STRING THEORY AND THE POMERON

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The duality between gauge theory and string theory allows us new methods for understanding the Pomeron. String theory on a curved space gives a unified description of the single-Pomeron-exchange amplitude in gauge theories, interpolating between the Regge trajectories at timelike momentum transfer and the BFKL region at large spacelike momentum transfer.

There is a long history, dating back to the 1970s, of attempts to show a connection between string theory and QCD, led by 't Hooft, Polyakov, and many others. In 1997 a precise conjecture was formulated by Maldacena [1], who claimed that string theory in ten dimensions (on an appropriate background space) and four-dimensional gauge theories are quantum-mechanically equivalent. Maldacena's conjecture is an example of a "duality", a *nonperturbative* quantum equivalence of classically different theories. (Note this is not related to twistors or any other issue in *perturbative* calculations.) Here a single physical theory has multiple descriptions: the first is gauge theory in 3+1 dimensions, while the second is string theory on a 3+1+1+5 dimensional space. I have divided the 10 dimensions up in this strange way for the following reason: the first 3+1 are the familiar ones; the last 5 will play no role here; but the 5th coordinate, "r", is important. Under duality, the fifth coordinate of the string theory corresponds roughly to the energy scale μ in the gauge theory. Both r and μ take values between 0 and ∞, with small (large) r encoding the infrared (ultraviolet) of the gauge theory.

As with any duality, the magical relation comes with a catch. In any particular physical regime, at most one of the multiple descriptions is simple. This catch is illustrated in Fig. 1, where it can be seen that the conjecture is most useful in the upper right, but unfortunately QCD lies in the lower left. So the stringy description of real QCD involves a string for which quantum effects are large, and whose precise form is in any case unknown. This sounds disastrous; why should we care about Maldacena duality if it is so completely useless for describing QCD?

There are two main reasons for maintaining interest in this subject. First, the $1 \ll g^2N \ll N$ theories are the best *toy models* for QCD that we have ever had. Like QCD, they are Lorentz

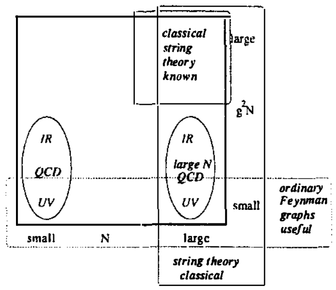

Figure 1: The Maldacena conjecture as a function of N (the number of colors) and the gauge coupling g.

invariant: they exist in 3+1 continuous *Minkowski* dimensions. They can exhibit infrared confinement and ultraviolet scaling, like QCD. Interesting non-perturbative dynamics inaccessible both to Feynman diagrams and to lattice gauge theory can be computed. They can help identify universal or nonuniversal aspects of confining theories, can be useful for supporting or disproving folk theorems and speculations that cannot otherwise be tested, and may someday be useful for developing new methodologies in QCD. Second, though they are not familiar, these $1 \ll g^2N \ll N$ theories are interesting and natural gauge theories in their own right. For instance, such a theory might be responsible for electroweak symmetry breaking, via technicolor-like or other compositeness dynamics. This is off our main topic, however, so let us simply note that study of these theories is well-justified on many counts.

String theory has gravity as its low-momentum limit, and there have already been a number of surprisingly successful applications of the *gravity limit* of these toy models to QCD and to pure Yang-Mills theory. But in this talk I will present an application that uses the *stringy physics* of these toy models. The topic in question [2] is the Pomeron, a phenomenon which involves the properties of fast hadrons, and the famous but little-understood work of BFKL [3] et al. The question to which the Pomeron is the answer is the following: when objects are boosted to very high energy, how do they change? We will address $2 \rightarrow 2$ elastic scattering, with fixed momentum-transfer t, and with $|t| \ll s \rightarrow \infty$. Our precise questions are the following: How do amplitudes grow with energy $\sqrt{s}$? and how do they fall with angle ($t < 0$)?

Let us first address this in a simple context: flat-space string theory itself. What happens to strings at large s and fixed t? The answer is well-known; strings in flat space become dense and grow! We can see this from the behavior of the 2–to–2 scattering amplitudes in closed string theory, which show Regge behavior $\mathcal{A} \sim \sum_i s^{J_i(t)}$. Here the sum is over different "Regge trajectories," each of the form $J_i(t) = \alpha_i(t) = \alpha_0^{(i)} + \alpha' t$, sharing the same "slope" α' but with different "intercepts" $\alpha_0^{(i)}$. The "leading trajectory", $i = 1$, has the largest intercept, which we will simply call α_0, dropping the superscript.

For positive t (the timelike region, unphysical for scattering) massive states with $m^2 = t$ and spin j are present wherever $J_i(t)$ is an integer j. Thus the Regge trajectories determine the spectrum. To interpret the amplitude at negative t, consider the Fourier transform from the momentum space variable t to the transverse position space variable $x_\perp^2$.

$$J(t) \sim \alpha_0 + \alpha' t \Rightarrow \tilde{\mathcal{A}} \sim s^{\alpha_0} \, \frac{\exp\left[-|\vec{x}|^2/\alpha' \ln s\right]}{\sqrt{\ln s}} \tag{1}$$

This shows that as s increases, the effective size of the string grows, $\langle |\vec{x}|^2 \rangle \sim \ln s$. This is a formula for random-walk diffusion, with a diffusion "time" proportional to the rapidity of the

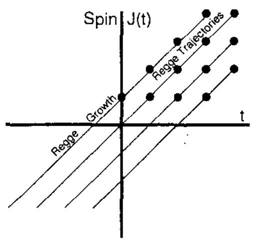

Figure 2: The Regge trajectories of string theory.

boost, $\tau \sim \ln s$. Exactly at $t = 0$ (forward scattering) one finds in string theory that the physics at large s is dominated by the largest intercept, which in flat-space string theory is precisely at $J_1(0) \equiv \alpha_0 = 2$. The corresponding massless resonance of spin two is, of course, the graviton. The corresponding cross-section then grows with s. (Note it must inevitably violate unitarity; the tree-level string theory formula must at some point be strongly corrected by loops.) These effects stem from a simple fact. A fixed observer will observe an increasingly boosted string for a shorter and shorter proper time, due to Lorentz time dilation. This allows the observer to resolve increasingly high-frequency fluctuations of the string, and these fluctuations make the string appear both larger in size and more dense in its core.

Now, what about hadrons in QCD; what happens to them at large s, fixed t? For t not too negative, what's true for strings is true for hadrons. Hadrons lie on Regge trajectories (as we know both from data and from lattice results.) Scattering amplitudes show the same Gaussian falloff with angle, $d\sigma/dt \sim s^{-\alpha'|t|}$, indicating that hadrons grow at large s. This is due to the huge number of "wee partons" — soft gluons — at small x. And the growing cross-sections, which threaten to violate unitarity, indicate that hadrons become dense when boosted. The dominant contribution to the scattering amplitude comes from the exchange not of a single hadronic resonance but of a coherent combination of colorless excitations called the "Pomeron", and in these regimes it resembles the leading trajectory of string theory.

Unfortunately, since it is both nonperturbative and highly Lorentzian in character, neither Feynman diagrams nor lattice methods can allow us to compute Pomeron exchange in the small $|t|$ region. Nor can we compute the very interesting transition from stringy behavior to the region $t \ll -1$ GeV^2, where (flat-space) strings and hadrons strongly differ. But at very large negative t, the Pomeron propagator can be computed. BFKL [3] calculate the relevant scattering amplitudes at large s by resumming perturbation theory in analogy with the renormalization group. In particular, they fix t, then sum all $(\alpha_s \ln s)^n$ terms at leading order in α_s.

If turns out to be simplest to compare BFKL results and string theory at $N \gg 1$, constant–α_s, $t = 0$. Of course this is quite far from QCD, where $N = 3$, the coupling runs, and the BFKL computation is most reliable at $t \ll -\Lambda^2$. But we will work our way toward QCD from this simpler beginning.

The BFKL computation involves summing all graphs that schematically look like Fig. 3. The BFKL kernel at $t = 0$, $K(s, k_\perp, k'_\perp)$ is a function of the momenta $k_\perp, k'_\perp$ flowing through the top and bottom gluon lines. The problem of including all leading $\ln s$ terms becomes a resummation of not merely ladder diagrams but diagrams with ladders of ladders of ladders of ladders of... After much effort one obtains a power of s times — again — a *diffusion kernel*. Again the diffusion time is proportional to rapidity $\ln s$, but strangely the "space" coordinate

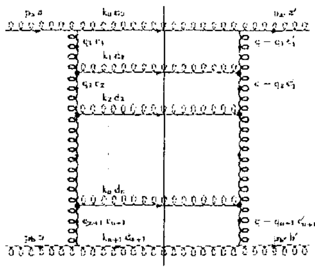

Figure 3: A graph contributing to the BFKL computation.

in which the diffusion occurs is $\ln k_\perp$ — compare with Eq. 1 —

$$\mathcal{A}_{2\to 2} = \int \frac{dk_\perp}{k_\perp} \int \frac{dk'_\perp}{k'_\perp} \Phi_1(k_\perp) K(s; k_\perp, k'_\perp) \Phi_2(k'_\perp) \tag{2}$$

$$K(s, k_\perp, k'_\perp) \approx s^{j_0} \frac{e^{-\left[(\ln[k'_\perp/k_\perp])^2/4\mathcal{D}\ln s\right]}}{\sqrt{4\pi\mathcal{D}\ln s}}, \tag{3}$$

where

$$j_0 = 1 + \frac{4\ln 2}{\pi}\alpha N \ , \quad \mathcal{D} = \frac{7\zeta(3)}{\pi}\alpha N \ . \tag{4}$$

are the leading intercept and diffusion constant, respectively.

At very small constant α_s, the BFKL calculation is *universal*: independent of N, matter content, etc. This gives us confidence that we can recalculate it in a theory with a string description, large N, and adjustable g^2N. In such a theory, we can compare two computations. First, at small g^2N, we can compute the amplitude using BFKL resummation — the infinite set of Feynman diagrams just discussed. Second, at large g^2N, we can compute the amplitude using string theory, deriving the answer from a single tree-level $2 \to 2$ string diagram, calculated on curved $3+1+1+5$ dimensional space.

To carry out the second computation, we need to know the curved space on which the strings propagate. Maldacena's conjecture requires that at large r, corresponding to the (almost) scaling ultraviolet region of the gauge theory, the curved space must be (almost) an $AdS_5 \times X$. The five-dimensional compact space X is of no consequence here, while AdS_5 includes the four ordinary Minkowski coordinates and the coordinate r. The metric in this region is

$$ds^2 = r^2 dx_\mu dx^\mu + \frac{dr^2}{r^2} + ds_X^2 \tag{5}$$

This captures the approximate ultraviolet conformal invariance of a QCD-like gauge theory. A rescaling in the gauge theory, $x \to \zeta x$, is an isometry of the metric if $r \to r/\zeta$. But recall $r \sim \mu$ in the string/gauge duality; thus we correctly learn that the energies in the gauge theory are rescaled by $1/\zeta$ when lengths are rescaled by ζ. Meanwhile, in the infrared we would like to have a confining theory. Experience with exact and approximate solutions for metrics related to confining gauge theory has shown that for many purposes (including ours) it suffices to model confinement in a simple way, as a lower cutoff on the coordinate r. Thus by placing a wall at $r_{\min}$, such that $r \sim \mu > r_{min} \sim \Lambda$, Λ the confinement scale, we will obtain a universal behavior which we claim will be true of any confining gauge theory with a dual string-theory description (at large g^2N). Thus our model metric is $AdS_5 \times X$ with r restricted to be greater than r_{min}.

Our computation begins by reconsidering the flat space scattering amplitude in the light of curved space. We note that t, which appears in the formula, is an eigenvalue of a Laplacian acting on one of the scattering strings.

$$\mathcal{A} \sim s^{J(t)} = s^{2+\alpha' t/2} \quad = s^{2+\alpha'\nabla^2/2} \tag{6}$$

Our claim (justified in our paper) is that the essential change from flat space is that the curved-space ∇^2 must be used. Thus in any weakly-curved space,

$$\mathcal{A} \sim s^2 e^{(\alpha' \ln s)\nabla^2/2} \equiv s^2 e^{-H\tau} \tag{7}$$

where we recognize the second factor as a diffusion kernel, where $\tau \propto \ln s$ is again a diffusion time, with a diffusion operator

$$H \propto -\nabla^2 = -\frac{1}{r^2}\nabla_{3+1} - \nabla_r^2 + \nabla_X^2 = -\partial_u^2 + (4 - e^{-2u}t/t_0) \tag{8}$$

where $\nabla_X^2 = 0$ here and $u = \ln r$. In order to gain intuition, it is also useful to view H as a Schrödinger operator in u with potential $V(u;t) = 4 - e^{-2u}t/t_0$. We are currently interested in $t = 0$, that is $V(u;0) = 4$, a constant potential, giving a trivially-solved Schrödinger problem.

$$H \propto -\partial_u^2 + 4 \Rightarrow \mathcal{A} \sim s^2 e^{-H\tau} \sim s^{j_0} e^{-\mathcal{D}\tau[-\partial_u^2]} \quad , \quad j_0 = 2 - \frac{2}{\sqrt{g^2N}} \; , \; \mathcal{D} = \frac{1}{2\sqrt{g^2N}} \tag{9}$$

(This value of j_0 was later obtained by a different method.[4]) Sandwiching this differential operator between the two scattering hadrons, and writing the kernel explicitly,

$$\mathcal{A} \sim \int \frac{dr}{r} \int \frac{dr'}{r'} \, \Phi_1(r) \; s^{j_0} \frac{e^{-\left[(\ln[r'/r])^2/4\mathcal{D}\ln s\right]}}{\sqrt{4\pi\mathcal{D}\ln s}} \; \Phi_2(r') \tag{10}$$

Compare this to Eq. 3. The amplitude is exactly of BFKL *form*, with $k_\perp \to r$. How and why did we get such similar answers?

The agreement of the form of the answer could have been predicted in advance, as some of its aspects follow from conformal invariance, once r and $k_\perp$ are identified. In the string calculation, the exchanged Pomeron — the graviton trajectory — is propagating in the curved 5th dimension, $u = \ln r$. Indeed, this is the string's ordinary Regge behavior; BFKL is Regge diffusion, Eq. 1, with time $\ln s$, and the space coordinate is $\ln r$ because of the metric, Eq. 5. The coefficients j_0 and $\mathcal{D}$ differ in the two calculations, but these quantities should depend on g^2N, so this is not surprising. Indeed $j_0 \to 1$ as $g^2N \to 0$, and $j_0 \to 2$ as $g^2N \to \infty$.

Now, let's move toward more realistic contexts. First, what about $t < 0$? It took 8 years to extend BFKL to $t < 0$, but in string theory it is very easy. The Schrödinger problem is only slightly more challenging; the diffusion kernel for the differential operator Eq. 8 can be easily analyzed. Next, how can we include confinement, and see the hadrons that should be present for integer J and $t > 0$? Again, this simply requires studying the spectrum of the same differential operator $H = -\partial_u^2 + V(u;t)$, now with $t > 0$, and with appropriate boundary conditions at r_{min} (or a more realistic metric.) Thus we can obtain the single-Pomeron exchange amplitude *at all* t — the BFKL behavior, the hadronic resonances, and the transition between them — from the analysis of the spectrum of a simple quantum mechanics Hamiltonian.

The logarithmically-running coupling can also be accommodated. In QCD, the order-α_s correction to BFKL, which includes the effect of the running, is large and negative. Thus BFKL is only reliable at large negative t, where $\alpha_s \ll 1$. There it is seen that the BFKL "cut" present for a constant coupling turns into a dense set of BFKL "poles". In string theory the running coupling simply alters the effective potential $V(u;t)$. However, here we are not restricted to

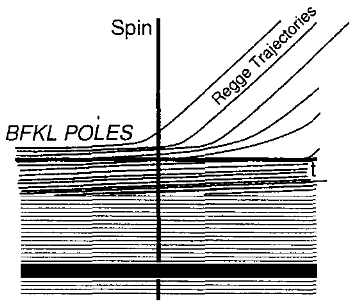

Figure 4: The positions of poles, as a function of t, in the Mellin transform from s to J of the Pomeron propagator, according to a string theory dual description of a confining gauge theory with a logarithmically running coupling.

large negative t. Instead, we can see from our Schrödinger problem that the Regge trajectories at positive t evolve smoothly in t to become the BFKL-like "poles" at negative t. This is shown in Fig. 4, which shows the position of the poles of the Mellin transform of the single-Pomeron-exchange contribution to 2–to–2 hadron elastic scattering.

It is natural to conjecture that the analytic structure seen in the figure is preserved for smaller g^2N, and applies even in QCD. But note the leading singularity is a pole that varies *monotonically* with t, which might seem to contradict data. However, data and theory are both ambiguous here; see our paper's final chapter [2] for further discussion of these subtle issues.

I hope that my audience is now persuaded that using gauge/string duality to learn about gauge theory is a worthwhile effort. The first applications of this formalism to issues of general theoretical importance in QCD are currently appearing. Here we found that the form of the BFKL result is reproduced in string theory, from Regge behavior in curved ten-dimensional space. Our (large-N) results extend to both positive and negative t, and show how the single Pomeron exchange amplitude incorporates both the BFKL-type kernel and the Regge trajectories in a single analytic structure. Much more remains to do, especially regarding the $t = 0$ region at ultra-high s.

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