_g} (\hat{H}_1 \cdot \hat{L}_g) \hat{e}_g - Y_{u_g} (\hat{H}_2 \cdot \hat{Q}_g) \hat{u}_g) + \mu H_2 H_1 \quad (2.1)$$

where the notation for L, Q, e, u, d is as in (1.1), but with the hat indicating superfields.

Finally, it has the contribution of the soft²⁰ breaking of the supersymmetry, which contains the great majority of the free parameters of the MSSM. There is an (unknown) sector of the theory that is responsible for the SUSY-breaking, and it is implemented by introducing operators that violate SUSY explicitly in order to parametrise in the most general possible way the ignorance of the underlying dynamics for such a breaking:

- Gaugino masses:

$$L_{\text{soft}}^{\text{MSSM}} \supset -\frac{1}{2} (M_1 \tilde{B} \tilde{B} + M_2 \tilde{W}_i \tilde{W}_i + M_3 \tilde{G} \tilde{G} + \text{h.c.}) \quad (2.2)$$

This summarizes the soft SUSY-breaking gaugino mass terms for the $U(1)_Y$, $SU(2)_L$, and $SU(3)_c$ gaugino fields $\tilde{B}$ (bino), $\tilde{W}_{1,2,3}$ (wino) and $\tilde{G}$ (gluino), with gaugino mass parameters M_1 , M_2 and M_3 .

- Soft masses for the scalar fields:

$$\begin{aligned} L_{\text{soft}}^{\text{MSSM}} \supset & -m_1^2 H_1^\dagger H_1 - m_2^2 H_2^\dagger H_2 - \sum_{g=1,2,3} [M_{\tilde{Q}_g}^2 \tilde{Q}_g^\dagger \tilde{Q}_g + M_{\tilde{L}_g}^2 \tilde{L}_g^\dagger \tilde{L}_g \\ & + M_{(\tilde{u}_g)_R}^2 (\tilde{u}_g)_R^* (\tilde{u}_g)_R + M_{(\tilde{d}_g)_R}^2 (\tilde{d}_g)_R^* (\tilde{d}_g)_R + M_{(\tilde{e}_g)_R}^2 (\tilde{e}_g)_R^* (\tilde{e}_g)_R] \end{aligned} \quad (2.3)$$

$$\tilde{Q}_g \equiv \begin{pmatrix} \tilde{u}_g \\ \tilde{d}_g \end{pmatrix}_L, \quad \tilde{L}_g \equiv \begin{pmatrix} \tilde{\nu}_g \\ \tilde{e}_g \end{pmatrix}_L$$

where the tilde denotes the scalar component of the corresponding superfield, m_1^2 , m_2^2 , $M_{\tilde{Q}_g}^2$, $M_{\tilde{L}_g}^2$, $M_{(\tilde{u}_g)_R}^2$, $M_{(\tilde{d}_g)_R}^2$ and $M_{(\tilde{e}_g)_R}^2$ are the soft-SUSY breaking mass parameters for the Higgs bosons, the squarks and the sleptons, respectively.

- There are the A terms which are given by

$$L_{\text{soft}}^{\text{MSSM}} \supset \sum_{g=1,2,3} (-Y_{e_g} A_{e_g} (H_1 \cdot \tilde{L}_g) \tilde{e}_g^* - Y_{d_g} A_{d_g} (H_1 \cdot \tilde{Q}_g) \tilde{d}_g^* + Y_{u_g} A_{u_g} (H_2 \cdot \tilde{Q}_g) \tilde{u}_g^* + \text{h.c.}) \quad (2.4)$$

²⁰ “Soft” supersymmetry breaking refers to a type of SUSY-breaking that preserves, however, its property of avoiding the appearance of ultraviolet divergences for scalar masses.

where A_{u_g} , A_{d_g} and A_{e_g} are the soft-SUSY breaking trilinear couplings of the squarks and sleptons, and they are complex numbers in general.

2.2.2 Problems with the MSSM

From the problems with the MSSM, most of them falling into understanding the parameters, e.g.:

- The “mu problem”: The higgsino mass parameter μ should have the same order of magnitude as the electroweak scale, many orders of magnitude smaller than that of the Planck scale, which is the natural cutoff scale. This brings about a problem of naturalness.
- Smallness of CP violating phases: since no CP violation additional to that predicted by the SM has been discovered so far, the additional terms in the MSSM Lagrangian must be, at least approximately, CP invariant.

The unconstrained MSSM has more than 100 parameters in addition to the Standard Model parameters. This makes any phenomenological analysis (e.g. finding regions in parameter space consistent with observed data) impractical. Under the three assumptions of no new source of CP -violation, no flavour changing neutral currents and first and second generation universality, one can reduce the number of additional parameters to the following 19 quantities:

Symbol	Description
μ	higgsino mass parameter
$M_{\tilde{Q}}, M_{\tilde{u}_R}, M_{\tilde{d}_R}$	first and second generation squark masses
$M_{\tilde{L}}, M_{\tilde{e}_R}$	first and second generation slepton masses
$M_{\tilde{Q}_3}, M_{\tilde{t}_R}, M_{\tilde{b}_R}$	third generation squark masses
$M_{\tilde{L}_3}, M_{\tilde{\tau}_R}$	third generation slepton masses
A_t, A_b, A_τ	third generation trilinear couplings
M_A	mass of the pseudoscalar Higgs boson
$\tan \beta$	ratio of the vacuum expectation values of the two Higgs doublets
M_1	bino mass parameter
M_2	wino mass parameter
M_3	gluino mass parameter

The light CP -even Higgs mass is a prediction of the model. There are no supersymmetric parameters to make the SM-like Higgs mass a free parameter in the MSSM. Nevertheless, the discovery of the Higgs with a mass of 125 GeV does not rule out the MSSM. The SM-like Higgs tree-level mass is limited to be less than the Z mass, but since supersymmetry is broken there are large radiative corrections that can increase this mass. The dominantly contributions at one-loop level, in the so-called “maximal stop mixing”, makes possible to push the Higgs mass to 125 GeV without decoupling the top squark or adding new dynamics to the MSSM (see chapter 5 for details). Still, as the Higgs was found at the LHC at around that mass value along with no superparticles, this also hints at new dynamics beyond the MSSM, such as the Next-to-Minimal Supersymmetric Standard Model (NMSSM).

2.3 Next-to-Minimal Supersymmetric Standard Model

The NMSSM²¹ is a supersymmetric extension to the SM that adds an additional singlet chiral superfield to the MSSM in order to dynamically generate the μ term, solving in this way the so-called μ -problem: the MSSM does not explain why the μ parameter in the superpotential term $\mu H_2 H_1$ is at the electroweak scale. The superpotential for the NMSSM is given by

$$W^{\text{NMSSM}} = W_{\text{Yuk}}^{\text{NMSSM}} + \lambda \hat{S} (\hat{H}_2 \cdot \hat{H}_1) + \frac{1}{3} \kappa \hat{S}^3 \quad (2.5)$$

where the Yukawa couplings for the (s)fermions are given as in the MSSM (see 2.1):

$$W_{\text{Yuk}}^{\text{NMSSM}} = Y_{e_g} (\hat{H}_1 \cdot \hat{L}_g) \hat{e}_g + Y_{d_g} (\hat{H}_2 \cdot \hat{Q}_g) \hat{u}_g + Y_{d_g} (\hat{H}_1 \cdot \hat{Q}_g) \hat{d}_g \quad (2.6)$$

with the left-handed quark and lepton superfields, $\hat{Q}_g, \hat{L}_g$, and the right-handed ones, $\hat{u}_g, \hat{d}_g, \hat{e}_g$, and the Higgs superfields $\hat{H}_1, \hat{H}_2$ and $\hat{S}$. The $SU(2)_L$ -invariant product is denoted by a dot. Since we will focus on the CP -conserving NMSSM, all the Yukawa-type couplings $Y_{u_g}, Y_{d_g}, Y_{e_g}$, as well as λ and κ , can be chosen as real parameters.

This is the most general supersymmetric Higgs superpotential, including all renormalizable couplings under the consideration of R-parity and CP -invariance [19], with two Higgs doublets and one singlet chiral superfields, with the addition of invariance under a discrete Z_3 tranformation (i.e. $\Phi \rightarrow e^{2\pi i/3} \Phi$, with Φ being the observable superfields, see [30] for

²¹ See [19] and [20] for exhaustive reviews.

motivations and discussion). Since the superpotential has mass dimension three, the couplings λ and κ are dimensionless, hence the μ -problem of the MSSM is solved –the superpotential of the NMSSM is scale invariant. The role of the λ term is then to generate an effective μ term via the scalar component of the singlet getting a vacuum expectation value $\langle S \rangle$:²²

$$\mu_{\text{eff}} = \lambda \langle S \rangle \quad (2.7)$$

The parameters λ and κ are genuine to the NMSSM and can be chosen freely. However, in order to keep the validity of perturbation theory up to the GUT scale, λ and κ are bound from above by [19,30]

$$\lambda^2 + \kappa^2 \lesssim 0.5 \quad . \quad (2.8)$$

Throughout this work this constraint will be assumed for the NMSSM, where $Y_t \approx 1 > \lambda$ is ensured.

Soft SUSY-breaking in the NMSSM gives rise to the real (in the CP -conserving case) trilinear soft SUSY-breaking parameters A_λ and A_κ , as well as to the soft-SUSY breaking mass term m_S^2 of the scalar singlet field. Together with the soft SUSY-breaking Lagrangian of the MSSM we have

$$L_{\text{soft}}^{\text{NMSSM}} = L_{\text{soft}}^{\text{MSSM}} - m_S^2 |S|^2 - (\lambda A_\lambda S (H_2 \cdot H_1) + \frac{1}{3} \kappa A_\kappa S^3 + \text{h.c.}) \quad (2.9)$$

where $L_{\text{soft}}^{\text{MSSM}}$ is the MSSM soft SUSY-breaking Lagrangian described in sect. 2.1.2.

Due to the additional singlet S , the NMSSM alters in general the phenomenology of both the Higgs sector and the neutralino sector compared to the MSSM. In the later we encounter five physical Higgs bosons; in the former we have two more Higgs bosons, that is, seven physical Higgs bosons in total. The Higgs sector is therefore much richer compared to the MSSM. In the case where CP is conserved (which we assume throughout this work), the NMSSM Higgs physical states can be classified as three CP -even Higgs bosons, $h_{1,2,3}$, two CP -odd Higgs bosons, $A_{1,2}$, and the charged Higgs boson pair $H^\pm$. The lightest but also the second lightest CP -even neutral Higgs boson can be interpreted as the signal observed at about 125 GeV [21]. On the other hand, the spin-1/2 SUSY partner of the singlet Higgs (called *singlino*) gives a fifth neutralino, compared to the four neutralinos

²² It is usual in the literature to use the notation “ μ_{eff} ” for this NMSSM quantity, in order to emphasize the fact that it is not an input parameter of the theory but a derived combination of them. In the rest of this work we will refer to it just as “ μ ”, however, in order to make more transparent the MSSM-limit. There is no other parameter with this same name in the NMSSM, so there is no possible confusion.

of the MSSM. The singlino does not couple to gauge bosons, gauginos, leptons, sleptons, quarks or squarks.

An overview about the differences in the parameters and the field contents between the MSSM and the NMSSM is given in tab. 2.1.

	MSSM	NMSSM
Higgs mix couplings	μ	λ, κ
soft SUSY Higgs mass terms	m_1^2, m_2^2	m_1^2, m_2^2, m_3^2
soft SUSY couplings	μB	A_λ, A_κ
CP -even scalars	h, H	h_1, h_2, h_3
CP -odd scalars	A	A_1, A_2
neutralinos	$\tilde{\chi}_{1,2,3,4}^0$	$\tilde{\chi}_{1,2,3,4,5}^0$
free parameters at tree-level	$\tan \beta, M_A$	$\tan \beta, \langle S \rangle, \lambda, \kappa, A_\lambda, A_\kappa$

Table 2.1: Additional couplings and particle spectrum for the CP -conserving NMSSM, compared to the MSSM with real parameters. Other sparticle masses and couplings are considered as in the MSSM.

MSSM Higgs masses

Experimental searches for the neutral MSSM Higgs bosons have been performed at LEP [64,65], placing important restrictions on the parameter space. At Run II of the Tevatron the search was continued, and now the LHC has taken the lead. Besides the discovery of a SM Higgs-like boson, the LHC searches already placed stringent bounds, in particular in the regions of small M_A and large $\tan\beta$ [66]. If supersymmetry is realized in nature and the charged Higgs boson mass is $M_{H^\pm} \lesssim 1.5 \text{ TeV}$, the additional Higgs bosons could be detectable at the LHC [67]. This would yield some initial data on the extended Higgs sector. Equally important, the additional Higgs bosons could also be produced at a future linear e^-e^+ collider such as the ILC [53,68,69] or CLIC [70,71], and a precise determination of the Higgs boson properties (either of the light Higgs boson at $\sim 125 \text{ GeV}$ or heavier MSSM Higgs bosons within the kinematic reach) will be possible [53,69,72,73]. Results on the combination of LHC and ILC results for the neutral MSSM Higgs sector has been discussed in [74,75].

The charged Higgs bosons of the MSSM (or a more general Two Higgs Doublet Model (THDM)) have also been searched for at LEP as well [76], yielding a bound of $M_{H^\pm} \gtrsim 80 \text{ GeV}$. The LHC places bounds on the charged Higgs mass, as for the neutral heavy MSSM Higgs bosons. For $m_{H^\pm} < m_t$ (with m_t denoting the mass of the top quark) the charged Higgs boson is mainly produced from top quarks and decays mainly as $H^\pm \rightarrow \tau \nu_\tau$. For $m_{H^\pm} > m_t$ the charged Higgs boson is mainly produced together with a top quark and the dominant decay channels are $H^\pm \rightarrow tb, \tau \nu_\tau$, where the latter is the main search channel. At the LC, for $M_{H^\pm} \lesssim \sqrt{s}/2$ a high-precision determination of the charged Higgs boson properties could be achieved [53,69,72,73].

For the MSSM the status of higher-order corrections to the masses and mixing angles in the neutral Higgs sector is quite advanced. The complete one-loop result within the MSSM is known [77-82]. The by far dominant one-loop contribution is the $O(\alpha_t)$ term due to top and stop loops (here $\alpha_f = Y_f^2/(4\pi)$, with Y_f denoting the fermion Yukawa coupling). Beyond the one-loop order, the computation of the two-loop corrections has meanwhile reached a stage where all the presumably dominant contributions are available. The corrections of $O(\alpha_t \alpha_s)$ [16,83-87] and $O(\alpha_t^2)$ [88,89], as well as the corresponding

corrections of $O(\alpha_b\alpha_s)$ and $O(\alpha_t\alpha_b)$ [90], are known since a few years already. The leading $O(\alpha_t\alpha_s)$ two-loop contributions for the three neutral Higgs bosons in the case of complex soft SUSY-breaking parameters have been obtained in [91]. For the (s)bottom corrections, which are mainly relevant for large values of $\tan\beta$, an all-order resummation of the $\tan\beta$ -enhanced term of $O(\alpha_b(\alpha_s \tan\beta)^n)$ is performed [92-96]. These corrections, together with a resummation of leading and subleading logarithms from the top/scalar-top sector [18] and momentum-dependent two-loop contributions [97,98], are included in FEYNHIGGS [13-18]. The remaining theoretical uncertainty on the lightest CP-even Higgs-boson mass within the MSSM from unknown higher-order contributions has been estimated to be at the level of about 3 GeV for scalar top masses at the TeV-scale, where the actual uncertainty depends on the considered parameter region [13,99,100].

Also the mass of the charged Higgs boson is affected by higher-order corrections. Because of the (*CP*-violating) mixing between all three neutral Higgs bosons, in the MSSM with complex soft SUSY-breaking parameters usually the charged Higgs mass is chosen as independent (on-shell) input parameter, which by construction does not receive any higher-order corrections. In the *CP*-conserving case, on the other hand, where usually M_A instead of $M_{H^\pm}$ is chosen as independent input parameter, the mass of the charged Higgs boson can be predicted in terms of the other parameters and thus receives a shift from the higher-order amplitudes, and the corresponding self-energy contribution can be utilized to obtain corrections of $O(\alpha_t\alpha_s)$ to the mass $M_{H^\pm}$. The status for the charged Higgs mass was somewhat less advanced as compared to the neutral Higgs bosons. First, in [101] leading corrections to the relation given in eq. (1.26) have been evaluated. The one-loop corrections from t/b and $\tilde{t}/\tilde{b}$ loops have been derived in [102,103]. A nearly complete one-loop calculation, neglecting the terms suppressed by higher powers of the SUSY mass scale, was presented in [104]. The first full one-loop calculation in the Feynman-diagrammatic approach has been performed in [105], but the numerical evaluation focused on particular parameter values, mostly excluded nowadays by the LEP Higgs searches [64,65,76]. It was re-evaluated later in [15].²³ Our results extended these previously known in the literature in several ways. We perform a more general numerical analysis in the *CP*-conserving case, including the full one-loop corrections. Furthermore, in our article [1] for the first time explicit two-loop corrections to $M_{H^\pm}$ were analysed, which will be briefly mention here.

²³ Also the $O(\alpha_t^2)$ two-loop corrections to the mass of the charged Higgs bosons in the real MSSM exist now [117] and have been included into FEYNHIGGS.

3.1 Higgs sector of the (complex) MSSM

The Higgs sector of the MSSM contains two doublets H_1, H_2 , of non-zero vacuum expectation values (vev's) v_1 and v_2 . They can be decomposed in the following way:

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1 - \rho i \chi_1) \\ -\rho \phi_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = e^{i\xi} \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2 + i\chi_2) \end{pmatrix} \quad (3.1)$$

being ϕ_i, χ_i real fields, while $\phi_i^- = (\phi_i^+)^*$. The Higgs potential reads [62]

$$\begin{aligned} V_{\text{Higgs}}^{\text{MSSM}} &= (m_1^2 + \mu^2) H_1^\dagger \cdot H_1 + (m_2^2 + \mu^2) H_2^\dagger \cdot H_2 - 2 \text{Re} (m_{12}^2 \varepsilon_{ab} H_1^a H_2^b) \\ &\quad + \frac{1}{8} (g_1^2 + g_2^2) (H_1^\dagger \cdot H_1 - H_2^\dagger \cdot H_2)^2 + \frac{1}{2} g_2^2 |H_1^\dagger \cdot H_2|^2 \\ &\equiv \text{const.} - \eta (T_{\phi_1}, T_{\phi_2}, T_{\chi_1}, T_{\chi_2}) (\phi_1, \phi_2, \chi_1, \chi_2)^T \\ &\quad + \frac{1}{2} (\phi_1, \phi_2, \chi_1, \chi_2) M_{\phi\phi\chi\chi}^2 (\phi_1, \phi_2, \chi_1, \chi_2)^T + (\phi_1^-, \phi_2^-) M_{\phi^\pm\phi^\pm}^2 (\phi_1^+, \phi_2^+)^T + \dots \end{aligned} \quad (3.2)$$

where m_1, m_2, m_{12} are soft SUSY-breaking parameters, $g_{1,2}$ are the gauge couplings of $SU(2)$ and $U(1)$, and ε_{ab} is a Levi-Civita antisymmetric tensor (sum is understood in the indexes a and b). We also introduce the parameters ρ, η in order to contemplate and trail the different conventions used in the literature, e.g. the ones from tab. 3.1, which correspond to two of the articles consulted in this work and which exemplified the two most common choices.²⁴

Paper	ρ	ε_{12}	η
[106]	-1	-1	-1
[15]	+1	+1	+1

Table 3.1: Typical conventions used in the literature for the sings $\rho, \varepsilon_{12}, \eta$ defined in eqs. (3.1) and (3.2).

The vev's define the parameter β :

$$v_1 = v \cos \beta, \quad v_2 = v \sin \beta, \quad \tan \beta = \frac{v_2}{v_1}, \quad 0 \leq \beta \leq \frac{\pi}{2} \quad (3.3)$$

²⁴ Note that $m_{12}^2 \varepsilon_{ab} H_1^a H_2^b$ contains only constant, linear and bilinear contributions in the fields, so that the sign of ε_{12} does not affect the Feynman rules for the vertexes.

$$\nu \equiv \sqrt{\nu_1^2 + \nu_2^2} = \frac{\sqrt{2}M_Z s_W c_W}{e} = \frac{\sqrt{2}M_W s_W}{e}$$

This vev ν can be use to express the couplings $g_{1,2}^2$ in function of the masses of the weak gauge bosons, in the same way as in the SM:

$$M_Z^2 = \frac{1}{2}(g_1^2 + g_2^2)\nu^2 \quad , \quad M_W^2 = \frac{1}{2}g_2^2\nu^2 \quad , \quad \cos\theta_W \equiv \frac{M_W}{M_Z} \quad (3.4)$$

From now on we will use the following notation for the trigonometric functions (the angles θ_W and $\theta_{\tilde{t}}$ will be defined later in this chapter):

$$\begin{aligned} s_{n\beta} &= \sin(n\beta) \quad , \quad c_{n\beta} = \cos(n\beta) \\ s_W &= \sin\theta_W \quad , \quad c_W = \cos\theta_W \\ s_{n\tilde{t}} &= \sin(n\theta_{\tilde{t}}) \quad , \quad c_{n\tilde{t}} = \cos(n\theta_{\tilde{t}}) \end{aligned} \quad (3.5)$$

3.1.1 Tadpoles

The coefficients for the terms linear in the fields are:

$$\begin{aligned} T_{\phi_1} &= -\eta \frac{\partial V_{\text{Higgs}}^{\text{cMSSM}}}{\partial \phi_1} \Big|_{\phi_{1,2}=\chi_{1,2}=0} = -\sqrt{2}\nu \eta (\tilde{m}_1^2 c_\beta + \frac{1}{4}\nu^2(g_1^2 + g_2^2)c_\beta c_{2\beta} - \varepsilon_{12}\mu B \cos \xi' s_\beta) \equiv \sqrt{2}\nu \bar{T}_{\phi_1} \\ T_{\phi_2} &= -\eta \frac{\partial V_{\text{Higgs}}^{\text{cMSSM}}}{\partial \phi_2} \Big|_{\phi_{1,2}=\chi_{1,2}=0} = -\sqrt{2}\nu \eta (\tilde{m}_2^2 s_\beta - \frac{1}{4}\nu^2(g_1^2 + g_2^2)s_\beta c_{2\beta} - \varepsilon_{12}\mu B \cos \xi' c_\beta) \equiv \sqrt{2}\nu \bar{T}_{\phi_2} \\ T_{\chi_1} &= -\eta \frac{\partial V_{\text{Higgs}}^{\text{cMSSM}}}{\partial \chi_1} \Big|_{\phi_{1,2}=\chi_{1,2}=0} = \sqrt{2}\nu \rho \eta \varepsilon_{12}\mu B \sin \xi' s_\beta \\ T_{\chi_2} &= -\eta \frac{\partial V_{\text{Higgs}}^{\text{cMSSM}}}{\partial \chi_2} \Big|_{\phi_{1,2}=\chi_{1,2}=0} = -\sqrt{2}\nu \eta \varepsilon_{12}\mu B \sin \xi' c_\beta \end{aligned} \quad (3.6)$$

being $B \equiv |m_{12}^2|/\mu$, $\xi' \equiv \xi + \arg(m_{12}^2)$, and $\tilde{m}_i^2 \equiv m_i^2 + \mu^2$. The tadpoles must be set to zero *at the end* of the calculation, in order to guarantee the vev's indeed correspond to a minimum of the Higgs potential. These tadpole quantities acquire, however, non-null corrections beyond tree-level.

The mixing angle α from the neutral sector will be defined later (see eq. (3.16) below), but we introduce them here to express the corresponding tadpoles in the physical neutral basis:

$$\begin{aligned}
T_H &= T_{\phi_1} c_\alpha + T_{\phi_2} s_\alpha \\
&= -\sqrt{2}\nu\eta(\tilde{m}_1^2 c_\alpha c_\beta + \tilde{m}_2^2 s_\alpha s_\beta + \frac{1}{4}\nu^2(g_1^2 + g_2^2)c_{\alpha+\beta}c_{2\beta} - \varepsilon_{12}\mu B \cos \xi' s_{\alpha+\beta}) \equiv \sqrt{2}\nu \bar{T}_H \\
T_h &= -T_{\phi_1} s_\alpha + T_{\phi_2} c_\alpha \\
&= \sqrt{2}\nu\eta(\tilde{m}_1^2 s_\alpha c_\beta - \tilde{m}_2^2 c_\alpha s_\beta + \frac{1}{4}\nu^2(g_1^2 + g_2^2)s_{\alpha+\beta}c_{2\beta} + \varepsilon_{12}\mu B \cos \xi' c_{\alpha+\beta}) \equiv \sqrt{2}\nu \bar{T}_h \\
T_G &= T_{\chi_1} c_\beta + \rho T_{\chi_2} s_\beta = 0 \\
T_A &= -\rho T_{\chi_1} s_\beta + T_{\chi_2} c_\beta = -\sqrt{2}\nu\eta\varepsilon_{12}\mu B \sin \xi' \equiv \sqrt{2}\nu \bar{T}_A
\end{aligned} \tag{3.7}$$

Inverting the expressions for $T_{\phi_1}, T_{\phi_2}, T_A$, we can express $\tilde{m}_2^2$ and m_{12}^2 in terms of the tadpoles:

$$\begin{aligned}
\tilde{m}_2^2 &= (\tilde{m}_1^2 c_\beta^2 + \frac{1}{4}\nu^2(g_1^2 + g_2^2)c_{2\beta} + \eta \bar{T}_{\phi_1} c_\beta - \eta \bar{T}_{\phi_2} s_\beta) / s_\beta^2 \\
\varepsilon_{12}\mu B \cos \xi' &= (\tilde{m}_1^2 c_\beta + \frac{1}{4}\nu^2(g_1^2 + g_2^2)c_{2\beta}c_\beta + \eta \bar{T}_{\phi_1}) / s_\beta \\
\varepsilon_{12}\mu B \sin \xi' &= -\eta \bar{T}_A
\end{aligned} \tag{3.8}$$

3.1.2 Bilinear contributions

The terms of this potential bilinear in the ϕ_i 's and χ_i 's give rise to the neutral Higgs mass matrix, which defines the mixing between these gauge eigenstates, and which (at LO) is given by

$$\mathbf{M}_{\phi\chi\chi}^2 = (\mathbf{M}_{\phi\phi\chi}^2)^T = \begin{pmatrix} m_{\phi_1\phi_1}^2 & m_{\phi_1\phi_2}^2 & m_{\phi_1\chi_1}^2 & m_{\phi_1\chi_2}^2 \\ * & m_{\phi_2\phi_2}^2 & m_{\phi_2\chi_1}^2 & m_{\phi_2\chi_2}^2 \\ * & * & m_{\chi_1\chi_1}^2 & m_{\chi_1\chi_2}^2 \\ * & * & * & m_{\chi_2\chi_2}^2 \end{pmatrix} \tag{3.9}$$

being $m_{\phi_i\phi_j}^2 \equiv \frac{\partial^2 V_{\text{Higgs}}^{\text{cMSSM}}}{\partial \phi_i \partial \phi_j} \bigg|_{\phi_{1,2}=\chi_{1,2}=0}$, so that

$$\begin{aligned}
m_{\phi_1\phi_1}^2 &= \tilde{m}_1^2 + \frac{1}{4}\nu^2(g_1^2 + g_2^2)(1 + 2c_{2\beta}) \\
m_{\phi_1\phi_2}^2 &= -\frac{1}{4}\nu^2(g_1^2 + g_2^2)s_{2\beta} - \varepsilon_{12}\mu B \cos \xi' \\
m_{\phi_1\chi_1}^2 &= 0 \\
m_{\phi_1\chi_2}^2 &= \varepsilon_{12}\mu B \sin \xi' \\
m_{\phi_2\phi_2}^2 &= \tilde{m}_2^2 + \frac{1}{4}\nu^2(g_1^2 + g_2^2)(1 - 2c_{2\beta}) \\
m_{\phi_2\chi_1}^2 &= -\rho \varepsilon_{12}\mu B \sin \xi'
\end{aligned}$$

$$\begin{aligned}
m_{\phi_2 \chi_2}^2 &= 0 \\
m_{\chi_1 \chi_1}^2 &= \tilde{m}_1^2 + \frac{1}{4} \nu^2 (g_1^2 + g_2^2) c_{2\beta} \\
m_{\chi_1 \chi_2}^2 &= -\rho \varepsilon_{12} \mu B \cos \xi' \\
m_{\chi_2 \chi_2}^2 &= \tilde{m}_2^2 - \frac{1}{4} \nu^2 (g_1^2 + g_2^2) c_{2\beta}
\end{aligned} \tag{3.10}$$

Note that if we consider the MSSM with real parameters, there is no mixing between the CP -even²⁵ fields $\phi_{1,2}$ and the CP -odd fields $\chi_{1,2}$, because $\sin \xi' = 0$.

The hermitian 2×2 -matrix $M_{\phi^\pm \phi^\pm}^2$ for the charged states $\phi_{1,2}^\pm$ contains the following elements:

$$M_{\phi^\pm \phi^\pm}^2 \equiv \begin{pmatrix} m_{\phi_1^- \phi_1^+}^2 & m_{\phi_1^- \phi_2^+}^2 \\ m_{\phi_2^- \phi_1^+}^2 & m_{\phi_2^- \phi_2^+}^2 \end{pmatrix} = \begin{pmatrix} \tilde{m}_1^2 + \frac{1}{4} \nu^2 (g_1^2 c_{2\beta} + g_2^2) & -\rho (\frac{1}{4} \nu^2 g_2^2 s_{2\beta} + \varepsilon_{12} \mu B e^{i\xi'}) \\ (m_{\phi_1^- \phi_2^+}^2)^* & \tilde{m}_2^2 - \frac{1}{4} \nu^2 (g_1^2 c_{2\beta} - g_2^2) \end{pmatrix} \tag{3.11}$$

3.1.3 Diagonalization and physical basis

The mass eigenstates in lowest order in the *charged* sector follow from unitary transformations on the original fields:

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = U_{c(0)}^T \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}, \quad U_{c(0)} = \begin{pmatrix} c_\beta & -\rho s_\beta \\ \rho s_\beta & c_\beta \end{pmatrix} \tag{3.12}$$

This rotation diagonalizes $M_{\phi^\pm \phi^\pm}^2$ once the tadpoles are set to zero:²⁶

$$\begin{aligned}
M_{G^\pm H^\pm}^2 &= (M_{G^\pm H^\pm}^2)^\dagger = U_{c(0)}^T M_{\phi^\pm \phi^\pm}^2 U_{c(0)} \equiv \begin{pmatrix} m_{G^\pm}^2 & m_{G^\pm H^\pm}^2 \\ m_{G^\pm H^\pm}^2 & m_{H^\pm}^2 \end{pmatrix} \\
&= \begin{pmatrix} \eta(-\bar{T}_{-H} c_{\alpha-\beta} + \bar{T}_{-h} s_{\alpha-\beta}) & \rho \eta(-\bar{T}_{-H} s_{\alpha-\beta} - \bar{T}_{-h} c_{\alpha-\beta} + i \bar{T}_{-A}) \\ \rho \eta(-\bar{T}_{-H} s_{\alpha-\beta} - \bar{T}_{-h} c_{\alpha-\beta} - i \bar{T}_{-A}) & m_{H^\pm}^2 \end{pmatrix}
\end{aligned} \tag{3.13}$$

$$\text{with } m_{H^\pm}^2 \equiv (\tilde{m}_1^2 + \frac{1}{4} \nu^2 (g_1^2 c_{2\beta} + g_2^2) + \eta \bar{T}_{\phi_1} c_\beta (1 + s_\beta^2) - \eta \bar{T}_{\phi_2} s_\beta c_\beta^2) / s_\beta^2$$

²⁵ It is said to be “ CP -even” about the particles/fields whose wave functions are invariant to the simultaneous transformation of charge conjugation and parity, whereas if they change sign due of those transformation they are said to be “ CP -odd”.

²⁶ This result match with eq. (20) in [106], where however the contributions proportional to $\bar{T}_A$ are missing.

where $m_{H^\pm}^2$ corresponds to the mass of the charged Higgs bosons at tree-level (which in turn is equal to $m_A^2 + M_W^2$, with m_A^2 the tree-level mass of the CP -odd Higgs boson, as will be shown in eq. (3.18) below). By setting $\bar{T}_{\phi_1} = \bar{T}_{\phi_2} = \bar{T}_A = 0$, from the non-null eigenvalues of $M_{\phi^\pm\phi^\pm}^2$ we get the square mass of the charged Higgs bosons $H^\pm$ at tree-level. Inverting (3.13) for $\tilde{m}_1^2$,

$$\tilde{m}_1^2 = m_{H^\pm}^2 s_\beta^2 - \frac{1}{4} \nu^2 (g_1^2 c_{2\beta} + g_2^2) - \eta \bar{T}_{-\phi_1} c_\beta (1 + s_\beta^2) - \eta \bar{T}_{-\phi_2} s_\beta c_\beta^2 \quad (3.14)$$

we can also express $M_{\phi^\pm\phi^\pm}^2$ as a function of $m_{H^\pm}^2$ and the tadpoles:²⁷

$$\begin{aligned} M_{\phi^\pm\phi^\pm}^2 = m_{H^\pm}^2 & \begin{pmatrix} s_\beta^2 & -\rho s_\beta c_\beta \\ -\rho s_\beta c_\beta & c_\beta^2 \end{pmatrix} - \eta \bar{T}_{-\phi_1} \begin{pmatrix} c_\beta (1 + s_\beta^2) & \rho s_\beta^3 \\ \rho s_\beta^3 & -s_\beta^2 c_\beta \end{pmatrix} \\ & - \eta \bar{T}_{-\phi_2} \begin{pmatrix} -s_\beta c_\beta^2 & \rho c_\beta^3 \\ \rho c_\beta^3 & s_\beta (1 + c_\beta^2) \end{pmatrix} + \eta \bar{T}_{-A} \begin{pmatrix} 0 & +\rho i \\ -\rho i & 0 \end{pmatrix} \end{aligned} \quad (3.15)$$

The diagonalization of *neutral* Higgs mass matrix connects the CP -even and CP -odd gauge eigenstates, $\phi_{1,2}$, $\chi_{1,2}$, with the physical states (mass eigenstates), H, h, G, A , via two simultaneous rotations in β and in an angle that is usually denoted by α , so that at LO we have:

$$\begin{aligned} M_{HhGA}^2 = U_{n(0)}^T M_{\phi\chi\chi}^2 U_{n(0)} & \equiv \begin{pmatrix} m_H^2 & m_{Hh}^2 & m_{HG}^2 & m_{HA}^2 \\ * & m_h^2 & m_{hG}^2 & m_{hA}^2 \\ * & * & m_G^2 & m_{GA}^2 \\ * & * & * & m_A^2 \end{pmatrix}, \quad U_{n(0)} = \begin{pmatrix} c_\alpha & -s_\alpha & 0 & 0 \\ s_\alpha & c_\alpha & 0 & 0 \\ 0 & 0 & c_\beta & -\rho s_\beta \\ 0 & 0 & \rho s_\beta & c_\beta \end{pmatrix} \\ M_{HhGA}^2|_{\bar{T}_{\phi_1}=\bar{T}_{\phi_2}=\bar{T}_A=0} & = \text{diag}\{m_H^2, m_h^2, m_G^2, m_A^2\} \end{aligned} \quad (3.16)$$

$$\begin{aligned} (H, h, G, A) &= (\phi_1, \phi_2, \chi_1, \chi_2) U_{n(0)} \\ (T_H, T_h, T_G, T_A) &= (T_{\phi_1}, T_{\phi_2}, T_{\chi_1}, T_{\chi_2}) U_{n(0)} \end{aligned}$$

$$V_{\text{Higgs}}^{\text{cMSSM}}|_{(\text{bi})\text{lineal}} = -\eta T_H H - \eta T_h h - \eta T_A A + \frac{1}{2} m_H^2 H^2 + \frac{1}{2} m_h^2 h^2 + \frac{1}{2} m_A^2 A^2 + m_{H^\pm}^2 H^- H^+$$

In the notation of eq. (3.16), the angles in $U_{n(0)}$ are such that the off-diagonal entries in M_{HhGA}^2 are zero for vanishing tadpoles. Moreover, the elements $m_{HG}^2, m_{HA}^2, m_{hG}^2, m_{hA}^2$ are exactly zero for the real case (they are proportional to $\bar{T}_A$, see eq. (3.18) below). The entries $m_H^2, m_h^2, m_G^2, m_A^2$ are the tree-level square masses for the corresponding physical fields.

²⁷ It is usually used T_A for written down these expressions even in gauge basis, because T_{χ_1} and T_{χ_2} always appear in the same combination.

From the relations in eq. (3.4), together with eqs. (3.7), (3.8) and (3.13), the entries in $M_{\phi\chi\chi}^2$ can be written in terms of tadpoles and physical parameters,

$$\begin{aligned}
m_{\phi_1\phi_1}^2 &= M_Z^2 c_\beta^2 + (m_{H^\pm}^2 - M_W^2) s_\beta^2 - \eta \bar{T}_{\phi_1} c_\beta (1 + s_\beta^2) + \eta \bar{T}_{\phi_2} s_\beta c_\beta^2 \\
m_{\phi_1\phi_2}^2 &= -(M_Z^2 + m_{H^\pm}^2 - M_W^2) s_\beta c_\beta - \eta \bar{T}_{\phi_1} s_\beta^3 - \eta \bar{T}_{\phi_2} c_\beta^3 \\
m_{\phi_2\phi_2}^2 &= M_Z^2 s_\beta^2 + (m_{H^\pm}^2 - M_W^2) c_\beta^2 + \eta \bar{T}_{\phi_1} s_\beta^2 c_\beta - \eta \bar{T}_{\phi_2} s_\beta (1 + c_\beta^2) \\
m_{\phi_1\chi_1}^2 &= 0 \\
m_{\phi_1\chi_2}^2 &= -\eta \bar{T}_A \\
m_{\phi_2\chi_1}^2 &= \rho \eta \bar{T}_A \\
m_{\phi_2\chi_2}^2 &= 0 \\
m_{\chi_1\chi_1}^2 &= (m_{H^\pm}^2 - M_W^2) s_\beta^2 - \eta \bar{T}_{\phi_1} c_\beta (1 + s_\beta^2) + \eta \bar{T}_{\phi_2} s_\beta c_\beta^2 \\
m_{\chi_1\chi_2}^2 &= -\rho((m_{H^\pm}^2 - M_W^2) s_\beta c_\beta + \eta \bar{T}_{\phi_1} s_\beta^3 + \eta \bar{T}_{\phi_2} c_\beta^3) \\
m_{\chi_2\chi_2}^2 &= (m_{H^\pm}^2 - M_W^2) c_\beta^2 + \eta \bar{T}_{\phi_1} s_\beta^2 c_\beta - \eta \bar{T}_{\phi_2} s_\beta (1 + c_\beta^2)
\end{aligned} \tag{3.17}$$

while the elements of the rotated matrix M_{HhGA}^2 reads

$$\begin{aligned}
m_H^2 &= M_Z^2 c_{\alpha+\beta}^2 + (m_{H^\pm}^2 - M_W^2) s_{\alpha-\beta}^2 - \eta \bar{T}_H c_{\alpha-\beta} (1 + s_{\alpha-\beta}^2) - \eta \bar{T}_h s_{\alpha-\beta} c_{\alpha-\beta}^2 \\
m_{Hh}^2 &= -M_Z^2 s_{\alpha+\beta} c_{\alpha+\beta} + (m_{H^\pm}^2 - M_W^2) s_{\alpha-\beta} c_{\alpha-\beta} + \eta \bar{T}_H s_{\alpha-\beta}^3 - \eta \bar{T}_h c_{\alpha-\beta}^3 \\
m_h^2 &= M_Z^2 s_{\alpha+\beta}^2 + (m_{H^\pm}^2 - M_W^2) c_{\alpha-\beta}^2 + \eta \bar{T}_H c_{\alpha-\beta} s_{\alpha-\beta}^2 + \eta \bar{T}_h s_{\alpha-\beta} (1 + c_{\alpha-\beta}^2) \\
m_{HG}^2 &= \rho \eta \bar{T}_A s_{\alpha-\beta} \\
m_{HA}^2 &= -\eta \bar{T}_A c_{\alpha-\beta} \\
m_{hG}^2 &= \rho \eta \bar{T}_A c_{\alpha-\beta} \\
m_{hA}^2 &= \eta \bar{T}_A s_{\alpha-\beta} \\
m_G^2 &= \eta(-\bar{T}_H c_{\alpha-\beta} + \bar{T}_h) s_{\alpha-\beta} \\
m_{GA}^2 &= -\rho \eta(\bar{T}_H s_{\alpha-\beta} - \bar{T}_h c_{\alpha-\beta}) \\
m_A^2 &= m_{H^\pm}^2 - M_W^2 .
\end{aligned} \tag{3.18}$$

The last line of (3.18) match with the relation (1.26) for the A boson mass at tree-level,²⁸ and it can be used to express the relations of eqs. (3.8) and (3.14), as well as the elements from (3.17) and (3.18), as a function of m_A^2 as input parameter instead of $m_{H^\pm}^2$, which is more suitable for the usual renormalisation procedure in the case of the real MSSM. Note

²⁸ Note that the relation $m_A^2 = m_{H^\pm}^2 - M_W^2$ does not depend on tadpoles coefficients, so that at higher order it does not receive corrections from tadpole counterterms.

for instance in (3.17) and (3.18) that the results are independent of the sign ε_{12} when written in terms mass and tadpole parameters, as it must be.

The tree-level tadpole coefficients can be set to zero at the end of the calculation.²⁹ In the $\phi_1 - \phi_2$ sector this ensures that $v_{1,2}$ are the vacuum expectation values, while in the $\chi_1 - \chi_2$ sector this corresponds to a redefinition of the phase of m_{12}^2 so that the phase $e^{i\xi}$ is absorbed [107]. Then we see from (3.18) that the matrix (3.16) diagonalizes the neutral Higgs sector at tree-level even for the complex MSSM, so there is no CP violation in the cMSSM Higgs sector at tree-level, and the fields h and H are decoupled from the fields A and G . However, the radiative corrections in $\delta\bar{T}_{-A}$ introduce a mixing between the CP -even and CP -odd sectors for the case complex of complex parameters ($\bar{T}_{-A} \propto \sin \xi' \neq 0$).

Rewriting m_A^2 as function of the original parameters in the Higgs potential, by using eqs. (3.13) and (3.4), and replacing the tadpoles according to (3.6), the eq. (30a) from [15] is recovered for $\varepsilon_{12} = +1$:

$$m_A^2 = \tilde{m}_1^2 s_\beta^2 + \tilde{m}_2^2 c_\beta^2 + \varepsilon_{12} \cos \xi' |m_{12}^2| s_{2\beta} - \frac{1}{4} \nu^2 (g_1^2 + g_2^2) c_{2\beta}^2 \quad (3.19)$$

But using eq. (3.8), we can write this expression also in the following way:

$$m_A^2 = \frac{\varepsilon_{12} \mu B \cos \xi' - \eta \bar{T}_{-\phi_1} s_\beta^3 - \eta \bar{T}_{-\phi_2} c_\beta^3}{s_\beta c_\beta} \quad (3.20)$$

We will use and compare this last result in chapter 4 for the case of the NMSSM.

Looking at the expression for m_{Hh}^2 in (3.18), we see that in order to M_{HhGA}^2 be diagonal (once the tadpoles are set to zero), it must be satisfied

$$M_Z^2 s_{2(\alpha+\beta)} = (m_{H^\pm}^2 - M_W^2) s_{2(\alpha-\beta)} \quad , \text{ i.e. } \quad \tan(2\alpha) = \tan(2\beta) \frac{m_A^2 + M_Z^2}{m_A^2 - M_Z^2} \quad , \quad -\frac{\pi}{4} \leq \alpha \leq \frac{\pi}{4} \quad (3.21)$$

which defines the mixing angle α . By replacing this result in (3.18), the tree-level CP -even Higgs masses are obtained:

$$m_{H,h}^2 = \frac{1}{2} (m_A^2 + M_Z^2 \pm \sqrt{(m_A^2 + M_Z^2)^2 - 4m_A^2 M_Z^2 c_{2\beta}^2}) \quad (3.22)$$

²⁹ In spite of that the tree-level tadpole coefficients must vanish, it is necessary to introduce and keep them explicitly as input parameters throughout the calculation since they will be needed for the renormalisation procedure in order to implement the higher-order tadpole contributions for the evaluation of the radiative corrections to the Higgs sector.

For the gauge-fixing, affecting terms in $V_{\text{Higgs}}|_{(\text{bi})\text{linear}}$ (cf. eq. (3.16)) involving the Goldstone fields, the entries for G and $G^\pm$ in our analysis are to be understood in the 't Hooft-Feynman gauge, and in the renormalisation we follow the usual approach where the gauge-fixing term receives no net contribution from the renormalisation transformations. Accordingly, the counterterms derived below arise only from the Higgs potential and the kinetic terms of the Higgs fields but not from the gauge-fixing term. This means that the only place where we take into account the gauge-fixing in our calculation is in the fact that the tree-level masses for the Goldstone fields must be set in accordance with the particles with respect to they behave as their longitudinal polarizations,

$$\begin{aligned} m_G^2 &= M_Z^2 \\ m_{G^\pm}^2 &= M_W^2 \end{aligned} \quad (3.23)$$

in spite this entries would vanish according to (3.13) and (3.18). This setting ensures that the divergences cancel out in our calculations at higher-order.

3.2 Sfermion sector

The sfermions are important in the Higgs calculations because of their appearance as virtual particles in the loops evaluations. The sfermions $\tilde{f}$ are the scalar particles that supersymmetry introduces in correspondence with the fermions f of the SM. The squarks associated with the top t and bottom b , for instance, are the so-called *stop* and *sbottom*, denoted by $\tilde{t}$ and $\tilde{b}$, respectively. The bilinear part of the $\tilde{t}$ and $\tilde{b}$ (N)MSSM Lagrangian,

$$L_{\tilde{t}/\tilde{b} \text{ mass}} = -\frac{1}{2}(\tilde{t}_L^\dagger, \tilde{t}_R^\dagger) M_{\tilde{t}}^2 \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix} - \frac{1}{2}(\tilde{b}_L^\dagger, \tilde{b}_R^\dagger) M_{\tilde{b}}^2 \begin{pmatrix} \tilde{b}_L \\ \tilde{b}_R \end{pmatrix} \quad (3.24)$$

contains the stop and sbottom squared mass matrices $M_{\tilde{t}}$ and $M_{\tilde{b}}$ for the squarks fields $\tilde{t}_{L,R}$ and $\tilde{b}_{L,R}$, given by

$$\begin{aligned} M_{\tilde{t}}^2 &= \begin{pmatrix} M_{Q_3}^2 + m_t^2 + M_Z^2 c_{2\beta} \left(\frac{1}{2} - \frac{2}{3} s_W^2\right) & m_t (A_t^* - \mu/t_\beta) \\ m_t (A_t - \mu^*/t_\beta) & M_{\tilde{t}_R}^2 + m_t^2 + \frac{2}{3} M_Z^2 c_{2\beta} s_W^2 \end{pmatrix} \equiv \begin{pmatrix} m_{\tilde{t}_L}^2 & m_t X_t^* \\ m_t X_t & m_{\tilde{t}_R}^2 \end{pmatrix} \\ M_{\tilde{b}}^2 &= \begin{pmatrix} M_{Q_3}^2 + m_b^2 + M_Z^2 c_{2\beta} \left(-\frac{1}{2} + \frac{1}{3} s_W^2\right) & m_b (A_b^* - \mu t_\beta) \\ m_b (A_b - \mu^* t_\beta) & M_{\tilde{b}_R}^2 + m_b^2 - \frac{1}{3} M_Z^2 c_{2\beta} s_W^2 \end{pmatrix} \equiv \begin{pmatrix} m_{\tilde{b}_L}^2 & m_b X_b^* \\ m_b X_b & m_{\tilde{b}_R}^2 \end{pmatrix} \end{aligned} \quad (3.25)$$

Here $M_{Q_3}^2, M_{t_R}^2, M_{b_R}^2$ are soft SUSY-breaking parameters, where $M_{Q_3}^2$ is the same for $M_{t_L}^2$ and $M_{b_L}^2$, the A_q are the trilinear soft-breaking parameters, and μ is the mixing parameter of the Higgs doublets in the superpotential for matter and Higgs fields of the MSSM. From these parameters, the A_q 's and μ are the ones that are complex in the case of the cMSSM.

Note that the presence of the (big) top mass in the non-diagonal elements of the matrix $M_{\tilde{t}}^2$ in (3.25) may lead eventually to a substantial mixing between $\tilde{t}_L$ and $\tilde{t}_R$. The corresponding mass eigenstates are obtained diagonalizing each mass matrix $M_{\tilde{q}}^2$ by rotation (unitary transformation) $U_{\tilde{q}}$, which can be parametrized by a mixing angle $\theta_{\tilde{q}}$:

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = U_{\tilde{q}} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}, \quad D_{\tilde{q}} = U_{\tilde{q}} M_{\tilde{q}} U_{\tilde{q}}^\dagger = \begin{pmatrix} m_{\tilde{q}_1}^2 & 0 \\ 0 & m_{\tilde{q}_2}^2 \end{pmatrix} \quad (3.26)$$

$$U_{\tilde{q}} = \begin{pmatrix} U_{11}^{\tilde{q}} & U_{12}^{\tilde{q}} \\ U_{21}^{\tilde{q}} & U_{22}^{\tilde{q}} \end{pmatrix} = \begin{pmatrix} \cos \theta_{\tilde{q}} & e^{-i \arg(X_q)} \sin \theta_{\tilde{q}} \\ -e^{+i \arg(X_q)} \sin \theta_{\tilde{q}} & \cos \theta_{\tilde{q}} \end{pmatrix}$$

3.2.1 Stops

Stop tree-level mass eigenvalues are given by³⁰

$$m_{\tilde{t}_{1,2}}^2 = \frac{1}{2}(m_{t_R}^2 + m_{t_L}^2 \mp \sqrt{(m_{t_R}^2 - m_{t_L}^2)^2 + 4m_t^2 |X_t|^2}) = m_t^2 + M_{\text{SUSY}}^2 \mp m_t |X_t| \sqrt{1 + \Xi^2} \quad (3.27)$$

where we have defined

$$\begin{aligned} M_{\text{SUSY}}^2 &\equiv \frac{1}{2}(m_{t_R}^2 + m_{t_L}^2) - m_t^2 \\ &= \frac{1}{2}(M_{t_R}^2 + M_{Q_3}^2) + \frac{1}{4}M_Z^2 c_{2\beta} \approx \frac{1}{2}(M_{t_R}^2 + M_{Q_3}^2) \\ \Xi &\equiv \frac{m_{t_R}^2 - m_{t_L}^2}{2m_t |X_t|} = \frac{M_{t_R}^2 - M_{Q_3}^2 - M_Z^2 c_{2\beta} (\frac{1}{2} - \frac{4}{3}s_W^2)}{2m_t |X_t|} \\ X_t &= A_t - \mu^* / t_\beta = |X_t| e^{+i \arg(X_t)} \end{aligned} \quad (3.28)$$

We see from (3.27) how the main shift between top and stop masses indeed comes from the SUSY-breaking parameters in M_{SUSY} . The stop mass matrix can now be expressed in terms of these physical parameters $m_{\tilde{t}_1}, m_{\tilde{t}_2}$ and the mixing angle $\theta_{\tilde{t}}$ as follows:

³⁰ It is assume/impose here that $m_{\tilde{t}_1} \leq m_{\tilde{t}_2}$.

$$\mathbf{M}_{\tilde{t}}^2 = \mathbf{U}_{\tilde{t}}^\dagger \begin{pmatrix} m_{\tilde{t}_1}^2 & 0 \\ 0 & m_{\tilde{t}_2}^2 \end{pmatrix} \mathbf{U}_{\tilde{t}} = \begin{pmatrix} c_{\tilde{t}}^2 m_{\tilde{t}_1}^2 + s_{\tilde{t}}^2 m_{\tilde{t}_2}^2 & e^{-i \arg(X_t)} s_{\tilde{t}} c_{\tilde{t}} (m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2) \\ e^{+i \arg(X_t)} s_{\tilde{t}} c_{\tilde{t}} (m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2) & s_{\tilde{t}}^2 m_{\tilde{t}_1}^2 + c_{\tilde{t}}^2 m_{\tilde{t}_2}^2 \end{pmatrix} . \quad (3.29)$$

Comparing with eq. (3.25), and by inverting the expression (3.27) in order to express $M_{\tilde{Q}_3, \tilde{t}_R}^2$ as a function of $m_{\tilde{t}_{1,2}}^2$, it can be seen that³¹

$$\begin{aligned} c_{\tilde{t}}^2 &= 1 - s_{\tilde{t}}^2 = \frac{1}{2} \left(1 + \frac{\Xi}{\sqrt{1 + \Xi^2}} \right) = \frac{1}{2} (1 + \text{sgn}(\Xi) \sqrt{1 - s_{2\tilde{t}}^2}) \\ s_{2\tilde{t}} &= \frac{2m_t |X_t|}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} = \frac{-1}{\sqrt{1 + \Xi^2}} \end{aligned} \quad (3.30)$$

If $m_{\tilde{t}_1} = m_{\tilde{t}_2}$ ($\Leftrightarrow m_{\tilde{t}_R}^2 - m_{\tilde{t}_L}^2 = X_t = 0$), the mixing angle $\theta_{\tilde{t}}$ can take any value, since in that case the mass matrix is proportional to the identity matrix, so that it is compatible with any degree of mixing. This angle enters in the calculation of the amplitudes evaluated for this work through the Feynman rules for the couplings between the stops and the Higgs fields.

Regarding definitions for mass eigenstates and mixing angles

The “no-mixing” or “vanishing $\tilde{t}$ mixing” condition for the stops sector is $|\Xi| \gg 1$:

$$\text{vanishing } \tilde{t} \text{ mixing : } |X_t| \ll \frac{|m_{\tilde{t}_R}^2 - m_{\tilde{t}_L}^2|}{m_t} \Leftrightarrow s_{\tilde{t}} c_{\tilde{t}} = 0 \Leftrightarrow |c_{2\tilde{t}}| = 1 \quad (3.31)$$

It is usually implemented in literature just by setting $X_t = 0$, so that the non-diagonal entries in the stop mixing matrix vanish. Note that it does not necessarily correspond to $c_{\tilde{t}}^2 \approx 1, s_{\tilde{t}}^2 \approx 0$. As was pointed out above, the case $s_{\tilde{t}}^2 = 0$ differs from the $c_{\tilde{t}}^2 = 0$ case just in the conventions adopted in the definitions of $\tilde{t}_1$ and $\tilde{t}_2$, or regarding if $m_{\tilde{t}_L}^2$ turns out to be bigger or smaller than $m_{\tilde{t}_R}^2$. From the convention $m_{\tilde{t}_1}^2 \leq m_{\tilde{t}_2}^2$, we can see from (3.30) that

$$c_{\tilde{t}}^2 \approx 1, s_{\tilde{t}}^2 \approx 0 \Leftrightarrow m_{\tilde{t}_R}^2 > m_{\tilde{t}_L}^2 \Leftrightarrow M_{\tilde{t}_R}^2 > M_{\tilde{Q}_3}^2 + M_Z^2 c_{2\beta} \left(\frac{1}{2} - \frac{4}{3} s_W^2 \right) . \quad (3.32)$$

Then, it cannot be said a priori that in the no $\tilde{t}$ -mixing case we get the identification $\tilde{t}_1 \sim \tilde{t}_L$ and $\tilde{t}_2 \sim \tilde{t}_R$.

Of course, one could relax the condition $m_{\tilde{t}_1}^2 \leq m_{\tilde{t}_2}^2$, so another possible convention would be to define the mass eigenstates in such a way as to force that $\tilde{t}_1 \sim \tilde{t}_L$, $\tilde{t}_2 \sim \tilde{t}_R$

³¹ The subtraction within the square roots in these expressions is not inconsistent with the fact that $s_{\tilde{t}}, c_{\tilde{t}}$ are real quantities, since from expression (3.27) it can be seen that in any case $m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2 \geq 2m_t |X_t|$.

($\Rightarrow c_{\tilde{t}}^2 \approx 1, s_{\tilde{t}}^2 \approx 0$) for vanishing $\tilde{t}$ -mixing, and in such a case we will have $m_{\tilde{t}_1}^2 < m_{\tilde{t}_2}^2$ if and only if $m_{\tilde{\nu}_R}^2 > m_{\tilde{t}_L}^2$.

On the other hand, the “maximal stop mixing” case is defined by the condition $|\Xi| < 1$, which is equivalent to any of these conditions:

$$\text{maximal } \tilde{t} \text{ mixing : } |X_t| \gg \frac{|m_{\tilde{t}_R}^2 - m_{\tilde{t}_L}^2|}{m_t} \Leftrightarrow |s_{\tilde{t}} c_{\tilde{t}}| = \frac{1}{2} \Leftrightarrow c_{2\tilde{t}} = 0 \quad (3.33)$$

These considerations and limits are summarized in the X_t - $(m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2)$ plane in fig. 3.1, where the specific values of $s_{\tilde{t}}, c_{\tilde{t}}$ are shown for real values of X_t , in which case the sing $e^{\pm i \arg(X_t)} = \text{sgn}(X_t)$ is absorbed in the definition of the angle in the rotating matrix (3.26):

$$X_t \text{ real} \rightarrow U_{\tilde{t}} \equiv \begin{pmatrix} \cos \theta_{\tilde{t}} & \sin \theta_{\tilde{t}} \\ -\sin \theta_{\tilde{t}} & \cos \theta_{\tilde{t}} \end{pmatrix}, \quad \sin \theta_{\tilde{t}} \cos \theta_{\tilde{t}} = \frac{m_t X_t}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} = -\frac{\text{sgn}(X_t)}{2\sqrt{1 + \Xi^2}} \quad (3.34)$$

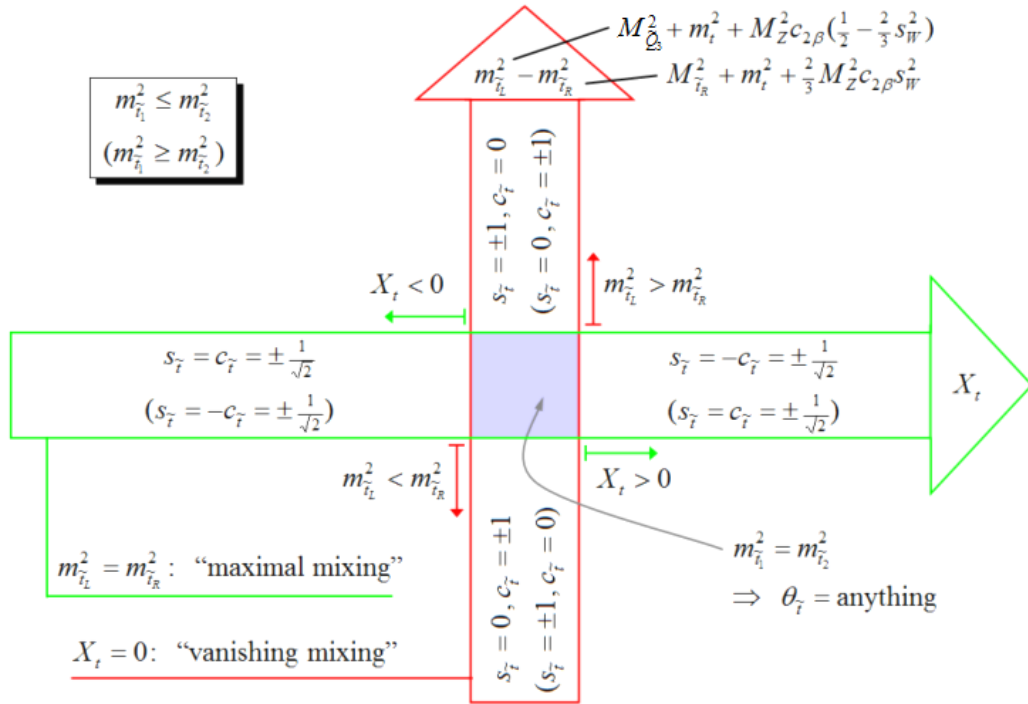


Figure 3.1: Schematic representation of the values adopted by the stop mixing angle for different limit cases, as a function of (real values of) X_t and the stop mass parameters.

In fig. 3.1, the following features can be observed:

- The case $s_{\tilde{t}} = 0$ is compatible with assuming $m_{\tilde{t}_1}^2 \leq m_{\tilde{t}_2}^2$ only if $m_{\tilde{t}_L}^2 \leq m_{\tilde{t}_R}^2$ is verified.

- The condition $X_t = 0$ takes into account also the particular case of $m_{\tilde{t}_1}^2 = m_{\tilde{t}_2}^2$, for which the mixing angle $\theta_{\tilde{t}}$ can take any value. The vanishing and maximal mixing coincides in this case (the gray intersection in the figure).
- The condition $X_t = 0$ is equivalent to $s_{\tilde{t}} c_{\tilde{t}} = 0$ (which in turn is equivalent to $|c_{2\tilde{t}}| = 1$).

A third possibility for the definitions of stops masses and mixing angles would be that, in case of $m_{\tilde{t}_R}^2 < m_{\tilde{t}_L}^2$, from the very beginning we exchanged in (3.26) and (3.29) the notation for the eigenvalues of $M_{\tilde{t}}^2$ in its diagonal decomposition:

$$\begin{pmatrix} \tilde{t}_2 \\ \tilde{t}_1 \end{pmatrix} \equiv U_{\tilde{t}} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}, \quad M_{\tilde{t}} \equiv U_{\tilde{t}}^\dagger \begin{pmatrix} m_{\tilde{t}_2}^2 & 0 \\ 0 & m_{\tilde{t}_1}^2 \end{pmatrix} U_{\tilde{t}} \quad (3.35)$$

Then, the relation $m_{\tilde{t}_1}^2 \leq m_{\tilde{t}_2}^2$ is guaranteed at the same time that $c_{\tilde{t}}^2 = 1 - s_{\tilde{t}}^2 \approx 1$ for no $\tilde{t}$ -mixing if we adopt the prescription of making the exchange $\tilde{t}_1 \leftrightarrow \tilde{t}_2$ in all expressions in case of $m_{\tilde{t}_R}^2 < m_{\tilde{t}_L}^2$.

In the (3.31) limit, the stop tree-level masses are

$$m_{\tilde{t}_1}^2 \approx m_{\tilde{t}_L}^2 - \frac{m_t^2 |X_t|^2}{m_{\tilde{t}_R}^2 - m_{\tilde{t}_L}^2}, \quad m_{\tilde{t}_2}^2 \approx m_{\tilde{t}_R}^2 + \frac{m_t^2 |X_t|^2}{m_{\tilde{t}_R}^2 - m_{\tilde{t}_L}^2} \quad (3.36)$$

while in the (3.33) case the result is just

$$m_{\tilde{t}_{1,2}}^2 \approx M_{\text{SUSY}}^2 + m_t^2 \mp m_t |X_t| \quad (3.37)$$

Analogous considerations apply for mass eigenstates and mixing angles definitions in the sbottom sector.

3.2.2 Sbottoms

For the sbottom mass eigenvalues, the exact tree-level result is given by

$$m_{\tilde{b}_{1,2}}^2 = \frac{m_{\tilde{b}_L}^2}{c_t^2 m_{\tilde{t}_1}^2 + s_t^2 m_{\tilde{t}_2}^2 - m_t^2} \left(\overbrace{M_{Q_3}^2 + M_Z^2 c_{2\beta} \left(\frac{1}{2} - \frac{2}{3} s_W^2 \right) - M_W^2 c_{2\beta} + m_b^2}^{m_{\tilde{b}_L}^2} + \frac{1}{2} (m_{\tilde{b}_R}^2 - m_{\tilde{b}_L}^2 \mp \sqrt{(m_{\tilde{b}_R}^2 - m_{\tilde{b}_L}^2)^2 + 4m_b^2 |X_b|^2}) \right) \quad (3.38)$$

where $m_{\tilde{b}_R}^2 - m_{\tilde{b}_L}^2 = M_{\tilde{b}_R}^2 - M_{\tilde{b}_L}^2 + M_Z^2 c_{2\beta} \left(\frac{1}{2} - \frac{2}{3} s_W^2 \right)$
 $X_b = A_b - \mu^* t_\beta$

It is usual in literature (see e.g. [106]) the following approximation for one of the sbottom masses,³² which is relevant for $O(m_t^4)$ and neglecting the bottom mass m_b :

$$m_{b(i)}^2 \approx c_t^2 m_{t_1}^2 + s_t^2 m_{t_2}^2 - m_t^2 \approx m_{b_L}^2 = M_{\text{SUSY}}^2 - 2m_t |X_t| \Xi = m_{t_L}^2 - m_t^2 \quad . \quad (3.39)$$

This approximation is based not only on considering $M_Z \ll M_{\tilde{Q}_3}$ (which implies $m_b \ll M_{\tilde{Q}_3}$), but also on assuming for instance that

$$\begin{aligned} \text{either} \quad & |m_{b_R}^2 - m_{b_L}^2| \ll m_b |X_b| \ll M_{\tilde{Q}_3}^2 \quad (\Rightarrow \text{maximal } \tilde{b} \text{ - mixing}) \\ \text{or} \quad & m_b |X_b| \ll |m_{b_R}^2 - m_{b_L}^2| \ll M_{\tilde{Q}_3}^2 \quad (\Rightarrow \text{no } \tilde{b} \text{ - mixing}) \end{aligned} \quad (3.40)$$

So that $m_{b(i)}^2 \approx m_{b_L}^2$ for some sbottom mass does not necessarily imply no-mixing in the sbottom sector.³³ In the (3.31) limit, this tree-level mass reduced to $m_{b_L}^2 \approx m_{t_L}^2 - m_t^2$, while $m_{b_L}^2 \approx M_{\text{SUSY}}^2$ is verified in the (3.33) case.

3.2.3 Squarks and sleptons

In order to be able to implement the virtual contributions to the self-energies from the other sfermion sectors of the theory, both for the evaluation of the “full” case or a “(s)fermion-contributions” approximation, the results from sections 3.2.1 and 3.2.2 can be translated to the other squarks and also the sleptons of the model from the generalization of the matrixes of eq. (3.25) and the diagonalization performed in (3.26):

$$M_{\tilde{\nu}_g}^2 = M_{L_g}^2 + \frac{1}{2} M_Z^2 c_{2\beta} = m_{(\tilde{\nu}_g)_1}^2 \quad (3.41)$$

$$\begin{aligned} M_{\tilde{e}_g}^2 &= \begin{pmatrix} M_{L_g}^2 + m_{e_g}^2 + M_Z^2 c_{2\beta} (-\frac{1}{2} + s_W^2) & m_{e_g} (A_{e_g}^* - \mu t_\beta) \\ m_{e_g} (A_{e_g} - \mu^* t_\beta) & M_{(\tilde{e}_g)_R}^2 + m_{e_g}^2 - M_Z^2 c_{2\beta} s_W^2 \end{pmatrix} = U_{\tilde{e}_g}^T \begin{pmatrix} m_{(\tilde{e}_g)_1}^2 & 0 \\ 0 & m_{(\tilde{e}_g)_2}^2 \end{pmatrix} U_{\tilde{e}_g} \\ M_{\tilde{u}_g}^2 &= \begin{pmatrix} M_{Q_g}^2 + m_{u_g}^2 + M_Z^2 c_{2\beta} (\frac{1}{2} - \frac{2}{3} s_W^2) & m_{u_g} (A_{u_g}^* - \mu / t_\beta) \\ m_{u_g} (A_{u_g} - \mu^* / t_\beta) & M_{(\tilde{u}_g)_R}^2 + m_{u_g}^2 + \frac{2}{3} M_Z^2 c_{2\beta} s_W^2 \end{pmatrix} = U_{\tilde{u}_g}^T \begin{pmatrix} m_{(\tilde{u}_g)_1}^2 & 0 \\ 0 & m_{(\tilde{u}_g)_2}^2 \end{pmatrix} U_{\tilde{u}_g} \\ M_{\tilde{d}_g}^2 &= \begin{pmatrix} M_{Q_g}^2 + m_{d_g}^2 + M_Z^2 c_{2\beta} (-\frac{1}{2} + \frac{1}{3} s_W^2) & m_{d_g} (A_{d_g}^* - \mu t_\beta) \\ m_{d_g} (A_{d_g} - \mu^* t_\beta) & M_{(\tilde{d}_g)_R}^2 + m_{d_g}^2 - \frac{1}{3} M_Z^2 c_{2\beta} s_W^2 \end{pmatrix} = U_{\tilde{d}_g}^T \begin{pmatrix} m_{(\tilde{d}_g)_1}^2 & 0 \\ 0 & m_{(\tilde{d}_g)_2}^2 \end{pmatrix} U_{\tilde{d}_g} \end{aligned}$$

where $g=1,2,3$ are the generations/families:

³² We don't say here that $\tilde{b}_{(i)}$ is $\tilde{b}_1$, because in such a case we would be defining $\tilde{b}_1$ such that it would not necessarily be verified that $m_{b_1}^2 \leq m_{b_2}^2$, although in this approximation the difference $|m_{b_1}^2 - m_{b_2}^2|$ is small.

³³ The no-mixing condition in the sbottom sector is needed, however, so that in the expressions for self-energies (and consequently for the masses) only the mass of one of the sbottoms intervenes.

$$\begin{aligned}
&\text{leptons} \left\{ \begin{array}{l} \nu_g = \nu_e, \nu_\mu, \nu_\tau \\ e_g = e, \mu, \tau \end{array} \right. \\
&\text{quarks} \left\{ \begin{array}{l} u_g = u, s, t \\ d_g = d, c, b \end{array} \right.
\end{aligned} \tag{3.42}$$

From these matrixes, masses and mixing results analogous to the ones from sections 3.2.1 and 3.2.2 for the remaining sfermions are straightforward.

3.3 Renormalisation and higher-orders contributions

Quantum corrections substantially modify the tree-level masses. Their Feynman-diagrammatic calculation at higher-order involves three steps:

- Renormalisation, in order to obtain the relations between the different counterterms and to fix (through appropriate renormalisation conditions) the finite parts of those corresponding to the fields and the quantities chosen as independent input parameters.
- Evaluation (via the implementation of some regularisation mechanism) of the amplitudes that take part in those renormalisation conditions as well as the two-point functions that contribute to the higher-order corrections for the mass matrix elements involved in the propagators.
- Determination of the higher-order expressions (in terms of the input parameters) for the (square) mass from the pole (in the external momentum) of the propagator for the corresponding field(s).

The *renormalised* self-energies bring together all the contributions of a given order that contribute to a given element of a mass matrix, and that's why they must be *finite* (by construction). In perturbation theory, each unrenormalised self-energy is expanded as follows:

$$\Sigma(p^2) = \Sigma^{(1L)}(p^2) + \Sigma^{(2L)}(p^2) + \dots \tag{3.43}$$

in terms of the i th-order contributions $\Sigma^{(i)}$, and analogously for the renormalised quantities $\hat{\Sigma}^{(i)}$. Details on the evaluation of the one-loop self-energies will be given in sects. 3.3.1-3.3.3, while the two-loop contributions will be briefly reviewed in sect. 3.3.4.

3.3.1 One-loop renormalisation of the neutral sector

The masses of the neutral Higgs bosons are determined as the real part of the p^2 values that solve the following equation:

$$\det(p^2 \mathbf{I} - \mathbf{M}_{\varphi\varphi\varphi\varphi}^2 + \hat{\Sigma}_{\varphi\varphi\varphi\varphi}(p^2)) = 0 \quad (3.44)$$

Here $\hat{\Sigma}_{\varphi\varphi\varphi\varphi}$ denotes the matrix with the corresponding renormalised self-energy corrections, and $\mathbf{I}$ is the 4×4 identity matrix. Because of the cyclic properties of the determinant, the same masses are obtain by solving (3.44) in gauge basis ($\varphi\varphi\varphi\varphi = \phi_1\phi_2\chi_1\chi_2$), physical basis ($\varphi\varphi\varphi\varphi = HhGA$) or any other basis defined through a unitary transformation, such as the mixed basis $\{\phi_1, \phi_2, G, A\}$ usually used in literature, being (c.f. eq. 3.16)

$$\begin{pmatrix} G \\ A \end{pmatrix} = \begin{pmatrix} c_\beta & \rho s_\beta \\ -\rho s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}. \quad (3.45)$$

The renormalised self-energies, $\hat{\Sigma}_{\varphi_i\varphi_j}(\varphi_i, \varphi_j = \phi_1, \phi_2, G, A)$, corresponding to the higher order corrections to the entries $m_{\varphi_i\varphi_j}^2$ for $\mathbf{M}_{\phi_1\phi_2GA}^2$ from (3.17),(3.18), are obtain at one-loop level from the unrenormalised self-energies, the field renormalisation constants and the mass counterterms insertions by doing³⁴

$$\hat{\Sigma}_{\varphi_i\varphi_j}(p^2) = \Sigma_{\varphi_i\varphi_j}(p^2) + (\delta_{\varphi_i\varphi_j} p^2 - m_{\varphi_i\varphi_j}^2) \frac{1}{2} (\delta Z_{\varphi_i} + \delta Z_{\varphi_j}) - \delta m_{\varphi_i\varphi_j}^2 \quad (3.46)$$

$$\delta \mathbf{M}_{\phi_1\phi_2GA}^2 = \delta \vec{\mathcal{P}} \cdot \partial_{\vec{\mathcal{P}}} \mathbf{M}_{\phi_1\phi_2GA}^2 \Big|_{T_{\phi_{1,2}} = T_A = 0}$$

being $\delta \mathbf{M}_{\phi_1\phi_2GA}^2$ the matrix of elements $\delta m_{\varphi_i\varphi_j}^2$, and where

$$\begin{aligned} \delta Z_{\phi_1} &\equiv \delta Z_{H_1} \\ \delta Z_{\phi_2} &\equiv \delta Z_{H_2} \\ \delta Z_G &\equiv c_\beta \delta Z_{H_1} + \rho s_\beta \delta Z_{H_2} \\ \delta Z_A &\equiv -\rho s_\beta \delta Z_{H_1} + c_\beta \delta Z_{H_2} \end{aligned}, \quad \delta_{\varphi_i\varphi_j} = \begin{cases} 1 & , \quad \varphi_i = \varphi_j \\ 0 & , \quad \varphi_i \neq \varphi_j \end{cases}. \quad (3.47)$$

³⁴ Equivalently, the same procedure can be performed in physical basis for $\mathbf{M}_{HhGA}^2$. We choose instead to work in gauge basis for the neutral sector just for simplicity, in order to avoid the presence of the mixing angle α in the expressions.

The field renormalisation is required for finite self-energies at arbitrary values of the external momentum p^2 . We assign one field-renormalisation constant for each Higgs doublet,

$$H_1 \rightarrow (1 + \frac{1}{2} \delta Z_{H_1}) H_1 \quad , \quad H_2 \rightarrow (1 + \frac{1}{2} \delta Z_{H_2}) H_2 \quad (3.48)$$

For the determination of the field renormalisation constants, we adopt the $\overline{\text{DR}}$ scheme:³⁵

$$\begin{aligned} \delta Z_{H_1} &= \delta Z_{H_1}^{\overline{\text{DR}}} = -\text{Re} \partial_{p^2} \Sigma_{\phi_1 \phi_1}^{\text{div}} \\ \delta Z_{H_2} &= \delta Z_{H_2}^{\overline{\text{DR}}} = -\text{Re} \partial_{p^2} \Sigma_{\phi_2 \phi_2}^{\text{div}} \end{aligned} \quad (3.49)$$

i.e. the renormalisation constants consist of divergent parts only (see the discussion in [15]). $\partial_{p^2} \Sigma_{\phi\phi}$ denotes the derivative of the unrenormalised self-energies of the neutral CP -even Higgs bosons with respect to the external momentum. As default value of the renormalisation scale we have chosen $\bar{\mu} = m_t$.

The “vector” of independent parameters, $\vec{P}$, is renormalised according to

$$\vec{P} \equiv \begin{pmatrix} m_{H^\pm}^2 \\ M_Z^2 \\ M_W^2 \\ \bar{T}_x \\ t_\beta \end{pmatrix}^{\text{bare}} \rightarrow \vec{P} + \delta \vec{P} = \begin{pmatrix} m_{H^\pm}^2 + \delta m_{H^\pm}^2 \\ M_Z^2 + \delta M_Z^2 \\ M_W^2 + \delta M_W^2 \\ \bar{T}_x + \delta \bar{T}_x \\ t_\beta (1 + \delta t_\beta) \end{pmatrix} \quad , \quad x = \phi_1, \phi_2, A \quad (3.50)$$

Note that the ν parameter only appears in the combination $\bar{T}_x = \frac{1}{\sqrt{\nu}} T_x$ for the entries in the Higgs mass matrixes, because of our choice of M_Z^2 , M_W^2 as input parameters. This fact allows us to avoid specifying an explicit renormalisation condition on $\delta \nu$ (or equivalently on the electric charge counterterm) for the MSSM Higgs sector, because the higher-order contributions to $\bar{T}_x$ are the same as for T_x , since the tree-level tadpole coefficients are set to zero at the end of the calculation:

$$\delta \bar{T}_x = \frac{1}{\sqrt{2\nu}} [\delta T_x - T_x \delta \nu / \nu]_{T_x=0} = \frac{1}{\sqrt{2\nu}} \delta T_x \quad , \quad (3.51)$$

so that $\delta \bar{T}_x$ will inherit just the condition imposed on δT_x (see eq. (3.62) below).

³⁵ The field renormalisation constants completely drop out in the determination of the Higgs-boson masses at one-loop order. They only enter via residual higher-order effects as a consequence of the iterative numerical determination of the propagator poles. The $\overline{\text{DR}}$ scheme for the field renormalisation constants is thus convenient in order to avoid the possible occurrence of unphysical threshold effects [118].

The following result for $\delta M_{\phi\chi\chi}^2$ is obtained:

$$\begin{aligned}
\delta m_{\phi_1\phi_1}^2 &= s_\beta^2 \delta m_A^2 + c_\beta^2 \delta M_Z^2 + \frac{1}{2} s_{2\beta}^2 (m_A^2 - M_Z^2) \tilde{\alpha}_\beta - c_\beta (1 + s_\beta^2) \eta \delta \bar{T}_{\phi_1} + s_\beta c_\beta^2 \eta \delta \bar{T}_{\phi_2} \\
\delta m_{\phi_1\phi_2}^2 &= -s_\beta c_\beta (\delta m_A^2 + \delta M_Z^2 + c_{2\beta} (m_A^2 + M_Z^2) \tilde{\alpha}_\beta) - s_\beta^3 \eta \delta \bar{T}_{\phi_1} - c_\beta^3 \eta \delta \bar{T}_{\phi_2} \\
\delta m_{\phi_2\phi_2}^2 &= c_\beta^2 \delta m_A^2 + s_\beta^2 \delta M_Z^2 - \frac{1}{2} s_{2\beta}^2 (m_A^2 - M_Z^2) \tilde{\alpha}_\beta + s_\beta^2 c_\beta \eta \delta \bar{T}_{\phi_1} - s_\beta (1 + c_\beta^2) \eta \delta \bar{T}_{\phi_2} \\
\delta m_{\phi_1 G}^2 &= -\rho s_\beta \eta \delta \bar{T}_A \\
\delta m_{\phi_1 A}^2 &= -c_\beta \eta \delta \bar{T}_A \\
\delta m_{\phi_2 G}^2 &= \rho c_\beta \eta \delta \bar{T}_A \\
\delta m_{\phi_2 A}^2 &= -s_\beta \eta \delta \bar{T}_A \\
\delta m_{GG}^2 &= -c_\beta \eta \delta \bar{T}_{\phi_1} - s_\beta \eta \delta \bar{T}_{\phi_2} \\
\delta m_{GA}^2 &= \rho (-s_\beta c_\beta m_A^2 \tilde{\alpha}_\beta + s_\beta \eta \delta \bar{T}_{\phi_1} - c_\beta \eta \delta \bar{T}_{\phi_2})
\end{aligned} \tag{3.52}$$

Note that the only trace in (3.52) of the complex nature of the parameters is in $\delta \bar{T}_A$, whose contribution induce CP -violation via the mixing from the counterterms $\delta m_{\phi_{1,2}G}^2, \delta m_{\phi_{1,2}A}^2$.

3.3.2 One-loop charged Higgs boson mass

The counterterms for the charged Higgs sector in physical basis are obtained by doing

$$\delta M_{\phi^\pm \phi^\pm}^2 = \delta \bar{\mathcal{P}} \cdot \partial_{\bar{\mathcal{P}}} M_{\phi^\pm \phi^\pm}^2 \Big|_{T_{\phi_1}=T_{\phi_2}=T_A=0}, \quad \delta M_{G^\pm H^\pm}^2 = U_{c(0)}^T \delta M_{\phi^\pm \phi^\pm}^2 U_{c(0)} \tag{3.53}$$

with $U_{c(0)}$ as in (3.12). Note that $\delta M_{G^\pm H^\pm}^2 \neq \delta \bar{\mathcal{P}} \cdot \partial_{\bar{\mathcal{P}}} M_{G^\pm H^\pm}^2 \Big|_{T_x=0}$, because the matrix $U_{c(0)}$ introduce for the elements in $M_{G^\pm H^\pm}^2$ some extra dependence in β that does not arise from the vev's in the Lagrangian but from the tree-level rotation procedure, and the dependence from $U_{c(0)}$ must not acquire counterterm corrections. Then, the mass counterterms for the charged Higgs sector read

$$\begin{aligned}
\delta m_{G^\pm}^2 &= -c_\beta \eta \delta \bar{T}_{\phi_1} - s_\beta \eta \delta \bar{T}_{\phi_2} \\
\delta m_{G^+ H^-}^2 &= (\delta m_{G^- H^+}^2)^* = \rho (-s_\beta c_\beta m_{H^\pm}^2 \tilde{\alpha}_\beta + s_\beta \eta \delta \bar{T}_{\phi_1} - c_\beta \eta \delta \bar{T}_{\phi_2} - i \eta \delta \bar{T}_A)
\end{aligned} \tag{3.54}$$

In determining the propagator poles one needs to take into account this mixing with the Goldstone bosons, as well as with the gauge bosons, both for the charged Higgs boson as for the neutral ones described in the previous section. However, the possible mixing with the Goldstone bosons used to be numerically negligible for the Higgs mass evaluation [15], and in particular they do not contribute in the $O(m_t^4)$ approximation that we describe below (see sect. 3.4.3) and to which we restrict ourselves in the rest of this chapter for the one-

loop case. At two-loop order, the contributions of G and Z to the neutral Higgs propagators appear via terms of the form $(\hat{\Sigma}_{\phi G}(p^2))^2$ and $p^2(\hat{\Sigma}_{\phi Z}(p^2))^2$, where $\phi = h, H, A$, which is negligible compared to the leading $O(\alpha_t \alpha_s)$ terms. For the charged Higgs sector, the contributions from the mixing with $G^\pm$ would contribute to the prediction at two-loop order onwards via terms of the form $(\hat{\Sigma}_{H^\pm G^\pm}^{(1L)})^2$, again subleading with respect to the leading terms at $O(\alpha_t \alpha_s)$ that we will consider later in sect. 3.3.4. Consequently, we neglect those Higgs-Goldstone or Higgs- Z mixing contributions throughout our analysis.

Therefore, the charged Higgs-boson pole mass, $M_{H^\pm}$, including higher-order contributions entering via the renormalised charged Higgs-boson self-energy $\hat{\Sigma}_{H^+H^-}$, is obtained by solving the equation

$$p^2 - m_{H^\pm}^2 + \hat{\Sigma}_{H^\pm H^\pm}(p^2) = 0 \quad . \quad (3.55)$$

This yields $M_{H^\pm}^2$ as the real part of the complex pole of eq. (3.55). The renormalised charged Higgs-boson self-energy, $\hat{\Sigma}_{H^+H^-}$, is composed of the unrenormalised self-energy $\Sigma_{H^+H^-}$, the field renormalisation constant and the mass counterterm:

$$\hat{\Sigma}_{H^\pm H^\pm}(p^2) = \Sigma_{H^\pm H^\pm}(p^2) + \delta Z_{H^\pm}(p^2 - m_{H^\pm}^2) - \delta m_{H^\pm}^2 \quad . \quad (3.56)$$

As will be explained in detailed in the next section, the mass for $H^\pm$ does not receive radiative corrections if we renormalized it on-shell, which means that $\delta m_{H^\pm}^2$ is fixed in order to $\hat{\Sigma}_{H^\pm H^\pm}(p^2)$ vanishes, so that $M_{H^\pm} = m_{H^\pm}$.³⁶ This will not be the case if the A boson mass is renormalized on-shell instead, in which case the renormalization condition will be imposed on δm_A^2 and therefore the (finite part of) $\delta m_{H^\pm}^2$ will be different from the $H^\pm$ on-shell case. In any case, one of this counterterms can be derived from the other by the relation

$$\delta m_{H^\pm}^2 = \delta m_A^2 + \delta M_W^2 \quad (3.57)$$

which follows from the last line in eq. (3.18).

3.3.3 Renormalisation conditions

Regarding the renormalisation scheme, we renormalise the W and Z bosons on-shell, yielding the mass counterterms

³⁶ Our notation for the Higgs masses in this paper is such that lowest-order mass parameters are written in lower case, e.g. $m_{H^\pm}^2$, while loop-corrected masses are written in upper case, e.g. $M_{H^\pm}^2$.

$$\delta M_W^2 = \text{Re} \Sigma_{WW}^T(M_W^2) \quad , \quad \delta M_Z^2 = \text{Re} \Sigma_{ZZ}^T(M_Z^2) \quad (3.58)$$

where $\Sigma_{WW}^T, \Sigma_{ZZ}^T$ are the transverse parts of the W and Z boson self-energies, respectively. For the charged Higgs field, eq. (3.48) implies

$$H^\pm \rightarrow (1 + \frac{1}{2} \delta Z_{H^\pm}) H^\pm \quad (3.59)$$

with

$$\delta Z_{H^\pm} = s_\beta^2 \delta Z_{H_1} + c_\beta^2 \delta Z_{H_2} \quad . \quad (3.60)$$

From these field counterterms, the one for the parameter $\tan \beta$ in the $\overline{\text{DR}}$ scheme are evaluated:³⁷

$$\frac{\delta t_\beta^{\overline{\text{DR}}}}{t_\beta} = \frac{1}{2} (\delta Z_{H_2} - \delta Z_{H_1}) \quad (3.61)$$

This $\overline{\text{DR}}$ renormalisation of t_β , which is manifestly process-independent, is convenient since there is no obvious relation of this parameter to a specific physical observable that would favor a particular on-shell definition. Furthermore, the $\overline{\text{DR}}$ renormalisation of t_β has been shown to yield stable numerical results, and is also gauge-independent at the one-loop level within the class of R_ξ gauges [108].

Finally, in order to have not to take into account the higher-order tadpole insertions in the evaluation of n -point functions, we set the tadpole counterterms in such a way that those insertions cancel out at one-loop order:

$$\begin{aligned} \hat{T}_x &= T_x^{(1L)} - \eta \delta T_x = 0 \quad , \quad x = \phi_1, \phi_2, A \\ \text{thus} \quad \delta \overline{T}_x &= \frac{1}{\sqrt{2}v} \eta T_x^{(1L)} \end{aligned} \quad (3.62)$$

where $T_{\phi_i}^{(1L)}$ represents the one-loop radiative contribution to the tadpole of the field ϕ_i .

We will show in the next sections the evaluation of the higher-order contributions to the Higgs boson masses derived from this renormalisation prescriptions, which were performed at first “by-hand” in the case of the leading $O(m_t^4)$ one-loop corrections, while for the computation of *all*-sector contributions to the self-energies as specified in eq. (3.56) we have evaluated the full one-loop result with the help of the tools FEYNARTS [22-24] and

³⁷ See also discussion on the derivation of this expression in sect. 4.2.2.

FORMCALC [25] complementing our own Mathematica code. The corresponding Feynman-diagrams for the charged Higgs boson (and similarly for the W boson, where additional diagrams with gauge boson and ghost loops contribute) are shown in fig. 3.2. The diagrams for the neutral Higgs bosons, entering δM_A^2 and $\delta Z_{H_1}, \delta Z_{H_2}$ (i.e. the neutral Higgs boson self-energies) are depicted in fig. 3.3.

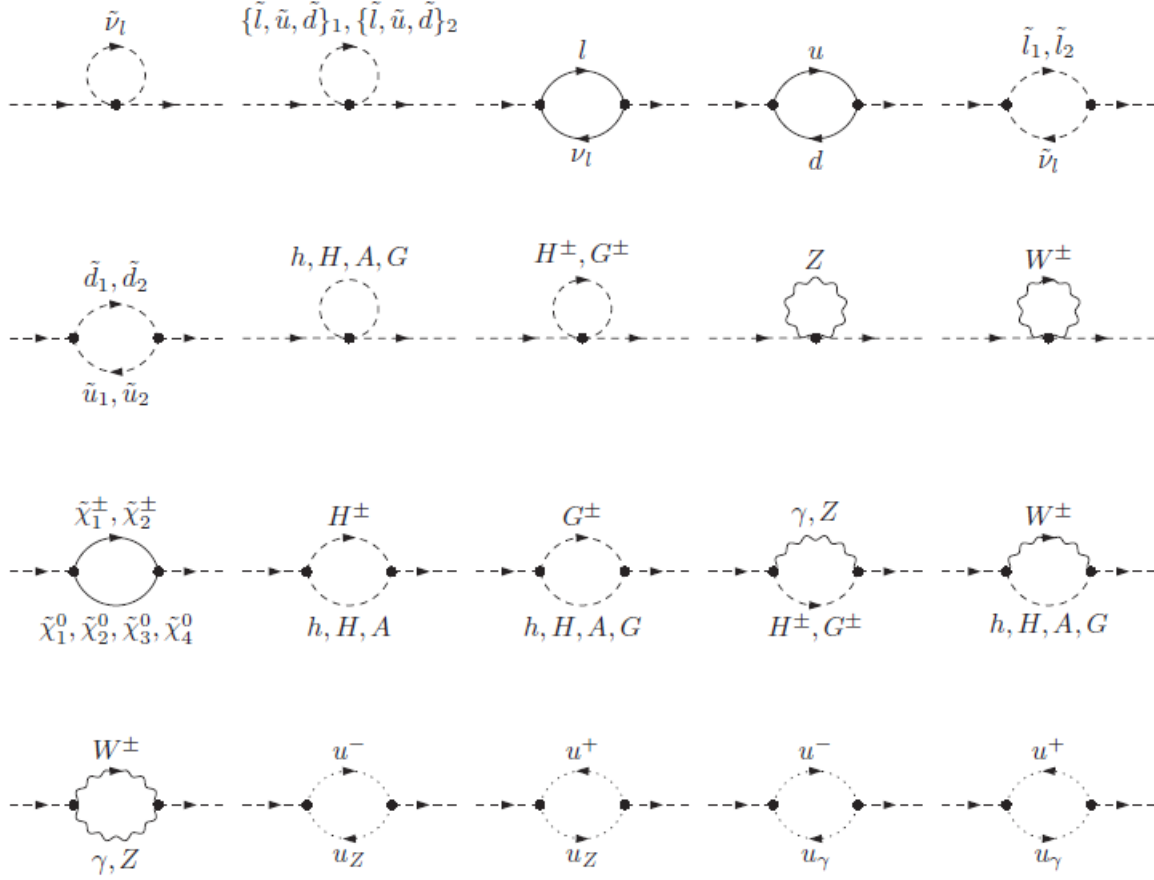


Figure 3.2. Generic Feynman diagrams for the $H^\pm$ self-energy ($l \in \{e, \mu, \tau\}$, $d \in \{d, s, b\}$, $u \in \{u, c, t\}$). Similar diagrams for the W boson self-energy are obtained by replacing the external Higgs boson by a W boson; not all combinations of particle insertions exist.

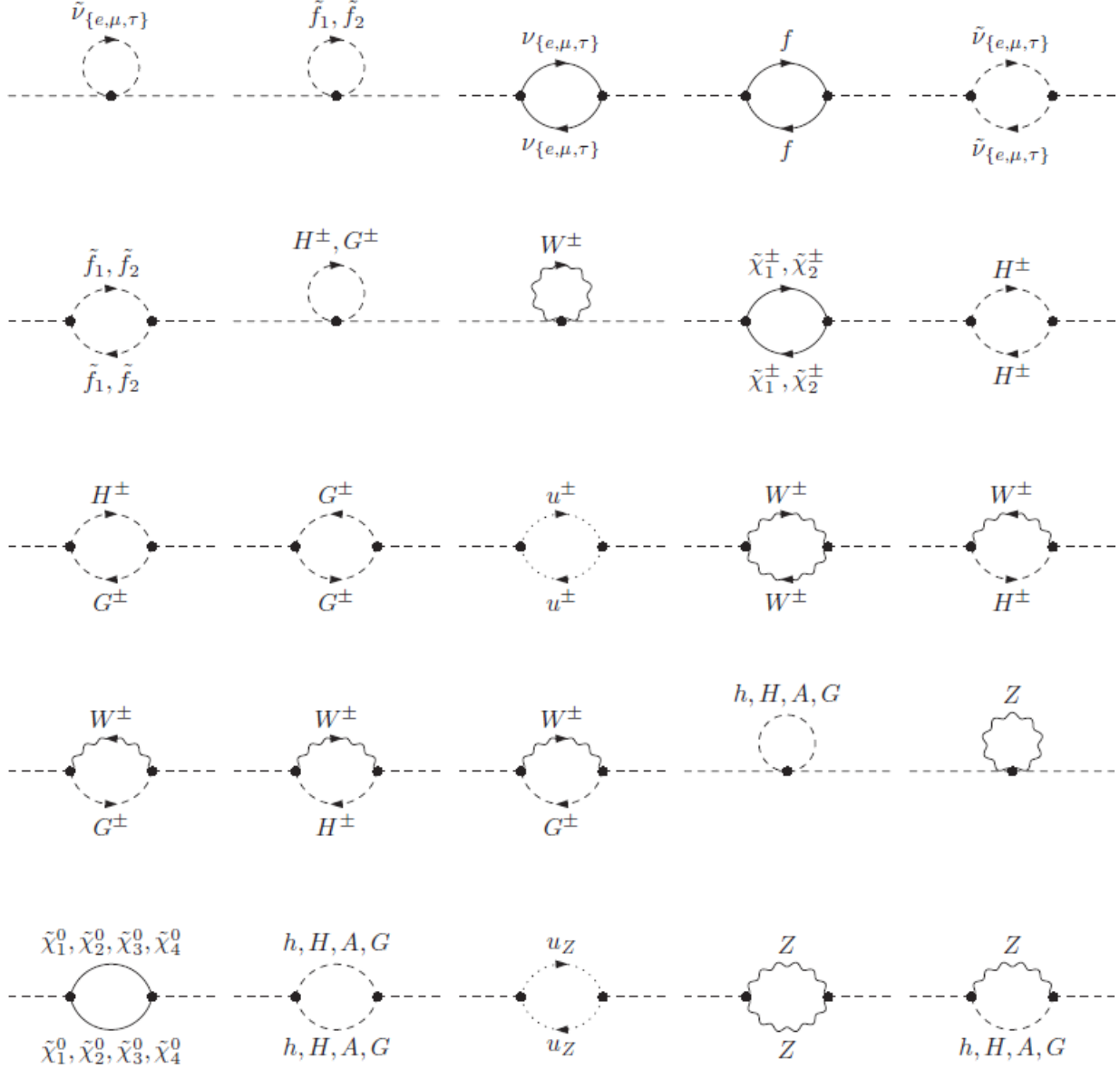


Figure 3.3. Generic Feynman diagrams for the h, H, A , self-energies ($f \in \{e, \mu, \tau, d, s, b, u, c, t\}$). Not all combinations of particle insertions exist for all neutral Higgs bosons.

The $O(m_t^4)$ approximation in the real and complex MSSM

We wish to compare here the expressions for M_A and $M_{H^\pm}$ with one or the other of these parameters renormalised as an on-shell quantity, in order that our result works for both the real case (“rMSSM”), where one conventionally chooses M_A as on-shell independent input parameter (so that the predictions for the neutral Higgs boson masses do not involve charged Higgs boson self-energies, and for the complex MSSM (“cMSSM”), where $M_{H^\pm}$

is renormalised on-shell (because the A field is no longer a mass eigenstate beyond tree-level).³⁸

Neglecting the radiative contributions from self-energies with external Goldstone bosons also in the neutral sector, expressions analogous to (3.55) and (3.56) apply to the A boson (it is derived from eq. (3.46) and the definition of the A field in (3.16)):

$$M_A^2 - m_A^2 + \hat{\Sigma}_{AA}(M_A^2) = 0 \quad (3.63)$$

$$\hat{\Sigma}_{AA}(M_A^2) = \Sigma_{AA}(M_A^2) + \delta Z_A(M_A^2 - m_A^2) - \delta m_A^2$$

where m_A^2, M_A^2 are the tree-level and one-loop A boson masses, respectively. If now one considers the $O(m_t^4)$ approximation, which consists on taking into account for the counterterms only the Yukawa contributions and evaluating the self-energies for vanishing external momentum, the following relations are obtained in both schemes for the corresponding higher-order contributions to the renormalised $H^\pm$ and A self-energies and the one-loop corrected values of the charged and the pseudo-scalar Higgs boson masses:

$$M_A \text{ on-shell:} \quad (3.64)$$

$$\begin{aligned} M_A^2 &= m_A^2 \quad , \quad \delta m_A^2 = \Sigma_{AA}(M_A^2) \approx \Sigma_{AA}(0) \\ \Delta M_{H^\pm}^2 &\equiv M_{H^\pm}^2 - m_{H^\pm}^2 \approx -\hat{\Sigma}_{H^\pm H^\pm}(0) \\ \hat{\Sigma}_{H^\pm H^\pm}(0) &\approx \Sigma_{H^\pm H^\pm}(0) - \Sigma_{AA}(0) \equiv \Delta_{H^\pm A} \end{aligned}$$

$$M_{H^\pm} \text{ on-shell:} \quad (3.65)$$

$$\begin{aligned} M_{H^\pm}^2 &= m_{H^\pm}^2 \quad , \quad \delta m_{H^\pm}^2 = \Sigma_{H^\pm H^\pm}(M_{H^\pm}^2) \approx \Sigma_{H^\pm H^\pm}(0) \\ \hat{\Sigma}_{AA}(0) &\approx \Sigma_{AA}(0) - \Sigma_{H^\pm H^\pm}(0) = -\Delta_{H^\pm A} \end{aligned}$$

where we have use that

$$\begin{aligned} \delta M_W^2 &= \delta M_Z^2 = \delta Z_{H_1} = \delta Z_{H_2} = 0 \\ (\Rightarrow \delta m_{H^\pm}^2 &= \delta m_A^2) \end{aligned} \quad (3.66)$$

in the aforementioned approximation. We will perform in section 3.4 the evaluation of the $O(m_t^4)$ contributions to $\Delta_{H^\pm A}$ (as well as the other leading self-energy contributions relevant to the calculation of the neutral Higgs masses) for the MSSM with complex parameters. The real limit of that result corresponds to the shift $-\Delta M_{H^\pm}^2$ in the expression

³⁸ In the complex MSSM, the A field is a physical field but only at tree-level, see eq. (3.18). The mixing between the CP -even and CP -odd Higgs sectors are induce by the higher-order contributions to δT_A .

for the charged Higgs boson mass in the scheme with M_A on-shell; we will use this result for the $O(m_t^4)$ case in our numerical analysis in sect. 3.5. The complex expression $\Delta_{H^\pm A}$, on the other hand, is just the $-\hat{\Sigma}_{AA}$ entry needed for the one-loop propagator in the neutral Higgs sector of the cMSSM (as was implemented for instance in [15]):³⁹

$$\begin{aligned} \text{rMSSM}, M_A^2 \text{ on-shell} &\Rightarrow \Delta M_{H^\pm}^2 = -\Delta_{H^\pm A}|_{\text{real}}, \quad M_A^2 = m_A^2 \\ \text{cMSSM}, M_{H^\pm}^2 \text{ on-shell} &\Rightarrow \Delta M_{H^\pm}^2 = 0, \quad \hat{\Sigma}_{AA} = -\Delta_{H^\pm A} \end{aligned} \quad (3.67)$$

Note that $M_{H^\pm}^2$ could be renormalised on-shell even for the real case (the fields $H^\pm$, defined as in (3.12), are physical degrees of freedom in both the real and complex MSSM). If this is the case, in addition to the relations in (3.65), $M_A^2 - m_A^2 \approx \Delta_{H^\pm A}$ holds. This implies that, *for both schemes* (3.64) and (3.65),⁴⁰ in the real MSSM the quantity $\Delta_{H^\pm A}$ is the shift from tree-level to one-loop order in the difference between the masses of A and $H^\pm$:

$$M_A^2 - M_{H^\pm}^2 = m_A^2 - m_{H^\pm}^2 + \Delta_{H^\pm A}|_{\text{real}} \quad \text{for rMSSM at } O(m_t^4) \quad (3.68)$$

3.3.4 Two-loop corrections to the charged Higgs mass

In order to reach a higher precision, two-loop corrections to $M_{H^\pm}^2$ should be included. They can be evaluated from the corrections of $O(\alpha_t^2)$ [88,89] and at $O(\alpha_t \alpha_s)$ by using the results of [91]. Here we include the corresponding discussion as given in ref. [1] for the rMSSM.

Again, we drop here the order-index for all Higgs boson and SM gauge boson self-energies and renormalisation constants, which are in this section understood to be of two-loop order. The $O(\alpha_t \alpha_s)$ terms are obtained in the limit of vanishing gauge couplings and neglecting the dependence on the external momentum [16], keeping only terms $\propto Y_t^2 \alpha_s$, with the top Yukawa coupling Y_t as defined in (2.1). We neglect the bottom Yukawa coupling in the two-loop Higgs-boson self-energies. In this approximation, the counterterms $\delta m_A^2, \delta M_W^2, \delta M_Z^2, \delta Z_{H^\pm}$ are determined as in eqs. (6.4),(6.6),

$$\delta m_A^2 = \Sigma_{AA}(0), \quad \delta M_W^2 = 0, \quad \delta Z_{H^\pm} = 0 \quad (3.69)$$

³⁹ The quantity called “ $\Delta_{H^\pm A}$ ” in eq. (67) from [15] differs in a factor c_β^2/c from the quantity $\Sigma_{H^\pm H^\pm}(0) - \Sigma_{AA}(0)$ that we define in (3.64), being “ c ” the factor defined here in eq. (3.83).

⁴⁰ The result (3.68) *must* be the same in both cases: m_A^2 and $m_{H^\pm}^2$ are tree-level parameters, so they are defined in the same way in any scheme; then, (3.68) is a relation between observables expressed in terms of the same set of parameters.

and consequently the expression for the two-loop renormalised $H^\pm$ self-energy can be written as

$$\hat{\Sigma}_{H^\pm H^\pm}(0) = \Sigma_{H^\pm H^\pm}(0) - \delta m_{H^\pm}^2 \quad \text{with} \quad \delta m_{H^\pm}^2 = \delta m_A^2 \quad . \quad (3.70)$$

From eq. (3.55) we get the two-loop correction to the charged Higgs-boson mass (see the discussion in [91]):

$$\Delta M_{H^\pm}^{2,2\text{-loop}} = \Sigma_{AA}(0) - \Sigma_{H^+ H^-}(0) \quad (3.71)$$

with the self-energies evaluated at the two-loop level.

One thus has to evaluate the $O(\alpha_t \alpha_s)$ contributions to the $H^\pm$ and A self-energies. Examples of generic Feynman diagrams for the $H^\pm$ self-energy are depicted in fig. 3.4, and in fig. 3.5 for the A boson self-energy.

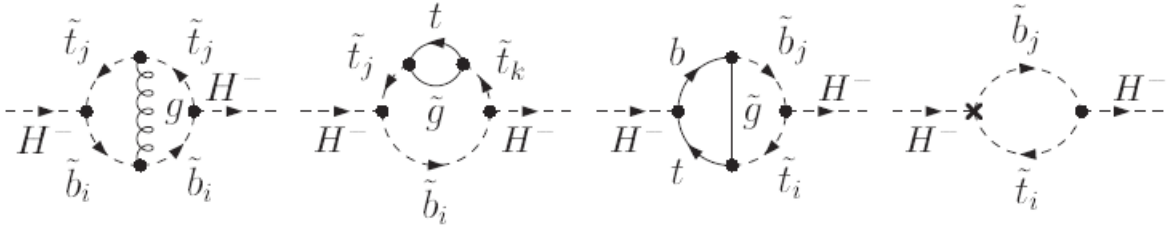


Figure 3.4. Examples of generic two-loop diagrams and diagrams with counterterm insertion for the charged Higgs-boson self-energy ($i,j,k = 1,2$).[91]

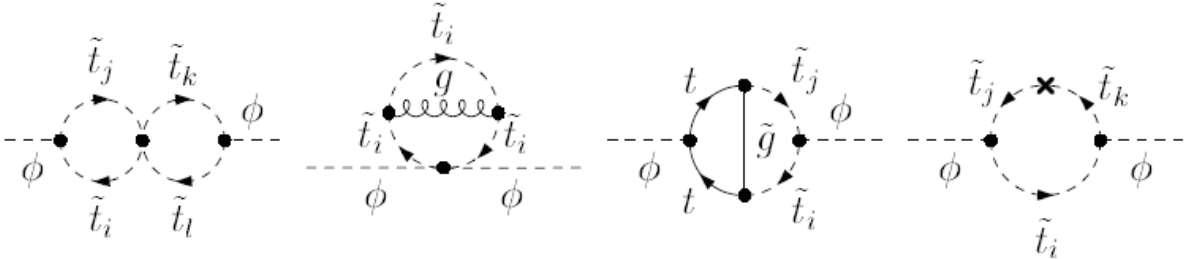


Figure 3.5. Examples of generic two-loop diagrams and diagrams with counterterm insertion for the A boson self-energy ($i,j,k,l = 1,2$).[91]

These approximations, together with the subloop renormalisation in the scalar top/bottom sector and the higher-order corrections in the $b/\tilde{b}$ sector from [1], provide the two-loop $O(\alpha_t \alpha_s)$ terms, which can be written as terms proportional to m_t^4 . For the neutral Higgs bosons this procedure yields the dominant part of the one-loop [77-82] and the two-loop corrections [16,83,84]. For the charged Higgs boson mass, $M_{H^\pm}$, the described procedure provides the analogous contribution to the mass shift as well,

$$\Delta M_{H^\pm}^2 \sim \frac{m_t^4}{v^2} \sim \frac{m_t^4}{M_W^2} \quad (3.72)$$

There are, however, other contributions of similar structure at the one-loop level [101-105]:

$$\Delta M_{H^\pm}^2 \sim \frac{m_t^2 m_b^2}{M_W^2} \quad \text{or} \quad \Delta M_{H^\pm}^2 \sim \frac{m_t^4}{M_W^2} \frac{M_W^2}{M_{\text{SUSY}}^2} \quad \text{or} \quad \Delta M_{H^\pm}^2 \sim \frac{m_t^4}{M_W^2} \frac{M_A^2}{M_{\text{SUSY}}^2} \quad (3.73)$$

which are not covered by our approximations for the two-loop terms because they would correspond to $m_b \neq 0$ (first), non-vanishing gauge-couplings (second), and $p^2 \neq 0$ in the A self-energy (third term). This is justified for large scalar-quark mass scales M_{SUSY} , where the second and third types of terms are suppressed. On the other hand, the term (3.72) extracted by our approximation can in general be large also for large M_{SUSY} , both at the one-loop and the two-loop level, as we will explain below.

3.4 Computation of renormalised self-energies

In this section we use the Feynman-diagrammatic (FD) approach⁴¹ by implementing the computation of the most relevant amplitudes for the one-loop corrections to the mass of the rMSSM CP -even Higgs bosons. The tree-level relationships for the masses of the Higgs bosons are strongly modified by radiative corrections involving the supersymmetric spectrum from the top sector [77-79,109,110]. The source of the major contributions to these corrections is the incomplete cancellations between the top and stops loops, reflecting the breaking of supersymmetry. Moreover, mass relations are affected by the potentially large mixing between $\tilde{t}_L$ and $\tilde{t}_R$ due to top Yukawa coupling. The dominant one-loop contributions for the mass matrix of eqs. (3.9),(3.17) can then be obtained by evaluating the contribution from the t - $\tilde{t}$ sector to the renormalised self-energies $\hat{\Sigma}_{\phi_i \phi_i}$ for zero external momentum ($p^2 = 0$)⁴² and only considering the Yukawa part of the theory (i.e. neglecting

⁴¹ Feynman-diagrammatic approach:

- Explicit evaluation of the relevant Feynman diagrams.
- The mass of a particle is the real part of the pole of its propagator.

The reference to this method is to differentiate it from other approaches that are not based on the direct calculation of the relevant amplitudes, such as the use of effective potentials.

⁴² The fact of being able to consider the external moment as null is due to the fact that we are interested in $O(m_t^4 / M_W^2)$ contributions, and the self-energies for arbitrary p^2 verify expansions of the form:

$$\Sigma_{\phi_i \phi_j}(p^2) = \Sigma_{\phi_i \phi_j}(0) + p^2 \partial_{q^2} \Sigma_{\phi_i \phi_j}(0) + \dots \sim \frac{m_t^4}{M_W^2} + p^2 \frac{m_t^2}{M_W^2} + \dots$$

the gauge couplings), and then introducing these self-energies in that matrix and implementing its diagonalization in order to obtain the one-loop corrected masses of the h and H bosons.

3.4.1 One-loop contributions to neutral Higgs masses

By applying (3.66) in (3.46) and to the counterterms from eq. (3.52), the leading $O(m_t^4)$ contributions to the one-loop renormalised self-energies for the neutral Higgs fields are obtained:

$$\begin{aligned}
\begin{pmatrix} \hat{\Sigma}_{\phi_1\phi_1} \\ \hat{\Sigma}_{\phi_1\phi_2} \\ \hat{\Sigma}_{\phi_2\phi_2} \end{pmatrix}_{p^2=0}^{(1L)} &= \begin{pmatrix} \Sigma_{\phi_1\phi_1} \\ \Sigma_{\phi_1\phi_2} \\ \Sigma_{\phi_2\phi_2} \end{pmatrix}_{p^2=0}^{(1L)} + \begin{pmatrix} -s_\beta^2 \\ s_\beta c_\beta \\ -c_\beta^2 \end{pmatrix} \delta m_A^{2,(1L)} + \begin{pmatrix} c_\beta(1+s_\beta^2) \\ s_\beta^3 \\ -s_\beta^2 c_\beta \end{pmatrix} \frac{e}{2M_W s_W} \eta T_{\phi_1}^{(1L)} + \begin{pmatrix} -s_\beta c_\beta^2 \\ c_\beta^3 \\ s_\beta(1+c_\beta^2) \end{pmatrix} \frac{e}{2M_W s_W} \eta T_{\phi_2}^{(1L)} \\
\begin{pmatrix} \hat{\Sigma}_{GG} \\ \hat{\Sigma}_{GA} \\ \hat{\Sigma}_{AA} \end{pmatrix}_{p^2=0}^{(1L)} &= \begin{pmatrix} \Sigma_{GG} \\ \Sigma_{GA} \\ \Sigma_{AA} \end{pmatrix}_{p^2=0}^{(1L)} + \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \delta m_A^{2,(1L)} + \begin{pmatrix} c_\beta \\ -\rho s_\beta \\ 0 \end{pmatrix} \frac{e}{2M_W s_W} \eta T_{\phi_1}^{(1L)} + \begin{pmatrix} s_\beta \\ \rho c_\beta \\ 0 \end{pmatrix} \frac{e}{2M_W s_W} \eta T_{\phi_2}^{(1L)} \\
\begin{pmatrix} \hat{\Sigma}_{\phi_1 G} & \hat{\Sigma}_{\phi_1 A} \\ \hat{\Sigma}_{\phi_2 G} & \hat{\Sigma}_{\phi_2 A} \end{pmatrix}_{p^2=0}^{(1L)} &= \begin{pmatrix} \Sigma_{\phi_1 G} & \Sigma_{\phi_1 A} \\ \Sigma_{\phi_2 G} & \Sigma_{\phi_2 A} \end{pmatrix}_{p^2=0}^{(1L)} + \begin{pmatrix} \rho s_\beta & c_\beta \\ -\rho c_\beta & s_\beta \end{pmatrix} \frac{e}{2M_W s_W} \eta T_A^{(1L)}
\end{aligned} \tag{3.74}$$

where, as was described in sect. 3.3.2,

$$\delta m_A^{2,(1L)} = \begin{cases} \Sigma_{AA}^{(1L)}(p^2=0) & , \quad M_A \text{ on-shell} \\ \Sigma_{H^\pm H^\pm}^{(1L)}(p^2=0) & , \quad M_{H^\pm} \text{ on-shell} \end{cases} . \tag{3.75}$$

For the real case with M_A on-shell, for instance, the diagrams that could possibly contribute to the CP -even sector in (3.74) at m_t^4 order are presented in fig. 3.6. We see from the last line in (3.74) that the CP -violating mixing are induced from the unrenormalised self-energies $\Sigma_{\phi_{1,2}G}, \Sigma_{\phi_{1,2}A}$ as well as from the A one-loop tadpole contribution, which are zero for the real-MSSM.

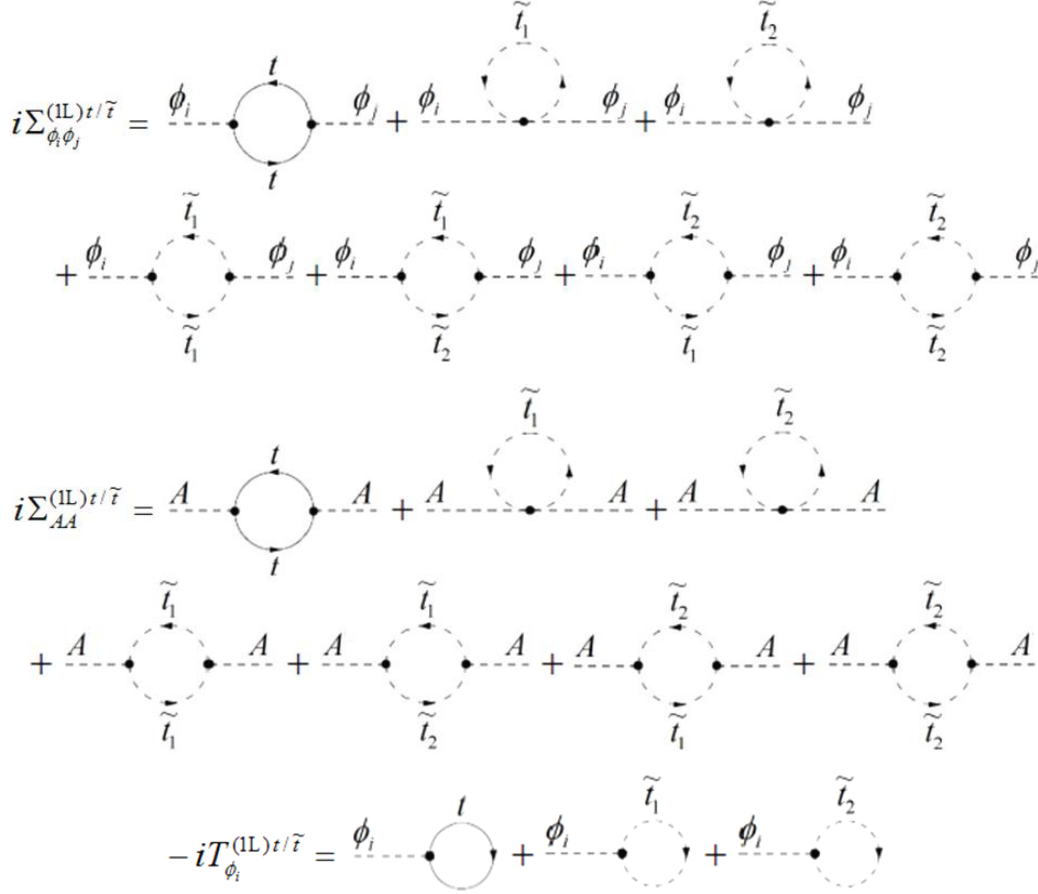


Figure 3.6: Diagrams that contribute to the evaluation at $O(m_t^4)$ of the renormalised one-loop self-energies, corresponding to virtual tops and stops. The Σ 's are unrenormalisation self-energies, whereas the T_i are tadpole-type contributions.

By performing the explicit evaluation of these Feynman diagrams, the $\propto m_t^4$ contributions to the renormalised self-energies are obtained:

$$\begin{aligned}
\hat{\Sigma}_{\phi_1\phi_1} &= -\frac{3G_F m_t^4}{\sqrt{2}\pi^2 s_\beta^2} \left[\frac{(\text{Re}(\mu X_t))^2 - (\text{Im}(\mu A_t))^2}{2(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \right] + s_\beta^2 \hat{\Sigma}_{AA} \\
\hat{\Sigma}_{\phi_1\phi_2} &= \frac{3G_F m_t^4}{\sqrt{2}\pi^2 s_\beta^2} \left[\frac{\text{Re}(\mu X_t)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \ln \frac{m_{\tilde{t}_1}}{m_{\tilde{t}_2}} + \frac{\text{Re}(\mu X_t) \text{Re}(A_t^* X_t) - (\text{Im}(\mu A_t))^2 / t_\beta}{2(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \right] - s_\beta c_\beta \hat{\Sigma}_{AA} \\
\hat{\Sigma}_{\phi_2\phi_2} &= -\frac{3G_F m_t^4}{\sqrt{2}\pi^2 s_\beta^2} \left[\ln \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} + \frac{\text{Re}(A_t^* X_t)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \ln \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} + \frac{(\text{Re}(A_t^* X_t))^2 - (\text{Im}(\mu A_t) / t_\beta)^2}{2(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \right] + c_\beta^2 \hat{\Sigma}_{AA} \\
\hat{\Sigma}_{\phi_1 A} &= \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 s_\beta^3} \frac{\text{Re}(\mu X_t) \text{Im}(\mu A_t)}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2)
\end{aligned}$$

$$\hat{\Sigma}_{\phi_2 A} = -\frac{3G_F m_t^4}{2\sqrt{2}\pi^2 s_\beta^3} \frac{\text{Im}(\mu A_t)}{m_{t_1}^2 - m_{t_2}^2} \left(\ln \frac{m_{t_1}^2}{m_{t_2}^2} + \frac{\text{Re}(A_t^* X_t)}{m_{t_1}^2 - m_{t_2}^2} g(m_{t_1}^2, m_{t_2}^2) \right)$$

$$\hat{\Sigma}_{\phi_1 G}(0) = \hat{\Sigma}_{\phi_2 G}(0) = \hat{\Sigma}_{GG}(0) = \hat{\Sigma}_{GA}(0) = 0 \quad (3.76)$$

where

$$g(m_{t_1}^2, m_{t_2}^2) \equiv 2 \left(1 - \frac{m_{t_1}^2 + m_{t_2}^2}{m_{t_1}^2 - m_{t_2}^2} \ln \frac{m_{t_1}}{m_{t_2}} \right) \quad (3.77)$$

and being G_F the Fermi constant:

$$G_F \equiv \frac{e^2}{4\sqrt{2}M_W^2 s_W^2} \quad (3.78)$$

It is seen in (3.76) how the mixing between A and the CP -even fields $\phi_{1,2}$ occurs only if $\text{Im}(\mu A_t) \neq 0$. In eq. (3.76), $\hat{\Sigma}_{AA}$ must be set to

$$\hat{\Sigma}_{AA} = \begin{cases} 0 & , \quad M_A^2 \text{ on-shell} \\ -\Delta_{H^\pm A} & , \quad M_{H^\pm}^2 \text{ on-shell} \end{cases} \quad (3.79)$$

where the explicit expression for the quantity $\Delta_{H^\pm A}$ is the one derived in sect. 3.4.3. By comparison with the full one-loop result for the real case [111], the contributions from (3.76) have in fact been shown to contain the main one-loop corrections: typically approximate the full result for the masses by less than 5 GeV.

3.4.2 One-loop masses of h and H at $O(m_t^4)$

Specially simple explicit results can be derive for the CP -even MSSM Higgs boson masses in the particular case of real parameters, as a function of the self-energies obtained in the previous section, which we briefly review here for completeness. Leading contributions beyond tree-level are obtained from the matrix

$$M_{\phi\phi}^{2,(1L)}(p^2) \equiv \begin{pmatrix} M_A^2 s_\beta^2 + M_Z^2 c_\beta^2 - \hat{\Sigma}_{\phi_1 \phi_1}^{(1L)}(p^2) & -(M_A^2 + M_Z^2) s_\beta c_\beta - \hat{\Sigma}_{\phi_1 \phi_2}^{(1L)}(p^2) \\ * & M_A^2 c_\beta^2 + M_Z^2 s_\beta^2 - \hat{\Sigma}_{\phi_2 \phi_2}^{(1L)}(p^2) \end{pmatrix} \quad (3.80)$$

While in the general case of considering the dependence on p^2 for the self-energies the masses are obtained from the zeros in p^2 of $\det(p^2 \mathbf{I} - \mathbf{M}_{\phi\phi}^{2,(1L)})$, for the leading $O(m_t^4)$ one-loop corrections the $\hat{\Sigma}_{\phi_i\phi_j}$ are evaluated at $p^2 = 0$, so physical fields $H^{(1L)}, h^{(1L)}$ can be defined from the diagonalization of $\mathbf{M}_{\phi\phi}^{2,(1L)}$ through the following orthogonal transformation:

$$\begin{pmatrix} H^{(1L)} \\ h^{(1L)} \end{pmatrix} \equiv \mathbf{D}^T(\alpha^{(1L)}) \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \mathbf{D}(\alpha^{(1L)}) = \begin{pmatrix} \cos \alpha^{(1L)} & \sin \alpha^{(1L)} \\ -\sin \alpha^{(1L)} & \cos \alpha^{(1L)} \end{pmatrix} \quad (3.81)$$

$$\mathbf{M}_{\phi\phi}^{2,(1L)} = \mathbf{D}(\alpha^{(1L)}) \begin{pmatrix} m_H^2 & 0 \\ 0 & m_h^2 \end{pmatrix} \mathbf{D}^T(\alpha^{(1L)})$$

$$\tan(2\alpha^{(1L)}) = \tan(2\beta) \frac{M_A^2 + M_Z^2 - 2\hat{\Sigma}_{\phi_1\phi_2}/s_{2\beta}}{M_A^2 - M_Z^2 + (\hat{\Sigma}_{\phi_1\phi_1} - \hat{\Sigma}_{\phi_2\phi_2})/c_{2\beta}}, \quad -\frac{\pi}{2} \leq \alpha^{(1L)} \leq \frac{\pi}{2}$$

For the mass eigenvalues we obtain:

$$\begin{aligned} m_{H,h}^2 = & \frac{1}{2}(M_A^2 + M_Z^2) - \frac{1}{2}(\hat{\Sigma}_{\phi_1\phi_1} + \hat{\Sigma}_{\phi_2\phi_2}) \pm \left[\left(\frac{1}{2}(M_A^2 + M_Z^2) \right)^2 - (M_A M_Z c_{2\beta})^2 \right. \\ & + ((M_A^2 - M_Z^2)c_{2\beta} + \frac{1}{2}(\hat{\Sigma}_{\phi_1\phi_1} - \hat{\Sigma}_{\phi_2\phi_2})) \frac{1}{2}(\hat{\Sigma}_{\phi_1\phi_1} - \hat{\Sigma}_{\phi_2\phi_2}) \\ & \left. + ((M_A^2 + M_Z^2)s_{2\beta} + \hat{\Sigma}_{\phi_1\phi_2}) \hat{\Sigma}_{\phi_1\phi_2} \right]^{\frac{1}{2}} \end{aligned} \quad (3.82)$$

3.4.3 Application to $M_{H^\pm}$

In the following, we use the factor “ c ” to simplify the notation:

$$c \equiv -\frac{3m_t^2}{16\pi^2 v^2 t_\beta^2} = -\frac{3e^2 m_t^2}{32\pi^2 s_W^2 M_W^2 t_\beta^2} = -\frac{3G_F m_t^2}{4\sqrt{2}\pi^2 t_\beta^2} \quad (3.83)$$

From analogous procedure to sect. 3.4.1, from the third (s)quark generation, with $m_b = 0$, one obtains the following explicit expressions:⁴³

⁴³ Unless otherwise specified, these quantities are understood to be one-loop quantities. Here the one-loop integral function $A_0(m)$, as well as the divergent quantity Δ in eqs. (3.87) and (3.88), follow from dimensional regularisation. See [41,42] for definitions and details.

$$\Sigma_{AA}(0)/c = 2A_0(m_t) - A_0(m_{t_2}) - A_0(m_{t_1}) - (A_t + \mu t_\beta)^2 \frac{A_0(m_{t_2}) - A_0(m_{t_1})}{m_{t_2}^2 - m_{t_1}^2} \quad (3.84)$$

$$\Sigma_{H^\pm H^\pm}(0) = c \left[2A_0(m_t) - A_0(m_{b_1}) - s_t^2 A_0(m_{t_1}) - c_t^2 A_0(m_{t_2}) \right. \\ \left. - (c_t m_t + s_t (A_t + \mu t_\beta))^2 \frac{A_0(m_{t_1}) - A_0(m_{b_1})}{m_{t_1}^2 - m_{b_1}^2} - (s_t m_t - c_t (A_t + \mu t_\beta))^2 \frac{A_0(m_{t_2}) - A_0(m_{b_1})}{m_{t_2}^2 - m_{b_1}^2} \right]$$

In the approximation of $m_b = 0$ and neglected gauge couplings, the mass of the left-handed sbottom is given by (see eq. (3.39))

$$m_{b_L}^2 \approx c_t^2 m_{t_1}^2 + s_t^2 m_{t_2}^2 - m_t^2 (\approx M_{Q_3}^2) \quad . \quad (3.85)$$

Using these relations results then in the following expression for $\Delta m_{H^\pm}^2$ as defined in (3.64):

$$\Delta_{H^\pm A}/c = m_{b_L}^2 \left[1 + \frac{((A_t + \mu t_\beta)s_t + m_t c_t)^2}{m_{b_L}^2 - m_{t_1}^2} + \frac{(m_t s_t - (A_t + \mu t_\beta)c_t)^2}{m_{b_L}^2 - m_{t_2}^2} \right] \ln \frac{m_{b_L}^2}{m_{t_1}^2} \\ + m_{t_2}^2 \left[-s_t^2 + \frac{(A_t + \mu t_\beta)^2}{m_{t_1}^2 - m_{t_2}^2} - \frac{(m_t s_t - (A_t + \mu t_\beta)c_t)^2}{m_{b_L}^2 - m_{t_2}^2} \right] \ln \frac{m_{t_2}^2}{m_{t_1}^2} \quad (3.86)$$

It is possible to eliminate the dependence on θ_t from this result. For instance, for the A boson self-energy we obtain

$$\Sigma_{AA}(0)/c = 2m_t^2(1 + \Delta - \ln m_t^2) - (m_{t_1}^2 + m_{t_2}^2 + |A_t + \mu^* t_\beta|^2) \left(1 + \Delta - \frac{m_{t_1}^2 \ln m_{t_1}^2 - m_{t_2}^2 \ln m_{t_2}^2}{m_{t_1}^2 - m_{t_2}^2} \right) \\ - \frac{2m_{t_1}^2 m_{t_2}^2}{m_{t_1}^2 - m_{t_2}^2} \ln \frac{m_{t_1}^2}{m_{t_2}^2} + \left(\frac{2m_t \text{Im}(\mu A_t)}{s_\beta c_\beta (m_{t_1}^2 - m_{t_2}^2)} \right)^2 \left(1 - \frac{m_{t_1}^2 + m_{t_2}^2}{m_{t_1}^2 - m_{t_2}^2} \ln \frac{m_{t_1}}{m_{t_2}} \right) \quad . \quad (3.87)$$

Accordingly, the expression for the $\Delta_{H^\pm A} \equiv \Sigma_{H^\pm H^\pm}(0) - \Sigma_{AA}(0)$ radiative corrections acquires the following final form:

$$\Delta_{H^\pm A} = -\frac{cm_t^2}{s_\beta^2 c_\beta^2} \left\{ \left(\frac{2 \text{Im}(\mu A_t)}{m_{t_1}^2 - m_{t_2}^2} \right)^2 \left(1 - \frac{m_{t_1}^2 + m_{t_2}^2}{m_{t_1}^2 - m_{t_2}^2} \ln \frac{m_{t_1}}{m_{t_2}} \right) \right. \\ \left. + \frac{|\mu|^2}{(m_{b_L}^2 - m_{t_1}^2)(m_{b_L}^2 - m_{t_2}^2)} \left(m_{b_L}^2 \ln \frac{m_{b_L}^2}{m_{t_1} m_{t_2}} - \frac{m_{t_1}^2 (m_{b_L}^2 - m_{t_2}^2) + m_{t_2}^2 (m_{b_L}^2 - m_{t_1}^2)}{m_{t_1}^2 - m_{t_2}^2} \ln \frac{m_{t_1}}{m_{t_2}} \right) \right\} \quad (3.88)$$

This shows explicitly the m_t^4 dependence of this contribution (keeping in mind the prefactor $c \propto m_t^2$ from eq. (3.83)), as well as the overall factor $|\mu|^2 / c_\beta^2$, which strongly determines the phenomenology of the $O(\alpha_t)$ charged Higgs boson mass corrections in the MSSM. Note that the result (3.88), as well as the renormalised self-energies from (3.76), are all independent of the sign convention adopted for $\rho, \varepsilon_{12}, \eta$ from tab. 3.1, as they must be.

As in the neutral sector, the renormalised self-energies related to the Goldstone fields vanish in this approximation:

$$\hat{\Sigma}_{G^\pm G^\pm}(0) = \hat{\Sigma}_{G^- H^+}(0) = \hat{\Sigma}_{G^+ H^-}(0) = 0 \quad (3.89)$$

as was anticipated in sect. 3.3.2. Therefore, $\Delta_{H^\pm A}$ can be directly associated to the charged Higgs mass. In the following of this section, we specify the analytic result assuming a common SUSY mass scale $M_{\tilde{Q}_3}^2 \approx M_{\tilde{t}_R}^2 \approx M_{\text{SUSY}}^2 \gg M_Z^2$. With this simplification one obtains

$$m_{\tilde{t}_1}^2 = M_{\text{SUSY}}^2 + m_t^2 - m_t |X_t|, \quad m_{\tilde{t}_2}^2 = M_{\text{SUSY}}^2 + m_t^2 + m_t |X_t|, \quad m_{\tilde{b}_L}^2 = M_{\text{SUSY}}^2 \quad (3.90)$$

and $s_t^2 = c_t^2 = 1/2$ in this case (i.e. maximal mixing, see sect. 3.2). This yields

$$\begin{aligned} \Delta M_{H^\pm}^2 &= \frac{cm_t^2 \mu^2}{\sin^2 \beta \cos^2 \beta} \frac{1}{2m_t^2 |X_t| (X_t^2 - m_t^2)} \\ &\times \left(m_t (M_{\text{SUSY}}^2 + m_t^2 - X_t^2) \ln \frac{M_{\text{SUSY}}^2 + m_t(m_t - |X_t|)}{M_{\text{SUSY}}^2 + m_t(m_t + |X_t|)} - M_{\text{SUSY}}^2 |X_t| \ln \frac{M_{\text{SUSY}}^4}{(M_{\text{SUSY}}^2 + m_t^2)^2 - m_t^2 X_t^2} \right) \end{aligned} \quad (3.91)$$

for the leading one-loop correction to the charged Higgs boson mass in the rMSSM with M_A on-shell. Expanding in inverse powers of M_{SUSY} and inserting the prefactor c from eq. (3.83), we find

$$\Delta M_{H^\pm}^2 \approx -\frac{3e^2 \mu^2}{64\pi^2 s_w^2 \sin^4 \beta} \frac{m_t^4}{M_W^2} \left(\frac{1}{M_{\text{SUSY}}^2} - \frac{2m_t^2}{3M_{\text{SUSY}}^4} + \frac{m_t^2(3m_t^2 + X_t^2)}{6M_{\text{SUSY}}^6} \right). \quad (3.92)$$

Thus one obtains the term proportional to m_t^4 / M_W^2 . If in addition $X_t = 0$ and restricting to the leading term in the expansion in inverse powers of M_{SUSY} , this reduces to

$$\Delta M_{H^\pm}^2 \approx -\frac{3e^2 \mu^2}{64\pi^2 s_w^2 \sin^4 \beta} \frac{m_t^4}{M_W^2 m_t^2} \sim (19 \text{ GeV})^2 \frac{\mu^2}{M_{\text{SUSY}}^2 m_t^2} (1 + 1/t_\beta^2)^2 \quad (3.93)$$

where $m_{\tilde{t}_1}^2 = m_{\tilde{t}_2}^2 \equiv m_{\tilde{t}}^2 = M_{\text{SUSY}}^2 + m_t^2$. If $|\mu| \approx m_{\tilde{t}}$, this term is not suppressed by large SUSY mass scales.

The two-loop case

The derivation of eqs. (3.92) and (3.93) shows that besides the m_t^2 in the prefactor arising from the Yukawa couplings, the second factor $\sim m_t^2$ stems from the stop mass matrix. In other words, it is induced by the $SU(2)$ breaking in the MSSM quark and squark sector. Thus, the derived term $\sim m_t^4 / M_W^2$ is related to the mass difference between top and bottom squarks resulting from $m_t / m_b \gg 1$. The diagrams playing the leading role here are the second and sixth Feynman diagram in figs. 3.2 and 3.3.

Eqs. (3.92) and (3.93) indicate which parameter combinations of A_t , μ and $\tan\beta$ can give rise to a sizable $O(\alpha_t)$ contribution to $M_{H^\pm}^2$ and possibly constitute a large part of the full one-loop corrections. For the corresponding parameter ranges it can be expected that also the two-loop corrections of $O(\alpha_t \alpha_s)$ are sizable and should be taken into account.

At the two-loop level, the $\sim m_t^4$ contributions are augmented by the corresponding term with a renormalised m_t parameter, leading to $\sim 4m_t^3 \delta m_t$. The source of these corrections is still related to the $SU(2)$ breaking inducing the mass difference for scalar tops and bottoms, which enters the two-loop level Higgs-boson self-energies through mass-counterterm insertions, as illustrated in the fourth diagram in fig. 3.5. The inserted one-loop counterterms for top-squarks and for bottom-squarks differ essentially by a term $2m_t \delta m_t$, which induces an effective mass splitting between the scalar top and bottom sector at the counterterm level. The full contribution $\sim \delta m_t$ can be obtained by renormalizing m_t in eqs. (3.92) and (3.93), or by an explicit extraction of this term. In the case of vanishing stop mixing, corresponding to eq. (3.93), it was checked in [1] that both calculations indeed agree. In this case they yield)

$$\begin{aligned} \Delta m_{H^\pm}^{2,2\text{-loop},\delta m_t} &\sim \frac{(A_t + \mu \tan\beta)^2}{m_t^2} \left(\delta m_t^2 \frac{m_t^2}{m_{\tilde{t}}^2} - (\delta m_{\tilde{t}}^2 - 2m_t \delta m_t) \ln \frac{m_{\tilde{t}}^2}{m_{\tilde{t}}^2 - m_t^2} \right) \\ &= \frac{\mu^2}{\sin^2 \beta \cos^2 \beta} \left(\frac{\delta m_{\tilde{t}}^2}{m_{\tilde{t}}^2} - \frac{\delta m_{\tilde{t}}^2 - 2m_t \delta m_t}{m_t^2} \ln \frac{m_{\tilde{t}}^2}{m_{\tilde{t}}^2 - m_t^2} \right) \\ &\approx \frac{\mu^2}{\sin^2 \beta \cos^2 \beta} \frac{2m_t \delta m_t}{m_{\tilde{t}}^2}. \end{aligned} \quad (3.94)$$

For the case of non-vanishing stop mixing, see eq. (3.92), one finds accordingly

$$\Delta m_{H^\pm}^{2,2\text{-loop},\delta m_t} \sim \frac{\mu^2}{\sin^2 \beta \cos^2 \beta} \left(\frac{2m_t}{M_{\text{SUSY}}^2} - \frac{2m_t^3}{M_{\text{SUSY}}^4} + \frac{m_t^3(4m_t^2 + X_t^2)}{2M_{\text{SUSY}}^6} \right) \delta m_t \quad (3.95)$$

In conclusion, although the two-loop corrections to $M_{H^\pm}^2$ covered by this approach are only part of the complete two-loop Yukawa corrections, they constitute a finite well-defined subset that can induce non-negligible mass shifts for the $H^\pm$ boson. Numerical examples will be given in the following section.

3.5 Numerical analysis for $M_{H^\pm}$

In this section we will show a numerical exploration based on some of the analytical results of the previous sections. We will circumscribe here to the MSSM with real parameters in order to match with the results implement in our article [1].⁴⁴

Higher-order corrected Higgs-boson sector has been implemented and evaluated here both by using our own Mathematica code and with the help of the Fortran code FEYNHIGGS (where our higher-order corrections had been implemented). The goal for the precision in predicting $M_{H^\pm}$ in the MSSM should be the prospective experimental resolution or better. For the LHC no dedicated study has been performed before. At the LC, for $M_{H^\pm} > m_t$ (but $M_{H^\pm} < \sqrt{s}/2$) a $\sim 1.5\%$ precision might be reachable using the $t\bar{b}$ decay mode [112].

Due to the large number of MSSM parameters, certain benchmark scenarios [11,113,114] have been used for the interpretation of MSSM Higgs boson searches at LEP, the Tevatron and the LHC. Since at tree level the Higgs sector of the MSSM with real parameters is governed by two parameters (in addition to the gauge couplings), the definition of the benchmarks is usually such that the two tree-level parameters, M_A and $\tan\beta$, are varied while the values of all other parameters are fixed at certain benchmark settings. The most commonly used benchmark scenario for the CP -conserving MSSM has been the “ $m_h^{\max}$ scenario”, and we therefore employ it in our analysis.⁴⁵ While the interpretation of the discovered Higgs-like state as the light MSSM Higgs boson is compatible with this scenario only within a strip at relatively low $\tan\beta$, it should be noted that changing the stop mixing parameter X_t from the “maximal” value of $X_t/M_{\text{SUSY}} \sim 2$ to slightly smaller values (with the other parameters fixed) yields $M_h \sim 125\text{ GeV}$ over large parts of the

⁴⁴ In case of consider the cMSSM, it should be taken into account that, in addition to the parameter μ and the A_q ’s, complex phases relevant to the Higgs sector enter also via the soft breaking terms M_1 and M_2 involved in the one-loop (and beyond) corrections.

⁴⁵ We employed in [1] also a scenario where the heavy CP -even Higgs boson is interpreted as the newly discovered particle at $\sim 125\text{ GeV}$. This scenario is meanwhile excluded via the search for charged Higgs bosons [119]. It has been replaced by a set of new scenarios [120], but no phenomenological analysis is available so far.

parameter space. Consequently, this scenario is expected to provide a good indication of the possible size of the radiative corrections to $M_{H^\pm}$. The scenario is defined as follows:

The $m_h^{\max}$ scenario:

The parameters are chosen such that the mass of the light CP -even Higgs boson acquires its maximum possible values as a function of $\tan\beta$ (for fixed M_{SUSY} and m_t , and M_A set to its maximum value, $M_A = 1 \text{ TeV}$). This was used in particular to obtain conservative $\tan\beta$ exclusion bounds at LEP [65]. The parameters are (including the most recent value for m_t [115]):

m_t	$\bar{\mu}$	M_{SUSY}	μ	M_2	X_t	A_b	$m_{\tilde{g}}$	
173.2 GeV	m_t	1000 GeV	200 GeV	200 GeV	$2 M_{\text{SUSY}}$	A_t	$0.8 M_{\text{SUSY}}$	(3.96)

$M_{\text{SUSY}} (\equiv M_{\tilde{Q}_3}^2 = M_{\tilde{t}_R}^2 = M_{\tilde{b}_R}^2)$ again denotes the diagonal soft SUSY-breaking parameters in the $\tilde{t}/\tilde{b}$ mass matrices, see eq. (3.25), that are all chosen to be equal. M_{SUSY} and X_t in this scenario correspond to the parameters used to express the m_t^4/M_W^2 corrections as given in eqs. (3.92),(3.95). In order to avoid conflicts with the LHC searches for squarks of the first and second generation, contrary to the original definition, M_{SUSY} should only be considered to fix the soft SUSY-breaking parameters for the squarks of the third generation, while the first two generations play a small role for the MSSM Higgs phenomenology. To fix a value for the squarks of the first two generations, for sake of simplicity, we kept the value of M_{SUSY} , but choosing higher values has a minor impact (see below). The gluino mass parameter, $m_{\tilde{g}}$, is in conflict with recent LHC SUSY searches. However, since also the impact of $m_{\tilde{g}}$ is relatively small, we keep its value at the original definition.

As discussed above, there are also potentially large corrections in the $b/\tilde{b}$ sector, depending on the value and sign of the parameter μ [114]. Consequently, besides analysing the $M_{H^\pm}$ dependence on M_A and $\tan\beta$, we also study the effect of a variation of μ , allowing both an enhancement and a suppression of the bottom Yukawa coupling. Concerning the $m_h^{\max}$ benchmark scenario, the Δ_b effects⁴⁶ are particularly pronounced.

The other MSSM parameters that are not specified above, such as the slepton masses, have only a minor impact on MSSM Higgs-boson phenomenology. In our numerical analysis below, we fix them such that all soft SUSY-breaking parameters in the diagonal entries of

⁴⁶ In order to include higher-order corrections from the (s)bottom sector in the predictions for $M_{H^\pm}$, the leading $\tan\beta$ -enhanced contribution in the limit of heavy SUSY masses can be expressed in terms of this quantity Δ_b and resummed to all orders using an effective Lagrangian approach. The corrections obtained in this way are described and characterized in our publication [1], and thus we include them in this numerical analysis for completeness.

the slepton mass matrices are set to M_{SUSY} , and the trilinear couplings for all sfermions are set to A_t , if not indicated differently for A_b ($= A_t$).

For our numerical evaluation the poles of the inverse propagator matrix are determined numerically. The algorithm for this procedure is the same as the one described in [116]:

«The mass eigenvalues are obtained iteratively. In order to obtain the lightest scalar Higgs boson mass e.g., in the first iteration the external momentum squared p^2 in the renormalised self-energies $\hat{\Sigma}$ is set equal to the lightest scalar tree-level mass squared. Then, $(p^2 I - M^2 + (\hat{\Sigma}))$ from eq. (3.44) is diagonalized, and the resulting mass eigenvalues squared are used in the next iteration where p^2 is set equal to the lightest of the obtained mass eigenvalues. Once again the mass eigenvalues are obtained. The procedure is repeated until the deviation between the lightest eigenvalue and the one of the previous iteration is less than 10^{-9} . The other Higgs mass eigenvalues are derived accordingly.»

3.5.1 One-loop corrections

We start with the analysis of the various one-loop contributions. In figs. 3.7 and 3.8 we show $\Delta M_{H^\pm} := M_{H^\pm} - m_{H^\pm}$, i.e. the difference between the result with radiative corrections and the tree-level value, in various approximations. The solid lines are the full one-loop result including the Δ_b resummation (see [1]). The first approximation to this is shown as short-dashed lines, where only the contributions from SM fermions and their SUSY partners (i.e. all squarks and sleptons) are taken into account, still including the Δ_b corrections. The next step of approximation is shown as dot-dashed lines, where only corrections from the t/b and $\tilde{t}/\tilde{b}$ sector are included, still with the Δ_b resummation. The penultimate step of the approximation is to leave out the Δ_b corrections, but using $\bar{m}_b$ (i.e. including the SM QCD corrections) in the Higgs boson couplings, shown as the long-dashed lines. The final step in the approximation is to drop the SM QCD corrections, i.e. replacing $\bar{m}_b$ by the bottom pole mass, $m_b = 4.8 \text{ GeV}$, in the Higgs Yukawa couplings, shown as the dotted lines.

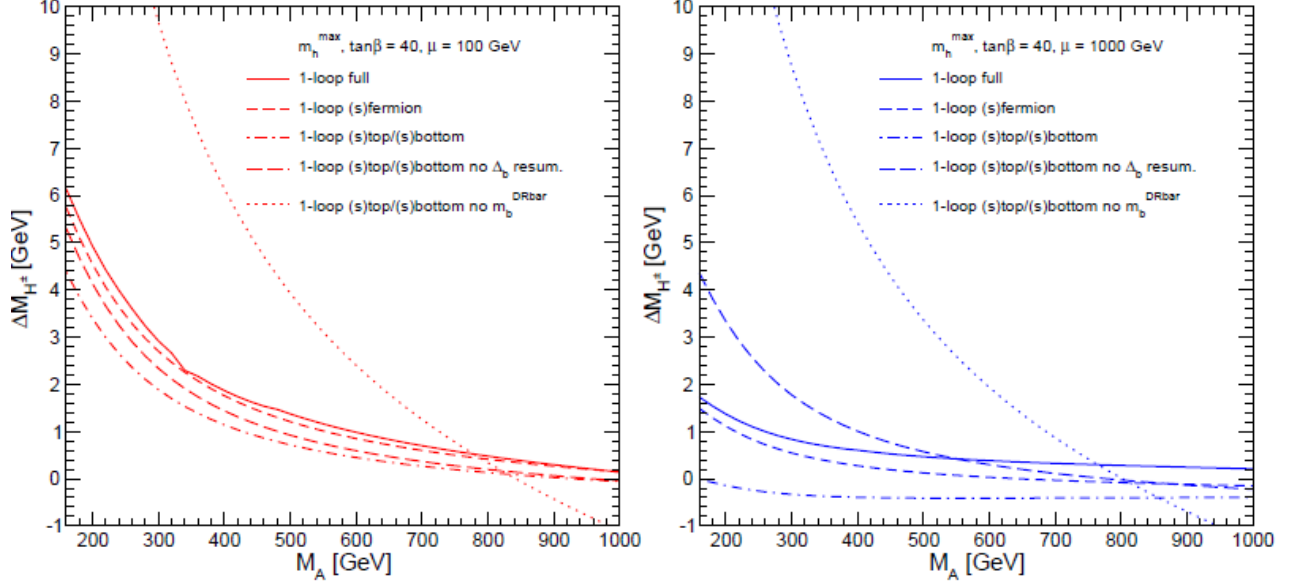


Figure 3.7. $\Delta M_{H^\pm} := M_{H^\pm} - m_{H^\pm}$ is shown in the $m_h^{\max}$ scenario as a function of M_A for $\mu = 100$ GeV (left) and $\mu = 1000$ GeV (right) and $\tan\beta = 40$, evaluated at the one-loop level. We show the full one-loop result including Δ_b corrections (solid lines), the pure SM fermion/sfermion contribution (short dashed), the $\tilde{t}/\tilde{b}$ contribution (dot-dashed), the $\tilde{t}/\tilde{b}$ corrections excluding the Δ_b corrections but using $\bar{m}_b$ (long dashed), and the $\tilde{t}/\tilde{b}$ corrections excluding the Δ_b resummation and using the bottom pole mass, m_b (dotted).

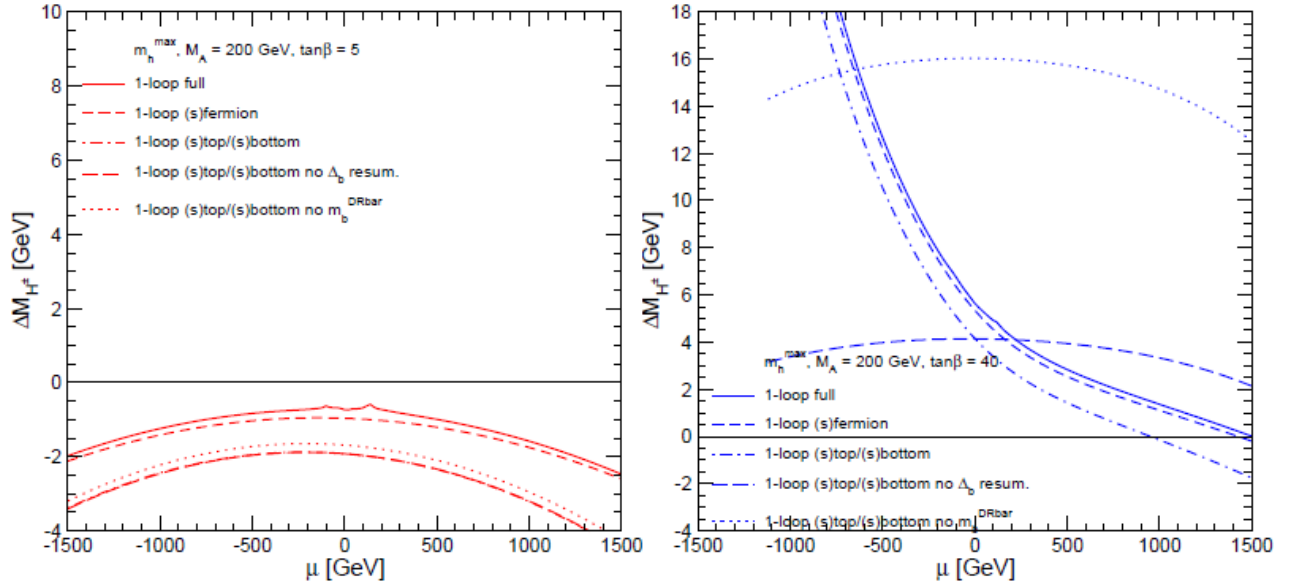


Figure 3.8. $\Delta M_{H^\pm} := M_{H^\pm} - m_{H^\pm}$ is shown in the $m_h^{\max}$ scenario as a function of μ for $\tan\beta = 5$ (left) and $\tan\beta = 40$ (right) and $M_A = 200$ GeV, evaluated at the one-loop level. The line coding is as in fig. 3.7.

First, in fig. 3.7, we analyse the dependence of $M_{H^\pm}$ on M_A in the $m_h^{\max}$ scenario. The left (right) plot of fig. 3.7 shows $\Delta M_{H^\pm}$ as a function of M_A for $\tan\beta = 40$ and

$\mu = 100(1000)$ GeV. It should be noted that the very low M_A values are by now ruled out by the LHC heavy MSSM Higgs boson searches [66] for this value of $\tan\beta$. However, in order to display the full parameter dependence we do not include these bounds here. The full result (solid lines) yields one-loop corrections between 1.5 GeV and 6.0 GeV for low M_A , becoming smaller for increasing M_A . The still allowed M_A values should give one-loop corrections of $O(2 \text{ GeV})$ in this scenario for small μ . The $f/\tilde{f}$ sector (short-dashed) gives a rather good approximation, better than 0.5 GeV. Going to the $t/b/\tilde{t}/\tilde{b}$ approximations (dot-dashed) yields a prediction that differs from the full result by up to ~ 2 GeV for low M_A . The $f/\tilde{f}$ corrections besides the ones from third generation squarks are roughly independent of the Yukawa couplings of the various (s)fermions and are of pure electroweak type, and can grow as $\log(M_{\text{SUSY}}/M_W)$ [104], and larger masses lead to larger corrections. Consequently, taking into account only the third generation (s)quark contribution can yield non-negligible uncertainties in the $M_{H^\pm}$ prediction. In the next step the Δ_b corrections are neglected, which are formally beyond the one-loop order, resulting in the long-dashed lines. The comparison between the dot-dashed and the long-dashed lines shows that the impact of the Δ_b corrections is small, below ~ 500 MeV for $\mu = 100$ GeV, but can be larger than 4 GeV for $\mu = 1000$ GeV. Finally we consider an approximation where the SM QCD corrections to the bottom Yukawa coupling are dropped, i.e. m_b is used instead of $\bar{m}_b$, resulting in the dotted lines. These contributions can be larger than all other steps of approximations in the region of large $\tan\beta$ considered here. Neglecting the SM QCD corrections in m_b shifts $\Delta M_{H^\pm}$ upwards by more than 10 GeV, depending on the scenario.

In order to analyse the dependence of the $M_{H^\pm}$ prediction on μ , we show in fig. 3.8 $\Delta M_{H^\pm}$ in the m_h^{max} scenario as a function of μ for $M_A = 200$ GeV and $\tan\beta = 5(40)$ in the left (right) plot. Again, the large $\tan\beta$ values are by now experimentally excluded by the LHC heavy MSSM Higgs searches for this value of M_A [66], but the two “extreme” $\tan\beta$ values are meant to give an idea about the possible variations. We start with the case of $\tan\beta = 5$, see the left plot of fig. 3.8. The $t/b/\tilde{t}/\tilde{b}$ corrections neglecting the SM QCD corrections (dotted line) are nearly symmetric in μ , ranging between -2 and -4 GeV. Including the SM QCD corrections (long-dashed) has a negligible impact. The same holds for the Δ_b corrections (dashed-dotted) due to the small value of $\tan\beta$, and the two lines lie on top of each other. Including the full (s)fermion corrections, on the other hand, has a sizable impact on the result. The contributions from the other s/fermions partially cancel the $t/b/\tilde{t}/\tilde{b}$ corrections. Including also the non-(s)fermionic contributions yields a total one-loop effect that stays below ~ -2 GeV.

The results look quite different for $\tan\beta = 40$ as shown in the right plot of fig. 3.8. For negative μ , the enhancement of the bottom Yukawa coupling can become very strong due to the large $\tan\beta$ value. In the m_h^{max} scenario, $\mu \lesssim -1200$ GeV yields $\Delta_b \rightarrow -1$, i.e. the

model enters the non-perturbative regime, and no evaluations in the Higgs sector are possible. Consequently, the corresponding curves in the right plot of fig. 3.8 stop at $\mu \approx -1100$ GeV. The pure $t/b/\tilde{t}/\tilde{b}$ corrections (dotted line) reach 13–16 GeV if they are evaluated with the bottom pole mass. Including the SM QCD corrections (long-dashed) into the effective bottom quark mass strongly reduces the effect to the level of 2–4 GeV. In the next step the Δ_b effects are included (dot-dashed line). Due to $\Delta_b \propto \mu \tan\beta$ the inclusion of Δ_b results in a strong asymmetry of $\Delta M_{H^\pm}$ with a larger correction to $M_{H^\pm}$ for negative μ (corresponding to an enhanced bottom Yukawa coupling) and a much smaller correction for positive μ (corresponding to a suppressed bottom Yukawa coupling). Including the full one-loop corrections the overall correction in the $m_h^{\max}$ scenario ranges from $\Delta M_{H^\pm} \gtrsim 18$ GeV for $\mu \lesssim -1000$ GeV to $\Delta M_{H^\pm} \approx 0$ for $\mu = +1500$ GeV.

The dependence on $\tan\beta$ is analysed in fig. 3.9. We show $\Delta M_{H^\pm}$ in the $m_h^{\max}$ scenario as a function of $\tan\beta$ for $M_A = 200$ GeV and as before for $\mu = 100(1000)$ GeV in the left(right) plot. It should be noted that values of $\tan\beta$ around 1 are excluded by LEP Higgs searches [65], whereas larger values are excluded by LHC Higgs searches for this value of M_A [66]. The sign and size of the one-loop correction to $M_{H^\pm}$ depends strongly on $\tan\beta$, which enters the Higgs couplings to (s)fermions as well as the Δ_b corrections. Negative corrections occur for $\tan\beta \lesssim 10$, while positive values of $\Delta M_{H^\pm}$ are obtained for large $\tan\beta$ values. In the phenomenologically allowed region of $\tan\beta$ the corrections stay within $\Delta M_{H^\pm} = \pm 2$ GeV. As in the plots of fig. 3.7, the effect of the non-sfermion sector in comparison with the $f/\tilde{f}$ contributions (short-dashed lines) is relatively small and stays below 0.5 GeV. The Yukawa coupling independent effects (dot-dashed lines) are ~ 2 GeV, largely independent of $\tan\beta$. The contribution from the Δ_b effects is negligible for $\tan\beta \lesssim 5$ and grows with increasing $\tan\beta$, reaching several GeV for large $\tan\beta$ and $\mu = 1000$ GeV. On the other hand, for $\mu = 100$ GeV these corrections stay very small even for the largest $\tan\beta$ values. The biggest effects again can arise from the inclusion of the SM QCD corrections to m_b for $\tan\beta \gtrsim 5$. Largely independently of the scenario and the choice for μ they reach 5–10 GeV.

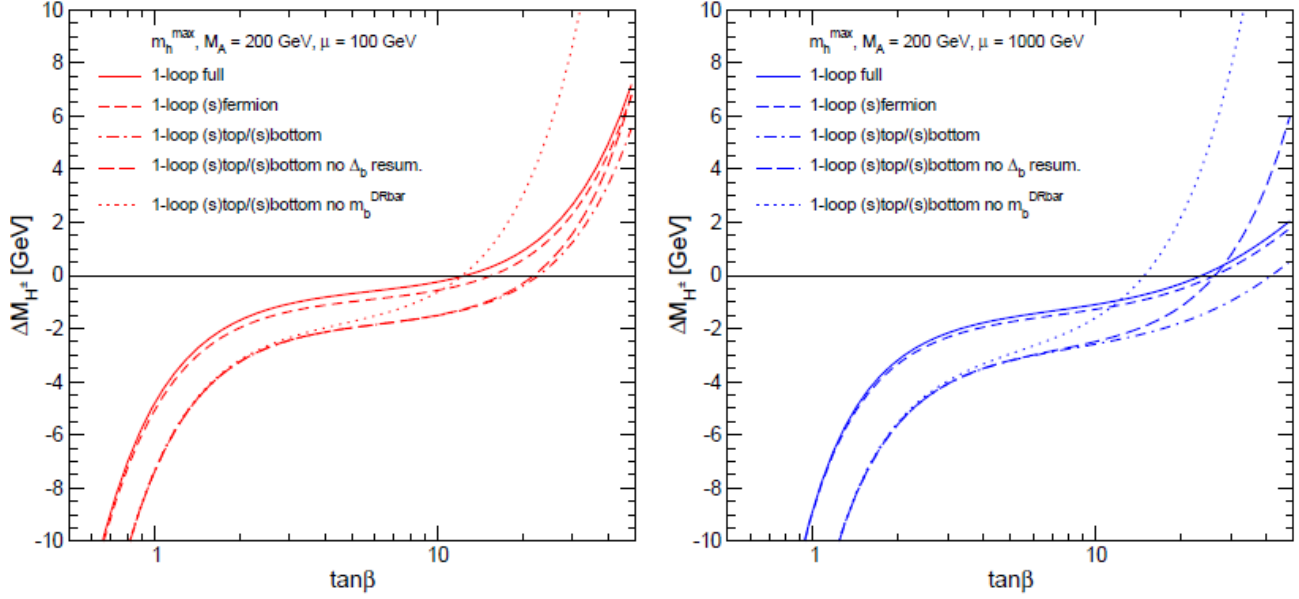


Figure 3.9. $\Delta M_{H^\pm} := M_{H^\pm} - m_{H^\pm}$ is shown in the $m_h^{\max}$ scenario as a function of $\tan\beta$ for $\mu=100$ GeV (left) and $\mu=1000$ GeV (right) and $M_A=200$ GeV, evaluated at the one-loop level. The line coding is as in fig. 3.7.

Next, in fig. 3.10 we show the dependence on M_{SUSY} . The SUSY mass scale (which we chose to be equal for all sfermions, see above) enters via contributions $\propto \log(M_{\text{SUSY}}/M_W)$ or $\propto M_W^2/M_{\text{SUSY}}^2$ into the charged Higgs-boson mass prediction [104], where several competing contributions have been identified. One is proportional to large Yukawa couplings from the top/bottom sector, while another one stems from the electroweak couplings of scalar fermions and is similar for all flavors.

In the left plot of fig. 3.10 we show $\Delta M_{H^\pm}$ as a function of M_{SUSY} in the $m_h^{\max}$ scenario for $\mu=1000$ GeV, $M_A=200$ GeV and $\tan\beta=5$. One can see that the $b/\tilde{b}$ contributions, which are influenced strongly by the bottom Yukawa coupling and the Δ_b corrections, do not play a prominent role as they change $\Delta M_{H^\pm}$ only weakly for small $\tan\beta$. The contributions from the lighter fermions (i.e. neither top nor bottom) and their SUSY partners, on the other hand, become very important for $M_{\text{SUSY}} \gtrsim 1000$ GeV. Without those corrections (dot-dashed line) rather large negative contributions to $M_{H^\pm}$ would occur for large M_{SUSY} , while including these corrections (short dashed line) $\Delta M_{H^\pm}$ flattens out for large M_{SUSY} , reaching ~ -1 GeV at $M_{\text{SUSY}}=2000$ GeV. The corrections from the non-(s)fermion sector are small and change $M_{H^\pm}$ by less than about 0.2 GeV. A qualitatively similar behaviour can be observed for $\tan\beta=20$, as shown in the right plot of fig. 3.10. Due to the larger value of $\tan\beta$ the $b/\tilde{b}$ corrections and Δ_b effects are much more pronounced. The contributions from the (s)fermion sector beyond $t/\tilde{t}/b/\tilde{b}$ are sizable for $M_{\text{SUSY}} \gtrsim 1000$ GeV. Due to numerical cancellations the full one-loop correction to $M_{H^\pm}^2$ is close to zero for this part of the SUSY parameter space. This is in agreement with the right

plot of fig. 3.9. In summary, for large M_{SUSY} especially the corrections from the *full* (s)fermion sector have to be taken into account.

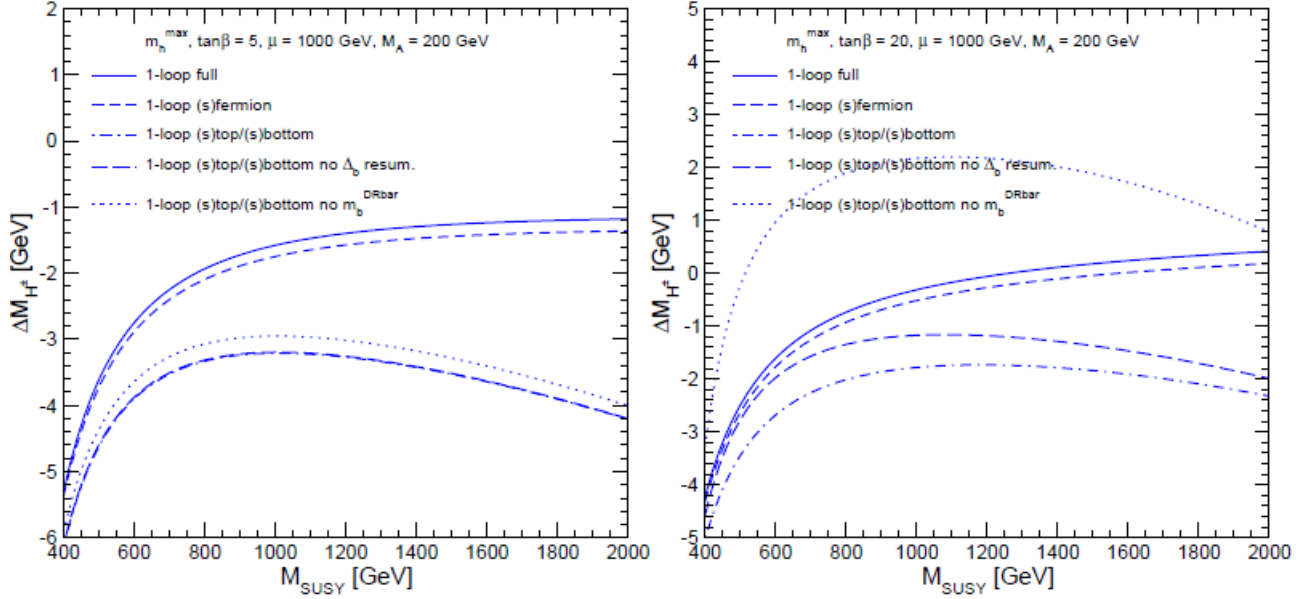


Figure 3.10. $\Delta M_{H^\pm} := M_{H^\pm} - m_{H^\pm}$ is shown in the m_h^{max} scenario as a function of M_{SUSY} for $\tan\beta = 5$ (left) and $\tan\beta = 20$ (right), $M_A = 200$ GeV and $\mu = 1000$ GeV, evaluated at the one-loop level. The line coding is as in fig. 3.7.

We finally analyse the size of the full one-loop corrections in the case of $A_t \neq A_b$ in fig. 3.11. We show the results in the $A_b - A_t$ plane for $M_A = 200(120)$ GeV in the top (bottom) row and $\tan\beta = 40(10)$ in the left(right) column. Again, the “extreme” choices for M_A and $\tan\beta$, by now excluded by LHC Higgs searches [66] indicate the range of the possible size of the corrections. The other parameters are $M_{\text{SUSY}} = 500$ GeV, $\mu = 1000$ GeV, $M_2 = 500$ GeV.

The color coding in the plots tells the value of $\Delta M_{H^\pm}$. At small $\tan\beta$, as can be seen in the upper right plot, the value of $\Delta M_{H^\pm}$ depends mainly on A_t , i.e. the convention $A_b = A_t$ in the m_h^{max} scenario does not have a relevant impact on the $M_{H^\pm}$ evaluation at the one-loop level for small $\tan\beta$. This changes for large $\tan\beta$ as can be observed in the two left plots of the figure. The main diagonal corresponds to $A_t = A_b$ and exhibits (for both M_A values) relatively small corrections up to ~ 3 GeV. The other extreme, $A_t = -A_b$, on the other hand, yields much larger corrections, exceeding $\Delta M_{H^\pm} = 10$ GeV for large $|A_t|$. Consequently, a full one-loop calculation, allowing for different values of A_t and A_b is crucial for a precise $M_{H^\pm}$ evaluation.

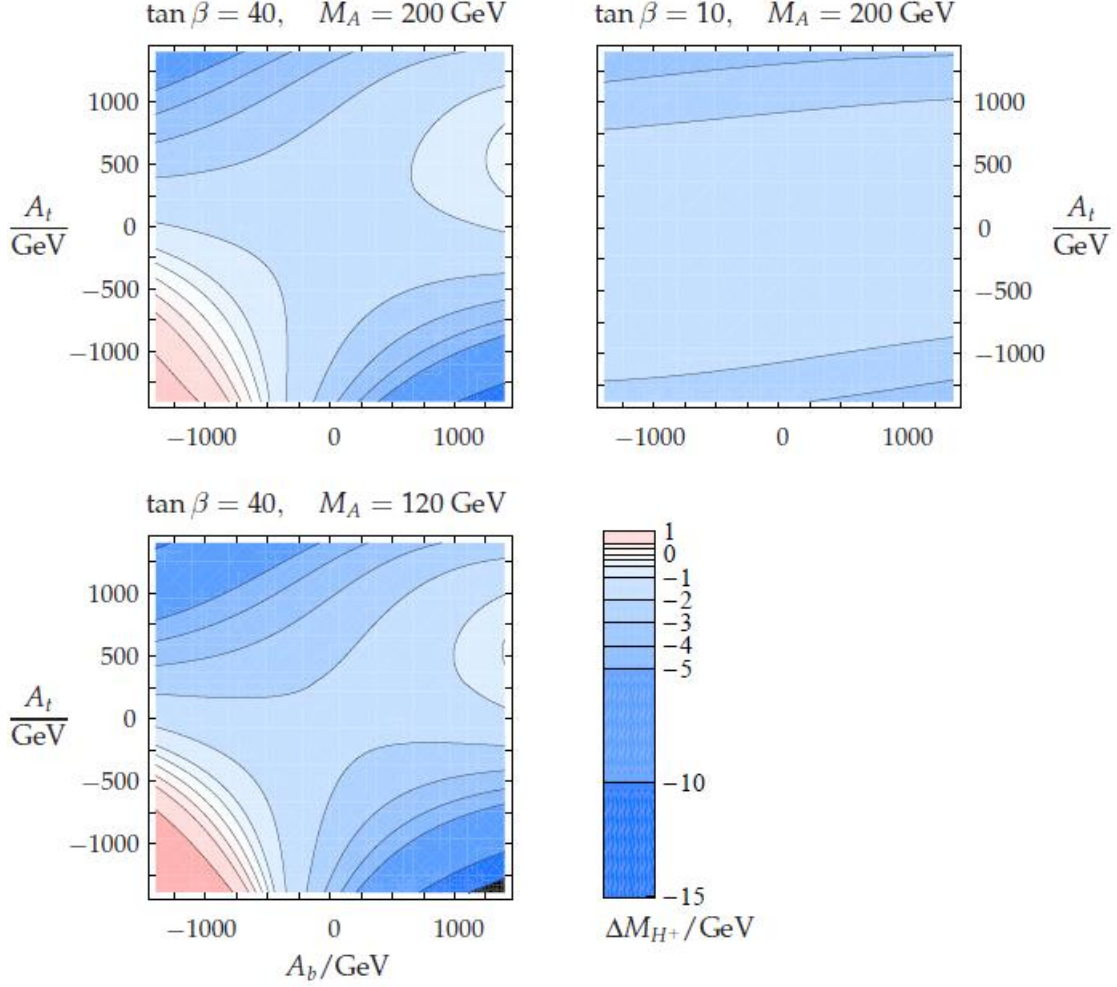


Figure 3.11. The size of the full one-loop correction $\Delta M_{H^\pm} := M_{H^\pm} - m_{H^\pm}$ is shown in the $A_b - A_t$ plane for $M_A = 200(120)$ GeV in the top (bottom) row and $\tan \beta = 40(10)$ in the left (right) column. The other parameters are $M_{\text{SUSY}} = 500$ GeV, $\mu = 1000$ GeV, $M_2 = 500$ GeV.

3.5.2 Two-loop corrections

As discussed before, in order to reach a sufficient accuracy leading two-loop corrections to $M_{H^\pm}$ should be included. Consequently we review here some of the results as given in [1]. These two-loop corrections are of $O(\alpha_t \alpha_s)$. In the plots we denote “2-loop” as the full one-loop corrections supplemented by these $O(\alpha_t \alpha_s)$ contributions, see sect. 3.3.4. In our numerical analysis we concentrate on cases where on the one hand the full one-loop contribution to $M_{H^\pm}^2$ is sizable, and on the other hand the $O(m_t^4 / M_W^2)$ corrections yield a relatively good approximation to the full one-loop result. For these cases it can be expected that the $O(\alpha_t \alpha_s)$ two-loop corrections also constitute a substantial part of the full two-loop contributions.

The focus in [1] was on relatively low $\tan\beta$, since it is known that at large $\tan\beta$ the bottom/sbottom one-loop corrections are sizable (see the previous subsection) and the $O(\alpha_t\alpha_s)$ terms cannot be expected to capture a leading piece of the two-loop contributions. As can be seen in eqs. (3.93),(3.92), the $O(m_t^4/M_W^2)$ terms going $\sim \mu$ are enhanced with $\tan\beta$. Therefore we review the two-loop $O(\alpha_t\alpha_s)$ corrections as a function of μ . We furthermore set $M_A=200\text{ GeV}$, which allows relatively large absolute higher-order corrections. The chosen parameters are thus mostly in agreement with the LHC heavy MSSM Higgs searches (and we will not address this issue in the rest of this subsection).

In the left plot of fig. 3.12 we show $\Delta M_{H^\pm} := M_{H^\pm} - m_{H^\pm}$ in the $m_h^{\max}$ scenario for $M_A=200\text{ GeV}$ and $\tan\beta=5$, as a function of μ for $M_{\text{SUSY}}=1000\text{ GeV}$. These are the same values implemented in the left plot of fig. 3.8, but now including the two-loop $O(\alpha_t\alpha_s)$ corrections (shown as the blue/dark gray solid line) from our article [1]. Also shown are the corresponding one-loop results of $O(m_t^4/M_W^2)$ (dashed line), the full one-loop corrections (red/light gray solid line) and the difference between the two-loop and the full one-loop result (dot-dashed line). One finds that the full one-loop corrections are well approximated by the $O(m_t^4/M_W^2)$ term. The corresponding two-loop corrections modify the full one-loop result by up to $\sim 1\text{ GeV}$ for $|\mu| \sim 1500\text{ GeV}$, i.e. the $O(\alpha_t\alpha_s)$ corrections can be sizable in this case (where our plot stops).

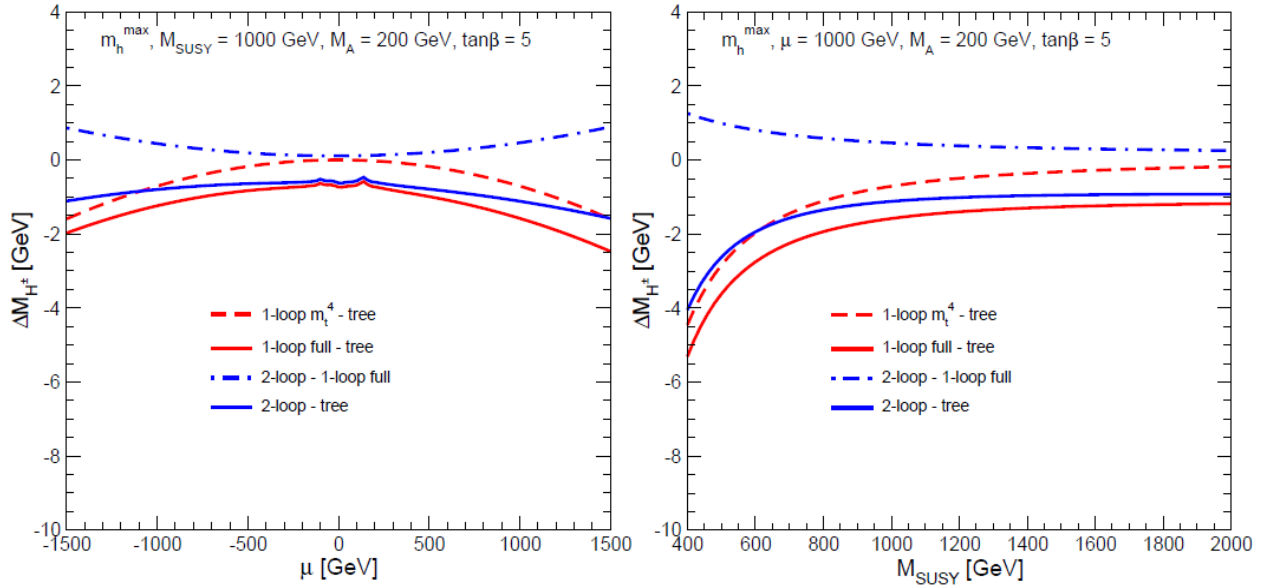


Figure 3.12. $\Delta M_{H^\pm} := M_{H^\pm} - m_{H^\pm}$ and $M_{H^\pm}^{2\text{-loop}} - M_{H^\pm}^{1\text{-loop}}$ are shown for $M_A=200\text{ GeV}$ and $\tan\beta=5$, as a function of μ for $M_{\text{SUSY}}=1000\text{ GeV}$ in the left plot, and as a function of M_{SUSY} for $\mu=1000\text{ GeV}$ in right plot. $M_{H^\pm}$ is evaluated at the two-loop level (blue/dark gray solid). Also shown are the corresponding one-loop results of $O(m_t^4/M_W^2)$ (dashed), the full one-loop corrections (red/light gray solid) and the difference between the two-loop and the full one-loop result (dot-dashed).

In the right plot of fig. 3.12, $\Delta M_{H^\pm}$ is shown as a function of M_{SUSY} , in analogy to the left plot of fig. 3.10, but again with the addition of the two-loop contributions as in [1]. We show the results for $\tan\beta=5$ in the m_h^{max} scenario (i.e. $X_t = 2M_{\text{SUSY}}$ and $m_{\tilde{g}} = 0.8M_{\text{SUSY}}$), as a function of M_{SUSY} for $\mu=1000$ GeV and $M_A=200$ GeV. The m_t^4/M_W^2 corrections approximate the full one-loop results fairly well. The largest deviations occur for large values of M_{SUSY} , where the other (s)fermion sectors become more relevant, see the discussion on fig. 3.10. For $M_{\text{SUSY}}=400$ GeV, the lowest value in our analysis, the two-loop $O(\alpha_t\alpha_s)$ corrections amount up to ~ 1 GeV. For large M_{SUSY} this correction goes down nearly to zero.

Our analysis in the m_h^{max} scenario is completed in fig. 3.13, where $M_{\text{SUSY}}=500$ GeV is used, but somewhat larger $\tan\beta$ values were chosen. We show $\Delta M_{H^\pm}$ for $\tan\beta=10$ and $M_A=200$ GeV in the left plot. The result look qualitatively similar to the case of $\tan\beta=5$: the m_t^4/M_W^2 approximation works well for the full one-loop result. The two-loop corrections go up to ~ 2 GeV for large values of $|\mu|$.

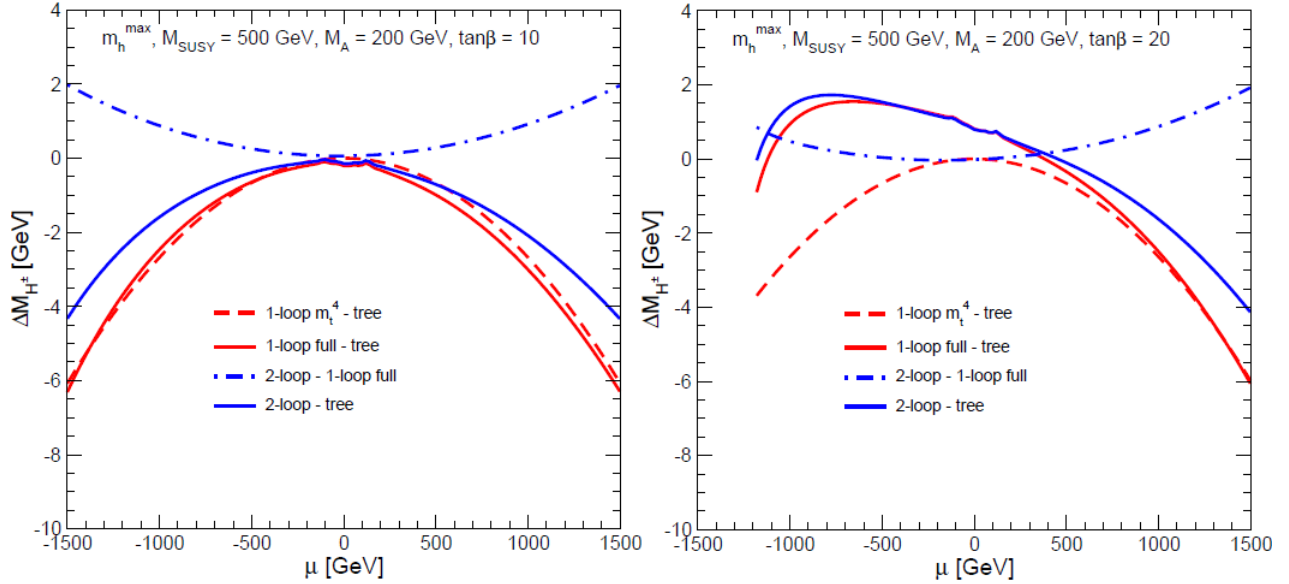


Figure 3.13. $\Delta M_{H^\pm} := M_{H^\pm}^{2\text{-loop}} - M_{H^\pm}^{1\text{-loop}}$ are shown for $\tan\beta=10$ (left plot) and $\tan\beta=20$ (right plot), $M_A=200$ GeV and $M_{\text{SUSY}}=500$ GeV as a function of μ in the m_h^{max} scenario. $M_{H^\pm}$ is evaluated at the two-loop level (blue/dark gray solid). Also shown are the corresponding one-loop results of $O(m_t^4/M_W^2)$ (dashed), the full one-loop corrections (red/light gray solid) and the difference between the two-loop and the full one-loop result (dot-dashed).

In the right plot in fig. 3.13, $\tan\beta=20$ is used. For large negative values of μ , $\mu \lesssim -1200$ GeV, the corrections become so large that an evaluation of the loop corrections to $M_{H^\pm}$ was not possible anymore (as was observed already in fig. 3.8). For negative μ the $O(m_t^4/M_W^2)$ corrections also start to deviate substantially from the full one-loop result, and

the corresponding two-loop corrections cannot be expected to yield a good approximation to the full two-loop result in the region of relatively large negative μ . For large and positive μ , however, the m_t^4/M_W^2 approximation works very well, so that in this region the $O(\alpha_t\alpha_s)$ corrections can be expected to provide a reasonable approximation of the full two-loop corrections. For $\mu=+1500$ GeV the two-loop corrections again are sizable and amount up to $\sim 2\text{--}3$ GeV.

NMSSM Higgs masses

The measured mass value of the observed Higgs signal at the LHC has already reached the level of a precision observable, with an experimental accuracy of better than 300 MeV [121], and by itself provides an important test for the predictions of models of electroweak symmetry breaking. In the MSSM, we saw that the masses of the Higgs bosons can be predicted at lowest order in terms of two SUSY parameters characterizing the MSSM Higgs sector, e.g. $\tan\beta$ (the ratio of the vacuum expectation values of the two doublets) and the mass of the CP -odd Higgs boson, M_A , or the charged Higgs boson, $M_{H^\pm}$. These relations, who in particular give rise to a tree-level upper bound on the mass of the light CP -even Higgs boson given by the Z -boson mass, receive large corrections from higher-order contributions. In the NMSSM,⁴⁷ the corresponding predictions are modified both at tree-level and the loop-level. In order to fully exploit the precision of the experimental mass value for constraining the available parameter space of the considered models, the theoretical predictions should have an accuracy that ideally is at the same level or even better than the one of the experimental value. The theoretical uncertainty, on the other hand, is composed of two sources, the parametric and the intrinsic uncertainty; the ones induced by the parametric errors of the input parameters are dominated by the experimental error of the top-quark mass, where the latter needs to include the systematic uncertainty from relating the measured mass parameter to a theoretically well-defined quantity [122]. However, the largest theoretical uncertainty at present arises from unknown higher-order corrections.

Within the NMSSM, beyond the one-loop results (see [116,123]) several codes exist that calculate the Higgs masses in the pure $\overline{\text{DR}}$ scheme with different contributions at higher-order. Amongst these codes, SPHENO [124,125] incorporates the most complete results at the two-loop level, including SUSY-QCD contributions from the fermion/sfermions of $O(\alpha_t\alpha_s, \alpha_b\alpha_s)$, as well as pure fermion/sfermion contributions of $O(\alpha_t^2, \alpha_b^2, \alpha_t\alpha_b, \alpha_\tau^2, \alpha_\tau\alpha_s)$ and mixed contributions from the latter and the Higgs/higgsino sectors of $O(\alpha_\lambda\alpha_t, \alpha_\lambda\alpha_b)$. The included Higgs/higgsino contributions are genuine to the NMSSM: they are proportional to the NMSSM parameters $\lambda^2 = 4\pi \cdot \alpha_\lambda$ and $\kappa^2 = 4\pi \cdot \alpha_\kappa$ (see sect. 2.3 for definitions of λ and κ parameters). The tool SOFTSUSY [126] includes NMSSM

⁴⁷ See sect. 2.3 for an introduction to the model.

corrections of $O(\alpha_t\alpha_s)$ and $O(\alpha_b\alpha_s)$ supplemented by certain MSSM corrections. NMSSMCALC [127,128] provides the option to perform the NMSSM Higgs mass calculation up to $O(\alpha_t\alpha_s)$ with the $\overline{\text{DR}}$ Renormalisation scheme applied to the top/stop sector, while in the electroweak sector at one-loop order on-shell conditions are used. Beyond the pure $\overline{\text{DR}}$ scheme, so far only the code NMSSMCALC provides a prediction in a mixed $\text{OS}/\overline{\text{DR}}$ scheme, where genuine two-loop contributions in the NMSSM up to $O(\alpha_t\alpha_s)$ have been incorporated. The resummation of logarithmic contributions beyond the two-loop level is not included so far in any of the public codes for Higgs-mass predictions in the NMSSM. Accordingly, at present the theoretical uncertainties from unknown higher-order corrections in the NMSSM are expected to be still larger than for the MSSM.

We perform here a full one-loop calculation of the Higgs-boson masses in the NMSSM, where the Renormalisation scheme and all parameters and conventions are chosen such that the well-known MSSM result of FEYNHIGGS [13-18] is obtained for the MSSM limit of the NMSSM. In order to complement the picture, we supplement the full one-loop result in the NMSSM by briefly mentioning the incorporation of all leading two-loop corrections of MSSM-type that are implemented in FEYNHIGGS, as have been obtained and discussed in our article [2], which are necessary to reach a better precision.⁴⁸ From our numerical evaluation, the full one-loop result in the NMSSM can be used to assess the quality of this approximation at the two-loop level. For a SM-like Higgs boson that is compatible with the detected signal at about 125 GeV, the approximation works indeed very well. We apply our results to one phenomenologically interesting scenario, and review the corresponding results for the CP -even Higgs boson masses. We also compare our predictions with the result in the MSSM limit. For the sectors that are identical for the calculation within the MSSM, the conventions as implemented in FEYNHIGGS are used in this chapter, as described in [15]. For the quantities genuine to the NMSSM, the details we developed here can be complemented, e.g., by the discussion from [19]. Several of our results shown in this chapter have been published in [2] and [129].

⁴⁸ In our article [2], a further comparison with the code NMSSMCALC can be found, where it is confirmed that the approximation in terms of MSSM-type contributions at the two-loop level induces an uncertainty that is numerically small if $\lambda < Y_t$, and that the approximation works best for the MSSM-like (doublet-like) fields, where the deviations stay below the level of 1 GeV even for the highest possible values of λ and in regions with a large admixture between double- and singlet-like states.

4.1 The relevant NMSSM sectors

The introduction of a singlet superfield alters both the Higgs and the neutralino sectors of the theory with respect to the MSSM. We give in this section a comprehensive detail of the main features of the NMSSM Higgs sector, and we also mention briefly the ones which are new for the neutralinos. In addition, we introduce the chargino mass matrix (which is the same as for the MSSM) and its rotation to physical states, because it will be relevant for our Renormalisation procedure for the μ parameter.

4.1.1 Higgs sector

From the superpotential introduced in sect. 2.3, the terms in the Lagrangian that involve only Higgs fields define the Higgs potential $V_{\text{Higgs}}^{\text{rNMSSM}}$, which can be written in powers of the fields,

$$\begin{aligned}
V_{\text{Higgs}}^{\text{rNMSSM}} &= m_1^2 H_1^\dagger \cdot H_1 + m_2^2 H_2^\dagger \cdot H_2 + m_S^2 |S|^2 + \lambda^2 |S|^2 (H_1^\dagger \cdot H_1 + H_2^\dagger \cdot H_2) \\
&\quad + |\lambda \varepsilon_{ab} H_1^a H_2^b - \kappa S^2|^2 - 2 \text{Re}(\lambda A_\lambda S \varepsilon_{ab} H_1^a H_2^b - \frac{1}{3} \kappa A_\kappa S^3) \\
&\quad + \frac{1}{8} (g_1^2 + g_2^2) (H_1^\dagger \cdot H_1 - H_2^\dagger \cdot H_2)^2 + \frac{1}{2} g_2^2 |H_1^\dagger \cdot H_2|^2 \\
&\equiv \text{const.} - T_{\phi_1} \phi_1 - T_{\phi_2} \phi_2 - T_{\phi_S} \phi_S + \frac{1}{2} (\phi_1, \phi_2, \phi_S) M_{\phi\phi}^2 (\phi_1, \phi_2, \phi_S)^T \\
&\quad + \frac{1}{2} (\chi_1, \chi_2, \chi_S) M_{\chi\chi}^2 (\chi_1, \chi_2, \chi_S)^T + (\phi_1^-, \phi_2^-) M_{\phi^\pm \phi^\pm} (\phi_1^+, \phi_2^+)^T + \dots
\end{aligned} \tag{4.1}$$

where the coefficients linear in the fields are the “tadpoles” $T_{\phi_{1,2,S}}$ and the bilinear in the fields are the mass matrices $M_{\phi\phi}$, $M_{\chi\chi}$ and $M_{\phi^\pm \phi^\pm}$. H_1 , H_2 and S are the scalar components of the Higgs doublet and singlet superfields introduced in eqs. (2.5),(2.6), who can be decomposed into CP -even and CP -odd neutral scalars $\phi_{1,2,S}$ and $\chi_{1,2,S}$, respectively, and charged states $\phi_{1,2}^\pm$,⁴⁹

$$H_1 = \begin{pmatrix} \nu_1 + \frac{1}{\sqrt{2}}(\phi_1 - i\chi_1) \\ -\phi_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} \phi_2^- \\ \nu_2 + \frac{1}{\sqrt{2}}(\phi_2 + i\chi_2) \end{pmatrix}, \quad S = \nu_S + \frac{1}{\sqrt{2}}(\phi_S + i\chi_S) \tag{4.2}$$

with the real vacuum expectation values for the doublet- and the singlet-fields, $\nu_{1,2}$ and ν_S .

⁴⁹ We follow in this section the sign convention as in [15] for the doublets, see tab. 3.1.

The potential (4.1) of the CP -conserving NMSSM contains additional dimensionless parameters λ and κ compared to the MSSM, while an explicit μ -term is absent. This term is effectively generated via the vacuum expectation-value of the singlet field:⁵⁰

$$\mu \equiv \lambda v_s \quad (4.3)$$

As in the MSSM, it is convenient to define

$$\tan \beta = \frac{v_2}{v_1} \quad , \quad v = \sqrt{v_1^2 + v_2^2} \quad . \quad (4.4)$$

Tadpoles coefficients

$$\begin{aligned} T_{\phi_1} &= -\sqrt{2}v(\tilde{m}_1^2 c_\beta + \frac{1}{4}v^2(g_1^2 + g_2^2)c_\beta c_{2\beta} - \mu B s_\beta + \lambda^2 v^2 s_\beta^2 c_\beta) \\ T_{\phi_2} &= -\sqrt{2}v(\tilde{m}_2^2 s_\beta - \frac{1}{4}v^2(g_1^2 + g_2^2)s_\beta c_{2\beta} - \mu B c_\beta + \lambda^2 v^2 s_\beta c_\beta^2) \\ T_{\phi_s} &= -\sqrt{2}v \left[\frac{\mu}{\lambda v} m_s^2 + \lambda v(\mu - (K\mu + B)s_\beta c_\beta) + \frac{K\mu^2}{\lambda v}(2K\mu + A_\kappa) \right] \end{aligned} \quad (4.5)$$

where $\tilde{m}_1^2 \equiv m_1^2 + \mu^2$ and $\tilde{m}_2^2 \equiv m_2^2 + \mu^2$. In order to reproduce the MSSM in the appropriate limit (see sect. 5.2), we've also defined

$$B \equiv A_\lambda + K\mu \quad . \quad (4.6)$$

Bilinear contributions

For the mass matrices, we express them here discriminating between the MSSM-like part and the genuine NMSSM contribution proportional to λ and/or λ^2 :

$$\begin{aligned} \mathbf{M}_{\phi\phi}^2 &= \begin{pmatrix} \tilde{m}_1^2 + \frac{1}{4}v^2(g_1^2 + g_2^2)(1 + 2c_{2\beta}) & -\frac{1}{4}v^2(g_1^2 + g_2^2)s_{2\beta} - \mu B & 0 \\ * & \tilde{m}_2^2 + \frac{1}{4}v^2(g_1^2 + g_2^2)(1 - 2c_{2\beta}) & 0 \\ * & * & m_s^2 + 2\frac{K}{\lambda}\mu(A_\kappa + 3\frac{K}{\lambda}\mu) \end{pmatrix} \\ &+ \lambda v \begin{pmatrix} 0 & 0 & 2\mu c_\beta - (B + \frac{K}{\lambda}\mu)s_\beta \\ * & 0 & 2\mu s_\beta - (B + \frac{K}{\lambda}\mu)c_\beta \\ * & * & 0 \end{pmatrix} + (\lambda v)^2 \begin{pmatrix} s_\beta^2 & s_{2\beta} & 0 \\ * & c_\beta^2 & 0 \\ * & * & 1 - \frac{K}{\lambda}s_{2\beta} \end{pmatrix} \equiv \begin{pmatrix} m_{\phi_1\phi_1}^2 & m_{\phi_1\phi_2}^2 & m_{\phi_1\phi_s}^2 \\ * & m_{\phi_2\phi_2}^2 & m_{\phi_2\phi_s}^2 \\ * & * & m_{\phi_s\phi_s}^2 \end{pmatrix} \end{aligned}$$

⁵⁰ See sect. 5.2 for a detailed discussion.

$$\begin{aligned}
\mathbf{M}_{\chi\chi\chi}^2 &= \begin{pmatrix} \tilde{m}_1^2 + \frac{1}{4}\nu^2(g_1^2 + g_2^2)c_{2\beta} & -\mu B & 0 \\ * & \tilde{m}_2^2 - \frac{1}{4}\nu^2(g_1^2 + g_2^2)c_{2\beta} & 0 \\ * & * & m_S^2 + 2\frac{\kappa}{\lambda}\mu(-A_\kappa + 3\frac{\kappa}{\lambda}\mu) \end{pmatrix} \\
&+ \lambda\nu(B - 3\frac{\kappa}{\lambda}\mu) \begin{pmatrix} 0 & 0 & -s_\beta \\ * & 0 & c_\beta \\ * & * & 0 \end{pmatrix} + (\lambda\nu)^2 \begin{pmatrix} s_\beta^2 & 0 & 0 \\ * & c_\beta^2 & 0 \\ * & * & 1 + \frac{\kappa}{\lambda}s_{2\beta} \end{pmatrix} \equiv \begin{pmatrix} m_{\chi_1\chi_1}^2 & m_{\chi_1\chi_2}^2 & m_{\chi_1\chi_S}^2 \\ * & m_{\chi_2\chi_2}^2 & m_{\chi_S\chi_S}^2 \\ * & * & m_{\chi_S\chi_S}^2 \end{pmatrix} \\
\mathbf{M}_{\phi^\pm\phi^\pm}^2 &= \begin{pmatrix} \tilde{m}_1^2 + \frac{1}{4}\nu^2(g_1^2c_{2\beta} + g_2^2) & -\frac{1}{4}\nu^2g_2^2s_{2\beta} - \mu B \\ * & \tilde{m}_2^2 - \frac{1}{4}\nu^2(g_1^2c_{2\beta} - g_2^2) \end{pmatrix} \\
&+ (\lambda\nu)^2 s_\beta c_\beta \begin{pmatrix} 0 & 1 \\ * & 0 \end{pmatrix} \equiv \begin{pmatrix} m_{\phi_1^-\phi_1^+}^2 & m_{\phi_1^-\phi_2^+}^2 \\ m_{\phi_2^-\phi_1^+}^2 & m_{\phi_2^-\phi_2^+}^2 \end{pmatrix} \quad (4.7)
\end{aligned}$$

The *'s mean that the matrices are symmetrical. Now we invert (4.5) in order to change variables $\tilde{m}_1^2$, $\tilde{m}_2^2$ and $\tilde{m}_S^2$ in function to the tadpoles:

$$\begin{aligned}
\tilde{m}_1^2 &= -\frac{1}{2}c_{2\beta}M_Z^2 + \mu B t_\beta - s_\beta^2(\lambda\nu)^2 - \frac{\bar{T}_{\phi_1}}{c_\beta} \\
\tilde{m}_2^2 &= \frac{1}{2}c_{2\beta}M_Z^2 + \mu B/t_\beta - c_\beta^2(\lambda\nu)^2 - \frac{\bar{T}_{\phi_2}}{s_\beta} \\
m_S^2 &= (\lambda\nu)^2 \left(s_\beta c_\beta \left(\frac{\kappa}{\lambda} + \frac{B}{\mu} \right) - 1 \right) - \frac{\kappa}{\lambda}\mu \left(A_\kappa + 2\frac{\kappa}{\lambda}\mu \right) - \frac{\lambda\nu}{\mu}\bar{T}_{\phi_S}
\end{aligned} \quad (4.8)$$

where we have use (CC), and with the “normalized tadpoles” defined as

$$\bar{T}_{\phi_i} \equiv \frac{T_{\phi_i}}{\sqrt{2}\nu} \quad , \quad i=1,2,S \quad . \quad (4.9)$$

In term of the tadpoles, the mass matrices read

$$\mathbf{M}_{\phi\phi\phi}^2 = \begin{pmatrix} c_\beta^2 M_Z^2 + s_\beta^2 m_A^2 & -s_\beta c_\beta (M_Z^2 + m_A^2) & 0 \\ * & s_\beta^2 M_Z^2 + c_\beta^2 m_A^2 & 0 \\ * & * & \frac{\kappa}{\lambda}\mu(A_\kappa + 4\frac{\kappa}{\lambda}\mu) \end{pmatrix}$$

$$\begin{aligned}
& + \lambda \nu \begin{pmatrix} 0 & s_{2\beta} \lambda \nu & 2c_\beta \mu - s_\beta (B + \frac{\kappa}{\lambda} \mu) \\ * & 0 & 2s_\beta \mu - c_\beta (B + \frac{\kappa}{\lambda} \mu) \\ * & * & s_\beta c_\beta \frac{\lambda \nu}{\mu} (B - \frac{\kappa}{\lambda} \mu) \end{pmatrix} + \left(\begin{array}{c|c} \mathbf{M}_{\text{Tadpole}}^{2,\text{MSSM}} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \hline 0 & 0 \end{array} \middle| -\frac{\lambda \nu}{\mu} \bar{T}_{\phi_s} \right) \\
\mathbf{M}_{\chi\chi\chi}^2 = & \begin{pmatrix} m_A^2 s_\beta^2 & -m_A^2 s_\beta c_\beta & 0 \\ * & m_A^2 c_\beta^2 & 0 \\ * & * & -3A_\kappa \frac{\kappa}{\lambda} \mu \end{pmatrix} \\
& + \lambda \nu \begin{pmatrix} 0 & 0 & -s_\beta (B - 3\frac{\kappa}{\lambda} \mu) \\ * & 0 & c_\beta (B - 3\frac{\kappa}{\lambda} \mu) \\ * & * & s_\beta c_\beta \frac{\lambda \nu}{\mu} (B + 3\frac{\kappa}{\lambda} \mu) \end{pmatrix} + \left(\begin{array}{c|c} \mathbf{M}_{\text{Tadpole}}^{2,\text{MSSM}} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \hline 0 & 0 \end{array} \middle| -\frac{\lambda \nu}{\mu} \bar{T}_{\phi_s} \right) \\
\mathbf{M}_{\phi^\pm \phi^\pm}^2 = & (m_A^2 + M_W^2 - (\lambda \nu)^2) \begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ * & c_\beta^2 \end{pmatrix} + \mathbf{M}_{\text{Tadpole}}^{2,\text{MSSM}} \tag{4.10}
\end{aligned}$$

with

$$\mathbf{M}_{\text{Tadpole}}^{2,\text{MSSM}} \equiv -\bar{T}_{\phi_1} \begin{pmatrix} c_\beta (1 + s_\beta^2) & s_\beta^3 \\ * & -s_\beta^2 c_\beta \end{pmatrix} - \bar{T}_{\phi_2} \begin{pmatrix} -s_\beta c_\beta^2 & c_\beta^3 \\ * & s_\beta (1 + c_\beta^2) \end{pmatrix} \tag{4.11}$$

and where in analogy with the MSSM we have defined (see eq. (3.20))

$$m_A^2 \equiv \frac{\mu B - \bar{T}_{\phi_1} s_\beta^3 - \bar{T}_{\phi_2} c_\beta^3}{s_\beta c_\beta} . \tag{4.12}$$

Again, the lower triangles in these matrices filled with the transposed matrix elements. In (4.10) the third rows and columns depend explicitly on μ . The numerical value of μ has an important impact on the singlet admixture after performing the rotation into the mass eigenstate basis. For instance, for values of μ large enough that

$$|(\mathbf{M}_{\phi\phi\phi}^2)_{33}| \gg |(\mathbf{M}_{\phi\phi\phi}^2)_{i3}| \quad , \quad i=1,2 \tag{4.13}$$

the mass of the singlet becomes decoupled from the doublet masses.

Diagonalization

The mixing into the eigenstates of mass and CP can be described at lowest order by the following unitary transformations:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = U_{e(0)} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_S \end{pmatrix}, \quad \begin{pmatrix} A_1 \\ A_2 \\ G \end{pmatrix} = U_{o(0)} \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_S \end{pmatrix}, \quad \begin{pmatrix} H^\pm \\ G^\pm \end{pmatrix} = U_{c(0)} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix} \quad (4.14)$$

$$\begin{aligned} M_{hhh}^2 &= U_{e(0)}^T M_{\phi\phi\phi}^2 U_{e(0)} & M_{AAG}^2 &= U_{o(0)}^T M_{\chi\chi\chi}^2 U_{o(0)} & M_{H^\pm G^\pm}^2 &= U_{c(0)}^T M_{\phi^\pm\phi^\pm}^2 U_{c(0)} \\ &= \text{diag}\{m_{h_1}^2, m_{h_2}^2, m_{h_3}^2\} & &= \text{diag}\{m_{A_1}^2, m_{A_2}^2, 0\} & &= \text{diag}\{m_{H^\pm}^2, 0\} \end{aligned}$$

$$V_{\text{Higgs}}^{\text{rNMSSM}} \Big|_{(\text{bi})\text{lineal}} = -T_{h_1} h_1 - T_{h_2} h_2 - T_{h_3} h_3 + \frac{1}{2} \sum_{i=1}^3 m_{h_i}^2 h_i^2 + \frac{1}{2} \sum_{j=1}^2 m_{A_j}^2 A_j^2 + m_{H^\pm}^2 H^- H^+$$

The matrices $U_{\{e,o,c\}(0)}$ transform the Higgs fields such that the mass matrices are diagonalised at tree level. The new fields correspond to the five neutral Higgs bosons h_i and A_j , the charged pair $H^\pm$, and the Goldstone bosons G^0 and $G^\pm$. The $U_{c(0)}$ matrix can be parametrized in the form

$$U_{c(0)} = \begin{pmatrix} -s_{\beta_c} & c_{\beta_c} \\ c_{\beta_c} & s_{\beta_c} \end{pmatrix} \quad (4.15)$$

The angle β_c is actually equal to β ,

$$M_{H^\pm G^\pm}^2 \Big|_{\substack{\bar{T}_{\phi_{1,2}}=0 \\ \beta_c=\beta}} = \begin{pmatrix} m_A^2 + M_W^2 - (\lambda\nu)^2 & 0 \\ * & 0 \end{pmatrix} \quad (4.16)$$

but naming it differently and replacing it with β just at the end of the calculation allow us to perform the charged Higgs sector Renormalisation in physical basis, i.e. by doing just

$$\delta M_{G^\pm H^\pm}^2 = \delta \bar{P} \cdot \partial_{\bar{P}} M_{G^\pm H^\pm}^2 \Big|_{\substack{\bar{T}_{\phi_{1,2}}=0 \\ \beta_c=\beta}} \quad (4.17)$$

instead of as in (VV). From (4.16) we identify the expression for the quantity $m_{H^\pm}^2$, the charged Higgs mass at tree-level:

$$m_{H^\pm}^2 = m_A^2 + M_W^2 - (\lambda\nu)^2 \quad (4.18)$$

from where we recover the MSSM result from the last line of (EE) in the limit $\lambda \rightarrow 0$. Note, however, that in (4.18) the quantity m_A^2 does not correspond to the tree-level mass of any particle, but it is just the combination of parameters defined in (4.12). Without replacing β_c by β , for the upper left element of the matrix in (4.16) we get

$$m_{H^\pm}^2(\beta_c) = c_{\beta-\beta_c}^2 (m_A^2 + M_W^2 - (\lambda\nu)^2) + s_{\beta-\beta_c} ((2s_\beta^2 s_{\beta_c} + c_\beta s_{\beta+\beta_c}) \bar{T}_{\phi_1} - (2c_\beta^2 c_{\beta_c} + s_\beta s_{\beta+\beta_c}) \bar{T}_{\phi_2}) \quad (4.19)$$

which is the expression we will use for the Renormalisation procedure in order to obtain $\delta m_{H^\pm}^2$ by doing $\delta \bar{P} \cdot \partial_{\bar{P}} m_{H^\pm}^2 \big|_{T_x=0, \beta_c=\beta}$, as explained in (4.17).

4.1.2 Neutralinos

The singlino, $\tilde{S}$, is the superpartner of the scalar singlet. It mixes with the wino $\tilde{B}$, the neutral bino $\tilde{W}_3$ and the higgsinos $\tilde{H}_{1,2}$, giving rise to a total of five neutralinos $\tilde{\chi}_{1,\dots,5}^0$. The corresponding 5×5 mass-matrix in the gauge basis $\{\tilde{B}, \tilde{W}_3, \tilde{H}_1, \tilde{H}_2, \tilde{S}\}$ reads

$$Y = \begin{pmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta & 0 \\ * & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta & 0 \\ * & * & 0 & -\mu & -\lambda \nu s_\beta \\ * & * & * & 0 & -\lambda \nu c_\beta \\ * & * & * & * & 2 \frac{\kappa}{\lambda} \mu \end{pmatrix} \quad (4.20)$$

This matrix is diagonalised by a unitary matrix

$$D_Y = N^* Y N^\dagger = \text{diag}\{m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}, m_{\tilde{\chi}_5^0}\} \quad (4.21)$$

Also in eq. (4.20) μ can have a significant influence on the mixing between the singlino and the doublet higgsino fields. For instance, for sufficiently large values of μ or sufficiently small values of λ such that

$$|(Y)_{55}| \gg |(Y)_{i5}| \quad , \quad i=3,4 \quad (4.22)$$

the singlino mass decouples from the masses of higgsinos and gauginos.

4.1.3 Charginos

Since the NMSSM does not add new charged fields with respect to de MSSM, the chargino sector is the same in both models. We introduce this sector here and not in the previous chapter, however, because a Renormalisation condition for the μ parameter is needed for the NMSSM Higgs sector, and for sake of simplicity we choose to implement it from the chargino sector, as we will show in eqs. (4.50),(4.51).

The mass eigenstates of the charginos can be determined from the matrix

$$X = \begin{pmatrix} M_2 & \sqrt{2}s_\beta M_W \\ \sqrt{2}c_\beta M_W & \mu \end{pmatrix}. \quad (4.23)$$

The rotation to the chargino mass eigenstates is done by transforming the original wino and higgsino fields with the help of two unitary 2×2 matrices U and V,⁵¹

$$\begin{pmatrix} \tilde{\chi}_1^+ \\ \tilde{\chi}_2^+ \end{pmatrix} = V \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_2^+ \end{pmatrix}, \quad \begin{pmatrix} \tilde{\chi}_1^- \\ \tilde{\chi}_2^- \end{pmatrix} = U \begin{pmatrix} \tilde{W}^- \\ \tilde{H}_1^- \end{pmatrix} \quad (4.24)$$

These rotations lead to the diagonal mass matrix

$$\begin{pmatrix} m_{\tilde{\chi}_1^\pm} & 0 \\ 0 & m_{\tilde{\chi}_2^\pm} \end{pmatrix} = U X V^T \quad (4.25)$$

From this relation, it becomes clear that the chargino masses $m_{\tilde{\chi}_1^\pm}$ and $m_{\tilde{\chi}_2^\pm}$ can be determined as the (real and positive) singular values of X. The singular value decomposition of X also yields results for U and V.

4.2 Expressions for the pole mass evaluation

Again, the masses of the Higgs bosons are obtained using diagrammatic methods. The masses of the *CP*-even Higgs bosons are obtained by determining the complex poles of the full propagator matrix. The inverse propagator is a 3×3 matrix that reads

⁵¹ The symmetry of Y in (4.20) permits the non-trivial condition of using only one matrix N for its diagonalization. This is not the case for the chargino sector, thus two matrixes U, V are needed.

$$\Delta^{-1}(p^2) = i(p^2 \mathbf{I}_{3 \times 3} - \mathbf{M}_{\phi\phi\phi}^2 + \hat{\Sigma}_{\phi\phi}(p^2)) \quad (4.26)$$

Here $\mathbf{M}_{\phi\phi\phi}^2$ again denotes the mass matrix of the Higgs fields at tree level, and $\hat{\Sigma}_{\phi\phi}$ is the matrix with their corresponding renormalised self-energy corrections, which at one-loop level are obtain as

$$\hat{\Sigma}_{ab}(p^2) = \Sigma_{ab}(p^2) + (\delta_{ab} p^2 - m_{ab}^2) \frac{1}{2} (\delta \mathbf{Z}_a + \delta \mathbf{Z}_b) - \delta m_{ab}^2 \quad . \quad (4.27)$$

$$\begin{aligned} \delta \mathbf{Z}_{\phi_1} &= \delta \mathbf{Z}_{\chi_1} = \delta \mathbf{Z}_{\phi_1^\pm} = \delta \mathbf{Z}_{H_1} \\ \delta \mathbf{Z}_{\phi_2} &= \delta \mathbf{Z}_{\chi_2} = \delta \mathbf{Z}_{\phi_2^\pm} = \delta \mathbf{Z}_{H_2} \\ \delta \mathbf{Z}_{\phi_S} &= \delta \mathbf{Z}_{\chi_S} = \delta \mathbf{Z}_S \end{aligned} \quad , \quad \delta_{ab} = \begin{cases} 1 & , \quad a = b^* \\ 0 & , \quad a \neq b^* \end{cases}$$

The three complex poles of the propagator in the CP -even Higgs sector consist of the masses M_{h_i} and the total widths Γ_{h_i} , and are given by the values of the external momentum for which the determinant of the inverse propagator-matrix vanishes:

$$\det \Delta^{-1}(p^2 = M_{h_i}^2 - i M_{h_i} \Gamma_{h_i}) = 0 \quad , \quad i = 1, 2, 3 \quad (4.28)$$

The real parts of the three poles are identified with the square of the Higgs-boson masses. The renormalised self-energy matrix $\hat{\Sigma}_{\phi\phi}$ is evaluated by taking into account the full contributions from the NMSSM at one-loop order, and also the dominant two-loop contribution can be added from the MSSM result as an approximation [2]:

$$\hat{\Sigma}_{\phi\phi}(p^2) \approx \hat{\Sigma}_{\phi\phi}^{(1L)}(p^2) \Big|_{p^2=0}^{\text{NMSSM}} + \hat{\Sigma}_{\phi\phi}^{(2L)}(p^2) \Big|_{p^2=0}^{\text{MSSM}} \quad (4.29)$$

It includes the MSSM contributions of $O(\alpha_t \alpha_s, \alpha_b \alpha_s, \alpha_t^2, \alpha_t \alpha_b)$ at vanishing external momentum⁵², and the resummation of leading and subleading logarithms induced by heavy SUSY particles from the top/scalar-top sector, taken over directly from the MSSM version of FEYNHIGGS. Identical procedure is performed for the CP -odd Higgs sector.

⁵² In the public version of FEYNHIGGS for the NMSSM, also the recent results for momentum-dependent two-loop contributions in the MSSM of [97,98] will be implemented.

4.2.1 Renormalisation scheme

In order to derive the counterterms entering the one-loop corrections to the Higgs-boson masses, the independent parameters appearing in the linear and bilinear terms of the Higgs potential in eq. (4.1) and also the gauge sector have to be renormalised. The set of independent input parameters from the Higgs sector used for the presented calculation is formed by

$$\begin{aligned} \text{MSSM-like :} & \quad T_{\phi_{1,2}}, \mu, M_{H^\pm}^2, \tan \beta, M_W^2, M_Z^2 \\ \text{genuine NMSSM:} & \quad T_{\phi_3}, \kappa, \lambda, A_\kappa, \nu = \sqrt{\nu_1^2 + \nu_2^2} \end{aligned} \quad (4.30)$$

Here T_{ϕ_i} denotes the tadpole coefficient for the field ϕ_i (as indicated by the subscript) in the mass eigenstate basis. Parameters that do not enter the MSSM calculation are considered as genuine of the calculation in the NMSSM. Although the vacuum expectation value ν is not a parameter genuine to the NMSSM, its appearance as an independent parameter is a specific feature of the NMSSM Higgs mass calculation, see below.

In order to facilitate the incorporation of the MSSM-like two-loop contributions from FEYNHIGGS, the Renormalisation scheme chosen for the NMSSM contributions closely follows the FEYNHIGGS conventions, and accordingly the stop masses are renormalised on-shell. We choose to renormalise the tadpoles to zero at all orders of perturbation theory. Rather than choosing the gauge couplings and the soft-breaking parameter A_λ as input parameters, the gauge-boson masses M_W, M_Z and the charged Higgs mass $M_{H^\pm}$ were chosen as independent parameters. The following Renormalisation schemes are chosen:

$$\begin{aligned} \text{on-shell :} & \quad M_W, M_Z, M_{H^\pm} \\ \overline{\text{DR}} : & \quad \lambda, \kappa, A_\kappa, \tan \beta, \mu, \nu \end{aligned} \quad (4.31)$$

For all parameters appearing only in the NMSSM-calculation, besides the additional tadpole coefficient, a $\overline{\text{DR}}$ -scheme is applied. This is a difference to the calculations performed in [116,123,127,128], where the electric charge e is renormalised instead of the parameter ν . Since the vacuum expectation value ν is directly related to the electromagnetic coupling constant charge e , a reparametrisation for the electric charge e is necessary in order to use a given numerical value for the electromagnetic coupling constant. In the following the value for e derived from Fermi's constant is used, which is the parametrisation that was chosen for the results that are implemented in FEYNHIGGS. These two parameters are related to each other by

$$\nu = \frac{\sqrt{2}s_w M_W}{e} \rightarrow \nu \left(1 + \frac{\delta\nu}{\nu}\right) = \nu \left(1 + \frac{1}{2} \left(\frac{\delta M_W^2}{M_W^2} + \frac{\delta s_w^2}{s_w^2} - 2\delta Z_e \right) \right) \quad (4.32)$$

with the Renormalisation constants for the W -boson mass, δM_W^2 , the sine of the weak mixing angle, δs_w^2 , where $s_w^2 = 1 - M_W^2 / M_Z^2$, and the electric charge renormalised as

$$e \rightarrow e(1 + \delta Z_e) \quad (4.33)$$

Considering δM_W^2 and δs_w^2 already fixed by on-shell conditions for the gauge-boson masses [15], either δZ_e or $\delta\nu$ in eq. (4.32) can be fixed by an independent Renormalisation condition (and the other counterterm is then a dependent quantity). The Renormalisation prescription, where δZ_e is fixed by renormalizing e in the static limit, results in a non- $\overline{\text{DR}}$ Renormalisation for $\delta\nu$. For the self-energies in the Higgs sector $\delta\nu$ enters the counterterms for the renormalised Higgs potential,

$$V_{\text{Higgs}}^{\text{rNMSSM}} \rightarrow V_{\text{Higgs}}^{\text{rNMSSM}} + \delta V_{\text{Higgs}}^{\text{rNMSSM}} \quad (4.34)$$

with coefficients involving λ and κ , like

$$\left. \frac{\delta^{(2)}}{\delta\phi_S \delta\phi_i} \delta V_{\text{Higgs}}^{\text{rNMSSM}} \right|_{\phi_\ell, \chi_m, \phi_n^\pm = 0} \supset -\kappa\mu\{\sin\beta, \cos\beta\}(\delta\nu + \dots) \quad (4.35)$$

for the self-energies with each an external doublet and singlet field. The ellipsis in eq. (4.35) denote other Renormalisation constants that are fixed in the $\overline{\text{DR}}$ -scheme and thus do not contribute with a finite part. However, a finite contribution from $\delta\nu$ would lead to a κ -dependence of all loop contributions entering via $\delta\nu$, in particular also of the corrections from the fermions and sfermions (while the fermion and sfermion contributions to the unrenormalised self-energy are κ -independent). A finite contribution from $\delta\nu$ would furthermore imply the rather artificial feature that a self-energy involving an external gauge singlet field would receive a counterterm contribution involving the Renormalisation constant δZ_e for the electric charge. We therefore prefer to use the $\overline{\text{DR}}$ -scheme for the Renormalisation of ν , which means that we use a scheme where δZ_e is a dependent counterterm. This leads to the relation

$$\delta Z_e^{\text{dep}} = \frac{1}{2} \left[\frac{\delta s_w^2}{s_w^2} + \frac{\delta M_W^2}{M_W^2} - \frac{\delta\nu^2}{\nu^2} \right] \quad (4.36)$$

which implies

$$\delta Z_e^{\text{dep}}|_{\text{fin}} = \frac{1}{2} \left[\frac{\delta s_W^2}{s_W^2} + \frac{\delta M_W^2}{M_W^2} \right]_{\text{fin}} \quad (4.37)$$

for the finite part of δZ_e^{dep} . In this scheme the numerical value for the electric charge e (and accordingly for the electromagnetic coupling constant α) is determined indirectly via eq. (4.37). In order to avoid a non-standard numerical value for α in our numerical results, we apply a two-step procedure: in the first step we apply a $\overline{\text{DR}}$ Renormalisation for ν as outlined above. As a second step we then reparametrise this result in terms of a suitably chosen expression for α . By default we use the same convention as for the MSSM result implemented in FEYNHIGGS, namely the expression for the electric charge in terms of the Fermi constant G_F , in order to facilitate the comparison between the FEYNHIGGS result in the MSSM and our new result for the NMSSM. Taking the MSSM-limit of our new NMSSM result, the MSSM result as implemented is recovered, since the described calculational differences are genuine NMSSM effects that vanish in this limit. See also the discussion in section 4.2.3.

4.2.2 Renormalisation conditions

As specified in (4.31), we impose for the masses of W and Z bosons and of the charged Higgs boson $H^\pm$ an on-shell Renormalisation. This means that we set the finite part of the counterterms in such a way that the radiative corrected expressions for the masses of those particles remain equal to the corresponding tree-level expressions. By doing this, those tree-level masses, which are chosen as input parameters (see (4.30)), match with the experimental mass values even at higher order.

Similar to the eqs. (4.26) and (4.28), the one-loop corrected expressions $m^{(1\text{L})}$ in these cases remain equal to their tree-level expression m if the finite part of the one-loop self-energy $\hat{\Sigma}^{(1\text{L})}$ evaluated in $p^2 = m^2$ are set to zero. In turn, similar to (4.27) it can be seen that this condition on $\hat{\Sigma}^{(1\text{L})}$ translates into the condition on (the finite part of) δm^2 :

$$\hat{\Sigma}^{(1\text{L})}(m^2) = 0 \quad \Leftrightarrow \quad m^{(1\text{L})} = m \quad \Leftrightarrow \quad \delta m^2 = \text{Re} \Sigma(m^2) \quad (4.38)$$

Then, the following conditions are set for the W , Z and $H^\pm$ mass counterterms:

$$\begin{aligned} \delta m_{H^\pm}^2 &= \text{Re} \Sigma_{H^\pm H^\pm}(m_{H^\pm}^2) \\ \delta M_W^2 &= \text{Re} \Sigma_{WW}(M_W^2) \\ \delta M_Z^2 &= \text{Re} \Sigma_{ZZ}(M_Z^2) \end{aligned} \quad (4.39)$$

Additionally,

$$\delta \overline{T}_{\phi_i} = \frac{T_{\phi_i}^{(1L)}}{\sqrt{2}v} \quad , \quad i = 1, 2, S \quad (4.40)$$

where $T_{\phi_i}^{(1L)}$ are the 1-loop radiative contributions to the tadpole of ϕ_i .

The rest of the quantities are renormalised them in the $\overline{DR}$ scheme, see (4.31). For t_β we do

$$t_\beta \rightarrow t_\beta + \delta t_\beta \quad . \quad (4.41)$$

On the other hand, as for the MSSM we assign one field Renormalisation constant for each Higgs doublet and one for the singlet,

$$H_1 \rightarrow (1 + \tfrac{1}{2} \delta Z_{H_1}) H_1 \quad , \quad H_2 \rightarrow (1 + \tfrac{1}{2} \delta Z_{H_2}) H_2 \quad , \quad S \rightarrow (1 + \tfrac{1}{2} \delta Z_S) S \quad (4.42)$$

so that the Renormalisation for the vev's is done in the following way:

$$\begin{aligned} \begin{pmatrix} \nu_1 + \frac{1}{\sqrt{2}}(\phi_1 - i\chi_1) \\ -\phi_1^- \end{pmatrix} &\rightarrow (1 + \tfrac{1}{2} \delta Z_{H_1}) \begin{pmatrix} \nu_1 + \delta \nu_1 + \frac{1}{\sqrt{2}}(\phi_1 - i\chi_1) \\ -\phi_1^- \end{pmatrix} \\ \begin{pmatrix} \phi_2^+ \\ \nu_2 + \frac{1}{\sqrt{2}}(\phi_2 + i\chi_2) \end{pmatrix} &\rightarrow (1 + \tfrac{1}{2} \delta Z_{H_2}) \begin{pmatrix} \phi_2^+ \\ \nu_2 + \delta \nu_2 + \frac{1}{\sqrt{2}}(\phi_2 + i\chi_2) \end{pmatrix} \\ \nu_s + \frac{1}{\sqrt{2}}(\phi_s + i\chi_s) &\rightarrow (1 + \tfrac{1}{2} \delta Z_S) (\nu_s + \delta \nu_s + \frac{1}{\sqrt{2}}(\phi_s + i\chi_s)) \end{aligned} \quad (4.43)$$

Therefore,

$$\begin{aligned} \nu_i &\rightarrow (1 + \tfrac{1}{2} \delta Z_{H_i}) (\nu_i + \delta \nu_i) = \nu_i + \delta \nu_i + \tfrac{1}{2} \delta Z_{H_i} \nu_i + O(\delta^2) \equiv \nu_i + \tilde{\delta} \nu_i \\ \Rightarrow \quad \frac{\tilde{\delta} \nu_i}{\nu_i} &= \frac{\delta \nu_i}{\nu_i} + \frac{1}{2} \delta Z_{H_i} \quad , \quad i = 1, 2, S \end{aligned} \quad (4.44)$$

where

$$\tilde{\delta} \nu_1 = \delta(\nu c_\beta) \quad , \quad \tilde{\delta} \nu_2 = \delta(\nu s_\beta) \quad , \quad \tilde{\delta} \nu_s = \delta(\mu / \lambda) \quad . \quad (4.45)$$

Then, by using the definition $t_\beta = \nu_2 / \nu_1$ we get

$$\frac{\delta t_\beta}{t_\beta} = \frac{\tilde{\delta} v_2}{v_2} - \frac{\tilde{\delta} v_1}{v_1} = \frac{\delta v_2}{v_2} - \frac{\delta v_1}{v_1} + \frac{1}{2}(\delta Z_{H_2} - \delta Z_{H_1}) \quad . \quad (4.46)$$

We will show in the sect. 4.3.2 how at one-loop level and tacking into account only (s)fermion contributions it must be

$$\frac{\delta v_1^{\text{div}}}{v_1} = \frac{\delta v_2^{\text{div}}}{v_2} \quad . \quad (4.47)$$

The validity of (4.47) for the full (i.e. all-sector contributions) one-loop result, in the case of t_β renormalised in a $\overline{\text{DR}}$ scheme, can be found in [81,82,130]. Then we get

$$\frac{\delta t_\beta^{\overline{\text{DR}}}}{t_\beta} = \frac{1}{2}(\delta Z_{H_2} - \delta Z_{H_1}) \quad (4.48)$$

In a similar way, $\delta\mu$ can be related with the vev's counterterms by using the fact that $v_s = \mu/\lambda = \mu v/\tilde{\lambda} = \mu\sqrt{v_1^2 + v_2^2}/\tilde{\lambda}$:

$$\begin{aligned} \frac{\tilde{\delta} v_s}{v_s} &= \frac{\delta\mu}{\mu} + \frac{v_1^2 \frac{\tilde{\delta} v_1}{v_1} + v_2^2 \frac{\tilde{\delta} v_2}{v_2}}{v^2} - \frac{\delta\tilde{\lambda}}{\tilde{\lambda}} \\ \Rightarrow \quad \frac{\delta\mu}{\mu} &= \frac{\delta v_s}{v_s} - c_\beta^2 \frac{\delta v_1}{v_1} - s_\beta^2 \frac{\delta v_2}{v_2} + \frac{1}{2}(\delta Z_s - c_\beta^2 \delta Z_{H_1} - s_\beta^2 \delta Z_{H_2}) + \frac{\delta\tilde{\lambda}}{\tilde{\lambda}} \end{aligned} \quad (4.49)$$

where $v_1 = v c_\beta$ and $v_2 = v s_\beta$ have been used. As we mentioned in sect. 4.1.3, we impose a $\overline{\text{DR}}$ Renormalisation condition on μ , implemented from the chargino sector. By solving $\delta m_{\tilde{\chi}_i^\pm} = (U \delta X V^T)_{ii}$ ($i = 1, 2$) for $\delta\mu$ and δM_2 , it follows that

$$\begin{aligned} \delta\mu &= \frac{1}{U_{11}U_{22}V_{11}V_{22} - U_{12}U_{21}V_{12}V_{21}} \times (U_{11}V_{11}\delta m_{\tilde{\chi}_2^\pm} - U_{21}V_{21}\delta m_{\tilde{\chi}_1^\pm} \\ &\quad + U_{11}U_{21}(V_{12}V_{21} - V_{11}V_{22})[\delta(\sqrt{2}s_\beta M_w)]^{\text{div}} + V_{11}V_{21}(U_{12}U_{21} - U_{11}U_{22})[\delta(\sqrt{2}c_\beta M_w)]^{\text{div}}) \end{aligned} \quad (4.50)$$

where we use the short-hand notation $U_{ij} \equiv (U)_{ij}$, $V_{ij} \equiv (V)_{ij}$ for the elements of the rotating matrixes defined in sect. 4.1.3. The $\overline{\text{DR}}$ condition on $\delta\mu$ is implemented by fixing the chargino mass counterterms in the following way:

$$\delta m_{\tilde{\chi}_i^\pm} = \text{Re} \Sigma_{\tilde{\chi}_i^\pm \tilde{\chi}_i^\pm} (m_{\tilde{\chi}_i^\pm}^2)^{\text{div}} = \text{Re} \left[\frac{1}{2} m_{\tilde{\chi}_i^\pm} (\Sigma_{\tilde{\chi}_i^\pm \tilde{\chi}_i^\pm}^L (m_{\tilde{\chi}_i^\pm}^2) + \Sigma_{\tilde{\chi}_i^\pm \tilde{\chi}_i^\pm}^R (m_{\tilde{\chi}_i^\pm}^2)) + \Sigma_{\tilde{\chi}_i^\pm \tilde{\chi}_i^\pm}^S (m_{\tilde{\chi}_i^\pm}^2) \right]^{\text{div}} \quad (4.51)$$

where the superscripts stand for the decomposition of the self-energies into left- and right-handed vector and scalar coefficients (see more details e.g. in [131]).

Finally, differentiating eq. (4.27) with respect to the external momentum we obtain the conditions for these field counterterms renormalised as $\overline{\text{DR}}$ quantities:

$$\begin{aligned}
\delta Z_{H_1} &= -\partial_{p^2} \Sigma_{\phi_1 \phi_1}^{\text{div}} (= -\partial_{p^2} \Sigma_{\chi_1 \chi_1}^{\text{div}}) \\
\delta Z_{H_2} &= -\partial_{p^2} \Sigma_{\phi_2 \phi_2}^{\text{div}} (= -\partial_{p^2} \Sigma_{\chi_2 \chi_2}^{\text{div}}) \\
\delta Z_S &= -\partial_{p^2} \Sigma_{\phi_S \phi_S}^{\text{div}} (= -\partial_{p^2} \Sigma_{\chi_S \chi_S}^{\text{div}})
\end{aligned} \tag{4.52}$$

4.2.3 Electromagnetic coupling reparametrisation

We need to link the Renormalisation condition (4.36), which defines the parameter e in our treatment, with the experimental value of the electric charge (or, equivalently, the Fermi constant G_F) in the Thomson limit. The couplings g^I and g^{II} in two different Renormalisation schemes are in general related to each other by

$$g^I (1 + \delta Z_g^I) = g^{II} (1 + \delta Z_g^{II}) \tag{4.53}$$

because of the equality of the bare couplings. The corresponding shift in the numerical values of the coupling definitions is obtained from the finite difference of the two counterterms, $\Delta_g \equiv g^{II} \delta Z_g^{II} - g^I \delta Z_g^I$. Accordingly, a reparametrisation from the numerical value of the coupling used in scheme I to the one of scheme II can be performed via

$$g^I = g^{II} + \Delta_g \tag{4.54}$$

Since Δ_g is of one-loop order, its insertion into a tree-level expression generates a term of one-loop order, etc.

In our calculation the reparametrisation of the electromagnetic coupling is only necessary up to the one-loop level, since all corrections of two-loop and higher order that we are going to incorporate have been obtained in the gauge-less limit (some care is necessary regarding the incorporation of the MSSM-type contributions of $O(\alpha_t^2)$, see [89]). At this order the shift Δ_g can simply be expressed as $\Delta_g = g^{II} (\delta Z_g^{II} - \delta Z_g^I)$. Specifically, for the reparametrisation of the electromagnetic coupling constant G_F the parameter shift Δ_{G_F} reads

$$\Delta_{G_F} = e(\delta Z_e - \delta Z_e^{\text{dep}} - \frac{1}{2} \Delta r^{\text{NMSSM}}) \quad (4.55)$$

Here δZ_e is the counterterm of the charge Renormalisation within the NMSSM according to the static (Thomson) limit,

$$\delta Z_e = \frac{1}{2} \Pi^{\gamma\gamma}(0) + \tan \theta_w \frac{\Sigma_{\gamma Z}^T(0)}{M_Z^2} \quad (4.56)$$

and $\Pi^{\gamma\gamma}(0), \Sigma_{\gamma Z}^T(0)$ are the derivative of the transverse part of the photon self-energy and the transverse part of the photon-Z self-energy at zero momentum transfer, respectively. The counterterm δZ_e^{dep} has been defined in eq. (4.36), and for the quantity Δr^{NMSSM} we use the result of [132]. The numerical value for the electromagnetic coupling e in this parametrisation is obtained from the Fermi constant in the usual way as $e = 2M_W s_W \sqrt{G_F \sqrt{2}}$.

4.3 Dominant and full contributions at one-loop order

As explained above, we will supplement our full one-loop result with all available higher-order contributions of MSSM type. In order to validate this approximation, we analyse at the one-loop level the size of genuine NMSSM corrections w.r.t. the MSSM-like contributions.

Since the corrections from the top/stop sector are usually the by far dominant ones, we start with a qualitative discussion of those contributions before we perform a numerical analysis in the following section. In the MSSM the leading corrections from the top/stop sector are commonly denoted as $O(\alpha_t)$, indicating the occurrence of two Yukawa couplings Y_t . In the limit where all other masses of the SM particles and the external momentum are neglected compared to the top-quark mass, for dimensional reasons the correction to the squared Higgs boson mass furthermore receives a contribution proportional to m_t^2 . This gives rise to the well-known coefficient $G_F m_t^4$ of the leading one-loop contributions. In the NMSSM the formally leading contributions either are of $O(Y_t^2)$ (involving two Yukawa couplings), of $O(\lambda Y_t)$ (involving one Yukawa coupling) or of $O(\lambda^2)$ (involving no Yukawa coupling). The various contributions from the top/stop sector are summarized in tab. 4.1. The contributions in the second column are the ones of MSSM-type, while the entries in the

third through fifth column represent the genuine NMSSM corrections, involving only scalar tops.⁵³

(i,j)	(1 2, 1 2)	(1, s)	(2, s)	(s, s)
order	$\mathcal{O}(Y_t^2)$	$\mathcal{O}(\lambda Y_t)$	$\mathcal{O}(\lambda Y_t)$	$\mathcal{O}(\lambda^2)$
fields	top/stop	stop	stop	stop
topologies				
				
				

Table 4.1: Topologies and their order in terms of the couplings in the top/stop sector that contribute to the self-energies of the CP -even fields ϕ_i at one-loop order in the gauge-less limit. The numbers 1 and 2 denote the doublet-states as external fields, while s denotes an external singlet. The internal lines depict either a top (solid) or a scalar top (dashed).

For the doublet fields, the couplings between the Higgs and stop fields in the gauge-less limit are proportional to the top quark Yukawa coupling,

$$i\Gamma_{\phi_2 \tilde{t}_i \tilde{t}_j} = i\sqrt{2}Y_t(A_t \cdot c_{ij}^{\phi_2}(\theta_t) + m_t \cdot (-1)^{1-i} \delta_{ij}) \quad , \quad i\Gamma_{\phi_1 \tilde{t}_i \tilde{t}_j} = i\sqrt{2}Y_t \mu \cdot c_{ij}^{\phi_1}(\theta_t) \quad (4.57)$$

while the corresponding coupling for the singlet field reads

$$i\Gamma_{\phi_s \tilde{t}_i \tilde{t}_j} = i\sqrt{2}\lambda(1/t_\beta)m_t \cdot c_{ij}^{\phi_s}(\theta_t) \quad (4.58)$$

The non-vanishing quartic Higgs–stop couplings read

$$i\Gamma_{\phi_2 \phi_2 \tilde{t}_i \tilde{t}_j} = -iY_t^2 \cdot \delta_{ij} \quad , \quad i\Gamma_{\phi_1 \phi_s \tilde{t}_i \tilde{t}_j} = -i\lambda Y_t \cdot c_{ij}^{\phi_1 \phi_s}(\theta_t) \quad (4.59)$$

Here functions of the mixing angle of the stop-sector, θ_t , are denoted by c with the appropriate indices and superscripts for the involved fields. These functions c can never be larger than 1. In the singlet–stop coupling we have explicitly spelled out a factor $\lambda \nu_1 Y_t = \lambda m_t \cot \beta$ to highlight the appearance of the factor m_t in eq. (4.58) instead of the usual factor $m_t / M_w \sim Y_t$ in eq. (4.57).

The genuine NMSSM couplings of a singlet to stops are seen to follow the pattern mentioned above, i.e. they give rise to contributions of $\mathcal{O}(\lambda Y_t)$ (third and fourth column in

⁵³ We discuss here only the Higgs boson self-energies. However, the same line of argument can be made for the tadpole contributions.

tab. 4.1) or $O(\lambda^2)$ (fifth column), whereas the MSSM-like contributions are of $O(Y_t^2)$ (second column). Those different patterns do not only indicate a distinction between the MSSM-like and the genuine NMSSM contributions, but also give rise to a significant numerical suppression⁵⁴ of the genuine NMSSM contributions w.r.t. the MSSM-like ones for $\lambda < Y_t$. As mentioned before, if one demands validity of perturbation theory up to the GUT scale, this relation is always fulfilled, since then λ and κ are bound from above by eq. (2.8), so that $\lambda \lesssim 0.7$, where the largest values are only allowed for vanishing κ . The size of the genuine NMSSM contributions will be discussed numerically in the following sections.

We deduce in sect. 4.3.1 the generic mass counterterms expressions for the neutral and charged Higgs sectors, in terms of the on-shell and $\overline{\text{DR}}$ counterterms of the input parameters as specified in (4.31). Explicit results for the corresponding renormalised self-energies derived from those counterterms will be given in sect. 4.3.2 in the leading $O(m_t^4)$ approximation, consisting (as in the MSSM) on evaluating the contributions from the (s)top sector for vanishing external momentum and taking into account the Yukawa part of the theory only (i.e. neglecting gauge couplings). Finally, explicit expressions will be derived for the $\overline{\text{DR}}$ input counterterms in the (s)fermion-contribution approximation and for the full case, in sects. 4.3.3 and 4.3.4, respectively.

4.3.1 Expressions for the counterterms

We introduce here the following notation:

$$\tilde{\lambda} \equiv \lambda v = \frac{\lambda}{e} \sqrt{2} M_W s_W \quad , \quad K \equiv \frac{\kappa}{\lambda} \quad (4.60)$$

We write the expressions in this section in function of these two parameters just in order to simplify the notation, but the $\overline{\text{DR}}$ Renormalisation conditions will be impose on λ and κ . The consequences of imposing such conditions on $\tilde{\lambda}$ and K instead will be explored in chapter 7.

Differentiating the expressions from sect. 4.1.1 in the same way as in (3.46), we obtain the counterterms for the elements of the CP -even Higgs mass matrix:⁵⁵

⁵⁴ For the trilinear couplings in eqs. (4.57),(4.58), comparing the Higgs singlet with the doublet, an additional potential suppression factor of $O(m_t / A_t)$ and/or $O(m_t / \mu)$ appears.

⁵⁵ In spite we are using $M_{H^\pm}^2$ as input on-shell parameter, we express these results in (4.61) and (4.62) in terms of δm_A^2 just for simplicity of notation, where $\delta m_A^2 = \delta m_{H^\pm}^2 - \delta M_W^2 + 2\tilde{\lambda} \delta \tilde{\lambda}$, see eq. (4.67) below.

$$\begin{aligned}
- \begin{pmatrix} \delta m_{\phi_1 \phi_1}^2 \\ \delta m_{\phi_1 \phi_2}^2 \\ \delta m_{\phi_2 \phi_2}^2 \\ \delta m_{\phi_1 \phi_S}^2 \\ \delta m_{\phi_2 \phi_S}^2 \\ \delta m_{\phi_S \phi_S}^2 \end{pmatrix} &= \begin{pmatrix} -t_\beta \\ 1 \\ -1/t_\beta \\ \frac{\tilde{\lambda}_{s_\beta}}{\mu} \\ \frac{\tilde{\lambda}_{c_\beta}}{\mu} \\ -\frac{\tilde{\lambda}_{s_\beta}}{\mu} \frac{\tilde{\lambda}_{c_\beta}}{\mu} \end{pmatrix} s_\beta c_\beta \delta m_A^2 + \begin{pmatrix} -1/t_\beta \\ 1 \\ -t_\beta \\ 0 \\ 0 \\ 0 \end{pmatrix} s_\beta c_\beta \delta M_Z^2 + \begin{pmatrix} (1+1/s_\beta^2)/t_\beta \\ 1 \\ -1/t_\beta \\ \frac{\tilde{\lambda}_{s_\beta}}{\mu} \\ \frac{\tilde{\lambda}_{c_\beta}}{\mu} \\ -\frac{\tilde{\lambda}_{s_\beta}}{\mu} \frac{\tilde{\lambda}_{c_\beta}}{\mu} \end{pmatrix} s_\beta^3 \delta \bar{T}_{\phi_1} + \begin{pmatrix} -t_\beta \\ 1 \\ (1+1/c_\beta^2)t_\beta \\ \frac{\tilde{\lambda}_{s_\beta}}{\mu} \\ \frac{\tilde{\lambda}_{c_\beta}}{\mu} \\ -\frac{\tilde{\lambda}_{s_\beta}}{\mu} \frac{\tilde{\lambda}_{c_\beta}}{\mu} \end{pmatrix} c_\beta^3 \delta \bar{T}_{\phi_2} \\
&+ \begin{pmatrix} -s_{2\beta}(m_A^2 - M_Z^2) \\ c_{2\beta}(m_A^2 + M_Z^2 - 2\tilde{\lambda}^2) \\ s_{2\beta}(m_A^2 - M_Z^2) \\ \frac{\tilde{\lambda}_{s_\beta}}{\mu}((c_\beta^2 + c_{2\beta})m_A^2 + 2\mu^2) + \frac{\tilde{\lambda}_{c_\beta}}{\mu} K\mu^2 \\ -\frac{\tilde{\lambda}_{c_\beta}}{\mu}((s_\beta^2 - c_{2\beta})m_A^2 + 2\mu^2) + \frac{\tilde{\lambda}_{s_\beta}}{\mu} K\mu^2 \\ ((\frac{\tilde{\lambda}_{s_\beta}}{\mu})^2 - (\frac{\tilde{\lambda}_{c_\beta}}{\mu})^2)(s_{2\beta}m_A^2 - K\mu^2) \end{pmatrix} c_\beta^2 \delta \bar{T}_\beta + \begin{pmatrix} 0 \\ -2s_{2\beta}\tilde{\lambda}^2 \\ 0 \\ \frac{\tilde{\lambda}_{c_\beta}}{\mu}(s_\beta^2 m_A^2 - 2\mu^2) + \frac{\tilde{\lambda}_{s_\beta}}{\mu} K\mu^2 \\ \frac{\tilde{\lambda}_{s_\beta}}{\mu}(c_\beta^2 m_A^2 - 2\mu^2) + \frac{\tilde{\lambda}_{c_\beta}}{\mu} K\mu^2 \\ -\frac{\tilde{\lambda}_{s_\beta}}{\mu} \frac{\tilde{\lambda}_{c_\beta}}{\mu}(s_{2\beta}m_A^2 - 2K\mu^2) \end{pmatrix} \frac{\delta \tilde{\lambda}}{\tilde{\lambda}} \\
&+ \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{\tilde{\lambda}_{c_\beta}}{\mu}(s_\beta^2 m_A^2 + 2\mu^2) + \frac{\tilde{\lambda}_{s_\beta}}{\mu} K\mu^2 \\ -\frac{\tilde{\lambda}_{s_\beta}}{\mu}(c_\beta^2 m_A^2 + 2\mu^2) + \frac{\tilde{\lambda}_{c_\beta}}{\mu} K\mu^2 \\ \frac{\tilde{\lambda}_{s_\beta}}{\mu} \frac{\tilde{\lambda}_{c_\beta}}{\mu} s_{2\beta}m_A^2 - (8K + \frac{A_\kappa}{\mu})K\mu^2 \end{pmatrix} \frac{\delta \mu}{\mu} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{\tilde{\lambda}_{s_\beta}}{\mu} \\ \frac{\tilde{\lambda}_{c_\beta}}{\mu} \\ \frac{\tilde{\lambda}_{s_\beta}}{\mu} \frac{\tilde{\lambda}_{c_\beta}}{\mu} - (8K + \frac{A_\kappa}{\mu}) \end{pmatrix} \mu^2 \delta K + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{\tilde{\lambda}}{\mu} \end{pmatrix} \delta \bar{T}_{-\phi_S} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \delta A_\kappa
\end{aligned} \tag{4.61}$$

Similarly for the CP -odd and charged Higgs sectors:

$$\begin{aligned}
- \begin{pmatrix} \delta m_{\chi_1 \chi_1}^2 \\ \delta m_{\chi_1 \chi_2}^2 \\ \delta m_{\chi_2 \chi_2}^2 \\ \delta m_{\chi_1 \chi_S}^2 \\ \delta m_{\chi_2 \chi_S}^2 \\ \delta m_{\chi_S \chi_S}^2 \end{pmatrix} &= \begin{pmatrix} -t_\beta \\ 1 \\ -1/t_\beta \\ \frac{\tilde{\lambda}_{s_\beta}}{\mu} \\ -\frac{\tilde{\lambda}_{c_\beta}}{\mu} \\ -\frac{\tilde{\lambda}_{s_\beta}}{\mu} \frac{\tilde{\lambda}_{c_\beta}}{\mu} \end{pmatrix} s_\beta c_\beta \delta m_A^2 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \delta M_Z^2 + \begin{pmatrix} (1+1/s_\beta^2)/t_\beta \\ 1 \\ -1/t_\beta \\ \frac{\tilde{\lambda}_{s_\beta}}{\mu} \\ -\frac{\tilde{\lambda}_{c_\beta}}{\mu} \\ -\frac{\tilde{\lambda}_{s_\beta}}{\mu} \frac{\tilde{\lambda}_{c_\beta}}{\mu} \end{pmatrix} s_\beta^3 \delta \bar{T}_{\phi_1} + \begin{pmatrix} -t_\beta \\ 1 \\ (1+1/c_\beta^2)t_\beta \\ \frac{\tilde{\lambda}_{s_\beta}}{\mu} \\ -\frac{\tilde{\lambda}_{c_\beta}}{\mu} \\ -\frac{\tilde{\lambda}_{s_\beta}}{\mu} \frac{\tilde{\lambda}_{c_\beta}}{\mu} \end{pmatrix} c_\beta^3 \delta \bar{T}_{\phi_2} \\
&+ \begin{pmatrix} -s_{2\beta}m_A^2 \\ c_{2\beta}m_A^2 \\ s_{2\beta}m_A^2 \\ \frac{\tilde{\lambda}_{s_\beta}}{\mu}(c_\beta^2 + c_{2\beta})m_A^2 - 3\frac{\tilde{\lambda}_{c_\beta}}{\mu} K\mu^2 \\ \frac{\tilde{\lambda}_{c_\beta}}{\mu}(s_\beta^2 - c_{2\beta})m_A^2 + 3\frac{\tilde{\lambda}_{s_\beta}}{\mu} K\mu^2 \\ ((\frac{\tilde{\lambda}_{s_\beta}}{\mu})^2 - (\frac{\tilde{\lambda}_{c_\beta}}{\mu})^2)(s_{2\beta}m_A^2 + 3K\mu^2) \end{pmatrix} c_\beta^2 \delta \bar{T}_\beta + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{\tilde{\lambda}_{c_\beta}}{\mu}s_\beta^2 m_A^2 - 3\frac{\tilde{\lambda}_{s_\beta}}{\mu} K\mu^2 \\ -\frac{\tilde{\lambda}_{s_\beta}}{\mu}c_\beta^2 m_A^2 + 3\frac{\tilde{\lambda}_{c_\beta}}{\mu} K\mu^2 \\ -\frac{\tilde{\lambda}_{s_\beta}}{\mu} \frac{\tilde{\lambda}_{c_\beta}}{\mu}(s_{2\beta}m_A^2 + 6K\mu^2) \end{pmatrix} \frac{\delta \tilde{\lambda}}{\tilde{\lambda}}
\end{aligned}$$

$$+ \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{\tilde{\lambda}c_\beta}{\mu}s_\beta^2m_A^2 - 3\frac{\tilde{\lambda}s_\beta}{\mu}K\mu^2 \\ \frac{\tilde{\lambda}s_\beta}{\mu}c_\beta^2m_A^2 + 3\frac{\tilde{\lambda}c_\beta}{\mu}K\mu^2 \\ \frac{\tilde{\lambda}s_\beta}{\mu}\frac{\tilde{\lambda}c_\beta}{\mu}s_{2\beta}m_A^2 + 3\frac{A_\kappa}{\mu}K\mu^2 \end{pmatrix} \frac{\delta\mu}{\mu} + 3 \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{\tilde{\lambda}s_\beta}{\mu} \\ \frac{\tilde{\lambda}c_\beta}{\mu} \\ -\frac{\tilde{\lambda}s_\beta}{\mu}\frac{\tilde{\lambda}c_\beta}{\mu} + \frac{A_\kappa}{\mu} \end{pmatrix} \mu^2 \delta K + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{\tilde{\lambda}}{\mu} \end{pmatrix} \delta \bar{T}_{-\phi_s} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 3K\mu \end{pmatrix} \delta A_\kappa \quad (4.62)$$

$$- \begin{pmatrix} \delta m_{\phi_1^-\phi_1^+}^2 \\ \delta m_{\phi_1^-\phi_2^+}^2 \\ \delta m_{\phi_2^-\phi_2^+}^2 \end{pmatrix} = \begin{pmatrix} (1+1/s_\beta^2)/t_\beta \\ 1 \\ -1/t_\beta \end{pmatrix} s_\beta^3 \delta \bar{T}_{\phi_1} + \begin{pmatrix} -t_\beta \\ 1 \\ (1+1/c_\beta^2)t_\beta \end{pmatrix} c_\beta^3 \delta \bar{T}_{\phi_2} + \begin{pmatrix} -s_{2\beta} \\ c_{2\beta} \\ s_{2\beta} \end{pmatrix} c_\beta^2 m_{H^\pm}^2 \delta \alpha_\beta + \begin{pmatrix} -t_\beta \\ 1 \\ -1/t_\beta \end{pmatrix} s_\beta c_\beta \delta m_{H^\pm}^2 \quad (4.63)$$

and $\delta m_{\phi_2^-\phi_1^+}^2 = \delta m_{\phi_1^-\phi_2^+}^2$. Note the presence in (4.61) and (4.62) of the quotients of vev's in the following combinations of parameters:

$$\frac{\tilde{\lambda}}{\mu} = \frac{\nu}{\nu_s}, \quad \frac{\tilde{\lambda}s_\beta}{\mu} = \frac{\nu_2}{\nu_s}, \quad \frac{\tilde{\lambda}c_\beta}{\mu} = \frac{\nu_1}{\nu_s} \quad (4.64)$$

In physical basis, we use that (cf. eq. (4.15), where now we can set $\beta_c = \beta$):

$$G^\pm = c_\beta \phi_1^\pm + s_\beta \phi_2^\pm, \quad H^\pm = -s_\beta \phi_1^\pm + c_\beta \phi_2^\pm \quad (4.65)$$

and then we get

$$\begin{aligned} \delta m_{G^\pm}^2 &= c_\beta^2 \delta m_{\phi_1^-\phi_1^+}^2 + s_\beta c_\beta (\delta m_{\phi_1^-\phi_2^+}^2 + \delta m_{\phi_2^-\phi_1^+}^2) + s_\beta^2 \delta m_{\phi_2^-\phi_2^+}^2 \\ &= -c_\beta \delta \bar{T}_{-\phi_1} - s_\beta \delta \bar{T}_{-\phi_2} \\ \delta m_{G^+H^-}^2 &= s_\beta c_\beta (\delta m_{\phi_2^-\phi_2^+}^2 - \delta m_{\phi_1^-\phi_1^+}^2) - s_\beta^2 \delta m_{\phi_1^-\phi_2^+}^2 + c_\beta^2 \delta m_{\phi_2^-\phi_1^+}^2 \\ = \delta m_{G^-H^+}^2 &= s_\beta c_\beta (\delta m_{\phi_2^-\phi_2^+}^2 - \delta m_{\phi_1^-\phi_1^+}^2) + c_\beta^2 \delta m_{\phi_1^-\phi_2^+}^2 - s_\beta^2 \delta m_{\phi_2^-\phi_1^+}^2 \\ &= s_\beta \delta \bar{T}_{-\phi_1} - c_\beta \delta \bar{T}_{-\phi_2} - m_{H^\pm}^2 c_\beta^2 \delta \alpha_\beta \\ \delta m_{H^\pm}^2 &= s_\beta^2 \delta m_{\phi_1^-\phi_1^+}^2 - s_\beta c_\beta (\delta m_{\phi_1^-\phi_2^+}^2 + \delta m_{\phi_2^-\phi_1^+}^2) + c_\beta^2 \delta m_{\phi_2^-\phi_2^+}^2 \end{aligned} \quad (4.66)$$

where (from eq. (4.18))

$$\delta m_{H^\pm}^2 = \delta m_A^2 + \delta M_W^2 - 2\tilde{\lambda}\delta\tilde{\lambda} \quad (4.67)$$

4.3.2 Explicit results for the self-energies at $O(m_t^4)$

The unrenormalised self-energies involving χ_s as external field vanish at $O(m_t^4)$, while the ones with ϕ_s external field(s) are finite and proportional to powers of both $\tilde{\lambda}$ and the stop mixing parameter X_t :

$$\begin{aligned}
\Sigma_{\phi_1\phi_s}^{m_t^4} &= 2\tilde{\lambda}cm_t^2 \frac{\mu X_t^2}{c_\beta(m_{t_1}^2 - m_{t_2}^2)^2} g(m_{t_1}^2, m_{t_2}^2) \\
\Sigma_{\phi_2\phi_s}^{m_t^4} &= -2\tilde{\lambda}cm_t^2 \frac{X_t}{c_\beta(m_{t_1}^2 - m_{t_2}^2)^2} \left((m_{t_1}^2 - m_{t_2}^2) \ln \frac{m_{t_1}^2}{m_{t_2}^2} + A_t X_t g(m_{t_1}^2, m_{t_2}^2) \right) \\
\Sigma_{\phi_s\phi_s}^{m_t^4} &= 2\tilde{\lambda}^2 cm_t^2 \frac{X_t^2}{(m_{t_1}^2 - m_{t_2}^2)^2} g(m_{t_1}^2, m_{t_2}^2) \\
\Sigma_{\chi_1\chi_s}^{m_t^4} &= \Sigma_{\chi_2\chi_s}^{m_t^4} = \Sigma_{\chi_s\chi_s}^{m_t^4} = 0
\end{aligned} \tag{4.68}$$

where c ($\propto m_t^2$) and $g(m_{t_1}^2, m_{t_2}^2)$ are defined in eqs. (3.83),(3.77). For the corresponding *renormalised* self-energies, by applying the Renormalisation scheme (4.31) to the counterterms of the previous section we get

$$\begin{pmatrix} \hat{\Sigma}_{\varphi_1\varphi_s} \\ \hat{\Sigma}_{\varphi_2\varphi_s} \\ \hat{\Sigma}_{\varphi_s\varphi_s} \end{pmatrix}_{(p^2)} = \begin{pmatrix} \Sigma_{\varphi_1\varphi_s} \\ \Sigma_{\varphi_2\varphi_s} \\ \Sigma_{\varphi_s\varphi_s} \end{pmatrix}_{(p^2)}^{\text{fin}} + \begin{pmatrix} s_\beta \\ ac_\beta \\ -\frac{\tilde{\lambda}}{\mu} s_\beta c_\beta \end{pmatrix} \frac{\tilde{\lambda}}{\mu} (\mu \delta A_\lambda^{\text{fin}}) + \begin{pmatrix} 0 \\ 0 \\ \frac{\tilde{\lambda}}{\mu} \end{pmatrix} \delta \bar{T}_{\phi_s}^{\text{fin}} \tag{4.69}$$

for both $\varphi = \phi, \chi$, with $a = +1$ for $\varphi = \phi$ and $a = -1$ for $\varphi = \chi$, and where

$$\mu \delta A_\lambda^{\text{fin}} = s_\beta c_\beta (\delta m_{H^\pm}^{2,\text{fin}} - \delta M_W^{2,\text{fin}}) + s_\beta^3 \delta \bar{T}_{\phi_1}^{\text{fin}} + c_\beta^3 \delta \bar{T}_{\phi_2}^{\text{fin}} \quad . \tag{4.70}$$

In the particular case the $O(m_t^4)$ approximation (where no singlet/singlino fields are involved as virtual contributions), from Renormalisation conditions (4.39),(4.40) the results for $\delta m_{H^\pm}^2, \delta M_W^2, \delta \bar{T}_{\phi_{1,2}}$ follow directly from the (s)top-contribution MSSM result,⁵⁶

$$\delta m_{H^\pm}^{2,t/\tilde{t}} = \Sigma_{H^\pm H^\pm}^{t/\tilde{t}} = \Delta_{H^\pm A} - \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 t_\beta^2} (1 + \Delta - \ln m_t^2) + O(m_t^2)$$

⁵⁶ A term like $\ln m$ may appear strange, since m is a dimensionfull quantity. Here we are using the definition $\Delta \equiv \frac{2}{4-D} - \gamma_E + \ln(4\pi\bar{\mu}^2)$ for the divergent part of the amplitudes in dimensional regularization, so that combinations of the form $\Delta - \ln m^2$ are just a compact notation for $\frac{2}{4-D} - \gamma_E + \ln(4\pi) - \ln \frac{m^2}{\bar{\mu}^2}$.

$$\begin{aligned}
\delta M_W^{2,t/\tilde{t}} &= \Sigma_{WW}^{t/\tilde{t}} = O(m_t^2) \\
\delta \overline{T}_{\phi_1}^{t/\tilde{t}} &= \frac{1}{\sqrt{2}\nu} T_{\phi_1}^{t/\tilde{t}} = O(m_t^2) \\
\delta \overline{T}_{\phi_2}^{t/\tilde{t}} &= \frac{1}{\sqrt{2}\nu} T_{\phi_2}^{t/\tilde{t}} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 s_\beta} (1 + \Delta - \ln m_t^2) + O(m_t^2)
\end{aligned} \tag{4.71}$$

with $\Delta_{H^\pm A}$ from eq. (3.88). In addition $\frac{1}{\sqrt{2}\nu} T_{\phi_s}^{t/\tilde{t}} = O(m_t^2)$ is verified for the one-loop contributions to the tadpole of the singlet field. Therefore we get that the expressions for the renormalised self-energies reduce to

$$\begin{pmatrix} \hat{\Sigma}_{\varphi_1 \varphi_s} \\ \hat{\Sigma}_{\varphi_2 \varphi_s} \\ \hat{\Sigma}_{\varphi_s \varphi_s} \end{pmatrix}^{m_t^4} = \begin{pmatrix} \Sigma_{\varphi_1 \varphi_s} \\ \Sigma_{\varphi_2 \varphi_s} \\ \Sigma_{\varphi_s \varphi_s} \end{pmatrix}^{m_t^4} + \begin{pmatrix} s_\beta \\ ac_\beta \\ -\frac{\tilde{\lambda}}{\mu} s_\beta c_\beta \end{pmatrix} \frac{\tilde{\lambda}}{\mu} s_\beta c_\beta \Delta_{H^\pm A} \quad . \tag{4.72}$$

The other renormalised self-energies involved in the neutral Higgs sector (i.e. the one with no external singlet field) at $O(m_t^4)$ are the same as specified in sect. 3.4 for the MSSM. Numerical results in this approximation will be explored in sect. 4.4.2.

4.3.3 (S)fermion contributions at one-loop order

In this section we perform a method to get explicit expressions in the (s)fermion-contribution approximation at one-loop order for the $\overline{\text{DR}}$ counterterms involved in the Higgs sector of the NMSSM, which requires the evaluation of the fields counterterms δZ 's only (which is very straightforward in this approximation). This allows not only to avoid the complexity inherent to the other amplitudes, but also to get a more profound understanding of the structure (or at least some characteristics) of the model.

Excluding one-loop tadpole and self-energy insertions (i.e. taking into account irreducible n -point functions only), one has for the one-loop contributions one possible topology for the tadpoles $T_X^{(1L)a}$, two for the self-energies $\Sigma_{XY}^{(1L)a/b}$, and four for the three-point functions $\Delta\Gamma_{XYZ}^{(1L)a/b/c}$, with virtual contributions from generical sectors a, b, c , as it is shown in fig. 4.1.

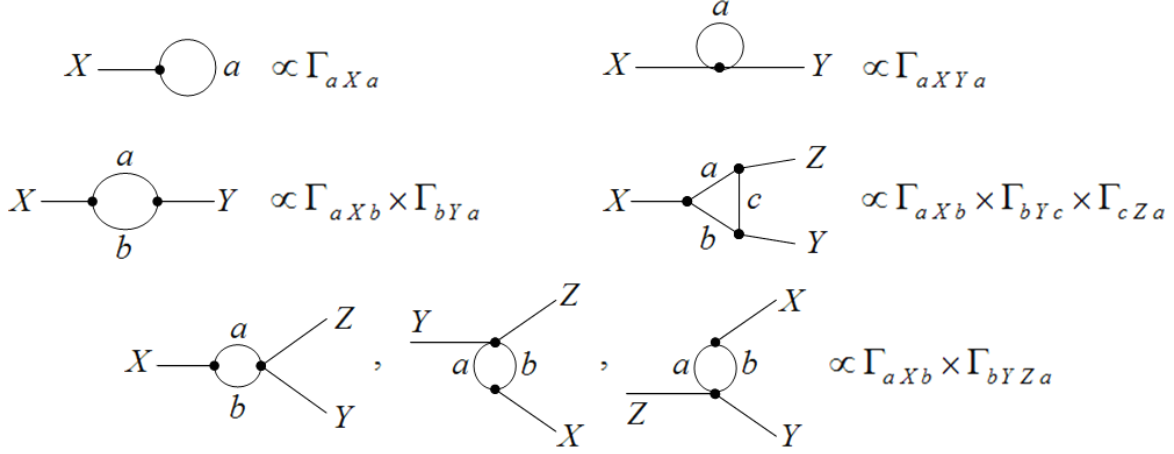


Figure 4.1: Possible topologies for irreducible 1-, 2- and 3-point functions at one-loop order. The couplings (tree-level vertexes) involved in the evaluation of the amplitudes from each of these Feynman diagrams are indicated.

On the other hand, the following are all the bilinear coefficients and vertices of the NMSSM that involve the parameters λ , κ and/or A_κ :

$$\begin{aligned}
m_{\varphi_{1,2}\varphi_{1,2}}^2 &\rightarrow \lambda \\
m_{\varphi_{1,2}\varphi_S}^2 &\rightarrow \lambda, \kappa \\
m_{\varphi_S\varphi_S}^2 &\rightarrow \lambda, \kappa, A_\kappa \\
\Gamma_{\tilde{e}_g\tilde{e}_g\varphi_S}, \Gamma_{\tilde{u}_g\tilde{u}_g\varphi_S}, \Gamma_{\tilde{d}_g\tilde{d}_g\varphi_S} &\rightarrow \lambda \\
\Gamma_{\tilde{\chi}^0\tilde{\chi}^0\varphi_{1,2}} &\rightarrow \lambda \\
\Gamma_{\tilde{\chi}^0\tilde{\chi}^0\varphi_S} &\rightarrow \lambda, \kappa \\
\Gamma_{\tilde{\chi}^0\tilde{\chi}^\pm\phi^\mp} &\rightarrow \lambda \\
\Gamma_{\tilde{\chi}^+\tilde{\chi}^-\varphi_S} &\rightarrow \lambda \\
\Gamma_{\varphi_1\varphi_1\varphi_{2,S}}, \Gamma_{\varphi_{1,S}\varphi_2\varphi_2} &\rightarrow \lambda \\
\Gamma_{\varphi_1\varphi_2\varphi_S}, \Gamma_{\varphi_{1,2}\varphi_S\varphi_S} &\rightarrow \lambda, \kappa \\
\Gamma_{\phi_S\varphi_S\varphi_S} &\rightarrow \lambda, \kappa, A_\kappa \\
\Gamma_{\varphi_{1,2}\phi^+\phi^-} &\rightarrow \lambda \\
\Gamma_{\varphi_S\phi^+\phi^-} &\rightarrow \lambda, \kappa \\
\Gamma_{\tilde{e}_g\tilde{e}_g\varphi_2\varphi_S}, \Gamma_{\tilde{u}_g\tilde{u}_g\varphi_1\varphi_S}, \Gamma_{\tilde{d}_g\tilde{d}_g\varphi_2\varphi_S} &\rightarrow \lambda \\
\Gamma_{\tilde{V}_g\tilde{e}_g\varphi_S\phi^\pm}, \Gamma_{\tilde{u}_g\tilde{d}_g\varphi_S\phi^\pm} &\rightarrow \lambda \\
\Gamma_{\varphi_1\varphi_1\varphi_2\varphi_2}, \Gamma_{\varphi_1\varphi_1\varphi_S\varphi_S}, \Gamma_{\varphi_2\varphi_2\varphi_S\varphi_S} &\rightarrow \lambda \\
\Gamma_{\varphi_S\varphi_S\varphi_1\varphi_2}, \Gamma_{\varphi_S\varphi_S\varphi_S\varphi_S}, \Gamma_{\varphi_S\varphi_S\phi^+\phi^-} &\rightarrow \lambda, \kappa \\
\Gamma_{\varphi_1\varphi_2\phi^+\phi^-}, \Gamma_{\phi^+\phi^+\phi^-\phi^-} &\rightarrow \lambda
\end{aligned} \tag{4.73}$$

where φ_i stands for both the neutral Higgs field ϕ_i and χ_i , and $\phi^\pm \sim G^\pm, H^\pm$. Now let's consider only virtual contributions due to leptons, sleptons, quarks and squarks. It is instructive to review which features of the theory are relevant as constraints for the determination of all possible allowed combinations of vertexes contributing to the amplitude topologies of fig. 4.1:

- Electric charge conservation.
- R-parity: only vertexes with even number of supersymmetric particles are allowed.
- Fermion flow: fermionic fields only appear in vertexes of the form *fermion-fermion-boson*.
- The Higgs singlet does not couple to vector bosons.
- $\Gamma_{\tilde{f}\tilde{f}\varphi_s\varphi_s} = 0$, where $\tilde{f}$ stands for both squarks and sleptons.
- $\Gamma_{\varphi_{1,2}\varphi_S\phi^+\phi^-} = \Gamma_{\varphi_{1,2}\varphi_{1,2}\varphi_{1,2}\varphi_S} = \Gamma_{\varphi_{1,2}\varphi_S\varphi_S\varphi_S} = 0$.
- The three-point amplitude with three internal lines from fig. 4.1 (the right diagram in the second row of the figure) is divergent only for fermion virtual contributions.⁵⁷

From this conditions, the topological analysis shows that the following amplitudes do not involve the parameters λ , κ nor A_κ in the (s)fermion-contributions approximation at one-loop order,

$$\begin{aligned} \partial_\lambda \Omega^{f\tilde{f}} &= \partial_\kappa \Omega^{f\tilde{f}} = \partial_{A_\kappa} \Omega^{f\tilde{f}} = 0 \\ \text{for } \Omega &= T_{\phi_{1,2}}^{(1L)}, \Sigma_{\varphi_{1,2}\varphi_{1,2}}, \Sigma_{H^\pm H^\pm}, \Sigma_{WW}, \Sigma_{ZZ}, \Delta\Gamma_{\varphi_{1,2}\varphi_{1,2}\phi_{1,2}} \end{aligned} \quad (4.74)$$

while these other amplitudes do depend on λ but do not on κ nor A_κ ,

$$\begin{aligned} \partial_\kappa \Omega^{f\tilde{f}}(\lambda) &= \partial_{A_\kappa} \Omega^{f\tilde{f}}(\lambda) = 0 \\ \text{for } \Omega &= T_{\phi_S}^{(1L)}, \Sigma_{\varphi_{1,2,S}\varphi_S}, \Delta\Gamma_{\varphi_{1,2}\varphi_{1,2,S}\varphi_S} \end{aligned} \quad (4.75)$$

and

$$\Delta\Gamma_{\varphi_S\varphi_S\phi_S}^{f\tilde{f}} = 0 \quad . \quad (4.76)$$

⁵⁷ Accordingly, in the case of neutral Higgs fields as external legs, we see from (4.73) that the divergent part of this kind of diagrams will be always independent of A_κ , and it will depend on λ and/or κ only in the case of charginos or neutralinos as virtual particles.

In the tab. 4.2 we gather summarize the consequent dependence with respect to λ , κ or A_κ for all the bilinear terms and three-point vertexes and all the self-energies and three-point one-loop amplitudes with neutral Higgs fields as external lines.

XY, XYZ	m_{XY}^2, Γ_{XYZ}	$\Sigma_{XY}^{f\tilde{f}(1L)}, \Delta\Gamma_{XYZ}^{f\tilde{f}(1L)}$
$\varphi_1\varphi_1\phi_1, \varphi_2\varphi_2\phi_2$	none	none
$\varphi_1\varphi_1\phi_S, \varphi_2\varphi_2\phi_S$	λ	λ
$\varphi_{1,2}\varphi_{1,2}, \varphi_1\varphi_1\phi_2, \varphi_2\varphi_2\phi_1$	λ	none
$\varphi_{1,2}\varphi_S, \varphi_1\varphi_2\varphi_S, \varphi_{1,2}\varphi_S\varphi_S$	λ, κ	λ
$\varphi_S\varphi_S$	$\lambda, \kappa, A_\kappa$	λ
$\varphi_S\varphi_S\varphi_S$	$\lambda, \kappa, A_\kappa$	$=0$

Table 4.2: Dependence of the bilinear terms and the 3-point vertexes from the Higgs sector, as well as of the self-energies and 3-point one-loop amplitudes with external neutral Higgs fields, w.r.t. $\lambda, \kappa, A_\kappa$. Here φ_i stand for ϕ_i and/or χ_i ($i=1,2,S$)

In addition, the field counterterms are defined as in eqs. (3.42),(3.52), with the identification

$$\begin{aligned}
\delta Z_{\phi_1} &= \delta Z_{\chi_1} = \delta Z_{H_1} \\
\delta Z_{\phi_2} &= \delta Z_{\chi_2} = \delta Z_{H_2} \\
\delta Z_{\phi_S} &= \delta Z_{\chi_S} = \delta Z_S
\end{aligned} \tag{4.77}$$

so one sees that the δZ 's inherits the same dependence as for Σ :

$$\begin{aligned}
\delta Z_{\varphi_i} &= -\partial_{p^2} \Sigma_{\varphi_i \varphi_i}^{\text{div}} \\
\text{thus } \partial_\alpha \Sigma_{XX} &= 0 \Rightarrow \partial_\alpha \delta Z_X = 0
\end{aligned} \tag{4.78}$$

Hence from tab. 4.2 we get the following properties for the field counterterms:⁵⁸

$$\partial_\lambda \delta Z_{\varphi_{1,2}}^{(1L)f/\tilde{f}} = \partial_\kappa \delta Z_{\varphi_{1,2,S}}^{(1L)f/\tilde{f}} = \partial_{A_\kappa} \delta Z_{\varphi_{1,2,S}}^{(1L)f/\tilde{f}} = 0 \tag{4.79}$$

As in (4.27), the development for the one-loop renormalised self-energies and three-point vertexes adopts the following form:⁵⁹

$$\hat{\Sigma}_{XY}(p^2) = \Sigma_{XY}(p^2) + (\delta_{XY} p^2 - m_{XY}^2) \frac{1}{2} (\delta Z_X + \delta Z_Y) - \delta m_{XY}^2$$

⁵⁸ In the MSSM, $\frac{\delta t_\beta}{t_\beta} = \frac{1}{2} (\delta Z_{H_2} - \delta Z_{H_1})$. We will not assume this here, and at the end we will find that this is also the case for the NMSSM.

⁵⁹ From now on, we drop the explicit the superscripts “ $f\tilde{f}$ ” or “(1L)”.

(4.80)

$$\hat{\Gamma}_{XYZ} = \Delta\Gamma_{XYZ} + \Gamma_{XYZ} \frac{1}{2}(\delta Z_X + \delta Z_Y + \delta Z_Z) + \delta\Gamma_{XYZ}$$

Therefore, we fix the six $\overline{\text{DR}}$ counterterms $\delta\lambda$, $\delta\kappa$, δA_κ , δt_β , $\delta\mu$, $\delta\nu$ (see (4.31)) by solving the following set of equations that follows from $\hat{\Sigma}^{\text{div}} = \hat{\Gamma}^{\text{div}} = 0$ and $\Delta\Gamma_{\varphi_S\varphi_S\phi_S} = 0$:

$$\triangleright 0 = -(\partial_\alpha m_{XY}^2) \frac{1}{2}(\delta Z_X + \delta Z_Y) - \partial_\alpha \delta m_{XY}^{2,\text{div}} \quad (4.81a)$$

$$\triangleright 0 = (\partial_\alpha \Gamma_{XYZ}) \frac{1}{2}(\delta Z_X + \delta Z_Y + \delta Z_Z) + \partial_\alpha \delta \Gamma_{XYZ}^{\text{div}} \quad (4.81b)$$

$$\triangleright 0 = \Gamma_{\varphi_S\varphi_S\phi_S} \frac{3}{2}\delta Z_S + \delta \Gamma_{\varphi_S\varphi_S\phi_S}^{\text{div}} \quad (4.81c)$$

with

$$\begin{aligned} \circ \quad & \alpha = \lambda, \kappa, A_\kappa \quad \text{for} \quad XYZ = \varphi_1\varphi_1\varphi_1, \varphi_2\varphi_2\varphi_2 \\ \circ \quad & \alpha = \kappa, A_\kappa \quad \text{for} \quad XY = \varphi_{1,2}\varphi_{1,2} \quad \text{and} \quad XYZ = \varphi_1\varphi_1\phi_{2,S}, \varphi_2\varphi_2\phi_{1,S} \\ \circ \quad & \alpha = A_\kappa \quad \text{for} \quad XY = \varphi_{1,2}\varphi_S \quad \text{and} \quad XYZ = \varphi_1\varphi_2\varphi_S, \varphi_{1,2}\varphi_S\varphi_S \\ & (\Rightarrow \partial_\alpha m_{XY}^2 = \partial_\alpha \Gamma_{XYZ} = \partial_\alpha \Sigma_{XY} = \partial_\alpha \Delta\Gamma_{XYZ} = 0) \end{aligned} \quad (4.82a)$$

$$\begin{aligned} \circ \quad & \alpha = \lambda \quad \text{for} \quad XY = \varphi_{1,2}\varphi_{1,2} \quad \text{and} \quad XYZ = \varphi_1\varphi_1\phi_2, \varphi_2\varphi_2\phi_1 \\ \circ \quad & \alpha = \kappa \quad \text{for} \quad XY = \varphi_{1,2,S}\varphi_S \quad \text{and} \quad XYZ = \varphi_1\varphi_2\varphi_S, \varphi_{1,2}\varphi_S\varphi_S \\ \circ \quad & \alpha = A_\kappa \quad \text{for} \quad XY = \varphi_S\varphi_S \\ & (\Rightarrow \partial_\alpha \Sigma_{XY} = \partial_\alpha \Delta\Gamma_{XYZ} = 0) \end{aligned} \quad (4.82b)$$

The δm^2 counterterms functionality in terms of the input (independent) parameters counterterms is detailed in sect. 4.3.1. For the vertexes, the Γ_{XYZ} 's are obtained as the coefficient of the trilinear terms in the Lagrangian,

$$\Gamma_{\varphi_i\varphi_j\varphi_k} = i \frac{\partial^3 V_{\text{Higgs}}^{\text{rNMSSM}}}{\partial\varphi_i\partial\varphi_j\partial\varphi_k} \bigg|_{\phi_{1,2}=\chi_{1,2}=0} \quad (4.83)$$

so the following vertexes are obtained:

$$\begin{aligned} s_\beta \Gamma_{\phi_1\phi_1\phi_1} &= c_\beta \Gamma_{\phi_2\phi_2\phi_2} = 3s_\beta \Gamma_{\chi_1\chi_1\phi_1} = 3c_\beta \Gamma_{\chi_2\chi_2\phi_2} = \frac{3i}{\sqrt{2}v} s_\beta c_\beta M_Z^2 \\ c_\beta \Gamma_{\phi_1\phi_1\phi_2} &= c_\beta \Gamma_{\chi_1\chi_1\phi_2} = s_\beta \Gamma_{\phi_2\phi_2\phi_1} = s_\beta \Gamma_{\chi_2\chi_2\phi_1} = \frac{i}{\sqrt{2}v} s_\beta c_\beta (2\lambda^2 v^2 - M_Z^2) \end{aligned}$$

$$\Gamma_{\phi_1\phi_1\phi_S} = \Gamma_{\phi_2\phi_2\phi_S} = \Gamma_{\chi_1\chi_1\phi_S} = \Gamma_{\chi_2\chi_2\phi_S} = \sqrt{2}i\lambda\mu$$

$$c_\beta \Gamma_{\chi_1\chi_S\phi_S} = -s_\beta \Gamma_{\chi_2\chi_S\phi_S} = \sqrt{2}i\lambda\kappa\nu s_\beta c_\beta$$

$$s_\beta (\Gamma_{\phi_1\phi_S\phi_S} + \Gamma_{\chi_1\chi_S\chi_S}) = c_\beta (\Gamma_{\phi_2\phi_S\phi_S} + \Gamma_{\chi_2\chi_S\chi_S}) = 2\sqrt{2}i\lambda^2\nu s_\beta c_\beta$$

$$c_\beta(\Gamma_{\phi_1\phi_S\phi_S} - \Gamma_{\phi_1\chi_S\chi_S}) = s_\beta(\Gamma_{\phi_2\phi_S\phi_S} - \Gamma_{\phi_2\chi_S\chi_S}) = -2\sqrt{2}i\lambda\kappa v s_\beta c_\beta \quad (4.84)$$

$$\begin{aligned} \Gamma_{\phi_1\phi_2\phi_S} + \Gamma_{\phi_1\chi_2\chi_S} &= \Gamma_{\chi_1\chi_2\phi_S} - \Gamma_{\chi_1\phi_2\chi_S} = -2\sqrt{2}i\kappa\mu \\ 3\Gamma_{\phi_1\phi_2\phi_S} - \Gamma_{\phi_1\chi_2\chi_S} &= 3\Gamma_{\chi_1\chi_2\phi_S} + \Gamma_{\chi_1\phi_2\chi_S} = -2\sqrt{2}i\frac{\lambda}{\mu}(s_\beta c_\beta(m_{H^\pm}^2 - M_W^2 + \lambda^2 v^2) + s_\beta^3 \bar{T}_{-\phi_1} + c_\beta^3 \bar{T}_{-\phi_2}) \\ \Gamma_{\phi_S\phi_S\phi_S} &= \sqrt{2}i\kappa(6\frac{\kappa}{\lambda}\mu + A_\kappa) \\ \Gamma_{\chi_S\chi_S\phi_S} &= \sqrt{2}i\kappa(2\frac{\kappa}{\lambda}\mu - A_\kappa) \end{aligned}$$

The corresponding counterterms, $\delta\Gamma_{XYZ}$, are obtained in a similar way as in (3.46):

$$\delta\Gamma_{\varphi_i\varphi_j\varphi_k} = \delta\vec{P} \cdot \partial_{\vec{P}} \Gamma_{\varphi_i\varphi_j\varphi_k} \Big|_{T_{\phi_{1,2,S}}=0} \quad (4.85)$$

Here $\vec{P} = (M_W^2, M_Z^2, m_{H^\pm}^2, \bar{T}_{-\phi_{1,2,S}}, \lambda, \kappa, A_\kappa, t_\beta, \mu, v)^T$ (see eq. (4.31)), but it's easy to see from eqs. (4.61),(4.62),(4.39),(4.40),(4.74),(4.75),(4.82),(4.84) that $\delta M_W^2, \delta M_Z^2, \delta m_{H^\pm}^2, \delta \bar{T}_{-\phi_{1,2,S}}$ don't take part (or their contributions cancel out) in (4.81).

From the two equations in (6c) (one for $\varphi_S\varphi_S\phi_S = \phi_S\phi_S\phi_S$ and one for $\varphi_S\varphi_S\phi_S = \phi_S\chi_S\chi_S$) we find that

$$\begin{aligned} \frac{\delta(\kappa A_\kappa)}{\kappa A_\kappa} &= \frac{\delta(\kappa^2 \mu / \lambda)}{\kappa^2 \mu / \lambda} = -\frac{3}{2} \delta Z_S \\ \text{i.e. } \frac{\delta A_\kappa}{A_\kappa} &= -\frac{\delta \kappa}{\kappa} - \frac{3}{2} \delta Z_S \quad \text{and} \quad \frac{\delta \mu}{\mu} = \frac{\delta \lambda}{\lambda} - 2\frac{\delta \kappa}{\kappa} - \frac{3}{2} \delta Z_S \end{aligned} \quad (4.86)$$

For the counterterms of the other input parameters, the cases (4.82a) allows us to infer their dependence with respect to the variables λ, κ and A_κ . That first group involve equations where not only the Σ_{XY} 's and the $\Delta\Gamma_{XYZ}$'s but also the m_{XY}^2 's and the Γ_{XYZ} 's do not depend on α (where α is λ, κ or A_κ as appropriate): $\partial_\alpha \delta m_{XY}^{2,\text{div}} = \partial_\alpha \delta \Gamma_{XYZ}^{\text{div}} = 0$. We get that both δt_β and δv^{div} are constant with respect to $\lambda, \kappa, A_\kappa$, while $\delta \lambda / \lambda$ and $\delta \kappa / \kappa$ can only depend on λ :

$$\delta t_\beta = c_{t_\beta}, \quad \delta v^{\text{div}} = c_v, \quad \delta \lambda = f_\lambda(\lambda), \quad \delta \kappa = \kappa f_\kappa(\lambda) \quad (4.87)$$

being c_{t_β}, c_v two quantities that are constant w.r.t. $\lambda, \kappa, A_\kappa$. On the other hand, in the set of equations from the cases (4.82b) Σ_{XY} and $\Delta\Gamma_{XYZ}$ don't depend on α , but m_{XY}^2 and Γ_{XYZ} do.

We solve with the help of result (4.87),⁶⁰ getting that we can write the six input $\overline{\text{DR}}$ parameter counterterms in terms of $\delta Z_{H_1, H_2, S}$:

$$\begin{aligned}\frac{\delta t_\beta}{t_\beta} &= -\frac{1}{2}(\delta Z_{H_1} - \delta Z_{H_2}) \\ \frac{\delta \lambda}{\lambda} &= -\frac{1}{2}(\delta Z_{H_1} + \delta Z_{H_2}) \\ \frac{\delta \kappa}{\kappa} &= -\delta Z_S \\ \frac{\delta v}{v} &= \frac{1}{2}(c_\beta^2 \delta Z_{H_1} + s_\beta^2 \delta Z_{H_2})\end{aligned}\tag{4.88}$$

and by using (9)

$$\begin{aligned}\frac{\delta A_\kappa}{A_\kappa} &= -\frac{1}{2}\delta Z_S \\ \frac{\delta \mu}{\mu} &= -\frac{1}{2}(\delta Z_{H_1} + \delta Z_{H_2} - \delta Z_S)\end{aligned}\tag{4.89}$$

At this point we must emphasize that the results from eqs. (4.88),(4.89) do not mean that $\delta t_\beta, \delta \lambda, \dots, \delta \mu$ are in fact dependent quantities. These relations are valid at one-loop order and in the (s)fermion contributions approximation only.

Comparing the result in (4.88) for δt_β , we see that $\delta v_1^{\text{div}}/\nu_1 = \delta v_2^{\text{div}}/\nu_2$, as it was anticipated in (4.47). Replacing in (4.49) the results from eqs. (4.88),(4.89) for $\delta \lambda$ and δv , and comparing it with the one for $\delta \mu$, it can be seen that also

$$\frac{\delta v_S^{\text{div}}}{\nu_S} = \frac{\delta v_1^{\text{div}}}{\nu_1} = \frac{\delta v_2^{\text{div}}}{\nu_2}\tag{4.90}$$

Eqs. (4.88),(4.89) anticipate a very simple explicit result for these counterterms, because the divergent parts of loops with only scalar internal lines do not depend on p^2 , so in our case $\delta Z_{\varphi_i} (= -\partial_{p^2} \Sigma_{\varphi_i \varphi_i}^{\text{div}})$ only depend on leptons and quarks sectors, not on the sfermions sectors, and therefore the counterterms from (4.88) and (4.89) do not depend on scalar masses nor on scalar mixing matrices. By direct calculation one finds that the field counterterms in $\overline{\text{DR}}$ Renormalisation and taking into account only (s)fermion contributions are

⁶⁰ Note that here we are solving simultaneously 28 equations with only 4 counterterms as unknowns to fix, so this is a calculation and a check at the same time.

$$\delta Z_{H_i} = \frac{3\Delta}{8\pi} \sum_{\substack{f=u_g, d_g, e_g \\ g=1,2,3}} \Gamma_{\phi_i, f, -f}^L \times \Gamma_{\phi_i, f, -f}^R \quad (4.91)$$

being Γ^L, Γ^R the factors that enter in the Feynman rule for the vertex between the field ϕ_i and two fermions f in the combination $\Gamma_{\phi_i, f, -f}^L \frac{1}{2}(1-\gamma_5) + \Gamma_{\phi_i, f, -f}^R \frac{1}{2}(1+\gamma_5)$ (here ϕ_i is any scalar real field pertaining to H_i , $i=1,2,S$ with $H_S \equiv S$). Accordingly, these are the explicit final results for the counterterms:⁶¹

$$\delta Z_{H_1} = -\frac{\Delta}{16\pi^2} \frac{3M_d^2 + M_e^2}{\nu_1^2}, \quad \delta Z_{H_2} = -\frac{\Delta}{16\pi^2} \frac{3M_u^2}{\nu_2^2}, \quad \delta Z_S = 0 \quad (4.92)$$

and hence

$$\begin{aligned} \frac{\delta t_\beta}{t_\beta} &= -\frac{\Delta}{32\pi^2} \left(\frac{3M_u^2}{\nu_2^2} - \frac{3M_d^2 + M_e^2}{\nu_1^2} \right) \\ \frac{\delta \mu}{\mu} &= \frac{\delta \lambda}{\lambda} = \frac{\Delta}{32\pi^2} \left(\frac{3M_u^2}{\nu_2^2} + \frac{3M_d^2 + M_e^2}{\nu_1^2} \right) \\ \frac{\delta \nu}{\nu} &= -\frac{\Delta}{32\pi^2} \frac{3(M_u^2 + M_d^2) + M_e^2}{\nu^2} \\ \delta \kappa &= \delta A_\kappa = 0 \end{aligned} \quad (4.93)$$

where

$$M_u^2 \equiv \sum_{q=u,c,t} m_q^2, \quad M_d^2 \equiv \sum_{q=d,s,b} m_q^2, \quad M_e^2 \equiv \sum_{\ell=e,\mu,\tau} m_\ell^2. \quad (4.94)$$

4.3.4 Explicit full results for the counterterms

In order to have a simple and elegant check of the chosen Renormalisation scheme from the finiteness of the relevant renormalised quantities, it is useful to have explicit expressions

⁶¹ It is easy and instructive to see why δZ_S must be zero:

As mention above, only leptons and quarks can contribute to the δZ 's, because sleptons and squarks are scalar fields and then they do not generate p^2 dependence in the divergent parts of the self-energies. But the Higgs singlet does not couple to leptons or quarks, and this is why: Both the Higgs doublets and the singlet belong of course to the Higgs sector and have non-zero vev's, but they have essentially distinct purposes in the model. The Higgs doublets and its vev's generate the mass terms in the model, so they couple to all particles which have a mass. But the purpose of the singlet and its vev is to generate the μ -terms dynamically, and we know from the MSSM that the fermion sectors are not affected by the μ parameter.

for the counterterms as performed in 4.3.3 but for the contributions from all sectors of the theory. In the case of $\delta\mu$, a direct evaluation from the chargino sector as specified in eqs. (4.50),(4.51) gives

$$\begin{aligned}\frac{\delta\mu}{\mu} &= \frac{\Delta}{16\pi^2} \left(\frac{1}{2} \left(\frac{3M_u^2}{\nu_2^2} + \frac{3M_d^2 + M_e^2}{\nu_1^2} \right) - 2e^2 \frac{2 + c_{2W}}{s_{2W}^2} + \lambda^2 \right) \\ &= \frac{\Delta}{16\pi^2 \nu^2} \left(\frac{1}{2} \left(\frac{3M_u^2}{s_\beta^2} + \frac{3M_d^2 + M_e^2}{c_\beta^2} \right) - M_Z^2 - 2M_W^2 + \tilde{\lambda}^2 \right) .\end{aligned}\quad (4.95)$$

In addition, compact expressions can be obtained for the divergent parts of the counterterms for M_W^2 and M_Z^2 , as well as for the field counterterms:

$$\begin{aligned}\frac{\delta M_W^2|^\text{div}}{M_W^2} &= -\frac{\Delta}{16\pi^2 \nu^2} (-12M_W^2 + M_e^2 + 3(M_u^2 + M_d^2)) \\ \frac{\delta M_Z^2|^\text{div}}{M_Z^2} &= -\frac{\Delta}{16\pi^2 \nu^2} (4M_Z^2(c_{2W} - c_{4W} - 3) + M_e^2 + 3(M_u^2 + M_d^2))\end{aligned}\quad (4.96)$$

$$\begin{aligned}\delta Z_{H_1} &= -\frac{\Delta}{16\pi^2} \left(\frac{3M_d^2 + M_e^2}{\nu_1^2} + \lambda^2 \right) = -\frac{\Delta}{16\pi^2 \nu^2} \left(\frac{3M_d^2 + M_e^2}{c_\beta^2} + \tilde{\lambda}^2 \right) \\ \delta Z_{H_2} &= -\frac{\Delta}{16\pi^2} \left(\frac{3M_u^2}{\nu_2^2} + \lambda^2 \right) = -\frac{\Delta}{16\pi^2 \nu^2} \left(\frac{3M_u^2}{s_\beta^2} + \tilde{\lambda}^2 \right) \\ \delta Z_S &= -\frac{\Delta}{16\pi^2} 2(\lambda^2 + \kappa^2) = -\frac{\Delta}{16\pi^2 \nu^2} 2(1 + K^2) \tilde{\lambda}^2\end{aligned}$$

Finally, the explicit counterterms for the $\overline{\text{DR}}$ quantities $\lambda, \kappa, A_\kappa$ are obtained:⁶²

$$\begin{aligned}\frac{\delta\lambda}{\lambda} &= \frac{\Delta}{16\pi^2} \left(\frac{1}{2} \left(\frac{3M_u^2}{\nu_2^2} + \frac{3M_d^2 + M_e^2}{\nu_1^2} \right) - 2e^2 \frac{2 + c_{2W}}{s_{2W}^2} + 2\lambda^2 + \kappa^2 \right) \\ &= \frac{\Delta}{16\pi^2 \nu^2} \left(\frac{1}{2} \left(\frac{3M_u^2}{s_\beta^2} + \frac{3M_d^2 + M_e^2}{c_\beta^2} \right) - M_Z^2 - 2M_W^2 + (2 + K^2) \tilde{\lambda}^2 \right) \\ \frac{\delta\kappa}{\kappa} &= \frac{\Delta}{16\pi^2} 3(\lambda^2 + \kappa^2) = \frac{\Delta}{16\pi^2 \nu^2} 3(1 + K^2) \tilde{\lambda}^2\end{aligned}\quad (4.97)$$

⁶² We thank Peter Drechsel [135] for performing the derivation of these expressions for $\lambda, \kappa, A_\kappa$, from an approach valid at one-loop level where he makes use of the fact that the gauge singlet do not couple to gauge bosons and their superpartners as well as to the SM fermions.

$$\delta A_\kappa = \frac{\Delta}{16\pi^2} 6(\lambda^2 A_\lambda + \kappa^2 A_\kappa) = \frac{\Delta}{16\pi^2 \nu^2} 6\left(\frac{s_{\beta^c} c_{\beta}}{\mu} (M_{H^\pm}^2 - M_W^2 + \tilde{\lambda}^2) - K\mu + K^2 A_\kappa\right) \tilde{\lambda}^2$$

4.4 Numerical analysis

We perform here a numerical exploration derived from the analytical development of the previous sections. A “canonical scenario” is used to exemplify some features of the several outlined approximations. A genuine NMSSM “sample scenario” will be also studied, which gives rise to a SM-like Higgs with a predicted mass at the two-loop level of around 125 GeV and a singlet-like Higgs field with a mass that can be above or below the one of the SM-like state. In order to investigate the influence of the extended Higgs and higgsino sector of the NMSSM compared to the MSSM, the parameter λ will be varied. In the limit $\lambda \rightarrow 0, \kappa \rightarrow 0$ and constant μ , all singlet fields decouple from the remaining field spectrum (see sect. 7.2 for a deduction of the MSSM limit). Increasing the value of λ directly translates to increasing the influence of genuine NMSSM effects. A detailed study of the one-loop result and the quality of approximations based on partial contributions will be presented here. In our discussion below we will just focus on the masses of the two lighter CP -even states: the effects discussed here turn out to be very small for the heaviest CP -even state, amounting to less than 1% for the sample scenario.

4.4.1 Tree-level masses and mixing

Some aspects of the introduction of a singlet in the Higgs sector of the model can be numerically explored even at lowest order. We performed this leading contributions analysis (and the one in the next section) for the following set of values (if not stated otherwise in the plots) for the relevant parameters:

canonical scenario							(4.98)
M_{SUSY}	$M_{H^\pm}$	$\tan\beta$	μ	A_κ	λ	κ/λ	
1000 GeV	250 GeV	5	250 GeV	−100 GeV	0.2	0.4	

This scenario does not follow from phenomenological reasons (for that we refer to the “sample scenario” in sects. 4.4.3 and 4.4.4) but just to allow us the analysis of some features of the different approximations described in the previous sections. We begin by showing in fig. 4.2 the singlet component $\langle\phi_s|h_i\rangle = (U_{e(0)})_{i3}$ (see eq. (4.14)) of the three

CP -even physical fields at tree-level, for several values of $M_{H^\pm}$ and $\tan\beta$. A cross-over behaviour is observed for $150\text{ GeV} \lesssim M_{H^\pm} \lesssim 300\text{ GeV}$, while for other values one of the three fields is dominantly singlet-like.

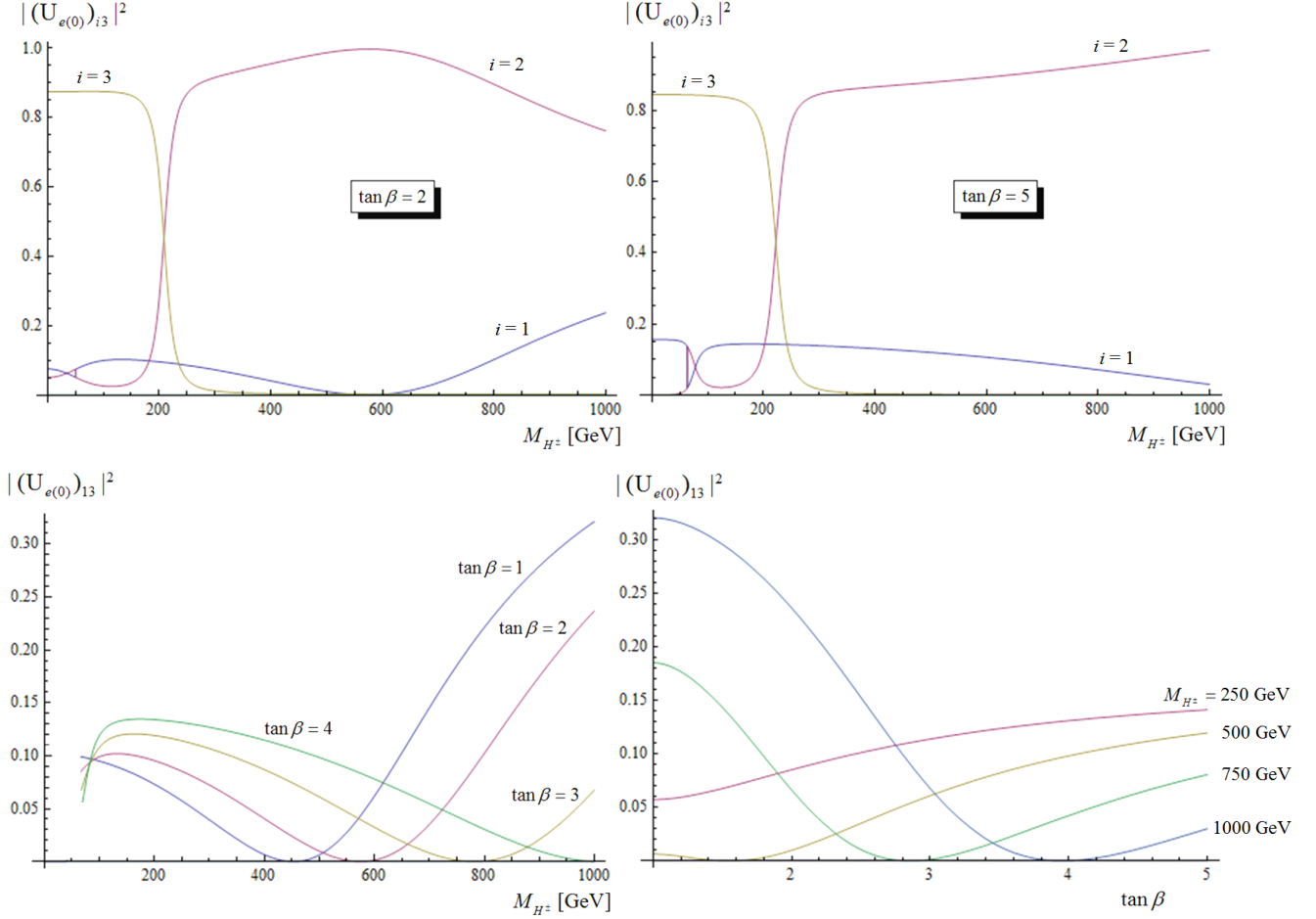


Figure 4.2: Singlet component of the CP -even physical fields at tree-level, in the scenario (4.98) and for several values of $M_{H^\pm}$ and $\tan\beta$. The singlet contribution only to the lightest CP -even Higgs field h_1 is exemplified in the bottom row.

The two lightest CP -even Higgs masses at tree-level are shown in fig. 4.3, for different values of $M_{H^\pm}$, $\tan\beta$, λ and $K (= \kappa/\lambda)$. We observe for $\lambda \approx 0$ that the lightest Higgs is always of MSSM-type for the cases considered here. On the other hand, the behaviour of the lightest Higgs mass w.r.t. λ it's seen to strongly depend on the specific values of $\tan\beta$ and $M_{H^\pm}$: while M_{h_1} decreases with λ in most cases in the six plots, an increasing behaviour is observed for the specific case of $M_{H^\pm} \approx 600\text{ GeV}$ and $\tan\beta \approx 2$.

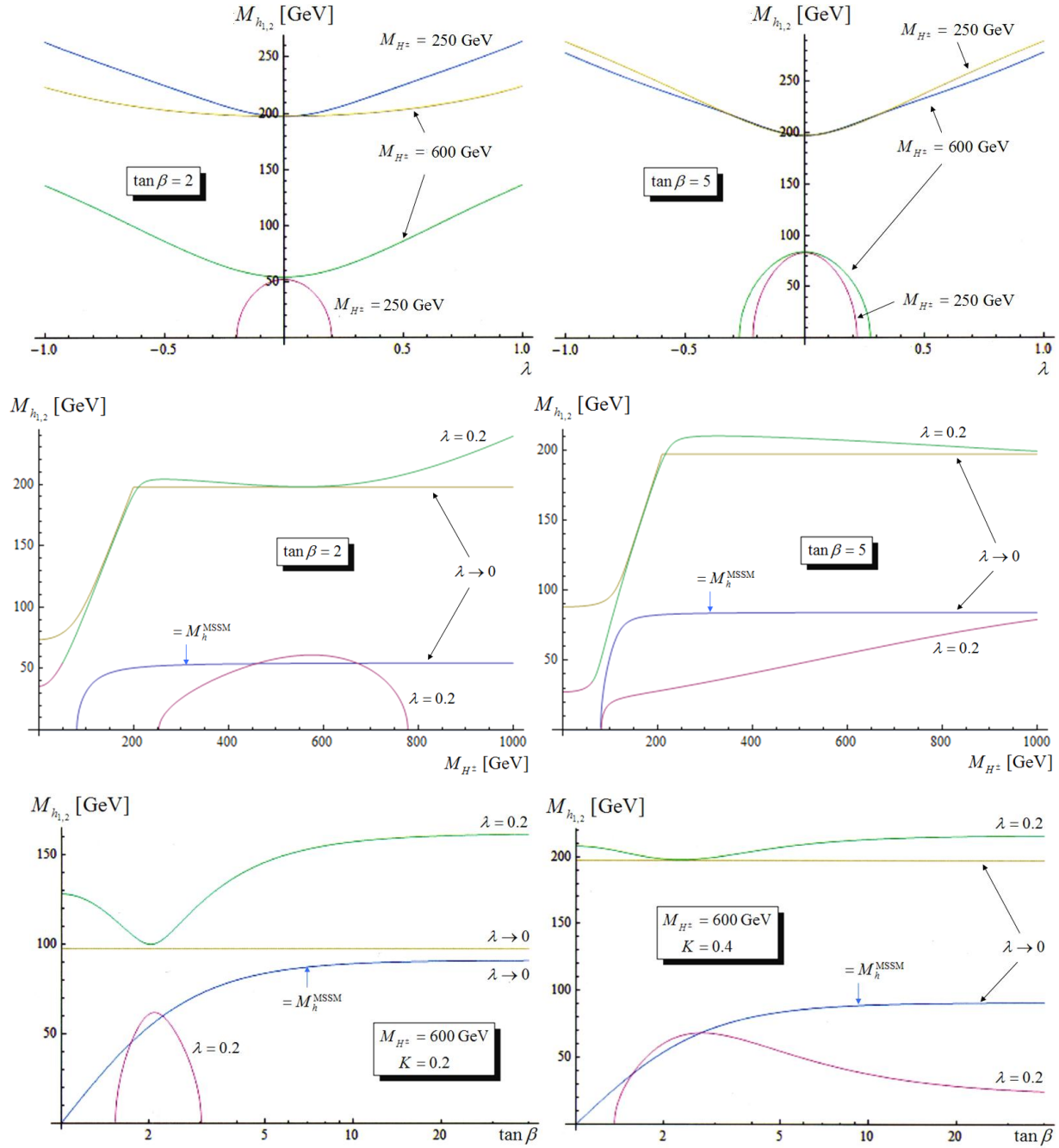


Figure 4.3: Lightest and second lightest CP -even Higgs masses in the scenario (4.98) at tree-level ($M_{h_i} = m_{h_i}$), varying the MSSM-like parameters $M_{H^\pm}$ and $\tan\beta$ for different values of the genuine-NMSSM parameters λ and $K (= \kappa/\lambda)$.

4.4.2 Leading contributions at one-loop order

In figs. 4.4 and 4.5 we compare the $O(m_t^4)$ result w.r.t. the (s)fermion approximation for the neutral Higgs fields renormalised self-energies, both in function of (real values of) the external momentum and with respect to X_t , for the same “canonical scenario” of the previous section. Only the self-energies that introduce sizeable corrections ($\gtrsim 3\%$) to the corresponding tree-level mass entries in the inverse propagator matrix, at least in some region of the considered ranges ($-4000 \text{ GeV} \leq X_t \leq 4000 \text{ GeV}$, $0 \leq \sqrt{p^2} \leq 1000 \text{ GeV}$), are shown.

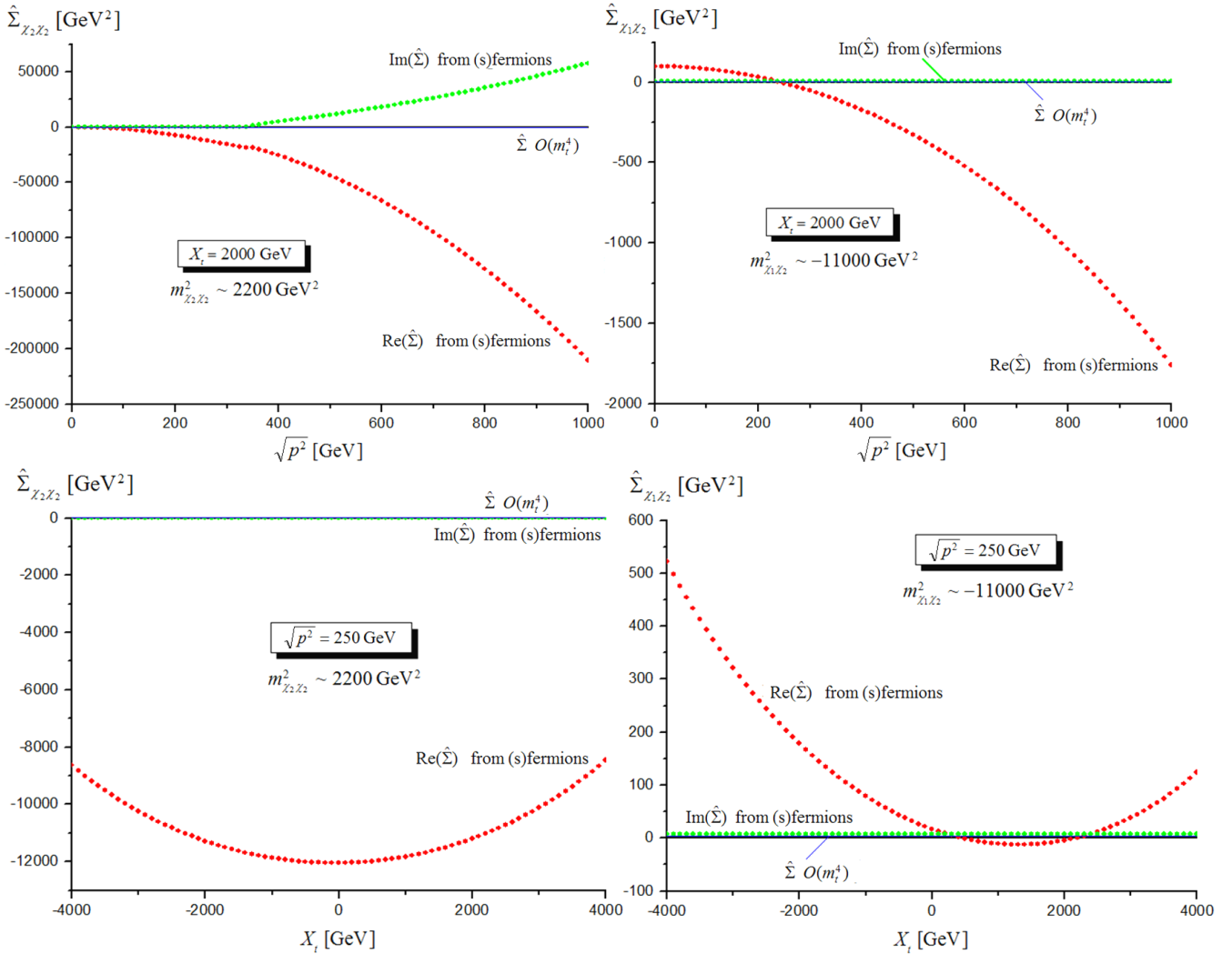


Figure 4.4: Comparison of CP -odd Higgs renormalised self-energies at $O(m_t^4)$ against the (s)fermion-approximation result, in the scenario (4.98), for different values of the external momentum and the stop mixing parameter X_t . The values of the corresponding tree-level mass entries in the inverse propagator matrix are indicated.

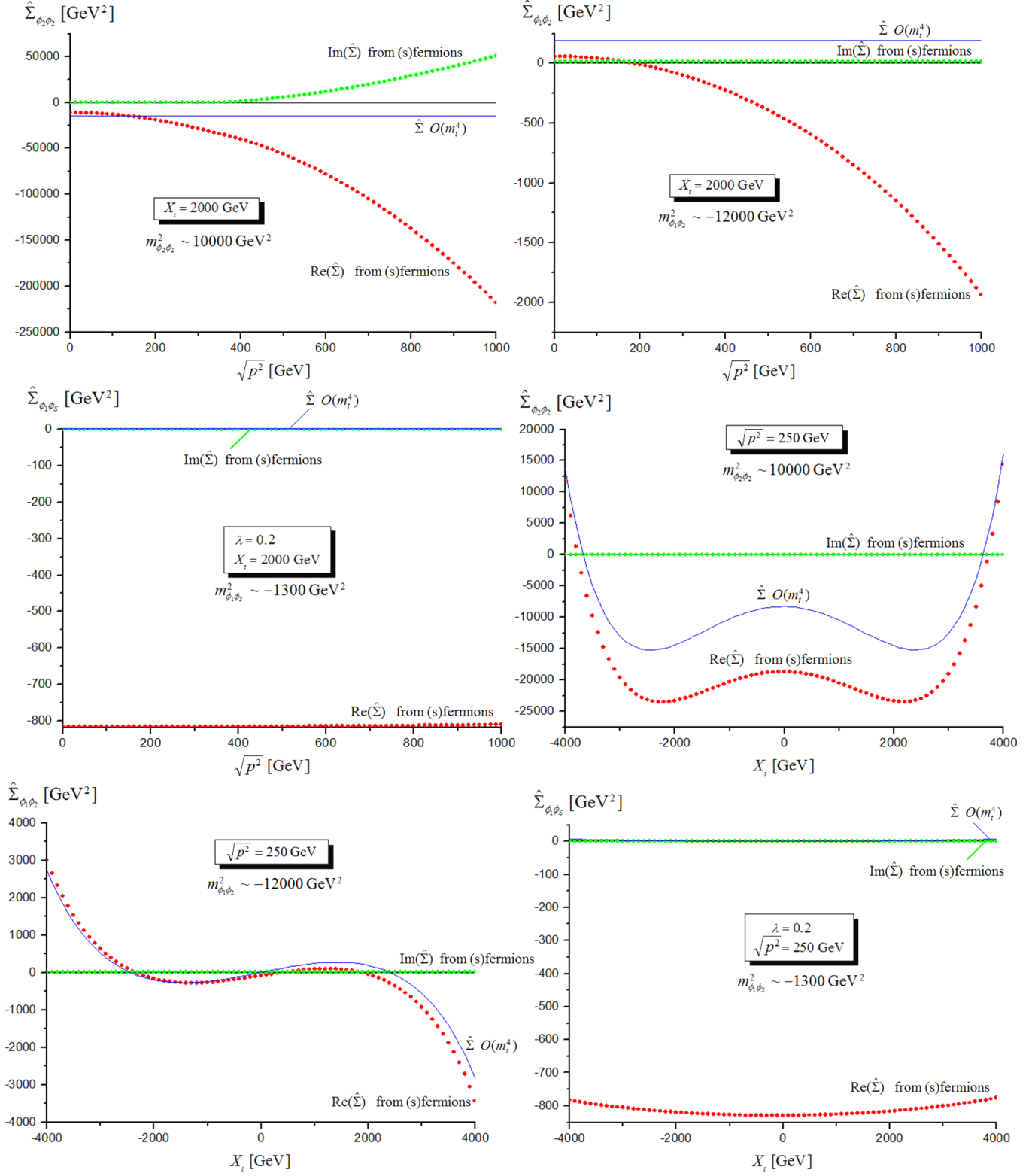


Figure 4.5: Comparison of CP -even Higgs renormalised self-energies at $O(m_t^4)$ against the (s)fermion-approximation result, in the scenario (4.98), for different values of the external momentum and the stop mixing parameter X_t . The values of the corresponding tree-level mass entries in the inverse propagator matrix are indicated.

In figs. 4.6-4.9, we compare the tree-level and the $O(m_t^4)$ results w.r.t. the (s)fermion approximation for the two lightest CP -even Higgs boson masses, for different values of the MSSM-like parameters $\tan\beta, X_t$ and the genuine-NMSSM parameters λ, A_κ . Also a comparison between the MSSM result (recovered in the limit of $\lambda \rightarrow 0$ by keeping $\kappa/\lambda = \text{constant}$)⁶³ w.r.t. genuine NMSSM results ($\lambda \neq 0$) can be observed.

The cases for $\lambda \rightarrow 0$ correspond to the MSSM-limit (because κ/λ is fixed), hence it reproduce the values of the CP -even MSSM Higgs bosons $M_{h,H}$ for the same scenario, as it is indicated in the figure (only the curves for M_h are shown). Moreover, as expected for the (s)fermion approximation, $\lambda \rightarrow 0$ not only reproduces the MSSM result, which means that the singlet sector decouples from the other sectors of the theory, but also makes that the singlet states do not acquire radiative corrections.⁶⁴

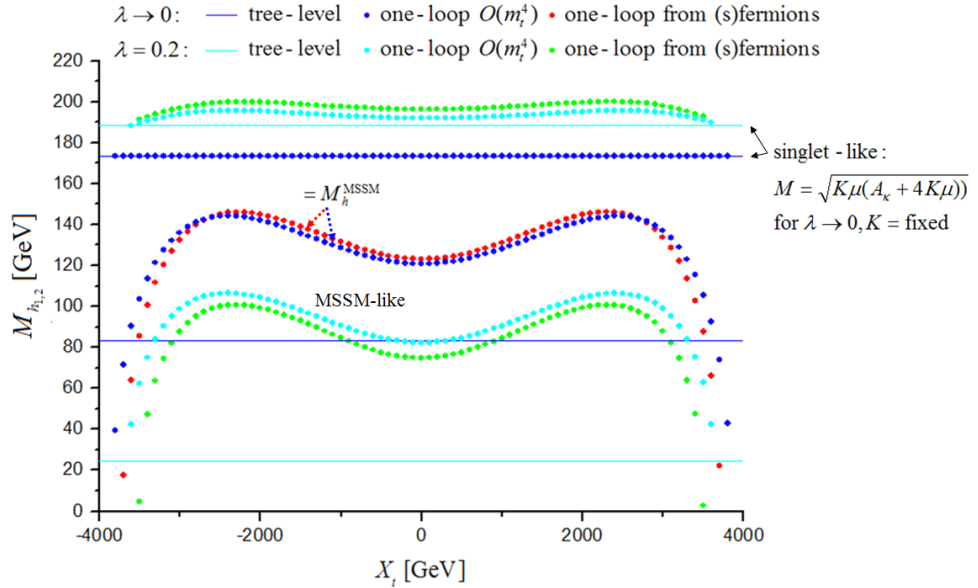


Figure 4.6: Comparison between the tree-level result, the one-loop $O(m_t^4)$ approximation and the one-loop (s)fermion contributions for the two lightest CP -even Higgs boson masses, in the scenario (4.98) and different values of X_t . The MSSM-limit and a genuine-NMSSM result are shown from two different values of λ .

⁶³ The details of the analytic derivation of the conditions for this limit will be discussed in the next chapter, see sect. 5.2.

⁶⁴ In the decoupling limit, this radiative corrections to singlet-like quantities could arise only from virtual contributions due to singlet or singlino fields, which is not allowed (not considered) in the (s)fermion approximation.

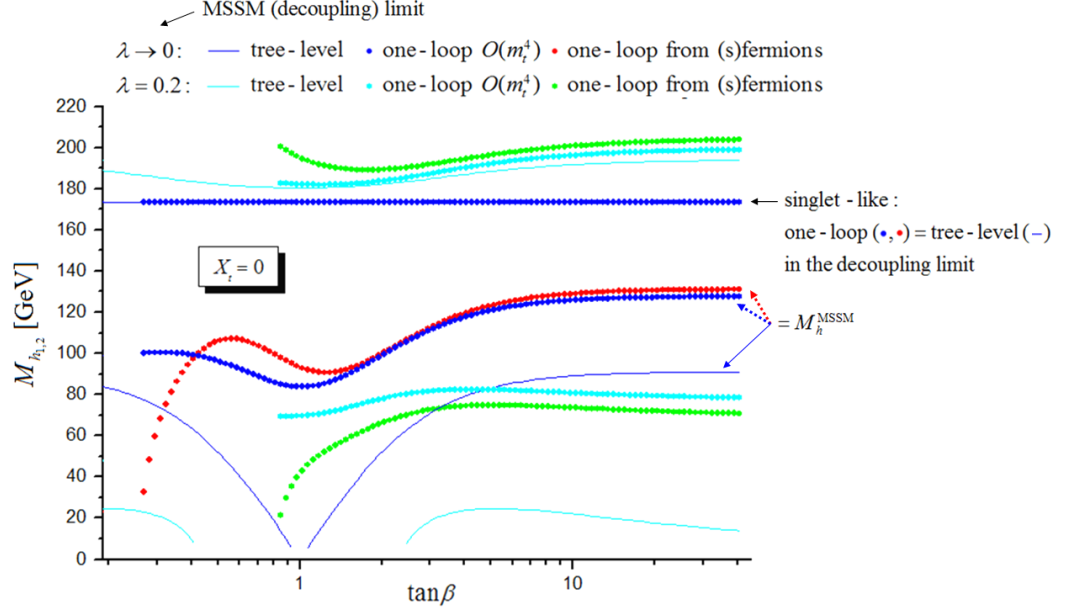


Figure 4.7: Comparison between the tree-level result, the one-loop $O(m_t^4)$ approximation and the one-loop (s)fermion contributions for the two lightest CP -even Higgs boson masses, in the scenario (4.98) and different values of $\tan\beta$. The MSSM-limit and a genuine-NMSSM result are shown from two different values of λ .

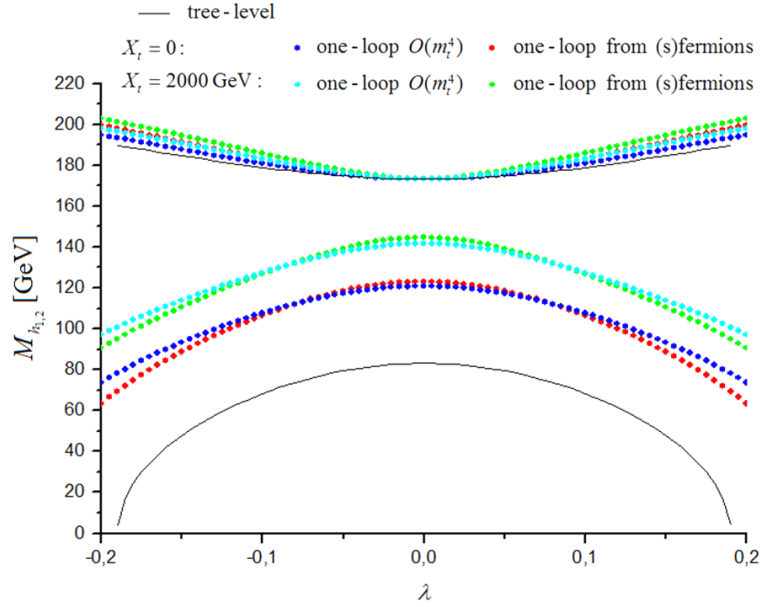


Figure 4.8: Comparison between the tree-level result, the one-loop $O(m_t^4)$ approximation and the one-loop (s)fermion contributions for the two lightest CP -even Higgs boson masses, in the scenario (4.98) and varying λ for two different mixing condition in the stop sector.

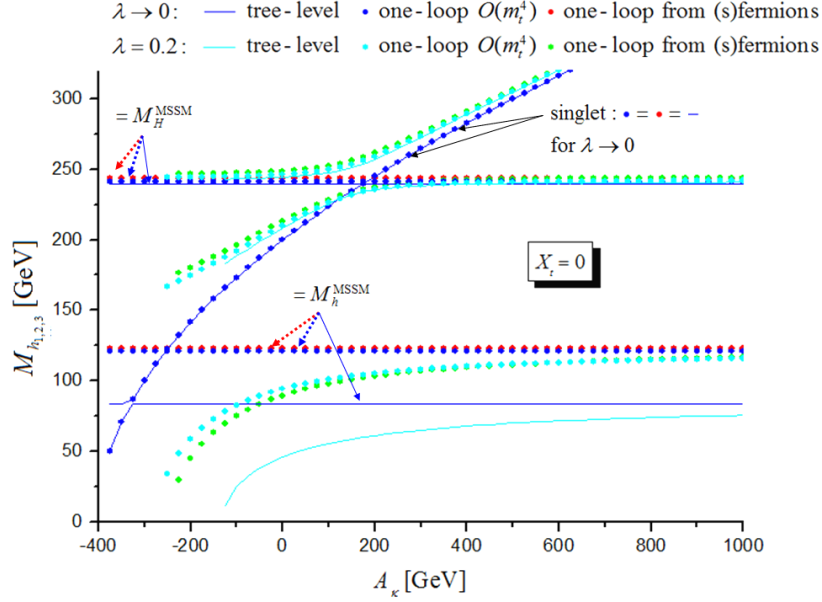


Figure 4.9: Comparison between the tree-level result, the one-loop $O(m_t^4)$ approximation and the one-loop (s)fermion contributions for the three CP -even Higgs boson masses, in the scenario (4.98) while varying the genuine-NMSSM parameter A_κ . The MSSM-limit and a genuine-NMSSM result are shown from two different values of λ .

Negligible variations were observed for the two CP -odd Higgs masses w.r.t. X_t and λ , while in the case of varying A_κ we get the behaviour shown in fig. 4.10. It can be seen that some λ -dependency is observed only around the cross-over point, while for the particular case of $\lambda = 0$ one of the two CP -odd masses is always equal to the MSSM value, M_A (which is constant w.r.t. A_κ).

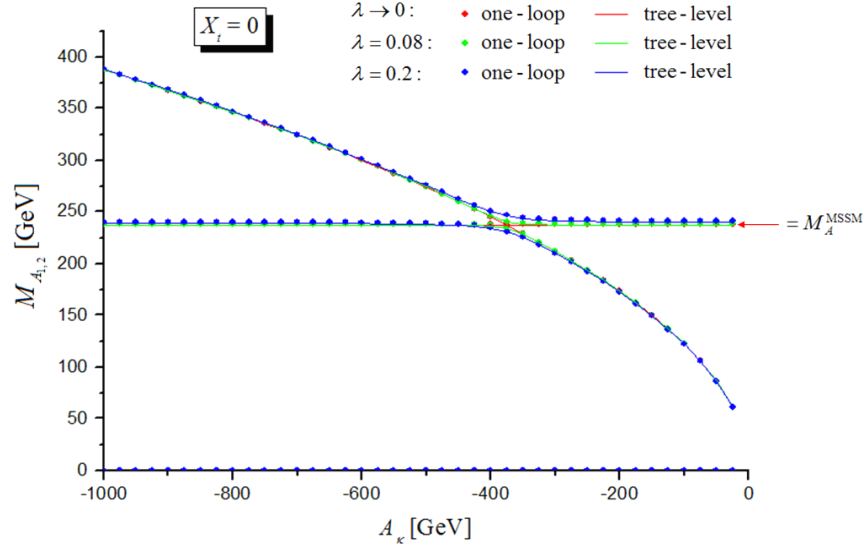


Figure 4.10: Comparison between the results at tree-level and one-loop order at $O(m_t^4)$ for the two CP -odd Higgs boson masses, in the scenario (4.98), varying the genuine-NMSSM parameter A_κ . Three different values of λ are exemplified.

4.4.3 A genuine NMSSM scenario

In our study for the rest of this chapter we will discuss a phenomenological well-motivated “sample scenario”, defined by the parameters given in tab. 4.3, while λ is varied. It is well suited to study the magnitude of the NMSSM-contributions and the behaviour of the employed approximation.⁶⁵ The choice for the top-quark mass in the loop contributions will be $m_t^{\overline{\text{MS}}}(m_t)$. Using the $\overline{\text{MS}}$ top-quark mass allows us to include the resummation of leading and next-to-leading logarithms implemented in FEYNHIGGS. The Renormalisation scale for the studies in this chapter will be fixed at the used value of the top-quark mass.

Higgs sector parameters					heavy fermion masses			
$M_{H^\pm}$	$\tan\beta$	μ	A_κ	κ	m_t^{OS}	$m_t^{\overline{\text{MS}}}(m_t)$	$m_b^{\overline{\text{MS}}}(m_b)$	m_τ
1000	8	125	−300	0.2	173.2	167.48	4.2	1.78

sfermion and gaugino parameters									
$M_{\tilde{Q}_g}^2, M_{(\tilde{u}_g)_R}^2, M_{(\tilde{d}_g)_R}^2$	$M_{\tilde{L}_g}^2, M_{(\tilde{e}_g)_R}^2$	A_t	A_{τ,u,s,d_g}	$A_{e,\mu}$	$M_1^{(\text{GUT})}$	M_2	M_3		
1500	200	−2000	−1500	−100	≈ 143	300	1500		

$M_{\tilde{Q}_g}^2, M_{(\tilde{u}_g)_R}^2, M_{(\tilde{d}_g)_R}^2$	squark mass breaking parameters
$M_{\tilde{L}_g}^2, M_{(\tilde{e}_g)_R}^2$	slepton mass breaking parameters
A_{u_g}, A_{d_g}	squark trilinear breaking parameters
A_{e_g}	slepton trilinear breaking parameters
$M_{1,2,3}$	gaugino mass breaking parameters

Table 4.3: Definition of the “sample scenario”. All dimensionful parameters are given in GeV. All $\overline{\text{DR}}$ -parameters are defined at $m_t^{\overline{\text{MS}}}(m_t)$. All stop-parameters are on-shell parameters. As indicated by the superscript “(GUT)”, M_1 is related to M_2 by the usual GUT relation, namely $M_1^{(\text{GUT})} = (5/3)t_w^2 M_2$.

⁶⁵ Two additional benchmark scenarios “P1” and “P9”, defined as in [136] but where the parameter λ is varied, were also analysed in our article [2]. These scenarios were motivated by the diphoton excess events at an invariant mass of about 750 GeV that was initially observed by data from both ATLAS [137] and CMS [138]. With the release of additional Higgs data by the ATLAS and CMS Collaborations [139], the final data obtained in the LHC Run I does not show however a significant enhancement over the SM prediction in the measured rate for the $h \rightarrow \gamma\gamma$ channel anymore, so this “heavy Higgs” scenario is actually much more restricted due to light charged Higgs boson searches. It was shown in [2] that the qualitative features of the scenarios P1 and P9 can nevertheless be understood from the discussion of the sample scenario.

For $\lambda \gtrsim 0.32$, the mass of the lightest Higgs state becomes tachyonic⁶⁶ at tree-level for this scenario, and therefore the analyses will be restricted to values of λ up to 0.32. The parameters $M_{\tilde{f}}$ and $A_{\tilde{f}}$ specify the universal sfermion mass and trilinear breaking-parameters, while $M_{\{1,2,3\}}$ are the gaugino mass breaking-parameters for $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$. All stop sector parameters are understood as on-shell parameters [16].

The viability of the discussed scenario was tested with the (then current) full set of experimental data implemented in the tool HIGGSBOUNDS 4.1.3 [133,134], for which in order to obtain the necessary input we made use of NMSSMTOOLS 4.4.0 [19]. While our calculation assumes an on-shell renormalised stop sector as in [15], the SLHA input file for NMSSMTOOLS needs $\overline{\text{DR}}$ -parameters for the stop sector. Thus a conversion from the on-shell into the $\overline{\text{DR}}$ scheme is necessary for the parameters of the sample scenario given in tab. 4.3. We only accounted for the dominant effect of these conversions that occurs for $X_t = A_t - \mu/\tan\beta$ by applying the on-shell to $\overline{\text{DR}}$ conversion outlined in. We found that the scenario is in agreement with the experimental limits implemented in HIGGSBOUNDS.

For the prediction in the MSSM, the top/stop sector contributions are numerically leading. In the studied scenario, given in tab. 4.3, the genuine NMSSM-corrections are suppressed w.r.t. the corresponding MSSM-like stop-corrections since $\lambda \lesssim \lambda_{\text{max}} < Y_t$, where $\lambda_{\text{max}} = 0.32$ in our scenario. Thus, the genuine NMSSM corrections from this sector are expected to be sub-leading.

In order to study the impact of the genuine NMSSM contributions, the masses of the two lighter CP -even Higgs fields are studied at one-loop order for the dominant contributions from top and stops. We compare the approximation based on the leading MSSM-type one-loop corrections in the gauge-less limit of $O(Y_t^2)$, labelled as “ $t/\tilde{t}$ -MSSM” in fig. 4.11, with the one where the genuine NMSSM corrections of $O(\lambda Y_t, \lambda^2)$ are incorporated.

⁶⁶ “Tachyonic” refers to negative square masses.

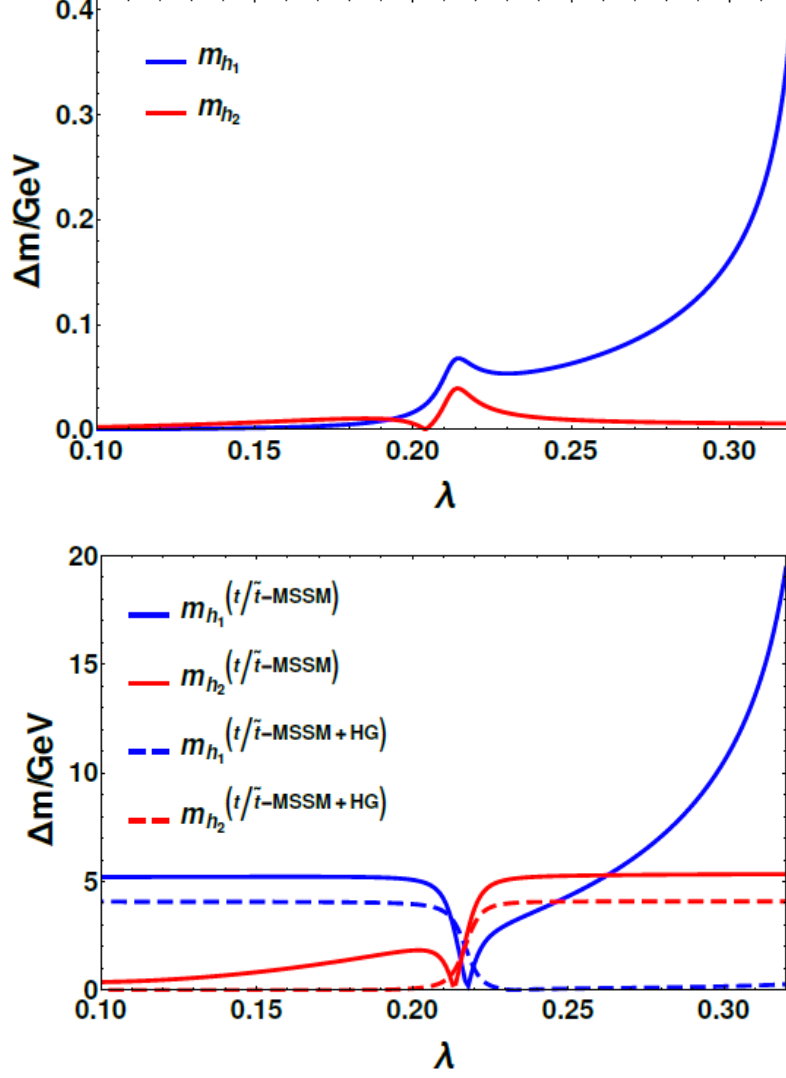


Figure 4.11: Absolute difference between partial and full results for the one-loop masses of the two lighter CP -even fields in the sample scenario. *Up*: Absolute difference between the mass predictions including and excluding the genuine NMSSM contributions from the stops of $O(\lambda Y_t, \lambda^2)$ for m_{h_1} and m_{h_2} . *Down*: Absolute differences between the mass predictions based on two different one-loop approximations and the full one-loop result. The solid lines, labelled as “ $t/\tilde{t}$ -MSSM”, depict the difference between the full result and the approximation based on the leading MSSM-type contributions from the top/stop sector, $\Delta m_{h_i} = |m_{h_i}^{(1L)} - m_{h_i}^{t/\tilde{t}\text{-MSSM}}|$. The dashed lines, labelled as “ $t/\tilde{t}$ -MSSM+HG”, show the corresponding result where the leading MSSM-type contributions from the top/stop sector are supplemented by the contributions from the Higgs-/higgsino- and gauge-/gaugino-sectors, $\Delta m_{h_i} = |m_{h_i}^{(1L)} - m_{h_i}^{t/\tilde{t}\text{-MSSM}+\text{HG}}|$.

The sharp increase of the corrections of $O(\lambda Y_t, \lambda^2)$ for the highest values of λ that is visible for the light singlet-like field in the up plot of fig. 4.11 indicates that the approximation for the stop sector of restricting to the MSSM-type contributions becomes

questionable for the singlet-like state in this region. However, as shown in the down plot of fig. 4.11, in this parameter region the stop sector as a whole ceases to provide a reliable approximation of the full one-loop contributions. In that plot the difference between the full one-loop mass prediction and the approximation based on including only the leading MSSM-type contributions of $O(Y_t^2, \lambda Y_t, \lambda^2)$ from the top/stop sector, $\Delta m_{h_i} = |m_{h_i}^{(1L)} - m_{h_i}^{t/\tilde{t}\text{-MSSM}}|$, is shown together with $\Delta m_{h_i} = |m_{h_i}^{(1L)} - m_{h_i}^{t/\tilde{t}\text{-MSSM+HG}}|$, where in the latter case the leading MSSM-type contributions from the top/stop sector are supplemented by the contribution from the Higgs/higgsino and gauge/gaugino sectors. While for the singlet-like state the deviation between the leading contributions from the top/stop sector and the full one-loop result becomes huge for the largest values of λ , reaching the level of 20 GeV, the deviations stay small, far below the level of 1 GeV, if the leading contributions from the top/stop sector are supplemented by the contributions from the Higgs-/higgsino- and gauge-/gaugino-sectors. This result for the singlet-like state can be understood from the fact that the gauge couplings of the singlet-like state are heavily suppressed and that therefore the leading contributions for large λ arise from the Higgs and higgsino sector. Thus, improving on the approximation of MSSM-type contributions in the stop sector requires the incorporation of the contributions from the Higgs and higgsino sector, while the genuine NMSSM contributions in the stop sector are of minor significance in this context.

For the doublet-like state, namely h_1 for values $\lambda \lesssim \lambda_c$ and h_2 for $\lambda \gtrsim \lambda_c$, the difference between the full one-loop result and the result based on the leading contributions from the top/stop sector amounts to a shift of about 5 GeV that is essentially independent of λ except for the region where the cross-over behaviour occurs. This nearly constant shift arises mainly from sub-leading contributions in the top/stop sector. As indicated by the dashed lines, the inclusion of the contributions from the Higgs and the gauge sector reduces the difference to the full one-loop result by about 1 GeV.

As a result of the comparison performed in this section, the MSSM-type top/stop sector contributions of $O(Y_t^2)$ have been verified as the leading one-loop contributions to MSSM-like fields. The genuine NMSSM top/stop sector contributions of $O(\lambda Y_t, \lambda^2)$ have the largest impact on singlet-like fields for large values of λ , where however an approximation based only on contributions from the fermion/sfermion sector is in any case insufficient. Our analysis at the one-loop level therefore shows that approximating the result for the top/stop sector by the leading MSSM-type contributions turns out to work well in the parameter regions where the top/stop sector itself yields a reasonable approximation of the full result.

4.4.4 Full results at higher-order

The full results for the tree-level, one- and two-loop Higgs-mass predictions in the discussed scenario defined in tabs. 4.3 are shown as a function of λ in fig. 4.12, for the two lighter CP -even physical fields. The term “full result” refers to all one-loop corrections in the NMSSM (including the full momentum dependence and also the reparametrisation of the electromagnetic coupling in terms of the Fermi constant) supplemented with all available contributions of $O(\alpha_t\alpha_s, \alpha_b\alpha_s, \alpha_t^2, \alpha_t\alpha_b)$ from the MSSM, and including the resummation of large logarithms.

The results for the masses of the two lighter Higgs fields in the sample scenario defined in tab. 4.3 are shown in fig. 4.12. For this study the parameter λ is varied between 0.1 and 0.32. The lower limit on the parameter λ has been chosen such that in the considered parameter region a cross-over type behaviour occurs only for the two smaller masses, m_{h_1} and m_{h_2} (for values $\lambda < 0.1$ there is another point with cross-over behaviour of the two larger Higgs-boson masses; however, because of the small values of λ this region is less suitable for studying the behaviour of the genuine NMSSM-corrections, which scale with λ).

The variation of the two masses with λ in fig. 4.12 clearly shows a cross-over type behaviour between the masses, which is correlated to their mixing character w.r.t. the singlet field and the doublet fields. For small values of λ the field h_1 is doublet-like in this scenario and, based on the prediction incorporating all available higher-order corrections, can be identified with the signal that was detected at the LHC at about 125 GeV. The prediction for m_{h_1} varies only very little with λ in this region. The field h_2 , on the other hand, is predominantly singlet-like in this parameter region, and its mass prediction falls steeply with increasing λ . The cross-over occurs at $\lambda_c^{(0)} \approx 0.26$ at tree-level, at $\lambda_c^{(1)} \approx 0.22$ at one-loop order, and at $\lambda_c^{(2)} \approx 0.23$ at two-loop order. Above the cross-over point, the behaviour of the two masses and the admixture of the fields h_1 and h_2 in terms of singlet and doublet fields are reversed. The two fields are evenly mixed between singlet- (i.e. genuine NMSSM-type) and doublet-field (i.e. MSSM-type) components for $\lambda_c^{(n)}$, with $n = 0, 1, 2$. The heaviest CP -even Higgs field, h_3 , is doublet-like in the depicted interval of λ , with a mass of $\approx 1\text{TeV}$. As in the MSSM, the larger masses (of doublet-like fields) are affected by higher-order corrections to a lesser extent than the lighter states. Since at $\lambda_c^{(n)}$ the MSSM-type and genuine NMSSM-type contributions enter at equal footing, the SM-like state is most sensitive to genuine NMSSM-type contributions in the region of the cross-over behaviour.

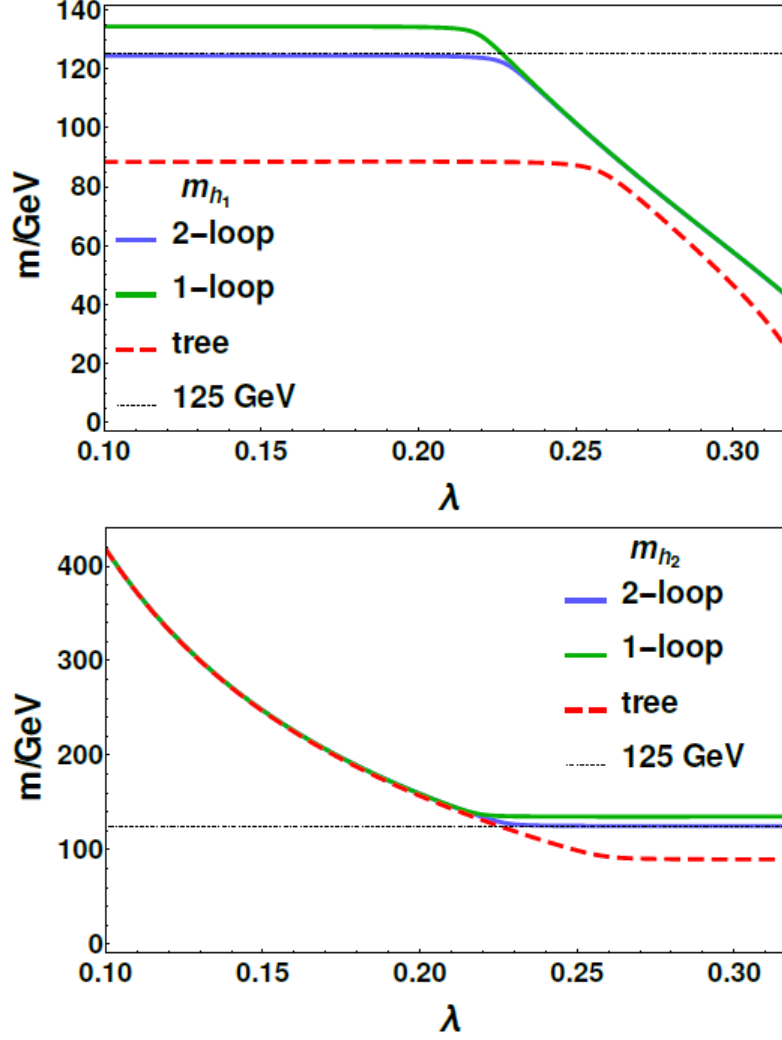


Figure 4.12: Mass of the lightest and next-to lightest CP -even Higgs-states, m_{h_1} (up) and m_{h_2} (down), at tree-level, one-loop and two-loop order for the sample scenario. At one-loop order all corrections of the NMSSM are included with their momentum-dependence. The two-loop corrections are approximated by the MSSM-type contributions of $O(\alpha_t \alpha_s, \alpha_b \alpha_s, \alpha_t^2, \alpha_t \alpha_b)$, including the resummation of the leading and next-to-leading logarithms, as implemented in [2]. The dotted line represents 125 GeV. The λ values for which a cross-over behaviour between the masses occurs are at the tree-level $\lambda_c^{(0)} \approx 0.26$, at one-loop order $\lambda_c^{(1)} \approx 0.22$ and at two-loop order $\lambda_c^{(2)} \approx 0.23$.

NMSSM Higgs with alternative renormalisation⁶⁷

In contrast to the MSSM, the renormalisation constant of the electric charge, δZ_e , enters already at one-loop level in the NMSSM. It appears artificially due to terms involving the vev's of the doublet Higgs fields, $v_{1,2}$, and their respective renormalisation constants in the form of (see eq. (4.10))

$$\begin{aligned} (M_{\phi\phi}^2)_{13} \supset -\kappa\mu s_\beta &\Rightarrow \delta(M_{\phi\phi}^2)_{13} \supset -\kappa\mu s_\beta \left(\frac{\delta\kappa}{\kappa} + \frac{\delta\mu}{\mu} + \frac{\delta v}{v} + \frac{\delta(s_\beta)}{s_\beta} \right) , \\ (M_{\phi\phi}^2)_{23} \supset -\kappa\mu c_\beta &\Rightarrow \delta(M_{\phi\phi}^2)_{23} \supset -\kappa\mu c_\beta \left(\frac{\delta\kappa}{\kappa} + \frac{\delta\mu}{\mu} + \frac{\delta v}{v} + \frac{\delta(c_\beta)}{c_\beta} \right) , \end{aligned} \quad (5.1)$$

where v is renormalised according to eq. (4.32):

$$v = \sqrt{v_1^2 + v_2^2} = \frac{\sqrt{2}s_w M_w}{e} \rightarrow \delta v = v \left(\frac{1}{2} \frac{\delta M_w^2}{M_w^2} + \frac{1}{2} \frac{\delta s_w^2}{s_w^2} - \delta Z_e \right) . \quad (5.2)$$

This gives rise to a κ dependency of all contributions to the self-energies, even with κ, μ, t_β renormalised as $\overline{\text{DR}}$ quantities, because of the finite part of δv . As a result, although the unrenormalised self-energy contributions from the fermions and sfermions are κ -independent, the renormalised self-energy (and therefore the masses) gains a dependency through the counterterms,

$$\frac{\partial}{\partial \kappa} \Sigma_{\phi\phi}^{f/\tilde{f}}(p^2) = 0 \quad , \text{ but } \quad \frac{\partial}{\partial \kappa} \hat{\Sigma}_{\phi\phi}^{f/\tilde{f}}(p^2) \neq 0 . \quad (5.3)$$

This behaviour is rather unphysical, in the sense that the Higgs-fermion and Higgs-sfermion couplings are all independent of κ . This is why in sect. 4 we chose to fix the renormalisation constant δv in a $\overline{\text{DR}}$ scheme, instead of imposing the usual Thomson limit prescription for the renormalisation condition on the electric charge in the Higgs sector.

⁶⁷ Our results from this chapter are not (yet) published in an article.

5.1 Some remarks on the renormalisation of ν and e

The reason for the presence of the δZ_e contributions to the Higgs self-energies is that, unlike for the MSSM, in the NMSSM Higgs bosons mass matrixes the electric charge e occurs not only in front of the tadpoles. However, once defining $\mu = \lambda \nu_s$ and $\bar{T}_\phi = T_\phi / \sqrt{2}\nu$, one can see that in the linear and bilinear terms in the Lagrangian the parameters λ, ν, κ only take part in the following two combinations:

$$\tilde{\lambda} \equiv \lambda \nu \quad , \quad K \equiv \frac{\kappa}{\lambda} \quad (5.4)$$

Thus, the problematic e -dependency can be eliminated by choosing to renormalise the (bi)linear Higgs sector using $\tilde{\lambda}$ and K as input parameters, instead of λ, κ, ν or λ, κ, e .⁶⁸ Then choice provides hence the following advantages:

- About choosing to work with $\tilde{\lambda}$:

We can avoid to worry about the specific renormalisation condition on the electric charge parameter (encode in the renormalisation condition on ν), because of the freedom on the choice of the finite part of $\delta \mathcal{L}$. The parameter ν can be written as a function of M_W^2, M_Z^2, e :

$$\nu = \frac{\sqrt{2M_W^2 / M_Z^2} \sqrt{M_Z^2 - M_W^2}}{e} \quad (5.5)$$

If we consider M_W^2, M_Z^2 and e as on-shell quantities, this fix the finite part of $\delta \nu$, which in turn is the only place where δZ_e takes part in the renormalisation of the (bi)linear Higgs sector. If we can choose to renormalise $\tilde{\lambda}$ as a $\overline{\text{DR}}$ quantity, this means that the finite part of $\delta \mathcal{L} / \lambda$ is chosen so as to cancel the finite part of $\delta \nu / \nu$:

$$\frac{\delta \tilde{\lambda}}{\tilde{\lambda}} \Big|_{\text{fin}} \stackrel{!}{=} 0 \quad \Rightarrow \quad \frac{\delta \mathcal{L}}{\lambda} \Big|_{\text{fin}} = - \frac{\delta \nu}{\nu} \Big|_{\text{fin}} \quad (5.6)$$

In this way the finite part of δZ_e is “absorbed” or “hidden”, in the sense that no explicit evaluation of $\delta \nu$ is needed, even though an on-shell condition on e is considered.

⁶⁸ We are NOT saying that the theory actually has an independent parameter less. The change of parameters $\lambda, \kappa, \nu \rightarrow \tilde{\lambda}, K$ or $\lambda, \kappa, e \rightarrow \tilde{\lambda}, K$ is possible for the computation of 2-point functions only. See sect. 5.1.2.

- About choosing to work with K :

The MSSM-limit requires that we set $\lambda \rightarrow 0$ and also $\kappa \rightarrow 0$, simultaneously, keeping $\lambda/\kappa = \text{constant}$. Therefore, the use of K is more natural and convenient: if we choose to write all our expressions in terms of K instead of κ , the MSSM-limit prescription becomes the simplest one: $\lambda \rightarrow 0$ (while keeping constant K and all other parameters), and that's it; the $\lambda \rightarrow 0$ case will always correspond to the MSSM result. Furthermore, we don't need to evaluate the finite part of $\delta\tilde{\lambda}$ ($\neq 0$ from imposing $\tilde{\lambda} = \tilde{\lambda}^{\overline{\text{DR}}}$), because it can be absorbed by fixing the finite part of $\delta\kappa$ such as $\lambda\delta\kappa|_{\text{fin}} = \kappa\delta\tilde{\lambda}|_{\text{fin}}$, which is nothing but implementing a $\overline{\text{DR}}$ renormalisation for K as well.

Let us refer to these prescriptions as “ $\tilde{\lambda}/K$ scheme”:

$$\tilde{\lambda}/K \text{ scheme:} \tag{5.7}$$

$$\begin{aligned} \delta(\lambda\nu) &\equiv \delta\tilde{\lambda} = \delta\tilde{\lambda}^{\overline{\text{DR}}} \\ \delta(\kappa/\lambda) &\equiv \delta K = \delta K^{\overline{\text{DR}}} \\ \delta\tilde{\lambda}|_{\text{fin}}, \delta\kappa|_{\text{fin}}, \delta\nu|_{\text{fin}} &\neq 0, \text{ in such a way that } \delta Z_e = \delta Z_e^{\text{on-shell}} \end{aligned}$$

As we pointed out before, an “unfortunate” choice of the renormalisation scheme could lead to the artificial possibility that the renormalised self-energies for the Higgs bosons do depend on the value of the parameter κ (and also A_κ) even in the (s)fermions-contribution approximation, where all the relevant coupling constants are κ -independent. Specifically, with $\tan\beta$, μ and A_κ as $\overline{\text{DR}}$ quantities, the way in which the κ -dependence could arise is always in the form

$$\frac{\partial}{\partial \kappa} \hat{\Sigma}_{\phi\phi, \chi\chi}^{f/\tilde{f}} = \dots \delta(\lambda\nu)|_{\text{fin}} + \dots \delta(\kappa/\lambda)|_{\text{fin}} \neq 0 \tag{5.8}$$

where $\dots$ stands for some factors. We ask for the quantity (5.8) to vanish in order to avoid the “unphysical” κ -dependency of these renormalised self-energies. Clearly, choosing $\delta\nu|_{\text{fin}} = 0$ in addition to $\delta\tilde{\lambda}|_{\text{fin}} = \delta\kappa|_{\text{fin}} = 0$ is a way of getting $\partial_\kappa \hat{\Sigma}_{\phi\phi, \chi\chi}^{f/\tilde{f}} = 0$; let's call “ λ/κ scheme” to this option.⁶⁹

$$\lambda/\kappa \text{ scheme:} \tag{5.9}$$

$$\begin{aligned} \delta\tilde{\lambda} &= \delta\tilde{\lambda}^{\overline{\text{DR}}} \\ \delta\kappa &= \delta\kappa^{\overline{\text{DR}}} \\ \delta\nu &= \delta\nu^{\overline{\text{DR}}}, \text{ so } \delta Z_e \neq \delta Z_e^{\text{on-shell}} \end{aligned}$$

⁶⁹ This is the scheme implemented in chapter 4.

But $\partial_{\kappa} \hat{\Sigma}_{\phi\phi,\chi\chi}^{f\bar{f}} = 0$ is also fulfilled within the $\tilde{\lambda}/K$ scheme, since it is enough with imposing $\delta(\lambda\nu)|^{\text{fin}} = \delta(\kappa/\lambda)|^{\text{fin}} = 0$, which in addition allows the possibility of considering δZ_e on-shell (i.e. we can therefore replace in our results the electric charge e by its Thomson-limit value, and there is no need to evaluate δZ_e explicitly!), as it is discussed above.

In summary, within the $\tilde{\lambda}/K$ scheme, simultaneously we are allowed to give the Thomson-limit value to the electric charge e , we do not need to evaluate δZ_e (or any shift for e), and we avoid any κ -dependence (or, equivalently, any K -dependence) for the renormalised self-energies at the (s)fermions-contributions approximation.

5.1.1 Numerical difference between schemes

The parameters λ, κ, ν appear in the Higgs boson mass matrixes only in the combinations $\lambda\nu$ and κ/λ , cf. eq. (5.4). On the other hand, both in the λ/κ -scheme and in the $\tilde{\lambda}/K$ -scheme the quantities $\delta(\lambda\nu)$ and $\delta(\kappa/\lambda)$ are purely divergent. This means that the only numerical difference between the renormalised self-energies from both schemes arises from the value given to e . In this sense, what the $\tilde{\lambda}/K$ -scheme says is that we can set $\delta(\lambda\nu) = \delta(\lambda\nu)^{\text{div}}$ and $\delta(\kappa/\lambda) = \delta(\kappa/\lambda)^{\text{div}}$ (thus avoiding the artificial κ -dependence discussed above) at the same time that we are enabled to assign the experimental value of the electric charge (no shift) to the e parameter.

5.1.2 Renormalised 3- and 4-point amplitudes

For the renormalisation of vertexes, some of the arguments in favour of the $\tilde{\lambda}/K$ scheme fail. By direct inspection of the Higgs potential (4.1) it can be seen that the dependence of the coefficient for the terms in the NMSSM Higgs potential is of the form

$$V_{\text{Higgs}}^{\text{NMSSM}} = \sum_i \overbrace{m_{\varphi_i}^{(3)}}^{-T_{-\varphi_i}} \varphi_i + \sum_{i,j} m_{\varphi_i\varphi_j}^{(2)} \varphi_i \varphi_j + \sum_{i,j,k} \overbrace{m_{\varphi_i\varphi_j\varphi_k}^{(1)}}^{\Gamma_{\varphi_i\varphi_j\varphi_k}} \varphi_i \varphi_j \varphi_k + \sum_{i,j,k,\ell} m_{\varphi_i\varphi_j\varphi_k\varphi_\ell}^{(0)} \varphi_i \varphi_j \varphi_k \varphi_\ell \quad (5.10)$$

$$\text{with } \underbrace{m_{\varphi_i\cdots\varphi_j}^{(4-n)}}_n = \nu^{2-n} \times f_{\varphi_i\cdots\varphi_j}(M_W^2, M_Z^2, t_\beta, \mu, A_\kappa, M_{H^\pm}^2, \bar{T}_{-\phi_{\{1,2,3\}}}, \tilde{\lambda}, K) \quad , \quad n=1, \dots, 4$$

where φ stands for both ϕ and/or χ , and the $f_{\varphi_i\cdots\varphi_j}$ are some functions. We see that the only case where we can avoid the explicit evaluation of the electric charge renormalisation by implementing the variables $\tilde{\lambda}$ and K is for $n=2$ (besides the case for $n=1$, where δZ_e neither occurs because it always appears multiplied for some tadpole $T_{-\phi_i}$, which is set

to zero at the end of the calculation, as it happens for the MSSM). For the 3- and 4-point functions (i.e. $n=3$ and $n=4$), despite the introduction of the variables $\tilde{\lambda}$ and K , an explicit dependence on e (or, equivalently, on ν) still remains. Therefore, unlike $n=2$ case, it is not possible to prevent any κ or A_κ dependence in the (s)fermions–contributions approximation for the renormalised 3- and/or 4-point functions without resigning an on-shell renormalisation for e .

5.2 The MSSM limit

Comparing the NMSSM Higgs potential from (4.1) with the MSSM one from (3.2) with real parameters, we find that

$$\begin{aligned} V_{\text{Higgs}}^{\text{NMSSM}} - V_{\text{Higgs}}^{\text{MSSM}} = & \lambda^2 |\varepsilon_{ab} H_1^a H_2^b|^2 \\ & + (\lambda^2 |S|^2 - \mu^2)(H_1^\dagger \cdot H_1 + H_2^\dagger \cdot H_2) \\ & + 2\text{Re}[(\mu B - \lambda \kappa (S^2)^* - \lambda A_\lambda S) \varepsilon_{ab} H_1^a H_2^b] \\ & + O(\phi_s, \chi_s) \end{aligned} \quad (5.11)$$

where $O(\phi_s, \chi_s)$ are terms that involve only singlet fields. Then, in the MSSM limit it must be

$$\begin{aligned} & \lambda^2 \stackrel{!}{=} 0 \\ & \lambda^2 |S|^2 - \mu^2 \stackrel{!}{=} 0 \\ & \mu B - \lambda \kappa (S^2)^* - \lambda A_\lambda S \stackrel{!}{=} 0 \quad . \end{aligned} \quad (5.12)$$

The first line impose that

$$\lambda \xrightarrow{\text{MSSM}} 0 \quad (5.13)$$

The second line justified our choice (4.3) in order to reproduce the μ -term:

$$\begin{aligned} \lambda^2 |S|^2 - \mu^2 = \lambda^2 (\nu_s^2 + \sqrt{2} \nu_s \phi_s + \tfrac{1}{2} (\phi_s^2 + \chi_s^2)) - \mu^2 & \xrightarrow{\lambda \rightarrow 0} 0 \quad \Rightarrow \quad \nu_s = \pm \frac{\mu}{\lambda} \\ \rightarrow \quad \text{we write } \nu_s = + \frac{\mu}{\lambda} \quad . \end{aligned} \quad (5.14)$$

Finally, from the third line in (5.12) we get the definition of the B parameter for the NMSSM from (4.6) in terms of A_λ , together with the condition on κ :

$$\begin{aligned} \mu B - \lambda \kappa (S^2)^* - \lambda A_\lambda S &\xrightarrow{\text{MSSM}} 0 \quad \Rightarrow \quad \mu B - \lambda \nu_S (\kappa \nu_S + A_\lambda), \lambda \kappa \nu_S, \lambda A_\lambda, \lambda \kappa \xrightarrow{\text{MSSM}} 0 \\ \text{thus } A_\lambda &= B - \frac{\kappa}{\lambda} \mu \\ \text{and } \kappa &\xrightarrow{\text{MSSM}} 0 \quad . \end{aligned} \quad (5.15)$$

In summary, once we express the results from the NMSSM in terms of MSSM-like parameters by implementing the identifications (4.3) and (4.6), the MSSM limit is summarized in the conditions (5.13) and (5.15), which in turn reduce just to the single $\lambda \rightarrow 0$ (or $\tilde{\lambda} \rightarrow 0$) condition in case of choosing to work with the K parameter (instead of κ) introduced in (5.4).

At this point we need to make clear an important observation. When one says, e.g., that “the limit $\lambda \rightarrow 0$ must be taken as a necessary condition for recovering the MSSM”, the immediate question is “ λ defined how?”, that is, “within which scheme for λ ?”. Strictly speaking, the comparison (5.11) has sense if we take into account the respective potentials (or the Lagrangians) written in terms of the *bare* quantities (who are univocally defined). This means that conditions (5.13-5.15) must hold for the bare parameters in order to reproduce the MSSM from the NMSSM in that limit. This implies that MSSM-like quantities such as μ and B must be renormalised in the same way (i.e. imposing the same renormalisation conditions to fix their counterterms)⁷⁰ for both models. In addition, for the renormalised genuine-NMSSM parameters λ and κ the limits $(\lambda + \delta\lambda) \rightarrow 0, (\kappa + \delta\kappa) \rightarrow 0$ must hold. However, in sect. 4.3.4 we explicitly saw that, for our scheme implemented in the that chapter,

$$\delta\lambda \xrightarrow{\lambda \rightarrow 0} 0 \quad , \quad \delta\kappa \xrightarrow{\kappa \rightarrow 0} 0 \quad . \quad (5.16)$$

Therefore, *that scheme* satisfies that the MSSM-limit reduces to the condition $\{\lambda, \kappa\} \rightarrow \{0, 0\}$ for the *renormalised* λ, κ parameters. From an inspection those same explicit results from sect. 4.3.4 and applying to the condition (5.7), a similar result we obtain for the $\tilde{\lambda}/K$ -scheme, where one sees that $\{\delta\lambda, \delta\kappa\} \xrightarrow{\tilde{\lambda} \rightarrow 0} \{0, 0\}$ (keeping $K = \text{const.}$) holds, so that in this case the MSSM-limit condition reduces just to $\tilde{\lambda} \rightarrow 0$ for the *renormalised* $\tilde{\lambda}$ parameter.

⁷⁰ Recall that in our approach of chapter 4 the condition for the B counterterm follows from the renormalization conditions imposed on $\delta M_{H^\pm}^2, \delta M_W^2, \delta\lambda, \delta\beta, \delta T_{\phi_{1,2}}, \delta\mu$, see eqs. (4.12), (4.67). Therefore, the MSSM-limit from the NMSSM requires the same renormalization scheme on $M_{H^\pm}^2, M_W^2, t_\beta, T_{\phi_{1,2}}, \mu$ for both models, and in addition the δB dependence w.r.t. the counterterm $\delta\lambda$ must vanish in that limit.

The specific renormalisation for the μ parameter is not relevant for the MSSM-limit, however, if we consider the Higgs sector at one-loop order only, because $\delta\mu$ does not take part in the one-loop renormalisation of the MSSM Higgs sector.

Note that it is not needed that the quotient κ/λ remain finite in the MSSM limit. This extra conditions is required, however, if we want to keep finite the singlet contributions for $\lambda \rightarrow 0$:

$$V_{\text{Higgs}}^{\text{rNMSSM}} \supset 2v_S^2 \kappa^2 \phi_S^2 = 2 \frac{\kappa^2}{\lambda^2} \mu^2 \phi_S^2 . \quad (5.17)$$

This is not a condition for recovering the MSSM, but in practice it is needed to avoid numerical instabilities in the evaluation of that limit, as well as for allow the analysis of the masses for values of $\lambda \approx 0$. The condition of κ/λ finite is, again, verified with the definitions from eq. (5.4) by keeping $K = \text{const.}$ in the MSSM limit.

The numerical analysis for the *sample scenario* in sects. 4.4.3 and 4.4.4 was made for a fixed positive value of κ . But the MSSM-limit requires $\lambda \rightarrow 0, \kappa \rightarrow 0$. Therefore, the mass values to which the curves in those plots converge for $\lambda \rightarrow 0$ do not correspond to any MSSM result. We show here in fig. 5.1 two plots for the sample scenario in function of λ but varying also κ accordingly, so as to keep the quotient κ/λ constant:⁷¹

See that for $\lambda = 0$ the mass value in fig. 5.1 does not depend on the specific value of κ/λ , as it must be because this corresponds to the MSSM-limit. It must be emphasized again, however, that the MSSM-limit only requires $\lambda \rightarrow 0, \kappa \rightarrow 0$: it is not needed to keep constant (not even finites!) the quotient κ/λ while taking this limit. The MSSM-limit is independent of the behaviour of κ/λ , and in fact the quantity κ/λ could diverge: this is not a problem in order to recover the MSSM. The condition $\kappa/\lambda < \infty$ must be satisfied if in addition we are interested in getting finite results also for the singlet-like quantities $\lambda \approx 0$.

⁷¹ Fig. 5.1 complement the numerical analysis performed in sects. 4.4.1 and 4.4.2 for the *canonical scenario*, where we reproduced the MSSM limit in several cases by using a fixed the value for κ/λ .

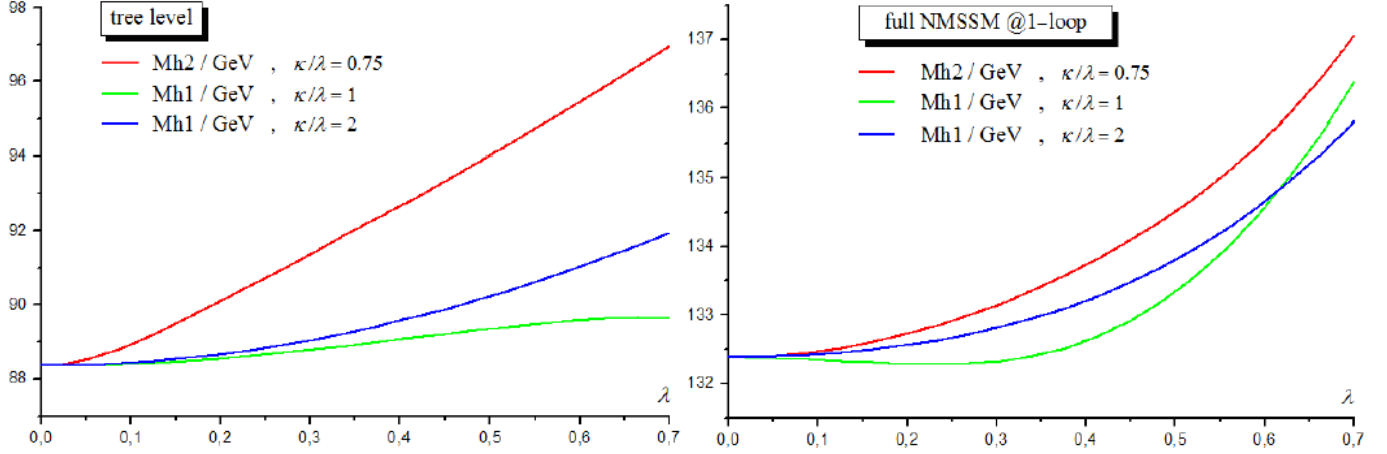


Figure 5.1: Comparison of the lightest CP -even MSSM-like boson masses for different values of κ/λ . The parameters not specified here are chosen according to the *sample scenario* described in sect. 4.4.3. *About the notation:* for $\lambda \rightarrow 0$, h_2 is the lightest MSSM-like CP -even boson when $\kappa/\lambda = 0.75$,⁷² while h_1 is that MSSM-like state when $\kappa/\lambda = 1$ and $\kappa/\lambda = 2$; this is why in these plots we show $M_{h_1}(\kappa/\lambda = 1)$, $M_{h_1}(\kappa/\lambda = 2)$ and $M_{h_2}(\kappa/\lambda = 0.75)$ (compare with the values in fig. 5.2, where the corresponding CP -even singlet-like masses are shown).

Effect of the μ renormalisation

In the decoupling limit, $\lambda \approx 0$, the behaviour shown fig. 5.2 is observed for the mass of the CP -even singlet-like physical state.

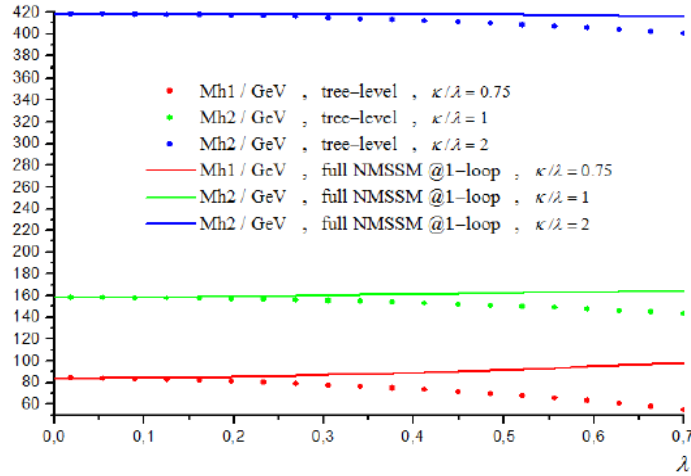


Figure 5.2: Behaviour of CP -even singlet-like mass in the decoupling limit. Here h_1 is the singlet-like state for $\kappa/\lambda = 0.75$, while h_2 is the singlet-like state for $\kappa/\lambda = 1$ and $\kappa/\lambda = 2$. The parameters not specified here are chosen according to the sample scenario described in sect. 4.4.3.

⁷² As always, we follow the convention $M_{h_1} \leq M_{h_2} \leq M_{h_3}$ for the notation.

We see that, for $\lambda = 0$, the singlet-like mass is always equal to its tree-level value. This means that all the renormalised self-energies for external singlet field(s) vanish for $\lambda = 0$ (and $\kappa/\lambda = \text{const.}$). This behaviour is a result of renormalizing μ as a $\overline{\text{DR}}$ quantity. $\delta\mu^{\text{fin}}$ has non-zero contributions to $\hat{\Sigma}_{\phi_S\phi_S, \chi_S\chi_S}$ in the decoupling limit; specifically:

$$\begin{aligned}\hat{\Sigma}_{\phi_S\phi_S}(\lambda \rightarrow 0, \kappa \rightarrow 0) &= \Sigma_{\phi_S\phi_S}^{\text{fin}}(\lambda \rightarrow 0, \kappa \rightarrow 0) - \frac{\kappa}{\lambda} \left(A_\kappa + 8 \frac{\kappa}{\lambda} \mu \right) \delta\mu^{\text{fin}} \\ \hat{\Sigma}_{\chi_S\chi_S}(\lambda \rightarrow 0, \kappa \rightarrow 0) &= \Sigma_{\chi_S\chi_S}^{\text{fin}}(\lambda \rightarrow 0, \kappa \rightarrow 0) + 3 \frac{\kappa}{\lambda} A_\kappa \delta\mu^{\text{fin}}\end{aligned}\quad (5.18)$$

where $\Sigma_{\phi_S\phi_S}^{\text{fin}}(\lambda \rightarrow 0, \kappa \rightarrow 0) = \Sigma_{\chi_S\chi_S}^{\text{fin}}(\lambda \rightarrow 0, \kappa \rightarrow 0) = 0$ if $\kappa/\lambda < \infty$.

5.3 The λ - κ symmetry

In Fig. 5.1 we show the curves for $\lambda \geq 0$. But in fact these results (i.e. setting $\kappa/\lambda = \text{const.}$) are completely symmetric in λ ; that is, the theory is symmetric under $\lambda \rightarrow -\lambda$ and $\kappa \rightarrow -\kappa$ simultaneously, or, equivalently, it would be λ -symmetric if, for instance, we chose to write it in terms of $K \equiv \kappa/\lambda$ instead of κ . In this section we derived this result and explore its implications.

Let's consider the following “ λ/κ parity transformation”:

$$\begin{aligned}\lambda &\rightarrow -\lambda \\ \kappa &\rightarrow -\kappa\end{aligned}\quad (5.19)$$

An inspection of the NMSSM Lagrangian (see for instance the Higgs potential in (4.1)) written in term of the input parameter as (4.30)⁷³ reveals that at tree-level this transformation is equivalent to changing the sign of the singlino $\tilde{S}$ and the singlet fields ϕ_S and χ_S simultaneously, which is nothing but a redefinition of the fields (a specific gauge transformation). Beyond tree-level, the symmetry still holds if the renormalisation scheme is chosen in such a way that in case of (5.19) the same change of sign is inherited by the corresponding counterterms: $\delta\lambda(\lambda, \kappa) = -\delta\lambda(-\lambda, -\kappa)$ and $\delta\kappa(\lambda, \kappa) = -\delta\kappa(-\lambda, -\kappa)$ (so that

⁷³ The invariance analysed here would be spoiled if, for instance, one chose to work with $T_\kappa \equiv \kappa A_\kappa$ instead of A_κ or $T_\lambda \equiv \lambda A_\lambda$ instead of A_λ , as some people do, or not to parametrise the singlet vev as $v_S \equiv \mu/\lambda$. Our analysis is, however, independent of considering A_λ or m_A^2 as input parameter instead of $m_{H^\pm}^2$, because $A_\lambda = s_\beta c_\beta m_A^2 / \mu - (\kappa/\lambda)\mu$ and $m_A^2 = m_{H^\pm}^2 - M_W^2 + (\lambda v)^2$, which are both invariant relations under (5.19).

$\lambda_B \rightarrow -\lambda_B$ and $\kappa_B \rightarrow -\kappa_B$ for the bare parameters).⁷⁴ This suggests that all the physical predictions of the theory must remain unchanged under (5.19), if the scheme is properly chosen. Let's examine this behaviour in detail for our calculation of the NMSSM Higgs masses at one-loop order.

As mentioned in (4.26) and (4.27), in order to obtain the masses (and decay widths) of the Higgs bosons, we have to obtain the poles of the inverse propagator. At one-loop level, these poles are the values of p^2 for which the determinants of the following matrixes vanish:

$$\begin{aligned}
 -i\Delta_{\phi\phi\phi}^{-1}(p^2) = & \begin{pmatrix} (p^2 - m_{\phi_1}^2)(1 + \delta Z_{\phi_1}) & -m_{\phi_1\phi_2}^2(1 + \frac{1}{2}(\delta Z_{\phi_1} + \delta Z_{\phi_2})) & -m_{\phi_1\phi_S}^2(1 + \frac{1}{2}(\delta Z_{\phi_1} + \delta Z_{\phi_S})) \\ * & (p^2 - m_{\phi_2}^2)(1 + \delta Z_{\phi_2}) & -m_{\phi_2\phi_S}^2(1 + \frac{1}{2}(\delta Z_{\phi_2} + \delta Z_{\phi_S})) \\ * & * & (p^2 - m_{\phi_S}^2)(1 + \delta Z_{\phi_S}) \end{pmatrix} \\
 & + \begin{pmatrix} \Sigma_{\phi_1\phi_1}(p^2) & \Sigma_{\phi_1\phi_2}(p^2) & \Sigma_{\phi_1\phi_S}(p^2) \\ * & \Sigma_{\phi_2\phi_2}(p^2) & \Sigma_{\phi_2\phi_S}(p^2) \\ * & * & \Sigma_{\phi_S\phi_S}(p^2) \end{pmatrix} - \begin{pmatrix} \delta m_{\phi_1}^2 & \delta m_{\phi_1\phi_2}^2 & \delta m_{\phi_1\phi_S}^2 \\ * & \delta m_{\phi_2}^2 & \delta m_{\phi_2\phi_S}^2 \\ * & * & \delta m_{\phi_S}^2 \end{pmatrix} \quad (5.20)
 \end{aligned}$$

and the corresponding one for CP -odd sector by doing $\phi \rightarrow \chi$. Equivalently, the masses (and decay widths) of the Higgs bosons annul the matrix

$$\begin{aligned}
 -i\Delta_{hhh}^{-1}(p^2) = & \begin{pmatrix} (p^2 - m_{h_1}^2)(1 + \delta Z_{h_1}) & 0 & 0 \\ * & (p^2 - m_{h_2}^2)(1 + \delta Z_{h_2}) & 0 \\ * & * & (p^2 - m_{h_3}^2)(1 + \delta Z_{h_3}) \end{pmatrix} \\
 & + \begin{pmatrix} \Sigma_{h_1h_1}(p^2) & \Sigma_{h_1h_2}(p^2) & \Sigma_{h_1h_3}(p^2) \\ * & \Sigma_{h_2h_2}(p^2) & \Sigma_{h_2h_3}(p^2) \\ * & * & \Sigma_{h_3h_3}(p^2) \end{pmatrix} - \begin{pmatrix} \delta m_{h_1}^2 & 0 & 0 \\ * & \delta m_{h_2}^2 & 0 \\ * & * & \delta m_{h_3}^2 \end{pmatrix} \quad (5.21)
 \end{aligned}$$

and analogously for the CP -odd sector. Note that (5.20) reduced to

$$-i(\Delta_{\phi\phi\phi}^{-1})_{ij} = p^2 \delta_{ij} - m_{\phi_i\phi_j}^2 + \Sigma_{\phi_i\phi_j}^{\text{fin}}(p^2) - \delta m_{\phi_i\phi_j}^{2,\text{fin}} \quad (5.22)$$

for the case of renormalizing the field counterterms in a $\overline{\text{DR}}$ scheme, cf. eq. (4.52) (we will use this result below).

⁷⁴ The eq. (5.19) is always understood here as a transformation on the *renormalised* parameters λ, κ .

In the following we will use the notation “ θ ” for representing fields in gauge basis (e.g. ϕ_1, ϕ_2, ϕ_s) and “ q ” for the physical basis (such as h_1, h_2, h_s) of a generic sector⁷⁵ with one singlet or singlino field “ θ_s ”. In order to show that the symmetry for the one-loop masses for that sector w.r.t the (5.19) transformation follows we need to prove that the elements of both $-i\Delta_{\phi\phi\phi}^{-1}$ and $-i\Delta_{\chi\chi\chi}^{-1}$ (and also the corresponding one for the neutralino sector) in gauge basis are of the form

$$(-i\Delta_{\theta\ldots\theta}^{-1})_{ij} \xrightarrow[\kappa \rightarrow -\kappa]{\lambda \rightarrow -\lambda} (-1)^{n_{s/\tilde{s}}[\theta_i, \theta_j]} (-i\Delta_{\theta\ldots\theta}^{-1})_{ij} \quad (5.23)$$

where we have defined $n_{s/\tilde{s}}[\theta_{i_1}, \dots, \theta_{i_n}]$ as the number of the fields $\theta_{i_1}, \dots, \theta_{i_n}$ that are singlet/singlino fields. This means that one can write those matrixes in the following form:

$$-i\Delta_{\theta\ldots\theta}^{-1} = \begin{pmatrix} a_{\theta_1\theta_1} & \cdots & a_{\theta_1\theta_{N-1}} & a_{\theta_1\theta_s}\lambda \\ \vdots & \ddots & \vdots & \vdots \\ a_{\theta_{N-1}\theta_1} & \cdots & a_{\theta_{N-1}\theta_{N-1}} & a_{\theta_{N-1}\theta_s}\lambda \\ a_{\theta_s\theta_1}\lambda & \cdots & a_{\theta_s\theta_{N-1}}\lambda & a_{\theta_s\theta_s}\lambda \end{pmatrix} \equiv M_{\theta\ldots\theta}^2, \text{ with all } a_{\theta_i\theta_j} \text{ (5.19)-invariant quantities} \quad (5.24)$$

The masses are obtained from the poles of these propagator, cf. eq. (4.28), and from (5.24) it follows that

$$\det(-i\Delta_{\theta\ldots\theta}^{-1}) = \frac{1}{\lambda^2} \det \begin{pmatrix} a_{\theta_1\theta_1} & \cdots & a_{\theta_1\theta_{N-1}} & a_{\theta_1\theta_s}\lambda^2 \\ \vdots & \ddots & \vdots & \vdots \\ a_{\theta_{N-1}\theta_1} & \cdots & a_{\theta_{N-1}\theta_{N-1}} & a_{\theta_{N-1}\theta_s}\lambda^2 \\ a_{\theta_s\theta_1}\lambda^2 & \cdots & a_{\theta_s\theta_{N-1}}\lambda^2 & a_{\theta_s\theta_s}\lambda^2 \end{pmatrix} \quad (5.25)$$

which is manifestly (5.19)-invariant.

The main result for proving (5.24) follows from an inspection of the Lagrangian of the model, which yields:

The “ λ/κ -parity” of a coupling or a bilinear coefficient, $C[\theta_{i_1}, \dots, \theta_{i_n}]$, with respect to (5.19) is determined by the number $n_{s/\tilde{s}}[\theta_{i_1}, \dots, \theta_{i_n}]$ of legs which correspond to (i.e. the fields $\theta_{i_1}, \dots, \theta_{i_n}$ that are) singlet or singlino fields:⁷⁶

⁷⁵ Although in (5.20) we show the explicit form of the CP -even and the CP -odd Higgs inverse propagators only, the analysis in this section applies to the neutralino sector too, where the singlino field takes part.

⁷⁶ If instead of the gauge fields $\theta_{i_1}, \dots, \theta_{i_n}$ we have some combination $q_{j_1}, \dots, q_{j_n}$ of singlino/singlet fields and other fields, $n_{s/\tilde{s}}[q_{i_1}, \dots, q_{i_n}]$ is not well-defined. The λ/κ -parity of $C[q_{i_1}, \dots, q_{i_n}]$ is still well-defined, however, if that combination of fields corresponds to the expansion of some physical field, see eq. (5.44).

$$C[\theta_{i_1}, \dots, \theta_{i_n}] \xrightarrow[\kappa \rightarrow -\kappa]{\lambda \rightarrow -\lambda} (-1)^{n_{S/\tilde{S}}[\theta_{i_1}, \dots, \theta_{i_n}]} C[\theta_{i_1}, \dots, \theta_{i_n}] \quad (5.26)$$

where

$$C[\theta_{i_1}, \dots, \theta_{i_n}] \equiv \left. \frac{\partial^n L}{\partial \theta_{i_1} \dots \partial \theta_{i_n}} \right|_{\text{all fields}=0} \quad (5.27)$$

This is an immediate consequence of the already mentioned fact that the tree-level NMSSM Lagrangian (written in terms of the input parameters (4.30)) it is invariant under (5.19) if in addition we do⁷⁷

$$\begin{aligned} \phi_S &\rightarrow -\phi_S \\ \chi_S &\rightarrow -\chi_S \\ \tilde{S} &\rightarrow -\tilde{S} \end{aligned} \quad (5.28)$$

once the tadpole coefficients are set to zero.

Eq. (5.26) means that at tree-level the only 2-, 3- and 4-point vertexes that are not invariant but change their sign under (5.19) are the ones that involve an odd number of singlino/singlet fields, namely:

$$\begin{array}{llll} \tilde{H}_i^0 \tilde{S} & \tilde{H}_i^0 \tilde{S} \varphi_j & \chi_2^\pm \phi_i^\pm \tilde{S} & \varphi_i \varphi_j \varphi_S \\ \tilde{H}_i^0 \tilde{S} & \tilde{H}_i^0 \tilde{H}_j^0 \varphi_S & \chi_2^\pm \phi_i^\pm \tilde{S} & \varphi_i \varphi_j \varphi_S \\ \varphi_i \varphi_S & \tilde{S} \tilde{S} \varphi_S & \chi_2^\pm \chi_2^\pm \varphi_S & \phi_i^\pm \phi_i^\pm \varphi_S \end{array} \quad \begin{array}{l} \tilde{f}_g^i \tilde{f}_g^j \varphi_S \\ \tilde{f}_g^i \tilde{f}_g^j \varphi_k \varphi_S \\ \tilde{f}_g^i \tilde{f}_g^j \phi_k^\pm \varphi_S \end{array} \quad (5.29)$$

where

$$\begin{aligned} i, j, k = 1, 2 & \quad , \quad \varphi = \phi, \chi \quad (\text{neutral Higgs fields}) \quad , \\ g = 1, 2, 3 & \quad , \quad \tilde{f} = \text{sleptons/squarks} \quad . \end{aligned} \quad (5.30)$$

With respect to the mixing matrixes that link the gauge basis with the physical basis, let's consider the diagonalization of the mass matrix $M_{\theta \dots \theta}^2$ in gauge basis:

$$U M_{\theta \dots \theta}^2 U^T = M_{q \dots q}^2 \quad , \quad U U^T = U^T U = I$$

⁷⁷ See the Higgs potential in eq. (4.1), and the neutralino mass matrix in (4.20).

$$\mathbf{M}_{q\dots q}^2 = \text{diag}\{m_{q_1}^2, \dots, m_{q_N}^2\} \quad , \quad \begin{pmatrix} q_1 \\ \vdots \\ q_{N-1} \\ q_N \end{pmatrix} = \mathbf{U} \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_{N-1} \\ \theta_{(S)} \end{pmatrix} \quad (5.31)$$

$$\begin{aligned} L &\supset \frac{1}{2}(\theta_1, \dots, \theta_N) \mathbf{M}_{\theta\dots\theta}^2 (\theta_1, \dots, \theta_N)^T = \frac{1}{2}(q_1, \dots, q_N) \mathbf{M}_{q\dots q}^2 (q_1, \dots, q_N)^T \\ &= \frac{1}{2} \sum_{k,\ell} \underbrace{(\mathbf{M}_{\theta\dots\theta}^2)_{k\ell}}_{\mathbf{C}[\theta_k, \theta_\ell]} \theta_k \theta_\ell = \frac{1}{2} \sum_{i,j} \underbrace{\sum_{k,\ell} U_{i,k} (\mathbf{M}_{\theta\dots\theta}^2)_{k\ell} U_{\ell,j}^T}_{\mathbf{C}[q_i, q_j]} q_i q_j \end{aligned}$$

The columns of $\mathbf{U}^T$ are the normalized eigenvectors $\mathbf{v}_i$ of the matrix $\mathbf{M}_{\theta\dots\theta}^2$:

$$\mathbf{M}_{\theta\dots\theta}^2 \mathbf{v}_i = m_{q_i}^2 \mathbf{v}_i \quad , \quad \mathbf{v}_i^T \mathbf{v}_j = \delta_{ij} \quad , \quad \mathbf{U}^T = (\mathbf{v}_1 \mid \mathbf{v}_2 \mid \dots \mid \mathbf{v}_N) \quad (5.32)$$

Being $\mathbf{M}_{\theta\dots\theta}^2$ of the form (5.24), it transforms as

$$\mathbf{M}_{\theta\dots\theta}^2 \xrightarrow[\kappa \rightarrow -\kappa]{\lambda \rightarrow -\lambda} \begin{pmatrix} a_{\theta_1\theta_1} & \cdots & a_{\theta_1\theta_{N-1}} & -a_{\theta_1\theta_S} \lambda \\ \vdots & \ddots & \vdots & \vdots \\ a_{\theta_{N-1}\theta_1} & \cdots & a_{\theta_{N-1}\theta_{N-1}} & -a_{\theta_{N-1}\theta_S} \lambda \\ -a_{\theta_S\theta_1} \lambda & \cdots & -a_{\theta_S\theta_{N-1}} \lambda & a_{\theta_S\theta_S} \end{pmatrix} \quad (5.33)$$

i.e.,

$$\mathbf{M}_{\theta\dots\theta}^2 \xrightarrow[\kappa \rightarrow -\kappa]{\lambda \rightarrow -\lambda} \text{diag}\{1, \dots, 1, -1\} \mathbf{M}_{\theta\dots\theta}^2 \text{diag}\{1, \dots, 1, -1\} \equiv \tilde{\mathbf{M}}_{\theta\dots\theta}^2 \quad . \quad (5.34)$$

Then,

$$\text{diag}\{1, \dots, 1, -1\} \mathbf{M}_{\theta\dots\theta}^2 \overbrace{\text{diag}\{1, \dots, 1, -1\} \text{diag}\{1, \dots, 1, -1\}}^{\mathbf{I}} \mathbf{v}_i = m_{q_i}^2 \text{diag}\{1, \dots, 1, -1\} \mathbf{v}_i \quad (5.35)$$

which means that the eigenvectors $\tilde{\mathbf{v}}_i$ of the transformed matrix $\tilde{\mathbf{M}}_{\theta\dots\theta}^2$ can be chosen as

$$\tilde{\mathbf{v}}_i = \text{diag}\{1, \dots, 1, -1\} \mathbf{v}_i \quad (5.36)$$

and hence the mixing matrix and the mass eigenstates transform as

$$\begin{aligned}
\mathbf{U} \equiv \begin{pmatrix} U_{1,1} & \cdots & U_{1,N-1} & U_{1,(S)} \\ \vdots & \ddots & \vdots & \vdots \\ U_{N-1,1} & \cdots & U_{N-1,N-1} & U_{N-1,(S)} \\ U_{N,1} & \cdots & U_{N,N-1} & U_{N,(S)} \end{pmatrix} \xrightarrow[\kappa \rightarrow -\kappa]{\lambda \rightarrow -\lambda} \tilde{\mathbf{U}} \equiv \begin{pmatrix} U_{1,1} & \cdots & U_{1,N-1} & -U_{1,(S)} \\ \vdots & \ddots & \vdots & \vdots \\ U_{N-1,1} & \cdots & U_{N-1,N-1} & -U_{N-1,(S)} \\ U_{N,1} & \cdots & U_{N,N-1} & -U_{N,(S)} \end{pmatrix} \\
\begin{pmatrix} q_1 \\ \vdots \\ q_{N-1} \\ q_N \end{pmatrix} \xrightarrow[\kappa \rightarrow -\kappa]{\lambda \rightarrow -\lambda} \tilde{\mathbf{U}} \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_{N-1} \\ \theta_{(S)} \end{pmatrix} = \mathbf{U} \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_{N-1} \\ -\theta_{(S)} \end{pmatrix}
\end{aligned} \tag{5.37}$$

Again, this is a manifestation of that the transformation (5.19) is equivalent (at least at tree level) to a redefinition $\tilde{S} \rightarrow -\tilde{S}, \phi_S \rightarrow -\phi_S, \chi_S \rightarrow -\chi_S$ of the singlet and singlino fields.

Analogous results as above but for physical states are needed in case of analysing the effect of the (5.19) transformation on the matrix (5.21) in physical basis, as well as to verify the λ/κ -symmetry of the tree-level masses. Let's consider two of the *physical* fields, q_i^A and q_j^B , from two (possibly different) sectors A and B , so their expressions in terms of gauge eigenstates are:

$$q_i^A = U_{i,S}^A \theta_S^A + \sum_{k \neq S} U_{i,k}^A \theta_k^A, \quad q_j^B = U_{j,S}^B \theta_S^B + \sum_{\ell \neq S} U_{j,\ell}^B \theta_\ell^B \tag{5.38}$$

As before, θ_S^A (θ_S^B) is the only (if any) singlet or singlino field of the sector $A(B)$.⁷⁸ From (5.31),

$$\sum_{k_1, \dots, k_n} \mathcal{C}[\theta_{k_1}, \dots, \theta_{k_n}] \theta_{k_1} \dots \theta_{k_n} = \sum_{i_1, \dots, i_n} \underbrace{\sum_{k_1, \dots, k_n} U_{i_1, k_1} \dots U_{i_n, k_n} \mathcal{C}[\theta_{k_1}, \dots, \theta_{k_n}]}_{\mathcal{C}[q_{i_1}, \dots, q_{i_n}]} q_{i_1} \dots q_{i_n} \tag{5.39}$$

thus for the coefficient of the bilinear term for $q_i^A q_j^B$ we get

$$\begin{aligned}
\mathcal{C}[q_i^A, q_j^B] = & \overbrace{U_{i,S}^A}^{-} \overbrace{U_{j,S}^B}^{-} \overbrace{\mathcal{C}[\theta_S^A, \theta_S^B]}^{+} + \sum_{\ell \neq S} \overbrace{U_{i,S}^A}^{-} \overbrace{U_{j,\ell}^B}^{+} \overbrace{\mathcal{C}[\theta_S^A, \theta_\ell^B]}^{-} \\
& + \sum_{k \neq S} \overbrace{U_{i,k}^A}^{+} \overbrace{U_{j,S}^B}^{-} \overbrace{\mathcal{C}[\theta_k^A, \theta_S^B]}^{-} + \sum_{k \neq S} \sum_{\ell \neq S} \overbrace{U_{i,k}^A}^{+} \overbrace{U_{j,\ell}^B}^{+} \overbrace{\mathcal{C}[\theta_k^A, \theta_\ell^B]}^{+}
\end{aligned} \tag{5.40}$$

⁷⁸ The expansions in (5.38) are very general expressions: if A and/or B are not from the neutral Higgs or neutralino sectors, then $Z_{i_A, S}^A = 0$ and/or $Z_{i_B, S}^B = 0$.

The indicated signs are the λ/κ -parity of the various factors, followed from (5.26) and (5.37). We see that the parity of the bilinear coefficient $C[q_i^A, q_j^B]$ is positive, which means that it is invariant under (5.19). It is straightforward to see that the same result holds for a 3-point vertex $C[q_i^A, q_j^B, q_k^C]$ or a 4-point vertex. Then, we obtain the following result:

$$\text{Any coupling or mass coefficient for physical fields are (5.19)-invariant.} \quad (5.41)$$

An immediate corollary is the following:

$$\text{Any amplitude for external physical fields are invariant under (5.19).} \quad (5.42)$$

The last sentence follows from that accordingly to (5.41) all tree-level masses are (5.19)-invariant, so do all Passarino-Veltman integrals.

We now need to extend the previous results to couplings involving fields in both gauge and physical basis, in order to analyse the effect of the (5.19) transformation on the radiative contributions. For an amplitude with external gauge eigenstates, it can always be evaluated (and for simplicity of calculation it is convenient to do so) considering only physical fields as internal lines, so we need to analyse the transformation of a coupling for a combination of gauge eigenstates and physical fields. For instance, let's consider a 3-point coupling $C[q_A, q_A, \theta_S^C]$ between the physical fields q_A and q_B from eq. (5.38) and a singlino/singlet field θ_S^C from a sector C :

$$\begin{aligned} C[q_i^A, q_j^B, \theta_S^C] &= \overbrace{U_{i,S}^A U_{j,S}^B}^{-} \overbrace{C[\theta_S^A, \theta_S^B, \theta_S^C]}^{-} + \sum_{\ell \neq S} \overbrace{U_{i,S}^A U_{j,\ell}^B}^{-} \overbrace{C[\theta_S^A, \theta_\ell^B, \theta_S^C]}^{+} \\ &\quad + \sum_{k \neq S} \underbrace{U_{i,k}^A}_{+} \underbrace{U_{j,S}^B}_{-} \underbrace{C[\theta_k^A, \theta_S^B, \theta_S^C]}_{+} + \sum_{k \neq S} \sum_{\ell \neq S} \underbrace{U_{i,k}^A}_{+} \underbrace{U_{j,\ell}^B}_{+} \underbrace{C[\theta_k^A, \theta_\ell^B, \theta_S^C]}_{-} \end{aligned} \quad (5.43)$$

$$\therefore C[q_i^A, q_j^B, \theta_S^C] \xrightarrow[\kappa \rightarrow -\kappa]{\lambda \rightarrow -\lambda} -C[q_i^A, q_j^B, \theta_S^C]$$

This result can be easily generalizable to any combination of fields from gauge and physical basis. We find that, in order to determining the λ/κ -parity of a coupling or bilinear coefficient, the physical fields as external legs do not matter, only the number of purely singlet and/or singlino fields do:

$$C[q_1, \dots, q_m, \theta_{m+1}, \dots, \theta_n] \xrightarrow[\kappa \rightarrow -\kappa]{\lambda \rightarrow -\lambda} (-1)^{n_S/\tilde{S}[\theta_{m+1}, \dots, \theta_n]} C[q_1, \dots, q_m, \theta_{m+1}, \dots, \theta_n] \quad (5.44)$$

This result is immediately transferred to the amplitudes. As we mentioned above, even when the external legs correspond to fields from some gauge basis, in the calculation all

virtual particles can (and it's preferable to) be considered in physical basis. In addition, we already pointed out that all the Passarino-Veltman integrals are (5.19)-invariant. Therefore:

The λ/κ -parity of an amplitude in gauge basis is just determined by the number of pure singlino/singlet fields as external legs.

(5.45)

Thus, for instance,⁷⁹

$$\begin{array}{ccc}
\Sigma_{\phi_1\phi_S} & \xrightarrow[\kappa \rightarrow -\kappa]{\lambda \rightarrow -\lambda} & -\Sigma_{\phi_1\phi_S} \\
\Sigma_{\chi_1\chi_1} & \longrightarrow & +\Sigma_{\chi_1\chi_1} \\
\Sigma_{\chi_S\chi_S} & \longrightarrow & +\Sigma_{\chi_S\chi_S} \\
\Delta\Gamma_{\phi_1\phi_1\phi_S} & \longrightarrow & -\Delta\Gamma_{\phi_1\phi_1\phi_S} \\
\Delta\Gamma_{\phi_1\chi_1\chi_1} & \longrightarrow & +\Delta\Gamma_{\phi_1\chi_1\chi_1} \\
\Delta\Gamma_{\phi_2\phi_2\phi_2} & \longrightarrow & +\Delta\Gamma_{\phi_2\phi_2\phi_2} \\
\Delta\Gamma_{\phi_2\chi_1\chi_S} & \longrightarrow & -\Delta\Gamma_{\phi_2\chi_1\chi_S} \\
\Delta\Gamma_{\phi_S\chi_S\chi_S} & \longrightarrow & -\Delta\Gamma_{\phi_S\chi_S\chi_S} \\
\Delta\Gamma_{\phi_S\chi_2\chi_S} & \longrightarrow & +\Delta\Gamma_{\phi_S\chi_2\chi_S} \\
\vdots & & \vdots
\end{array} \tag{5.46}$$

Returning to expression (5.22), we see that the last piece of our analysis is to prove the following λ/κ -parity behaviour for the $\delta m_{\phi_i\phi_j}^{2,\text{fin}}$ counterterms:

$$\delta m_{\phi_i\phi_j}^{2,\text{fin}} \xrightarrow[\kappa \rightarrow -\kappa]{\lambda \rightarrow -\lambda} (-1)^{n_{S/\tilde{S}}[\phi_i,\phi_j]} \delta m_{\phi_i\phi_j}^{2,\text{fin}} \tag{5.47}$$

and analogously for the CP -odd sector. According to the quantities that we don't renormalised in a $\overline{\text{DR}}$ scheme (both in the λ/κ - and the $\tilde{\lambda}/K$ -scheme), c.f. (4.31), and the renormalisation conditions of sect. 6.2.2, we have

$$\begin{aligned}
\delta m_{\phi_i\phi_j}^{2,\text{fin}} = & (\partial_{m_{H^\pm}^2} m_{\phi_i\phi_j}^2) \text{Re} \Sigma_{H^\pm H^\pm}^{\text{fin}}(m_{H^\pm}^2) + (\partial_{M_Z^2} m_{\phi_i\phi_j}^2) \text{Re} \Sigma_{ZZ}^{\text{fin}}(M_Z^2) \\
& + (\partial_{M_W^2} m_{\phi_i\phi_j}^2) \text{Re} \Sigma_{WW}^{\text{fin}}(M_W^2) + \frac{1}{\sqrt{2}v} \sum_{k=1,2,S} (\partial_{\bar{T}_{\phi_k}} m_{\phi_i\phi_j}^2) T_{\phi_k}^{(1L),\text{fin}}
\end{aligned} \tag{5.48}$$

The parity of the coefficients $\partial_{\sigma} m_{\phi_i\phi_j}^2$ can be readable directly from (4.61) and (4.62):

⁷⁹ These results are independent of the external momentum.

$$\begin{aligned}
\partial_{\sigma} m_{\phi_i \phi_j}^2 &\xrightarrow[\kappa \rightarrow -\kappa]{\lambda \rightarrow -\lambda} (-1)^{n_{S/\bar{S}}[\phi_i, \phi_j]} \partial_{\sigma} m_{\phi_i \phi_j}^2, \quad \sigma = m_{H^{\pm}}^2, M_Z^2, M_W^2 \\
\partial_{\bar{T}_{\phi_k}} m_{\phi_i \phi_j}^2 &\longrightarrow (-1)^{n_{S/\bar{S}}[\phi_i, \phi_j, \phi_k]} \partial_{\bar{T}_{\phi_k}} m_{\phi_i \phi_j}^2
\end{aligned} \tag{5.49}$$

while for the amplitudes in (5.48) we use the results (5.42) and (5.45):

$$\begin{aligned}
\Sigma_{\sigma\sigma} &\xrightarrow[\kappa \rightarrow -\kappa]{\lambda \rightarrow -\lambda} +\Sigma_{\sigma\sigma}, \quad \sigma = H^{\pm}, Z, W \\
T_{\phi_k}^{(1L), \text{fin}} &\longrightarrow (-1)^{n_{S/\bar{S}}[\phi_k]} T_{\phi_k}^{(1L), \text{fin}}
\end{aligned} \tag{5.50}$$

which prove that (5.47) is verified. Note again that this result depends on the “right” choice of the renormalisation scheme.

This confirms the λ/κ -parity behaviour (5.24) for the NMSSM Higgs inverse propagators, which prove the invariance of the one-loop masses with respect to the (5.19) transformation. There is no definite parity (even nor odd) in the theory, however, if we performed $\lambda \rightarrow -\lambda$ or $\kappa \rightarrow -\kappa$ but not both at the same time; in fact, in most cases the lightest CP -odd Higgs mass turn out to be tachyonic at tree level for $\kappa/\lambda < 0$. This again suggests the choice of (5.4) as input parameters instead of λ, κ as a preferred/improved choice for the NMSSM phenomenology analysis, because the theory is $\tilde{\lambda} \rightarrow -\tilde{\lambda}$ invariant (within the appropriate renormalisation scheme), while keeping K to a fixed and positive value avoids those negative square mass results for $\kappa/\lambda < 0$.

Summary and Conclusions

In this work we performed precise calculations for the Higgs boson masses with radiative corrections in the context of supersymmetric extensions of the Standard Model of particle physics. The most significant achievements include a new accuracy on predictions within the Minimal Supersymmetric Standard Model (MSSM), specifically for the charged Higgs boson mass, and the *CP*-conserving Next-to-Minimal Supersymmetric Standard Model (NMSSM), consisting on the evaluation of the full one-loop corrections for the relevant sectors, including derivation of the renormalisation. The implementation of the leading two-loop contributions developed in our publications [1,2,129] was briefly discussed as well. Where available, for both models we verified that the analytical expressions and numerical results obtained from our own computation were in agreement with previous literature and consistent with other public codes.

The Higgs mechanism satisfies the need of gauge theories for some form of spontaneous electroweak symmetry breaking to provide mass to the corresponding gauge bosons. The MSSM is one of the most popular theoretical models that implement this mechanism, containing in its spectrum for the Higgs sector two charged scalar bosons and three neutral ones. Supersymmetric theories such as the MSSM extend the SM in order to solve the problem of the appearance of quadratically divergent quantum corrections for the mass of the Higgs particle. In the framework of this model, the full numerical analysis for the physical masses of the neutral Higgs bosons confirms that the virtual fermions and sfermions generate the most important contributions to the radiative corrections through the Yukawa couplings, being the top and stop loops who give rise to the dominant terms that grows as the fourth power of the top quark mass.

We have evaluated the dominant contributions at $O(m_t^4)$ from the (s)top sector, as well as the (s)fermion approximation and the full result, for the MSSM Higgs sector structure at one-loop order, deriving and discussing the analytical results for the masses and mixing of the Higgs bosons. In particular, we have analysed the details of the MSSM Higgs and sfermion sectors, concerning the relations between the physical and the nonphysical parameters, as well as the implementation of the explicit calculation of the amplitudes needed for the evaluation of the relevant renormalised self-energies.

We have presented a detailed analysis of the prediction for the charged Higgs boson mass, $M_{H^\pm}$, within the real MSSM, on the basis of a complete one-loop calculation. The incorporating of the two-loop contributions of $O(\alpha_t \alpha_s)$ in [1] was also reviewed. We find relatively large mass shifts at the one-loop level. In particular, we have analysed the dependence of $M_{H^\pm}$ on the trilinear couplings A_t and A_b . For the case $A_t = A_b$, which is assumed in the so-called $m_h^{\max}$ benchmark scenario, corrections to $M_{H^\pm}$ of several GeV are found. The opposite case, $A_t = -A_b$, can yield much larger shifts exceeding $\Delta M_{H^\pm} = 10 \text{ GeV}$ for large $|A_t|$. In general, the full one-loop corrections are negative for small $\tan\beta$ and positive for large $\tan\beta$ in the $m_h^{\max}$ benchmark scenario.

Pronounced effects on $M_{H^\pm}$ in the region of large $\tan\beta$ originate from the standard QCD corrections to the bottom Yukawa coupling, formally a contribution beyond one-loop order. Similarly important are the shifts from the inclusion of Δ_b effects, leading to a strong dependence of $M_{H^\pm}$ on the size and the sign of μ . The contributions from the (s)fermion sector beyond $t/\tilde{t}/b/\tilde{b}$ are sizable for $M_{\text{SUSY}} \gtrsim 1000 \text{ GeV}$ and can exceed $\sim 2 \text{ GeV}$.

The new two-loop corrections of $O(\alpha_t \alpha_s)$ [91] in most of the considered cases are of opposite sign to the one-loop corrections. The induced shifts in $M_{H^\pm}$ can be of several GeV for small M_A and $\tan\beta$ and large values of $|\mu|$, and are thus of a size that may be probed at the LHC and the LC. The set of two-loop contributions considered are expected to be particularly relevant for those MSSM parameter regions where the m_t^4/M_W^2 terms yield a good approximation to the full one-loop result, i.e. in particular for relatively low values of $\tan\beta$. For the general case, a more comprehensive higher-order calculation would be required.

We have implemented our results for the charged Higgs-boson mass into the public Fortran code FEYNHIGGS. The code also contains the evaluation of the charged Higgs-boson decays and the main charged Higgs-boson production channels at the LHC. It can be obtained from www.feynhiggs.de.

We have presented predictions for the Higgs boson masses also in the NMSSM, obtained again within the Feynman-diagrammatic approach. The Higgs-mass predictions include the full one-loop calculation in the NMSSM. They have been supplemented by all available MSSM-like two-loop contributions extracted from [2], where the validity of this approximation in the perturbative regime for λ and κ has been confirmed. The dominant and sub-dominant two-loop corrections of MSSM-type include the contributions at $O(\alpha_t \alpha_s, \alpha_b \alpha_s, \alpha_t^2, \alpha_t \alpha_b)$, as well as a resummation of leading and subleading logarithms from the top/scalar top sector. While the full one-loop contributions are evaluated with an own code, making use of the FEYNARTS and FORMCALC packages and a model file generated with SARAH, the leading two-loop corrections correspond to the output from FEYNHIGGS. In order to enable a direct comparison with the corresponding results in the

MSSM, the renormalisation scheme and all parameters and conventions have been chosen such that the well-known MSSM result is recovered in the appropriate limit.

In our phenomenological analysis we have first investigated a “sample scenario” where, depending on the value of λ , either the lightest or the next-to-lightest neutral Higgs state can be identified with a SM-like Higgs boson at about 125 GeV. The lightest neutral Higgs state can be identified with a SM-like Higgs boson at ~ 125 GeV for low and moderate values of λ . The state that can be identified with the observed Higgs signal is doublet-like in all cases, i.e. it receives only relatively small contributions from the singlet state of the NMSSM. The inclusion of the higher-order contributions which are known for the MSSM were crucial in this context in order to obtain an accurate prediction for the mass spectrum.

We have investigated different approximations at the one-loop level in comparison with our full one-loop result for the NMSSM. We have found that the approximation of the result for the top/stop sector in terms of the leading MSSM-type contributions works well in the parameter regions where the top/stop sector itself yields a reasonable approximation of the full result. It therefore appears to be well motivated to make use the same approximation at the two-loop level. The genuine NMSSM top/stop sector contributions of $O(Y_t \lambda, \lambda^2)$ can be significant for singlet-like fields if λ is large. For such large values of λ , however, the improvement achieved by including those genuine NMSSM contributions from the top/stop sector is by far overshadowed by the fact that contributions from the Higgs and higgsino sectors become more and more important for a singlet-like Higgs field.

Finally, we have explored some additional theoretical aspects of the NMSSM. We reproduce the analytical conditions for recovering the MSSM results in some appropriate limit for the NMSSM parameters, and we discussed the impact of the chosen renormalization scheme in that limit. These results were confirmed in our numerical analysis, for the canonical scenario in chapter 4 and the sample scenario in chapter 5. An additional scheme (slightly different from the one implemented in chapter 4) for the Higgs sector were proposed in chapter 5, motivated (among other things) on showing that we could use the Thomson limit as the value for the electric charge parameter in our results from chapter 4 (no shift) at one-loop order. The schemes from chapters 4 and 5 were compared, and precisely the value for the e parameter ultimately resulted in the only relevant numerical difference between them. We made explicit the MSSM-limit prescriptions for both, and in addition the implice of the chosen scheme on the parity behaviour of the results from the theory with respect to the signs of the λ and κ parameters was analysed.

This thesis contributes to the effort of providing phenomenological insights for the Higgs bosons within supersymmetric extensions of the Standard Model, which might open a new perspective for the ongoing Higgs and Beyond SM search at the LHC and future colliders.

While we have already implemented our original results for the MSSM in FEYNHIGGS, the work presented here will be used also as a basis for the extension of this code to the NMSSM, so as to improve the possibility of making phenomenological predictions for the NMSSM Higgs bosons and comparing it with the MSSM and other models with Higgs sectors of non-standard structure.

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