Chapter 13

SUSY adjoint $SU(5)$ grand unified model with S_4 flavor symmetry

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Abstract

We construct a supersymmetric (SUSY) $SU(5)$ model with the flavor symmetry $S_4 \times Z_3 \times Z_4$. Three generations of adjoint matter fields are introduced to generate the neutrino masses via the combined type I and type III see-saw mechanism. The first two generations of the the $\mathbf{10}$ dimensional representation in $SU(5)$ are assigned to be a doublet of S_4, the second family $\mathbf{10}$ is chose as the first component of the doublet, and the first family as the second component. Tri-bimaximal mixing in the neutrino sector is predicted exactly at leading order, charged lepton mixing leads to small deviation from the tri-bimaximal mixing pattern. Subleading contributions introduce corrections of order λ_c^2 to all three lepton mixing angles. The model also reproduces a realistic pattern of quark and charged lepton masses and quark mixings.

13.1. Introduction

So far there is convincing evidence that the so-called solar and atmospheric anomaly can be well explained through the neutrino oscillation. A very good approximation for the lepton mixing matrix is provided by the so-called Tri-bimaximal (TB) pattern [1], which suggests the following values of the mixing angles

$$\sin^2\theta_{12}^{TB} = \frac{1}{3}, \quad \sin^2\theta_{23}^{TB} = \frac{1}{2}, \quad \sin\theta_{13}^{TB} = 0 \tag{13.1}$$

which agrees at about the 1σ level with the data. We note that recently new data from T2K collaboration [3] and corresponding fits [3,10] indicate a 3σ evidence of a non-vanishing θ_{13} with a relatively "large" central value. The simple structure of the TB mixing matrix suggests that there may be some symmetry underlying the lepton sector. In the past years, it is found that the flavor symmetry based on finite discrete groups particularly A_4 is particularly suitable to reproduce this constant texture. S_4 is claimed to be the minimal group which can predict the TB mixing without ad hoc assumptions, from the group theory point of view [5]. In this work, we shall present a model combining the S_4 flavor symmetry with the $SU(5)$ grand unified theory (GUT) [6].

13.2. The model

Fields	T_3	$(T_2, T_1)^T$	F	A	H_5	H_{45}	$H_{\overline{5}}$	$H_{\overline{45}}$	H_{24}	χ	φ	ζ	ϕ	η	Δ	ξ
$SU(5)$	$\mathbf{10}$	$\mathbf{10}$	$\mathbf{\overline{5}}$	$\mathbf{24}$	$\mathbf{5}$	$\mathbf{45}$	$\mathbf{\overline{5}}$	$\mathbf{\overline{45}}$	$\mathbf{24}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
S_4	$\mathbf{1_1}$	$\mathbf{2}$	$\mathbf{3_1}$	$\mathbf{3_1}$	$\mathbf{1_1}$	$\mathbf{1_1}$	$\mathbf{1_1}$	$\mathbf{1_1}$	$\mathbf{1_1}$	$\mathbf{3_1}$	$\mathbf{2}$	$\mathbf{1_2}$	$\mathbf{3_1}$	$\mathbf{2}$	$\mathbf{3_1}$	$\mathbf{1_1}$
Z_3	1	ω	1	1	1	1	ω	1	1	1	1	$\mathbf{1}$	ω^2	ω^2	ω	ω
Z_4	1	i	$-i$	i	1	-1	1	-1	1	-1	-1	-1	i	i	i	i
$U(1)_R$	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0

Table 13.1
Fields and their transformation properties under the symmetry groups $SU(5)$, S_4, Z_3 and Z_4, where $\omega = e^{i2\pi/3} = (-1+i\sqrt{3})/2$.

The flavor symmetry group of the model is $S_4 \times Z_3 \times Z_4$, where the auxiliary symmetry $Z_3 \times Z_4$ plays an important role in eliminating unwanted couplings, ensuring the needed vacuum alignment and reproducing the observed fermion mass hierarchies. We introduce three generations chiral superfields A in the adjoint $\mathbf{24}$ representation in addition to $\mathbf{\overline{5}}$ matter fields denoted by F and the tenplet $\mathbf{10}$ dimensional matter fields denoted by $T_{1,2,3}$. The neutrino masses are generated through the combination of type I and type III see-saw mechanism in the present model. In the Higgs sector H_{24}, H_5, $H_{\overline{5}}$, H_{45} and $H_{\overline{45}}$ are included. Moreover, flavon fields are introduced to spontaneously break the S_4 flavor symmetry. The transformation properties of all the fields under $SU(5)$, S_4, Z_3 and Z_4 are summarized in Table 13.1. We note that the first two generations of the tenplet are assigned to be a doublet of S_4, the second family $\mathbf{10}$ is taken to be the first component of the doublet, and the first family as the second component. If we reverse the assignment, the down quark and strange quark masses would be of the same order without fine tuning unless some special mechanisms are introduced such as Ref. [7]. Using the standard driving field method, we can show that the scalar components of the flavon fields acquire vacuum expectation values (VEV) according to the following scheme [6],

$$\langle\chi\rangle = \begin{pmatrix} v_\chi \\ v_\chi \\ v_\chi \end{pmatrix}, \quad \langle\varphi\rangle = \begin{pmatrix} v_\varphi \\ v_\varphi \end{pmatrix}, \quad \langle\zeta\rangle = 0, \quad \langle\phi\rangle = \begin{pmatrix} 0 \\ v_\phi \\ 0 \end{pmatrix}, \quad \langle\eta\rangle = \begin{pmatrix} 0 \\ v_\eta \end{pmatrix},$$
$$\langle\Delta\rangle = \begin{pmatrix} v_\Delta \\ 0 \\ 0 \end{pmatrix}, \qquad \langle\xi\rangle = v_\xi \tag{13.2}$$

In order to produce the observed ratios of up quarks and down quarks and charged lepton masses, the VEVs (scaled by the cutoff Λ) v_χ/Λ, v_φ/Λ, v_ϕ/Λ, v_η/Λ, v_Δ/Λ and v_ξ/Λ should be of the same order of magnitude about λ_c^2 with $\lambda_c \simeq 0.22$ being the Cabibbo angle, and we will parameterize the ratio VEV/Λ by the parameter ε.

13.2.1. Neutrino sector

The LO superpotential which contributes to the neutrino masses is given by

$$w_\nu = y_\nu (FA)_{1_1} H_5 + \lambda_1 (AA)_{3_1}\chi + \lambda_2 (AA)_2 \varphi \tag{13.3}$$

It is well-known that there are both $SU(2)$ triplet ρ_3 and singlet ρ_0 with hypercharge $Y=0$ in the decomposition of the adjoint matter field A with respect to the standard model. From Eq.(13.3) the neutrino Dirac mass matrices read as

$$M^D_{\rho_3}=\frac{1}{2}y_\nu v_5\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix},\quad M^D_{\rho_0}=\frac{\sqrt{15}}{10}y_\nu v_5\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} \tag{13.4}$$

The last two terms in Eq.(13.3) lead to the Majorana mass matrices of ρ_3 and ρ_0

$$M^M_{\rho_3}=\begin{pmatrix}2\lambda_1 v_\chi & -\lambda_1 v_\chi+\lambda_2 v_\varphi & -\lambda_1 v_\chi+\lambda_2 v_\varphi\\ -\lambda_1 v_\chi+\lambda_2 v_\varphi & 2\lambda_1 v_\chi+\lambda_2 v_\varphi & -\lambda_1 v_\chi\\ -\lambda_1 v_\chi+\lambda_2 v_\varphi & -\lambda_1 v_\chi & 2\lambda_1 v_\chi+\lambda_2 v_\varphi\end{pmatrix},\quad M^M_{\rho_0}=M^M_{\rho_3} \tag{13.5}$$

The light neutrino mass matrix is the sum of type I and type III see-saw contributions

$$\begin{aligned}M_\nu &= -(M^D_{\rho 3})^T(M^M_{\rho_3})^{-1}M^D_{\rho_3}-(M^D_{\rho 0})^T(M^M_{\rho_0})^{-1}M^D_{\rho_0}\\ &= \begin{pmatrix}\frac{-a-b}{5b(3a-b)} & \frac{-a+b}{5b(3a-b)} & \frac{-a+b}{5b(3a-b)}\\ \frac{-a+b}{5b(3a-b)} & \frac{-3a^2-4ab+b^2}{5b(9a^2-b^2)} & \frac{-3a^2+2ab-b^2}{5b(9a^2-b^2)}\\ \frac{-a+b}{5b(3a-b)} & \frac{-3a^2+2ab-b^2}{5b(9a^2-b^2)} & \frac{-3a^2-4ab+b^2}{5b(9a^2-b^2)}\end{pmatrix}y^2_\nu v^2_5\end{aligned} \tag{13.6}$$

where $a\equiv\lambda_1 v_\chi$ and $b\equiv\lambda_2 v_\varphi$. This light neutrino mass matrix M_ν is exactly diagonalized by the TB mixing matrix

$$U^T_\nu M_\nu U_\nu=\mathrm{diag}(m_1,m_2,m_3) \tag{13.7}$$

where $m_{1,2,3}$ are the light neutrino masses, in unit of $\frac{2}{5}y^2_\nu v^2_5$ they are

$$m_1=\frac{1}{|3a-b|},\quad m_2=\frac{1}{2|b|},\quad m_3=\frac{1}{|3a+b|} \tag{13.8}$$

The unitary matrix U_ν is given by

$$U_\nu=U_{TB}\,\mathrm{diag}(e^{-i\alpha_1/2},e^{-i\alpha_2/2},e^{-i\alpha_3/2}) \tag{13.9}$$

U_{TB} is the well-known TB mixing matrix

$$U_{TB}=\begin{pmatrix}\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}\end{pmatrix} \tag{13.10}$$

The phases α_1, α_2 and α_3 are

$$\alpha_1=\arg(-y^2_\nu v^2_5/(3a-b)),\quad \alpha_2=\arg(-y^2_\nu v^2_5/b),\quad \alpha_3=\arg(-y^2_\nu v^2_5/(3a+b)) \tag{13.11}$$

The light neutrino mass spectrum can be both normal hierarchy and inverted hierarchy. Taking into account the experimentally measured mass square differences Δm^2_{sol} and Δm^2_{atm}, we obtain the following constraints on the lightest neutrino mass

$$\begin{aligned}&m_1\geq 0.011\mathrm{eV},\quad \text{for normal hierarchy}\\ &m_3\geq 0.028\mathrm{eV},\quad \text{for inverted hierarchy}\end{aligned} \tag{13.12}$$

13.2.2. Charged leptons and quark sector

The LO superpotential giving rise to the masses of the up type quarks after S_4 and $SU(5)$ symmetry breaking, is given by

$$\begin{aligned} w_u &= y_t T_3 T_3 H_5 + \sum_{i=1}^{4} \frac{y_{ci}}{\Lambda^2} TT\mathcal{O}_i^{(1)} H_5 + \frac{y_{ut1}}{\Lambda^2} TT_3(\phi\chi)_2 H_5 + \frac{y_{ut2}}{\Lambda^2} TT_3(\eta\varphi)_2 H_5 \\ &\quad + \frac{y_{ut3}}{\Lambda^2} TT_3\eta\zeta H_5 + \frac{y_{ct}}{\Lambda} TT_3\eta H_{45} \end{aligned} \quad (13.13)$$

with $\mathcal{O}^{(1)} = \{(\phi\phi)_{1_1}, (\phi\phi)_2, (\eta\eta)_{1_1}, (\eta\eta)_2\}$. The superpotential generating the masses of down quarks and charged lepton is

$$\begin{aligned} w_d &= \frac{y_b}{\Lambda} T_3 F\phi H_{\overline{5}} + \frac{y_{s1}}{\Lambda^2}(TF)_{3_1}(\Delta\Delta)_{3_1} H_{\overline{45}} + \frac{y_{s2}}{\Lambda^2}(TF)_{3_1}\Delta\xi H_{\overline{45}} + \sum_{i=1}^{9} \frac{y_{di}}{\Lambda^3} T_3 F\mathcal{O}_i^{(2)} H_{\overline{5}} \\ &\quad + \sum_{i=1}^{6} \frac{x_{di}}{\Lambda^3} T_3 F\mathcal{O}_i^{(3)} H_{\overline{45}} + \sum_{i=1}^{7} \frac{z_{di}}{\Lambda^3} TF\mathcal{O}_i^{(4)} H_{\overline{5}} + \ldots \end{aligned} \quad (13.14)$$

where dots stand for higher dimensional operators.

$$\begin{aligned} &\mathcal{O}^{(2)} = \{\chi^2\phi, \chi^2\eta, \varphi\chi\phi, \varphi\chi\eta, \varphi^2\phi, \chi\phi\zeta, \chi\eta\zeta, \varphi\phi\zeta, \phi\zeta^2\} \\ &\mathcal{O}^{(3)} = \{\phi^3, \phi^2\eta, \phi\eta^2, \Delta^3, \Delta^2\xi, \Delta\xi^2\} \\ &\mathcal{O}^{(4)} = \{\phi^2\chi, \phi^2\varphi, \phi^2\zeta, \eta\phi\chi, \eta\phi\varphi, \eta\phi\zeta, \eta^2\chi\} \end{aligned} \quad (13.15)$$

With the vacuum alignment in Eq.(13.2), we can straightforwardly derive the mass matrix as follows

$$\begin{aligned} M_u &= \begin{pmatrix} 0 & 0 & 4(y_{ut1}\frac{v_\phi v_\chi}{\Lambda^2} + y_{ut2}\frac{v_\eta v_\varphi}{\Lambda^2})v_5 \\ 0 & 8(y_{c2}\frac{v_\phi^2}{\Lambda^2} + y_{c4}\frac{v_\eta^2}{\Lambda^2})v_5 & 8y_{ct}\frac{v_\eta}{\Lambda}v_{45} + 4y_{ut1}\frac{v_\phi v_\chi}{\Lambda^2}v_5 \\ 4(y_{ut1}\frac{v_\phi v_\chi}{\Lambda^2} + y_{ut2}\frac{v_\eta v_\varphi}{\Lambda^2})v_5 & -8y_{ct}\frac{v_\eta}{\Lambda}v_{45} + 4y_{ut1}\frac{v_\phi v_\chi}{\Lambda^2}v_5 & 8y_t v_5 \end{pmatrix} \\ M_d &= \begin{pmatrix} y_{11}^d\varepsilon^3 v_{\overline{5}} & y_{12}^d\varepsilon^3 v_{\overline{5}} & y_{13}^d\varepsilon^3 v_{\overline{5}} + 2y_{13}^{d'}\varepsilon^3 v_{\overline{45}} \\ y_{21}^d\varepsilon^3 v_{\overline{5}} & 2y_{22}^d\varepsilon^2 v_{\overline{45}} + y_{22}^{d'}\varepsilon^3 v_{\overline{5}} & y_{23}^d\varepsilon^3 v_{\overline{5}} \\ 2y_{22}^d\varepsilon^2 v_{\overline{45}} + y_{31}^{d'}\varepsilon^3 v_{\overline{5}} & y_{32}^d\varepsilon^3 v_{\overline{5}} & y_{33}^d\varepsilon v_{\overline{5}} \end{pmatrix} \\ M_\ell &= \begin{pmatrix} y_{11}^d\varepsilon^3 v_{\overline{5}} & y_{21}^d\varepsilon^3 v_{\overline{5}} & -6y_{22}^d\varepsilon^2 v_{\overline{45}} + y_{31}^{d'}\varepsilon^3 v_{\overline{5}} \\ y_{12}^d\varepsilon^3 v_{\overline{5}} & -6y_{22}^d\varepsilon^2 v_{\overline{45}} + y_{22}^{d'}\varepsilon^3 v_{\overline{5}} & y_{32}^d\varepsilon^3 v_{\overline{5}} \\ y_{13}^d\varepsilon^3 v_{\overline{5}} - 6y_{13}^{d'}\varepsilon^3 v_{\overline{45}} & y_{23}^d\varepsilon^3 v_{\overline{5}} & y_{33}^d\varepsilon v_{\overline{5}} \end{pmatrix} \end{aligned} \quad (13.16)$$

where the factor of 3 difference in the (13), (22) and (31) elements between M_d and M_ℓ is the so-called Georgi-Jarlskog factor [10], which is induced by the Higgs $H_{\overline{45}}$. Diagonalizing these mass matrices, we find that the CKM matrix elements are as follows

$$\begin{aligned} &V_{ud} \simeq V_{cs} \simeq V_{tb} \simeq 1 \\ &V_{us}^* \simeq -V_{cd} \simeq \frac{y_{21}^d}{2y_{22}^d}\frac{v_{\overline{5}}}{v_{\overline{45}}}\varepsilon + \frac{1}{2}\frac{y_{ct}(y_{ut1}v_\phi v_\chi + y_{ut2}v_\eta v_\varphi)v_5 v_{45}}{y_t(y_{c2}v_\phi^2 + y_{c4}v_\eta^2)v_5^2 + y_{ct}^2 v_\eta^2 v_{45}^2}\frac{v_\eta}{\Lambda} \\ &V_{ub}^* = 2\frac{y_{22}^d}{y_{33}^d}\frac{v_{\overline{45}}}{v_{\overline{5}}}\varepsilon + \frac{y_{31}^{d'}}{y_{33}^d}\varepsilon^2 - \frac{y_{ut1}}{2y_t}\frac{v_\phi v_\chi}{\Lambda^2} - \frac{y_{ut2}}{2y_t}\frac{v_\eta v_\varphi}{\Lambda^2} + \frac{1}{2}\frac{y_{ct}^2(y_{ut1}v_\phi v_\chi + y_{ut2}v_\eta v_\varphi)v_{45}^2}{y_t^2(y_{c2}v_\phi^2 + y_{c4}v_\eta^2)v_5^2 + y_t y_{ct}^2 v_\eta^2 v_{45}^2}\frac{v_\eta^2}{\Lambda^2} \\ &V_{cb}^* \simeq -V_{ts} \simeq \frac{y_{ct}v_{45}}{y_t v_5}\frac{v_\eta}{\Lambda} \\ &V_{td} = -2\frac{y_{22}^d}{y_{33}^d}\frac{v_{\overline{45}}}{v_{\overline{5}}}\varepsilon - \frac{y_{31}^{d'}}{y_{33}^d}\varepsilon^2 + \frac{y_{ut1}}{2y_t}\frac{v_\phi v_\chi}{\Lambda^2} + \frac{y_{ut2}}{2y_t}\frac{v_\eta v_\varphi}{\Lambda^2} + \frac{y_{ct}y_{21}^d}{2y_t y_{22}^d}\frac{v_{45}}{v_5}\frac{v_{\overline{5}}}{v_{\overline{45}}}\frac{v_\eta}{\Lambda}\varepsilon \end{aligned} \quad (13.17)$$

In order to produce the Cabibbo mixing angle between the first and the second family, for the parameters y_{21}^d and y_{22}^d of order $\mathcal{O}(1)$ we could choose $v_{\overline{45}} \sim \lambda_c v_{\overline{5}}$. We note that the observed mass hierarchies of quarks and charged lepton are produced. Moreover, we find the following relations between down quarks and charged lepton masses

$$m_\tau \simeq m_b, \quad m_\mu \simeq 3m_s \tag{13.18}$$

These are the well-known bottom-tau unification and the Georgi-Jarlskog relation [10] respectively. Taking into the non-trivial mixing in the charged lepton sector, the lepton mixing angles are given by

$$\sin\theta_{13} =\simeq \Big|\frac{y_{12}^d}{6\sqrt{2}y_{22}^d}\frac{v_{\overline{5}}}{v_{\overline{45}}}\varepsilon\Big|, \ \sin^2\theta_{12} \simeq \frac{1}{3} + \frac{1}{18}\Big[\frac{y_{12}^d}{y_{22}^d}\frac{v_{\overline{5}}}{v_{\overline{45}}}\varepsilon + (\frac{y_{12}^d}{y_{22}^d}\frac{v_{\overline{5}}}{v_{\overline{45}}}\varepsilon)^*\Big], \ \sin\theta_{23}^2 \simeq \frac{1}{2} + \frac{1}{144}\Big|\frac{y_{12}^d}{y_{22}^d}\frac{v_{\overline{5}}}{v_{\overline{45}}}\varepsilon\Big|^2$$

Taking into account the results for quark mixing shown in Eq.(13.17), we have $|V_{us}| \simeq |\frac{y_{21}^d}{2y_{22}^d}\frac{v_{\overline{5}}}{v_{\overline{45}}}\varepsilon| \sim \lambda_c$. As a result, the model predicts the deviation of the lepton mixing from the TB pattern as follows

$$\sin\theta_{13} \sim \frac{\lambda_c}{3\sqrt{2}} \simeq 2.97^\circ, \quad |\sin^2\theta_{12} - \frac{1}{3}| \sim \frac{2}{9}\lambda_c, \quad |\sin^2\theta_{23} - \frac{1}{2}| \sim \frac{\lambda_c^2}{36} \tag{13.19}$$

The lepton mixing angles are predicted to be in agreement at 3σ error with the experimental data. It is remarkable that Eq.(13.19) belongs to a set of well-known leptonic mixing sum rules [9], and the same results have been obtained in Ref.[7]. The above LO predictions for the fermion masses and flavor mixing patterns are correction by the next to leading order high dimensional operators allowed by the symmetry of the model. Detailed analysis has shown that the successful LO predictions for the order of magnitudes of both the CKM matrix elements and the quark masses are not spoiled by the subleading corrections, and all the three leptonic mixing angles receive corrections of λ_c^2 [6], they are still compatible with the current experimental data.

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