# The microscopic black hole production at the LHC with the CMS experiment

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Abstract. CMS (Compact Muon Solenoid) is one of the two general-purpose detectors at the LHC. The physics programme of the experiment ranges from checking the Standard Model to searching for a new physics. One of the most interesting directions in the new physics is the extra dimensions. In modern theoretical ideas related to the concept of extra dimensions and multidimensional low-energy gravity, one can expect to observe the creation of the microscopic black holes at the LHC in proton-proton collisions at currently available energies.

#### 1. Introduction

The Standard Model (SM) is an exceptionally successful theory that completely describes the experimental data in particle physics at energies up to several TeV. Being low-energy effective quantum field theory, it describes three fundamental interactions (strong, weak, electromagnetic), but does not include gravity. At very high energies, it is expected that the intensity of all four fundamental interactions will be of the same order, and it becomes possible to combine all interactions into one. The Planck energy (the fundamental four-dimensional scale of gravity), at which gravity becomes strong, is  $10^{19}$  GeV, which is 17 orders of magnitude higher than the electroweak scale (246 GeV). The divergence of scales by such a large order is known as the hierarchy problem [1]. From a technical point of view, the hierarchy problem arises from the small Higgs boson mass compared to the Planck scale. In fact, the large quantum corrections to the squared Higgs boson mass would inevitably make this mass enormous (of the order of  $M_{Pl}$ , if not the fine-tuning used in the Standard Model. There are numbers of theories which try to solve the hierarchy problem, among them the most preferential ones are supersymmetry and extra dimensions.

In this article, we will consider the most interesting consequences for a class of models in which, by introducing n extra dimensions, one can obtain strong gravity in (4+n) -dimensional space and make a fundamental multidimensional scale of gravity  $M_D$  of the order of electroweak scale. The main two classes of models discussed here are the Arkani-Hamed, Dimopoulos and Dvali (ADD) model [2] and the Randall–Sundrum type 1 (RS1) model [3]. In these theoretical models, scenarios with three-dimensional branes, where SM fields are localised, embedded in a full multidimensional volume called 'bulk', and with the possibility for a graviton to be in a multidimensional volume, are considered.

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Since the fundamental scale of gravity  $M_D$  is of the order of TeV, in the models of TeV-scale gravity in the region above this scale the production of the microscopic multidimensional black holes is theoretically possible. In particular, such objects can be observed in cosmic rays and at the LHC. Before the launch of the LHC lower limits on the minimum mass of the black holes were available only from experiments on high energy cosmic neutrino scattering in the Earth's atmosphere (by the lack of the neutrino flux due to the reaction  $\nu N \rightarrow BH$ ). The fundamental scale of gravity  $M_D$  (for  $M_D \leq M_{min}^{BH} \leq 3M_D$ ) was limited by values 1.0 - 1.4 TeV for the number of extra dimensions n > 4 [4]. The searches during the first and second stages of the LHC operation (Run 1 and Run 2) in pp collisions at a center-of-mass energies 7, 8 and 13 TeV with the data sample corresponding to the integrated luminosities of 4.7, 12 and 36 fb<sup>-1</sup> recorded with CMS experiment excluded black holes with masses below 3.8 to 5.3 TeV, 4.3 to 6.2 TeV and below 10 TeV respectively [5]–[11]. These studies will be ongoing for the next 2 years with the analysis of the full Run 2 LHC data and the the physical modeling of the microscopic black hole production necessary for comparison with the experimental data is described in the current work.

## 2. Black holes in TeV-scale gravity

#### 2.1. TeV-scale gravity models: ADD and RS1

The ADD model [2] contains n additional spatial dimensions, and the geometry of the full (4+n) multidimensional space is flat. In this construction, extra dimensions should be compact to avoid contradictions with the observed four-dimensional picture of the world, but they can still be quite large compared to the Planck length. In modern experiments at accelerators, strong and electroweak processes have been tested up to 1 - 3 TeV (according to LHC data), so the energy scale of a new physics should be greater than these values. In the effective description of ADD the brane is considered to be rigid up to an energy of about  $M_D$ , that is, having no fluctuations and having zero tension.

Planck's mass, which is the fundamental scale of gravity in the case of a 4D-space, in the multidimensional theory becomes a derived value, rather than a fundamental value, associated with a new, multidimensional gravity scale  $M_D$  of the order of TeV by relation [2]

$$M_{Pl} = M_D^{1+n/2} R^{n/2}.$$
 (1)

Here n is the number of extra dimensions, R is a compactification radius.

The compactification radius of extra dimensions is not an independent parameter of the model; it can be expressed in terms of a single control parameter, the multidimensional scale  $M_D$ , and the number of extra dimensions:

$$R \approx M_D^{-1} \times (M_{Pl}/M_D)^{2/n} \approx 10^{32/n} \times 10^{17} \ cm.$$
 (2)

For the geometrical implementation of the flavour mixing, not one but usually several spatially separated packages of branes are considered, so that the fermions of different flavours sit on different branes of the package, and their mixing is given by small distances between branes of the same package.

Unlike the ADD model, the RS1 [3] scenario considers only one extra dimension of the Planck size. The full five-dimensional space is an anti-de Sitter space  $AdS_5$  constrained by two branes. Two branes are located at a finite distance from each other (RS1) or are divided into infinity (RS2), and one of them reproduces our world with the fields of the Standard Model, and the second — the hidden sector from which interactions can be transmitted.

2.2. Kerr-Newman solution for neutral rotating black holes in a flat multidimensional space Let's consider multidimensional space-time of dimension D = d + 1 = 4 + n, where d is the number of spatial dimensions, n is the number of compactified extra dimensions. In a flat (d+1)-dimensional space there is analogue of Kerr-Newman solution, which in Boyer-Lindquist coordinates for the rotating neutral black hole with angular momentum J looks like:

$$ds^{2} = \left(1 - \frac{\mu r^{1-n}}{\Sigma(r,\Theta)}\right) dt^{2} - \sin^{2}\Theta \left(r^{2} + a^{2} \left(1 + \sin^{2}\Theta \frac{\mu r^{1-n}}{\Sigma(r,\Theta)}\right)\right) d\phi^{2} + 2asin^{2}\Theta \frac{\mu r^{1-n}}{\Sigma(r,\Theta)} dt d\phi - \frac{\Sigma(r,\Theta)}{\Delta} dr^{2} - \Sigma(r,\Theta) d\Theta^{2} - r^{2}cos^{2}\Theta d^{n}\Omega.$$
(3)

## 2.3. Formation, evolution and characteristics of black holes

Since gravity in models like ADD and RS1 becomes strong at a TeV-scale, above these value we can expect the formation of microscopic multidimensional black holes in proton-proton collisions.

According to the Thorne's hoop conjecture [12], two colliding partons with center of mass energy  $E_{cm} = \sqrt{s}$  can form microscopic multidimensional black hole, if impact parameter is less than horizon of *D*-dimensional black hole of mass  $M = E_{cm}$ :

$$b < 2r_h(n, M, J),\tag{4}$$

where  $r_h$  is BH radius, which depends on the dimension of the space, the BH mass and its angular momentum J. It is necessary to take into account the fact that the black hole event horizon must be less than the compactification radius of the extra dimensions in order for the object to be treated as multidimensional (the black hole should be in the extra dimensions).

The simplest estimate of the probability of the Schwarzschild black hole formation for a cross section gives a geometric value – 'black disk' with an area of  $\pi r_s^2$  [13], where the Schwarzschild radius is defined by the expression [13] :

$$r_s = \frac{1}{\sqrt{\pi}M_D} \left( \frac{M_{BH}}{M_D} \left( \frac{8\Gamma\left(\frac{n+3}{2}\right)}{n+2} \right) \right)^{\frac{1}{n+1}}.$$
(5)

The evolution of the microscopic multidimensional black hole, like its astronomical fourdimensional analogue, can be divided into the following stages:

- 'balding phase', where black hole sheds its asymmetries through the emission of gravitational radiation and also loses any gauge field charges arising from the particles which formed it;
- 'spin-down phase', where black hole loses both mass and angular momentum, at the end is no longer rotating;
- 'Swarzschild stage', in which the radiation is continued by the Hawking mechanism with regard to spin-dependent corrections known as 'grey-body factors';
- 'final Planck stage', where the effects of quantum gravity should appear and model approaches are needed to describe the final states.

In order for a black hole to be interpreted as a classical object, it must have a sufficiently large mass  $M_{BH} \gg M_D$ . The criterion for the semiclassicality of this object is entropy, and for its sufficiently large values, a black hole can really be considered as a classical object radiating by the Hawking mechanism. The entropy and Hawking temperature of a microscopic black hole II Trans-Siberian School on High Energy Physics

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is given by the following relations [13]:

$$S_{BH} = \frac{4\pi}{n+2} \frac{M_B H}{M_D} \frac{\frac{n+2}{n+1}}{\left(\frac{2^n \pi^{\frac{n-3}{2}} \Gamma\left(\frac{n+3}{2}\right)}{n+2}\right)^{\frac{1}{n+1}}}{n+2} = \frac{1+n}{2+n} \frac{M_{BH}}{T_H},$$
(6)

$$T_H = M_D \left(\frac{M_D}{M_{BH}} \frac{n+2}{8\Gamma\left(\frac{n+3}{2}\right)}\right)^{\frac{1}{n+1}} \frac{n+1}{4\sqrt{\pi}} = \frac{n+1}{4\pi r_S}.$$
(7)

In order to treat the microscopic black hole as a semiclassical object the value of the entropy is assumed to be  $S_{BH} > 25$  for the case of the LHC. This assumption sets the lower black hole mass value to  $M_{min} = 5M_D$  for the ADD model. Typical Hawking temperatures for decay particles from a microscopic black hole at the LHC are  $T_H \sim 200 - 500$  GeV. Hawking temperature will grow at each next radiation step until it reaches the value of the fundamental multidimensional scale of gravity  $M_D$ . However, at such small masses of the black holes (masses are only several times larger than  $M_D$  under LHC conditions), simulations of the processess of Hawking radiation in generators are usually made with the assumption that all radiation acts occur simultaneously, so that a microscopic black hole decays simultaneously and almost immediately after formation.

#### 2.4. Black hole production cross section at colliders

If the elementary (at the parton level) cross section for the production of black holes is determined by the geometrical approximation as a black disk area, and the initial interaction energy  $M_{BH}(z) = \sqrt{\hat{s}}$  is trapped under the horizon, then in *pp*-collisions the differential cross-section in the leading order approximation can be written as a convolution of the parton densities of the particles involved in the hard process with this elementary cross section [14]:

$$\frac{d\sigma(pp \to BH + X)}{dM_{BH}} = \frac{dL}{dM_{BH}}\hat{\sigma}(ij \to BH)|_{\hat{s}=M_{BH}^2},\tag{8}$$

where 'differential luminosity' is expressed as follows:

$$\frac{dL}{dM_{BH}} = \frac{2M_{BH}}{s} \sum_{i,j} \int_{M_{BH}^2/s}^1 \frac{dx_i}{x_i} f_i(x) f_j(\frac{M_{BH}^2}{sx_i}).$$
(9)

Here,  $\sqrt{s}$  is the energy of the colliding protons in center-of-mass frame,  $\hat{s} = x_i x_j s$ ,  $x_{i,j}$  are fractions of the collision energy carried away by the partons *i* and *j*, the sum is taken for all types of initial partons. It is also possible that the part of the initial collision energy leak away during the horison formation. This effect is known as the loss by the Yoshino–Rychkov mechanism [15]. If the fraction of energy leaving the horizon is called the 'inelasticity coefficient' y(z), which depends on  $z = b/b_{max}$ , here *b* is the impact parameter of the collision,  $b_{max}$  corresponds to the radius of the apparent horizon in the Yoshino-Rychkov method, and always  $z \leq 1$ , then taking into account the Yoshino–Rychkov loss,  $M_{BH}$  is written as

$$M_{BH}(z) = y(z)\sqrt{\hat{s}}.$$
(10)

Due to the losses the cross section 8 is modified as:

$$\sigma^{pp}(\sqrt{\hat{s}}, x_{min}, n, M_D) = \int_0^1 2z dz \int_{\frac{x_{min}M_D^2}{y^{2s}}}^1 du \times \int_u^1 \frac{dv}{v} f(n) \times \pi r_s^2(u\sqrt{\hat{s}}, n, M_D) \times \sum_{i,j}^N f_i(v, Q^2) f_j(u/v, Q^2).$$
(11)

Here  $x_{min} = M_{min}^{BH}/M_D$ , *n* is the number of extra dimensions,  $\nu$  and *u* are fractions of momentum in PDF and f(n) is defined as:

$$f(n) = \left[2^n \pi^{(n-3)/2} \frac{\Gamma\left((n+3)/2\right)}{n+2}\right]^{1/(n+1)}$$
(12)

## 3. Results of the simulations of the microscopic black hole production

In order to study the production of the microscopic black holes, first of all we need to simulate signal samples of the physics we are interested in. For that purpose the black hole event generators with built-in parametrizations of matrix elements BlackMax [16] and Charybdis2 [17] were used. These generators contain all the above stages of the black hole evolution and acts out the formation of black holes in accordance with formulas 8 or 11 in case of the processes with losses by Yoshino-Rychkov mechanism and setting the percentage of losses manually. The black hole formation was simulated with the set of parton distribution functions (PDF) MSTW2008lo68cl [18].



Figure 1. (a) The cross section vs  $\sqrt{s}$  plot of the rotating black holes without losses, simulated in Charybdis2 (solid lines) and BlackMax (dash-dotted lines) for the value of fundamental scale  $M_D = 2$  TeV, minimum mass of black hole  $M_{min} = 4$  TeV and the number of extra dimensions n = 2, 4, 6 (from bottom to top). (b) The cross section vs  $M_{min}^{BH}$  plot, simulated in Charybdis2, for the rotating black holes without losses (C1) and with losses (C4) for the value of fundamental scale  $M_D = 4$  TeV and the number of extra dimensions n = 4 at  $\sqrt{s} = 13$  TeV.

The cross section in dependence of the collision energy  $\sqrt{s}$  of the rotating black holes, simulated in both – BlackMax and Charybdis2 for the value of fundamental scale  $M_D = 2$  TeV, minimum mass of the black hole  $M_{min} = 4$  TeV and the number of extra dimensions n = 2, 4, 6 is shown in Fig. 1,*a*. The difference in the values of cross sections simulated in BlackMax and Charybdis2 for the same type of black holes makes up  $\approx 30 - 50\%$ . The cross section gets larger values with the increase of the energy and the number of the extra dimensions n.

The basic scenarios are slightly differently implemented in BlackMax and Charybdis2. The cross section in Charybdis2 is defined as  $\pi r_S^2$ , where  $r_S$  is the Schwarzschild radius, and in BlackMax as  $\pi r_h^2$ , where  $r_h$  is gravitational radius. As  $r_h$  depends on the number of extra dimensions and for n > 2 is always greater than  $r_S$ , the BlackMax returns greater values of the cross section. These generators allow to realize the same scenarios, but also offer possibilities to generate different scenarios, for example various options in the final stage, such as the 'non-observable stable residual' that does not interact with the detector system or 'boiling model' – near-threshold transition of a black hole into a string ball and complicated models with



Figure 2. (a) The cross section vs  $M_{min}^{BH}$  plot, simulated in BlackMax, for the rotating black holes without losses and with fractional losses as 10 %, 15 %, 20 % and 25 % of the initial mass, momentum and angular momentum and with Yoshino-Rychkov mechanism for the value of fundamental scale  $M_D = 2$  TeV and the number of extra dimensions n = 4 at  $\sqrt{s} = 13$  TeV (from top to bottom). (b) The ratio of the cross sections of the rotating black holes with fractional losses as 10 %, 15 %, 20 % and 25 % of the initial mass, momentum and angular momentum and with Yoshino-Rychkov mechanism to the cross section of the rotating black holes without losses vs  $M_{min}^{BH}$  plot, simulated in BlackMax, for the value of fundamental scale  $M_D = 2$  TeV and the number of extra dimensions n = 4 at  $\sqrt{s} = 13$  TeV (from top to bottom).

'packages' of branes at different points in the multidimensional space or manual setting of losses during formation. So the scenarios are separately predicted and compared separately with the experiment.

Fig. 1(b) presents the cross section dependence on minimum mass of the black hole  $M_{min}^{BH}$ , simulated in Charybdis2, for the rotating black holes without losses (C1) and with losses (C4) for the value of fundamental scale  $M_D = 4$  TeV and the number of extra dimensions n = 4. The losses are more significant for the black hole with large masses.

The cross section in dependence of the minimum mass of the black hole  $M_{min}^{BH}$ , simulated in BlackMax, for the rotating black holes without losses and with fractional losses as 10 %, 15 %, 20 % and 25 % of the initial mass, momentum and angular momentum and with Yoshino-Rychkov mechanism for the value of fundamental scale  $M_D = 2$  TeV and the number of extra dimensions n = 4 is shown in Fig. 2,a. It is easy to notice that the largest losses correspond to the simulation by the Yoshino-Rychkov mechanism, and the losses described in this way always exceed considerably 25% of the initial energy of the collisions. The ratio of the cross sections of the rotating black holes with different type of losses mentioned above to the rotating black holes without losses as a function of the minimum mass of the black hole  $M_{min}^B{}^H$  is presented in Fig. 2,b and demonstrates the importance of taking losses into account in the modeling of the black hole events. The black hole production cross section calculated with losses by the Yoshino-Rychkov mechanism may differ from the processes without losses by several orders of magnitude. The effects of losses during the formation is dominant in calculating and numerical modeling of cross sections.

The cross section in dependence of the minimum mass of black hole  $M_{min}^{BH}$ , simulated in BlackMax, for different set of fundamental scale  $M_D = 2 - 9$  TeV and the number of extra dimensions n = 6 for the rotating black holes without losses is shown in Fig. 3,*a*. The same plot for the rotating black holes with Yoshino-Rychkov losses, simulated in Charybdis2, is shown in Fig. 3,*b*.



Figure 3. (a) The cross section vs  $M_{min}^{BH}$  plot, simulated in BlackMax, for the rotating black holes without losses for the value of fundamental scale  $M_D = 2 - 9$  TeV (from top to bottom) and the number of extra dimensions n = 6 at  $\sqrt{s} = 13$  TeV. (b) The cross section vs  $M_{min}^{BH}$  plot, simulated in Charybdis2, for the rotating black holes with Yoshino-Rychkov losses for the value of fundamental scale  $M_D = 2 - 9$  TeV (from top to bottom) and the number of extra dimensions n = 6 at  $\sqrt{s} = 13$  TeV.

With the increase of the minimum mass of a black hole  $M_{min}^{BH}$ , the cross section decreases significantly. This is due to the fact that the PDF decreases very rapidly with increasing energy (the transferred four-momentum), and the production of more massive objects is strongly suppressed. With the luminosity and statistics of the collected data on the first and second stages of the LHC (Run 1 and Run 2)<sup>-1</sup>, the observation of the processes of the microscopic black holes production with cross sections up to 0.1 fb is theoretically possible. The current studies of the CMS experiment show that such objects remain hypothetical [11].

## 4. Conclusions

One way to eliminate the hierarchy of scales is to solve this problem in a geometrical way – by introducing extra dimensions. Currently, the main two classes of models are ADD and RS1 with the concept of three-dimensional branes with the Standard Model fields are localised on it and a multidimensional volume, where only gravitons can propagate. In this approach, the multidimensional scale of gravity can be of the order of  $M_D \sim M_{EW}$ . If this is the case, then microscopic black holes in proton-proton collisions can be formed at the LHC. After formation the microscopic black holes quickly evaporate into particles of the Standard Model and graviton emission into a multidimensional volume is also possible. To study such processes, it is necessary to simulate the production of multidimensional black holes under LHC conditions. This is implemented using physical generators BlackMax and Charybdis2. The results of calculating the production cross sections of semiclassical black holes, depending on various parameters, show that the cross section can vary up to six orders of magnitude. With the luminosity and statistics of the collected data on the first and second stages of the LHC operation (Run 1 and Run 2), it is theoretically possible to observe the processes of the microscopic black hole production with cross section values up to 0.1 fb.

<sup>1</sup>  $L_{int} \approx 27 \text{ fb}^{-1} \text{ Run 1}$  (at  $\sqrt{s} = 7 \text{ TeV}$ , 8 TeV) and  $L_{int} \approx 150 \text{ fb}^{-1} \text{ Run 2}$  (at  $\sqrt{s} = 13 \text{ TeV}$ ).

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