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BEYOND EINSTEIN: GRAVITATIONAL WAVES FROM EXTENDED GRAVITY

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Abstract

We show that linearizing Extended Theories of Gravity, further gravitational modes emerge. Besides massless spin-2, also spin-0 and spin-2 massive and ghost fields have to be considered as soon as one is considering the full curvature budget of generic metric theories of gravity. Such additional modes give rise to further polarizations that could be of interest for direct detection by the forthcoming Advanced LIGO-VIRGO and other collaborations.

1 Introduction

The recent discovery of gravitational waves [1] pointed out several new perspectives for some key questions of fundamental physics, astrophysics and cosmology. They range from the validity of Equivalence Principle, to black hole physics to the nature of dark energy and dark matter. Combined gravitational-wave, neutrino and electromagnetic observations can be used to understand the main characteristics of several astrophysical systems and to map in detail the observed Universe. Furthermore, observations indicate consistent upper bounds on the graviton mass [1] allowing the possibility that further metric theories of gravity can be investigated besides General Relativity (GR).

Given these facts and the lack of a final self-consistent theory of Quantum Gravity, alternative theories of gravity can be pursued as part of a semi-classical approach where GR and its positive results should be retained. The approach of Extended Theories of Gravity (ETG), based on corrections and enlargements of the Einstein theory, has become a sort of paradigm in the study of the gravitational interaction. These theories have received a lot of interest in cosmology since they "naturally" exhibit inflationary and dark energy behaviors [2]. At a fundamental level, detecting new gravitational modes could be a sort of *experimentum crucis* in order to discriminate among competing models since this possible detection could be the "signature" that GR should be enlarged, modified or retained as it is [3].

In this report, we discuss the problem of gravitational waves in ETG, showing that new polarizations are derived besides the two standard ones of GR. The theoretical set up of the approach is reported together with some consideration on the actual detectability of such new modes.

2 Gravitational waves in Extended Gravity

Let us generalize the action of GR by adding curvature invariants other than the standard Ricci scalar. Specifically, we are considering the action [1]

$$S = \int d^4x\sqrt{-g}f(R, P, Q) \tag{1}$$

where

$$P \equiv R_{ab}R^{ab}\,, \qquad Q \equiv R_{abcd}R^{abcd} \tag{2}$$

In other words, we are taking into account the full curvature budget of generic metric theories of gravity. Varying with respect to the metric, one gets the field equations:

[1] Conventions: $g_{ab} = (-1, 1, 1, 1)$, $R^a_{bcd} = \Gamma^a_{bd,c} - \Gamma^a_{bc,d} + ...$, $R_{ab} = R^c_{acb}$, $G_{ab} = 8\pi G_N T_{ab}$ and all indices run from 0 to 3.

$$
\begin{aligned}
FG_{\mu\nu} \quad = \quad & \frac{1}{2} g_{\mu\nu} \left(f - R\ F\right) - (g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu)F - 2\left(f_P R^a_\mu R_{a\nu} + f_Q\ R_{abc\mu} R^{abc}{}_\nu\right) \\
& - g_{\mu\nu} \nabla_a \nabla_b (f_P R^{ab}) - \Box(f_P R_{\mu\nu}) + 2\nabla_a \nabla_b \left(f_P\ R^a{}_{(\mu} \delta^b{}_{\nu)} + 2 f_Q\ R^a{}_{(\mu\nu)}{}^b\right)
\end{aligned} \tag{3}
$$

where we have defined

$$
F \equiv \frac{\partial f}{\partial R}, \quad f_P \equiv \frac{\partial f}{\partial P}, \quad f_Q \equiv \frac{\partial f}{\partial Q} \tag{4}
$$

and $\Box = g^{ab}\nabla_a \nabla_b$ is the d'Alembert operator. The notation $T_{(ij)} = \frac{1}{2}(T_{ij} + T_{ji})$ denotes symmetrization with respect to the indices (i, j). Considering the trace of eq. (3), we find:

$$
\begin{aligned}
& \Box\left(F + \frac{2}{3}(f_P + f_Q)R\right) = \\
& = \frac{1}{3}[2f - RF - 2R^{ab}\nabla_a \nabla_b (f_P + 2f_Q) - R\Box(f_P + 2f_Q) - 2(f_P P + f_Q Q)]
\end{aligned} \tag{5}
$$

If we define

$$
\Phi \equiv F + \frac{2}{3}(f_P + f_Q)R \quad \text{and} \quad \frac{dV}{d\Phi} \equiv \text{RHS} \quad \text{of} \quad (6)
$$

we get a Klein-Gordon equation for the scalar field Φ:

$$
\Box\Phi = \frac{dV}{d\Phi} \tag{6}
$$

In order to find the gravitational modes as perturbations, we need to linearize the field around the Minkowski background:

$$
g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \text{and} \quad \Phi = \Phi_0 + \delta\Phi \tag{7}
$$

From eq. (6), we get

$$
\delta\Phi = \delta F + \frac{2}{3}(\delta f_P + \delta f_Q)R_0 + \frac{2}{3}(f_{P0} + f_{Q0})\delta R \tag{8}
$$

where $R_0 \equiv R(\eta_{\mu\nu}) = 0$ and similarly $f_{P0} = \frac{\partial f}{\partial P}|_{\eta_{\mu\nu}}$ (note that the 0 indicates the value around the Minkowski metric) which is either constant or zero. The first term of eq. (8) is

$$
\delta F = \frac{\partial F}{\partial R}|_0\ \delta R + \frac{\partial F}{\partial P}|_0\ \delta P + \frac{\partial F}{\partial Q}|_0\ \delta Q \tag{9}
$$

However, since δP and δQ are second order, we get $\delta F \simeq F_{,R0}\ \delta R$ and

$$\delta\Phi = \left(F_{,R0} + \frac{2}{3}(f_{P0} + f_{Q0})\right)\delta R \tag{10}$$

Finally, from eq. (6), we get the Klein-Gordon equation for the scalar perturbation $\delta\Phi$

$$\begin{aligned}\Box\delta\Phi &= \frac{1}{3}\frac{F_0}{F_{,R0} + \frac{2}{3}(f_{P0} + f_{Q0})}\delta\Phi - \frac{2}{3}\delta R^{ab}\partial_a\partial_b(f_{P0} + 2f_{Q0}) - \frac{1}{3}\delta R\Box(f_{P0} + 2f_{Q0}) \\ &= m_s^2\delta\Phi\end{aligned} \tag{11}$$

The last two terms in the first line are actually zero since the terms f_{P0}, f_{Q0} are constants and we have defined the scalar mass as $m_s^2 \equiv \frac{1}{3}\frac{F_0}{F_{,R0}+\frac{2}{3}(f_{P0}+f_{Q0})}$. Perturbing the field equations (3) and working in Fourier space [2], we can rewrite the metric perturbation as

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{\bar{h}}{2}\ \eta_{\mu\nu} + \eta_{\mu\nu}h_f \tag{12}$$

and use the gauge freedom to demand that the standard conditions $\partial_\mu\bar{h}^{\mu\nu} = 0$ and $\bar{h} = 0$ hold. The first of these conditions implies that $k_\mu\bar{h}^{\mu\nu} = 0$ while the second that

$$h_{\mu\nu} = \bar{h}_{\mu\nu} + \eta_{\mu\nu}h_f \quad \text{and} \quad h = 4h_f \tag{13}$$

With these considerations in mind, after some algebra, we get:

$$\frac{1}{2}\left(k^2 - k^4\frac{f_{P0} + 4f_{Q0}}{F_0}\right)\bar{h}_{\mu\nu} = (\eta_{\mu\nu}k^2 - k_\mu k_\nu)\frac{\delta\Phi}{F_0} + (\eta_{\mu\nu}k^2 - k_\mu k_\nu)h_f \tag{14}$$

Defining $h_f \equiv -\frac{\delta\Phi}{F_0}$, the equation for the perturbations is

$$\left(k^2 + \frac{k^4}{m_{spin2}^2}\right)\bar{h}_{\mu\nu} = 0 \tag{15}$$

[2] It is convenient to work in Fourier space so that, for example, $\partial_\gamma h_{\mu\nu} \to ik_\gamma h_{\mu\nu}$ and $\Box h_{\mu\nu} \to -k^2h_{\mu\nu}$.

where we have defined $m^2_{spin2} \equiv -\frac{F_0}{f_{P0}+4f_{Q0}}$, while from eq. (11), one obtains:

$$\Box h_f = m_s^2 h_f \tag{16}$$

From equation (15) it is easy to see that a modified dispersion relation is achieved. It corresponds to a massless spin-2 field ($k^2 = 0$) and a massive spin-2 ghost mode $k^2 = \frac{F_0}{\frac{1}{2}f_{P0}+2f_{Q0}} \equiv -m^2_{spin2}$ with mass m^2_{spin2}. To see this, note that the propagator for $\bar{h}_{\mu\nu}$ can be rewritten as

$$G(k) \propto \frac{1}{k^2} - \frac{1}{k^2 + m^2_{spin2}} \tag{17}$$

Clearly the second term has the opposite sign, which indicates the presence of a ghost mode. Also, as a sanity check, we can see that for the Gauss-Bonnet term $\mathcal{L}_{GB} = Q - 4P + R^2$ we have $f_{P0} = -4$ and $f_{Q0} = 1$. Then, eq. (15) simplifies to $k^2\bar{h}_{\mu\nu} = 0$ and, in this case, we have no ghosts as expected. The solution to eqs. (15) and (16) can be written in terms of plane waves

$$\bar{h}_{\mu\nu} \quad = \quad A_{\mu\nu}(\overrightarrow{p}) \cdot exp(ik^\alpha x_\alpha) + cc \tag{18}$$

$$h_f \quad = \quad a(\overrightarrow{p}) \cdot exp(iq^\alpha x_\alpha) + cc \tag{19}$$

where

$$\begin{aligned} k^\alpha &\equiv (\omega_{m_{spin2}}, \overrightarrow{p}), \qquad \omega_{m_{spin2}} = \sqrt{m^2_{spin2} + p^2} \\ q^\alpha &\equiv (\omega_{m_s}, \overrightarrow{p}), \qquad \omega_{m_s} = \sqrt{m_s^2 + p^2}. \end{aligned} \tag{20}$$

and where m_{spin2} is zero (non-zero) in the case of massless (massive) spin-2 mode. The polarization tensors $A_{\mu\nu}(\overrightarrow{p})$ can be found in Ref. [4]. Eqs. (15) and (18) mean that the standard waves of GR [5] can be obtained, while eqs. (16) and (19) represent further massive gravitational modes [6, 7].

3 Polarization states of gravitational waves

Considering the above equations, we can note that there are two conditions for eq. (11) that depend on the value of k^2. In fact, we have a $k^2 = 0$ mode that corresponds to a massless spin-2 field with two independent polarizations plus a scalar mode, while if we have $k^2 \neq 0$ we have a massive spin-2 ghost mode and there are five independent polarization tensors plus a scalar mode. Taking $\overrightarrow{p}$ in the z direction, a gauge where only A_{11}, A_{22}, and $A_{12} = A_{21}$ are different to zero can be chosen. The condition $\bar{h} = 0$ gives $A_{11} = -A_{22}$. In this frame we can take the bases of polarizations defined as[3]

$$e^{(+)}_{\mu\nu} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad e^{(\times)}_{\mu\nu} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$e^{(B)}_{\mu\nu} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad e^{(C)}_{\mu\nu} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$e^{(D)}_{\mu\nu} = \frac{\sqrt{2}}{3} \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix}, \qquad e^{(s)}_{\mu\nu} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and the amplitude can be written in terms of the 6 polarization states as

$$\begin{aligned} & h_{\mu\nu}(t,z) = A^{+}(t - v_{G_{s2}} z) e^{(+)}_{\mu\nu} + A^{\times}(t - v_{G_{s2}} z) e^{(\times)}_{\mu\nu} \\ & + B^{B}(t - v_{G_{s2}} z) e^{(B)}_{\mu\nu} + C^{C}(t - v_{G_{s2}} z) e^{(C)}_{\mu\nu} \\ & + D^{D}(t - v_{G_{s2}} z) e^{(D)}_{\mu\nu} + h_s(t - v_G z) e^{s}_{\mu\nu}. \end{aligned} \tag{21}$$

where $v_{G_{s2}}$ is the group velocity of the massive spin-2 field. The terms $A^{+}(t - z)e^{(+)}_{\mu\nu} + A^{\times}(t - z)e^{(\times)}_{\mu\nu}$ describe the two standard polarizations of gravitational waves which arise from GR, while the other terms arise from the generic extended models, involving any curvature invariants, that we considered here.

The first two polarizations are the same as in the massless case, inducing tidal deformations on the x-y plane. In Fig.1, we illustrate how each GW polarization affects test masses arranged on a circle.

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[3] The polarizations are defined in our 3-space, not in a spacetime with extra dimensions. Each polarization mode is orthogonal to another one and it is normalized as $e_{\mu\nu}e^{\mu\nu} = 2\delta$. Note that other modes are non-traceless, in contrast to the ordinary plus and cross polarization modes of GR.

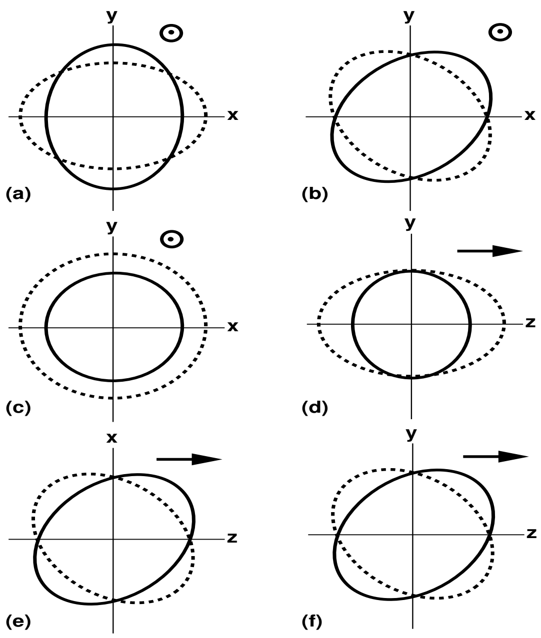

Figure 1: The six polarization modes of gravitational waves. The picture shows the displacement that each mode induces on a sphere of test particles at the moments of different phases by π. The wave propagates out of the plane in (a), (b), (c), and it propagates in the plane in (d), (e) and (f). Where in (a) and (b) we have respectively the plus mode and cross mode, in (c) the scalar mode, in (d), (e) and (f) the D, B and C mode.

4 Conclusions

We considered a generic gravitational Lagrangian with any possible combination of curvature invariants. The only assumption is that the gravitational Lagrangian is analytic. We have linearized the field equations around the Minkowski background and found that, besides the massless spin-2 field, there are also spin-0 and spin-2 massive modes with the latter being, in general, ghosts. Then, we have classified the additional polarization modes. However, a point has to be stressed. If the interferometer is directionally sensitive and we also know the orientation of the source (and of course if the source is coherent) the situation is straightforward. In this case, the massive modes would induce longitudinal displacements along the direction of propagation which should be detectable and the amplitude due to the scalar mode would be a possible "new" detectable signal [6]. The other modes should be disentangled according to particular features of the sources [7]. As a final remark, it is worth noticing that detecting further gravitational modes, besides the two standard of GR, could be a formidable challenge for gravitational physics in view to select the final theory of gravity. In this perspective, Advanced Virgo-LIGO, and the other running GW experiments should be correlated in a sort of global interferometer to investigate polarizations other than the two standard of GR.

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