Cosmology with the 21cm signal

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HI intensity mapping (IM) is a novel technique capable of mapping the large-scale structure of the Universe in three dimensions and delivering exquisite constraints on cosmology, by using HI as a biased tracer of the dark matter density field. This is achieved by measuring the intensity of the redshifted 21cm line over the sky in a range of redshifts without the requirement to resolve individual galaxies. Using this technique, the SKA will provide an unique (and transformational) window on Cosmology (from dark energy to Gravity and the primordial Universe). Moreover, a similar technique at higher redshifts (> 6) is being used to probe the Epoch of Reionization, when the first stars and galaxies formed.

1 Introduction

The cosmic microwave background (CMB) has been one of the main observational tools for cosmology in recent years. Although basically only giving 2-dimensional information, we were able to constrain the standard cosmological model with great accuracy ¹. This "high precision cosmology" is particularly true for the "vanilla" model with 6 parameters. More parameters or non-standard models can lead to degeneracies and limit the constraining power of the CMB (for instance the w_0/w_a non-flat model). The next step towards precision cosmology and exploring novel models will need to use extra information. In particular, due to its huge information content, measurements of the 3-dimensional large-scale structure of the Universe across cosmic time will be an invaluable tool. One of the most accessible methods to probe this is through large galaxy surveys to trace the underlying dark matter distribution. Several surveys are now under way or in preparation, such as BOSS (SDSS-III), DES, eBOSS, DESI, 4MOST, LSST, and the Euclid satellite. These surveys are based on imaging of a large number of galaxies at optical or near-infrared wavelengths combined with redshift information to provide a 3-dimensional position of the galaxies.

Galaxy surveys are threshold surveys in that they set a minimum flux above which galaxies can be individually detected. Instead we could consider measuring the integrated line emission of several galaxies in one angular pixel on the sky and for a given frequency resolution. For a reasonably large 3d pixel we expect to have several galaxies in each pixel so that their combined emission will provide a larger signal. Since Cosmology relies on scales much larger than galaxy sizes, using these large pixels should not affect the quality of the measurements. Moreover we can use statistical techniques, similar to those that have been applied for instance to CMB experiments, to measure quantities in the low signal to noise regime. By not requiring the detection of individual galaxies, the specification requirements imposed on the telescope will be much less demanding. This is what has been commonly called an "intensity mapping" experiment. It is similar to what is being planned for experiments aimed at probing the Epoch of Reionization (at z > 6), such as the ones using the radio telescopes LOFAR, MWA and PAPER. By not requiring the detection of individual galaxies, the specification requirements imposed on the telescope will be much less demanding. This way, the intensity mapping technique transfers the problem to one of foreground cleaning: how to develop cleaning methods to remove everything that is not the HI signal at a given frequency ^{2,3,4}. This in turn also impacts on the calibration requirements of the instrument.

Several lines could be considered for intensity mapping: CO, CII, Lyman- α , H- α , etc. In this proceeding, we concentrate on the 21cm HI line. Although weaker than other lines, it has several advantages: Hydrogen is the most abundant element in the Universe and a good tracer of dark matter. It is observed at low radio frequencies and so it has very little contaminants from other lines at low redshifts. Moreover, because observations are done at long wavelengths, radio telescopes have naturally low resolutions and large fields of view, which matches perfectly the requirements for intensity mapping (while at the same time, having naturally high frequency/redshift resolution). Telescopes probing the sky between a rest frequency of 1420 MHz and 250 MHz will be able to detect the signal up to redshift 5. In particular, we focus on what can be achieved with a large survey using phase 1 of the Square Kilometre Array⁵.

2 The HI signal

After reionization, most neutral hydrogen will be found in dense systems inside galaxies, e.g. Damped Lyman-alpha Absorbers (DLAs). In terms of the brightness temperature, the average signal over the sky can be written as:

$$\overline{T}_b(z) \approx 566h\left(\frac{H_0}{H(z)}\right) \left(\frac{\Omega_{\rm HI}(z)}{0.003}\right) (1+z)^2 \ \mu {\rm K},\tag{1}$$

where the neutral hydrogen density fraction is given by

$$\Omega_{\rm HI}(z) \equiv (1+z)^{-3} \rho_{\rm HI}(z) / \rho_{c,0}, \tag{2}$$

 $\rho_{\rm HI}(z)$ is the proper HI density and $\rho_{c,0}$ the critical density of the Universe at redshift zero. Figure 1 shows constraints on $\Omega_{\rm HI}(z)$ from different experiments. For a recent summary of observed trends we refer to Padmanabhan et al.⁶.

Assuming the signal is linear with respect to the underlying dark matter fluctuations, the total brightness temperature at a given position on the sky and frequency will be

$$T_b(\nu,\Omega) \approx \overline{T}_b(z) \Big[1 + b_{\rm HI} \delta_m(z) - \frac{1}{H(z)} \frac{dv}{ds} \Big].$$
(3)

The signal will then be completely specified once we find a prescription for the HI density and bias function $(b_{\rm HI})$. This can be obtained by making use of the halo mass function, $\frac{dn}{dM}$ and halo bias, while relying on a model for the amount of HI mass in a dark matter halo of mass M, e.g. $M_{\rm HI}(M)$.

For the mass function, we decided to consider a simple power law:

$$M_{\rm HI}(M) = AM^{\alpha},\tag{4}$$

which is independent of redshift. We found that a value of $\alpha \sim 0.6$ fits both the low z and high z data reasonably well. This can be seen in figure 1 (left), that shows the $\Omega_{\rm HI}(z)$ measurements and the evolution obtained from this model (solid line). The constant A is normalised to the results from Switzer et al. ⁸ at $z \sim 0.8$. The right panel shows the redshift evolution for both the linear and power law model of the temperature multiplied by the bias, which is the figure of merit for the strength of the power spectrum used in the forecasts.



Figure 1 – Left: Current constraints on the HI density fraction as a function of redshift, partially based on the compilation in Duffy et al.⁷. DLA observations are shown in blue, cross-correlations in orange, other observations in red, and simulations in green. The thick black line shows $\Omega_{\rm HI}(z)$ from the fiducial model power law used throughout this chapter. Right: Evolution of the brightness temperature times bias with redshift for the linear (red curve) and our fiducial power-law model (blue).

3 Current and planned experiments

First attempts at using intensity mapping have been promising, but have highlighted the challenge of calibration and foreground subtraction. The Effelsberg-Bonn survey ⁹ has produced a data cube covering redshifts out to z = 0.07, while the Green Bank Telescope (GBT) has produced the first (tentative) detection of the cosmological signal through IM by cross-correlating with the WiggleZ redshift survey ^{10,8,11}. As probes to constrain cosmological parameters these measurements are, as yet, ineffective, but they do point the way to a promising future.

We can basically divide the intensity mapping experiments into two types: single dish surveys and interferometers. In single dish surveys (e.g. using auto-correlations) each pointing of the telescope gives us one single pixel on the sky (though more dishes or feeds can be used to increase the field of view). This has the advantage of giving us the large scale modes by scanning the sky. Since brightness temperature is independent of dish size we can achieve the same sensitivity with a smaller dish although that will in turn limit the angular resolution of the experiment (a 30 arc min resolution at $z \sim 1$ would require a dish of about 50 m in diameter). One example is the GBT telescope as described above. BINGO¹² is a proposed 40m multi-receiver single-dish telescope to be situated in South America and aimed at detecting the HI signal at $z \sim 0.3$.

Interferometers basically measure the Fourier transform modes of the sky. They have the advantage of easily providing high angular resolution as well being less sensitive to systematics that can plague the auto-correlation power. On the other hand, the minimum angular scale they can probe is set by their shortest baseline which can be a problem when probing the BAO scales. One example of a purpose built interferometer for intensity mapping is CHIME, a proposed array, aimed at detecting BAO at $z \sim 1$, made up of 20 × 100m cylinders, based in British Columbia, Canada. TIANLAI, set in China, follows a similar approach. A different setup is used in HIRAX, to be set in South Africa: an interferometer with about 1,000 highly packed 6m dishes.

The next generation of large dish arrays can also potentially be exploited for HI intensity mapping measurements. Such is the case of MeerKAT and ASKAP. However, these interferometers do not provide enough baselines on the scales of interest (5m to 80m) so that their sensitivity to BAO will be small. The option is to use instead the auto-correlation information from each dish, e.g. make a survey using the array in single dish mode. The large number of dishes available with these telescopes will guarantee a large survey speed for probing the HI signal. The great example of this approach will be SKA1, the first phase of the SKA telescope, to be built in 2018. An HI intensity mapping survey will turn SKA phase 1 into a state of the art cosmological probe. In particular, the huge volume available with such a survey will surpass any other large experiment such as Euclid or LSST. In the following sections I will summarise what can be achieved with SKA1, assuming 133 15m dishes plus 64 13.5m MeerKAT dishes, a band 2 from 950 MHz to 1420 MHz (0 < z < 0.5) and a band 1 from 350 MHz to 1050 MHz (0.35 < z < 3.06) and a survey size of 25,000 deg² over 10,000 hours.

4 High precision cosmology with an SKA1-MID HI intensity mapping survey

Surveys of large-scale structure are a rich source of information about the geometry and expansion history of the Universe. The baryon acoustic oscillations (BAO) are a preferred clustering scale imprinted in the galaxy distribution, originating from the time when photons and baryonic matter were coupled together in the early Universe. By using them as a statistical 'standard ruler', one can obtain constraints on the expansion rate, H(z), and (angular) distance-redshift relation, $D_A(z)$, as functions of redshift, as has been done successfully with recent large galaxy redshift surveys such as BOSS and WiggleZ. Measuring these functions is vital for testing theories of dark energy which seek to explain the apparent acceleration of the cosmic expansion, as they constrain its equation of state, $w = P/\rho$, and thus its physical properties. Shedding light on the behaviour of dark energy – especially whether w deviates from -1 and whether it varies in time – is one of the foremost problems in cosmology.

Intensity mapping (IM) has a few major advantages over conventional galaxy surveys for this task. IM surveys can map a substantial fraction of the sky with low angular resolution in a short period of time. Combined with the wide bandwidths of modern radio receivers, this makes it possible to cover extremely large survey volumes and redshift ranges in a relatively short time, helping to beat down sample variance. Figure 2 (left) summarises the expected constraints from the SKA HI IM surveys for the BAO scale at $k \sim 0.074 \text{ Mpc}^{-1}$. Although the real power of the SKA1 IM survey will be on very large scales, we see that even at BAO scales, SKA1-MID present constraints not far from Euclid while only using a ~ 2 year survey (the full Euclid requires about 5 years). In fact, as shown in Bull¹³, the high sensitivity of the SKA1 survey at low redshifts will allow it to surpass contemporary spectroscopic galaxy surveys such as DESI and Euclid in terms of constraints on modified gravity parameters. This is aided by the ability of an SKA1 IM survey to achieve sub-1% measurements of $f\sigma_8$, where f(z) is the linear growth rate, which can be measured from the degree of anisotropy of the redshift-space correlation function (or power spectrum). The growth rate is directly related to the strength of gravity, and so is an extremely useful tool for probing possible deviations from general relativity that have been invoked as an alternative to dark energy to explain cosmic acceleration (see Figure 2 - right).

5 Probing very large scales with a SKA1 HI intensity mapping survey

The study of the Universe on ultra-large scales is one of the major science cases for the SKA. On ultra-large cosmic scales, two key effects become significant: primordial non-Gaussianity and relativistic corrections to cosmological observables. Moreover, if late-time acceleration is driven not by dark energy but by modifications to general relativity, then such modifications should become apparent near and above the horizon scale. As a result, the SKA is forecast to deliver transformational constraints on non-Gaussianity and to probe gravity on super-horizon scales for the first time. Figure 3 (left) summarises the expected constraints from the SKA HI IM surveys for a very large scale, past the equality peak at $k \sim 0.01 \text{ Mpc}^{-1}$. We see the huge



Figure 2 – Left: Constraints (noise over signal) from SKA HI IM surveys for BAO scales ($k \sim 0.074 \text{ Mpc}^{-1}$) as a function of redshift. Dashed line shows the BAO detection threshold. The lower green curve shows what would be expected from a SKA2 IM survey (in interferometer mode) optimised for high-z. The grey curve shows what can be expected for a two-year H_{α} galaxy survey with similar depth as Euclid but over a smaller sky area. Right: Predicted constraints from SKA on the unparameterized growth function $f\sigma_8$ from the SKA1 (galaxy and IM) and the SKA2 galaxy survey, compared with predicted constraints coming from the Euclid galaxy survey. Both constraints include Planck+BOSS priors.

constraining power of these surveys (see Camera et al.¹⁴ for a more in depth discussion).

In Camera et al. ¹⁵, an analysis is given of the constraining power of IM surveys over non-Gaussianity, showing that errors on $f_{\rm NL}$ can be taken down towards $\sigma_{f_{\rm NL}} \lesssim 3$ with SKA1, which is more than three times better than the current constraint from Planck. In terms of testing Einstein's theory of general relativity on horizon scales, one of the most interesting effects is the correction to the standard Newtonian approximation for the observed galaxy overdensity. It turns out that these relativistic corrections are very hard to detect using IM (although the same is true for other single tracers) due to cosmic variance. The way forward is the use of the multi-tracer technique. By combining an HI IM survey from SKA1 with a galaxy survey such as from Euclid or LSST, it is possible to obtain exquisite constraints on these large scale relativistic corrections as well as primordial non-Gaussianity ¹⁶ (Figure 3 - right). Moreover, these novel cross-correlation between different surveys will give a better handle on systematics and foreground issues.



Figure 3 – Left: Constraints (noise over signal) from SKA HI IM surveys for large scales, past the equality peak $(k \sim 0.01 \text{ Mpc}^{-1})$ as a function of redshift. A value below 1 would imply a detection. Dashed line indicates what can be achieved with SKA0 (50% of SKA1) which is quite similar to SKA1. Right: Joint constraints on primordial non-Gaussianity and relativistic corrections by combining an HI IM survey with a LSST type survey using the multi-tracer technique and assuming two different types of galaxy bias.

6 Foregrounds and Technical challenges

One of the most important challenges facing HI intensity mapping is the presence of foregrounds (both galactic and extra-galactic) with amplitudes several orders of magnitude larger than the signal to be measured. Because the frequency structure as well as other statistical properties of the foregrounds are significantly different from those of the cosmological signal, it is not unreasonable to hope that they can be successfully subtracted ^{17,18,19,20,21,22,23,24,25,26,27,28,29}. Although a lot of work in terms of simulations and testing cleaning techniques has already been done, we still face huge challenges ahead, in particular if we want to use this signal for high precision cosmology. Increasingly realistic large simulations should be developed to try to test the limitations of the intensity mapping measurements. This should include as many instrumental effects as possible in order to account for possible contamination from the calibration process. Ultimately, we will need to start analysing real data in order to improve and build up our knowledge towards the SKA.

The problem of foregrounds has been addressed in the literature mainly within the EoR regime. The different algorithms that have been proposed to date can be classified into blind 21,8,30 and *non-blind* 31,29,32 methods, depending on the kind of assumptions made about the nature of the foregrounds (e.g. whether only generic properties such as spectral smoothness and degree of correlation are assumed or whether a more intimate knowledge of the foreground statistics is required). The poor observational constraints on the foregrounds in the relevant range of frequencies justifies considering the use of blind methods. Recently, Wolz et al.²⁸ studied the effectiveness of independent component analysis (in particular the implementation of FastICA³³) for intensity mapping. By propagating the foreground removal residuals into the cosmological analysis, they showed that, while foreground cleaning may induce a residual bias on large angular scales, which could prevent a full analysis based on the shape of the temperature power spectrum, robust features like the BAO scale should remain unaffected. This result is reasonable: most relevant foregrounds are (fortunately) exceptionally smooth and therefore it should be possible to distinguish them from the much "noisier" cosmological signal. Any foreground residual will probably be dominated by galactic synchrotron emmission, which is most relevant on large angular scales.

With regards to calibrating single dish experiments, this is a source of major concern. Major systematic effects to be tackled are spillover and sidelobe pickup as well as gain drifts. Again, these are issues that have been tackled successfully in the analysis of CMB data although novel approaches can be envisaged. So, for example, the BINGO experiment ¹² propose to use a partially illuminated aperture and a fixed single dish, minimising the problems that arise from moving parts. Another intriguing possibility is, for a cluster of single dishes working in autocorrelation mode, to use the cross correlation data for calibrating off known sources. This means that in principle, calibrating the gains should be straightforward using the interferometer data since the high resolution will allow access to a good sky model.

7 Conclusions

HI intensity mapping is set to become a leading cosmology probe during this decade. One of the key instruments that can be used for this purpose is phase I of the SKA. A large sky survey with this telescope (in total power array) should be able to provide stringent constraints on the nature of dark energy, modified gravity models and the curvature of the Universe. Moreover, it will open up the possibility to probe BAO at high redshifts as well as ultra-large scales, beyond the horizon size, which can be used to constrain effects such as primordial non-Gaussianity or potential deviations from large-scale homogeneity and isotropy. The combination of this signal with galaxy surveys using the multi-tracer technique will provide revolutionary constraints on non-Gaussianity and relativistic corrections on large scales.

Several challenges will have to be overcome, however, if we want to use IM for cosmological

purposes. In particular, cleaning of the huge foreground contamination, removal of any systematic effects and calibration of the system. Foreground cleaning methods have already been tested with relative success taking advantage of the foreground smoothness across frequency but novel methods need to be explored in order to deal with more complex foregrounds. Other contaminants, such as some instrumental noise bias that shows up in the auto-correlation signal, can in principle be dealt with the same methods. Ultimately, we should deal with the cleaning of the signal and the map making at the same time. This will require even more sophisticated statistical analysis methods and it will be crucial to take on such an enterprise in the next few years in order to take full advantage of this novel observational window for cosmology.

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