Modifications to the Baryonic Regge Trajectories

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Introduction

Understanding of hadronic spectrum has been a very important challenge in nuclear and particle physics. Hadrons are well described by the string model [1]. According to the model, the mass (M) of a hadron with massless and spinless quarks is given by

$$M = 2\int_0^{l/2} \frac{K\,dr}{\sqrt{(1 - v^2/c^2)}} = \frac{\pi K l}{2}, \qquad (1)$$

and the angular momentum(J) is given by

$$J = 2\int_0^{l/2} \frac{K r v\,dr}{\sqrt{(1 - v^2/c^2)}} = \frac{\pi K l^2}{8}. \qquad (2)$$

These strings are characterised by the Regge trajectories of hadrons (i.e., the relation between M and J). Such Regge trajectory is described by,

$$J = \alpha' M^2 + \alpha_0 \qquad (3)$$

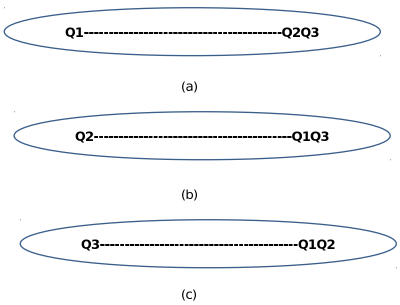

FIG. 1: Different equally probable quark-configurations of a baryon.

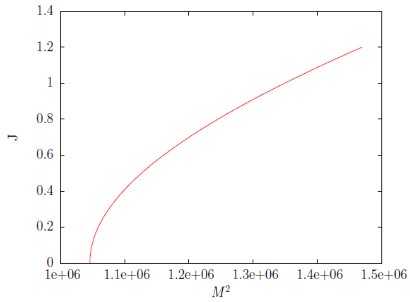

FIG. 2: The Regge trajectory for baryons when the speed of quark '1' is varying and it's mass remains same.

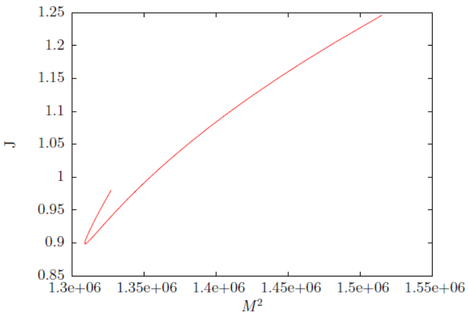

FIG. 3: The Regge trajectory for baryon when the mass of quark '1' is varying and it's speed remains same.

where, α_0 is a constant known as the Regge intercept and α' is the Regge slope parameter which is given by $\alpha' = 1/(2\pi K)$. Here, K is the linear energy density of the string, 'l' is length of the the hadronic string, and 'c' is the speed of light in vacuum. Throughout our calculations, we have considered the natural units, i.e., $c = 1$ or, $fc = f$.

The non-linearity in the Regge trajectories with physical realisation is still not understood [2–4]. In this paper, we have analysed string model of baryons with massive quarks

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to study non-linear Regge trajectories.

Formalism and Calculations

For a baryon which is made of three different kind of massive but spinless quarks, let us assume that the quarks are separated by the string of length l. In string model, a baryon can be described by three equally probable configurations as shown in Fig 1. The baryonic string will rotate about its center of mass. The modified expression for classical mass(M_a) of a baryon for configuration (a) is given as

$$M_a = \frac{Kl(m_b + m_c)}{f(m_a + m_b + m_c)} \left(sin^{-1} f + sin^{-1} \frac{m_a f}{m_b + m_c} \right) + \gamma_0 m_a + \gamma_a (m_b + m_c) \quad (4)$$

where m_a, m_b, m_c are masses of quarks '1', '2', and '3' respectively and $\gamma_0 = \frac{1}{\sqrt{1-f^2}}, \& \gamma_a = \frac{1}{\sqrt{1-\left(\frac{m_a f}{m_b+m_c}\right)^2}}$.

The modified expression of angular momentum (J_a) for configuration (a) is obtained as

$$J_a = \frac{\pi}{sin^{-1} f + sin^{-1} \frac{m_a f}{m_b+m_c}} \left(1 - \frac{sin(2 sin^{-1} f) + sin\left(2 sin^{-1} \frac{m_a f}{m_b+m_c}\right)}{2\left(sin^{-1} f + sin^{-1} \frac{m_a f}{m_b+m_c}\right)}\right) \times \alpha' \{M_a - \gamma_0 m_a - \gamma_a (m_b + m_c)\}^2 + \frac{m_a f l}{m_a + m_b + m_c} \{\gamma_a m_a + \gamma_0 (m_b + m_c)\} \quad (5)$$

Similarly for configurations (b) and (c), the corresponding masses and angular momenta can be obtained. The modified mass (M_{mod}) angular momentum of a baryon (J_{mod}) will be average of them. We have assumed that 'f' and 'l' are same in all probable configurations; in configuration (a) the quark '1' rotates with speed 'f', in configuration (b) the quark '2' rotates with speed 'f', and in configuration (c) the quark '3' rotates with speed 'f', and $0 < f \leq 1$. The general structure of J_i's (where i=a,b,c) are given by

$$J_i = C_i^2 (M_i - a)^2 + \gamma m_i f l \quad (6)$$

where C_i's are functions of 'f', 'l', and m_a, m_b, m_c and 'a' is a constant. Now it can be proved that it is not possible to write the expression for J_{mod} in the form of Eq 3.

In the expressions of J_i's, we have encountered terms $sin^{-1} \frac{m_i f}{\sum_j m_j - m_i}$ where i,j can take values 1,2,3. Since $sin\theta \leq 1$ therefore leads to the condition $f \leq \frac{\sum_j m_j}{m_i} - 1$ which along with $f \leq 1$ should satisfy simultaneously.

Results and Conclusions

In computations, we have taken m_a=1.5MeV(u quark), m_b=3.0MeV(d quark), m_c=3.0MeV(d quark), K=0.2GeV^2, from Particle Data Group (PDG) [5]. The string length is taken as $l = 1fm$. Fig2 shows the baryonic Regge trajectory for different quark speeds and same quark mass. In the figure in low velocity range (non-relativistic range) they show non-linear behavior. Fig3 shows that in low mass range for one value of baryonic mass two values of angular momenta are possible. In fact, these two angular momenta are for two different kind of baryons having same mass.

Finally, we conclude that for baryons it is not possible to write the Regge trajectory in its usual form (i.e. Eq 3). In low mass and angular momentum region two baryons with different quark compositions can have same mass and angular momentum.

Acknowledgments

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