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A DEVICE TO PRODUCE HIGH CENTER-OF-MASS ENERGY e + e COLLISIONS — ACCELERATOR BEAM COLLIDING WITH A STORED BEAM*

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ABSTRACT

If 20-GeV electrons from the Stanford Linear Accelerator collide with 2-GeV electrons (or positrons) circulating in the storage ring now under construction at SLAC, then the reaction center-of-mass energy will be $E_{cm} = 12.6$ GeV. The luminosity of this device is calculated to be about $2.4 \times 10^{29}\ cm^{-2}\ sec^{-1}$, and the number of $e + e \rightarrow e + e + X$ reactions at this energy is estimated to be about 60 $hour^{-1}$ when $X = \mu^+ + \mu^-$, about 10 $hour^{-1}$ when $X = \pi^+ + \pi^-$, and about 1 $hour^{-1}$ when $X = \pi^0$ or η.

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I. INTRODUCTION

Two methods for inducing high center-of-mass energy collisions are commonly accepted today. (1) A stationary target is bombarded by a beam produced in a high energy accelerator. (2) Two circulating beams of particles are stored and collided with each other in storage rings. The second method is less expensive provided that the particles to be stored are charged, long lived, and their energy is not too high. If the particles are uncharged, then of course they cannot be stored in the usual manner; if they have a short lifetime, it is difficult to achieve high enough densities in the stored beam; if the particles are electrons and their energy is too high, then very large storage rings are needed to avoid unacceptable energy losses in the form of synchrotron radiation. Due to these reasons only protons and muons, and electrons with moderate energy can be stored economically today.

A method intermediate between (1) and (2) mentioned above, would consist of colliding an accelerator beam with a stored beam. By using directly a high energy beam of those particles which cannot be stored efficiently, one could induce in this manner a wider variety of reactions with reasonable counting rates than by method (2), although not as many as by method (1). On the other hand, the reaction center-of-mass energy which can be reached without an increase in the accelerator energy and with moderate radiation losses from the stored beam would be higher than what is accessible when the first method is used, but not as high as when both beams are stored at the accelerator beam energy. The method of colliding a high energy beam with a stored beam was proposed earlier[1,2,3] and it was found that acceptable counting rates could be obtained.

Up to the present time the design of electron storage rings has concentrated on the achievement of high luminosities at center-of-mass energies of 6 GeV and below, extension to higher energies being generally left as an option for future development.

The main reason for this, besides the large amounts of radiofrequency power needed to operate high energy storage rings, was the prevalent belief that the interesting production cross sections are a fast decreasing function of the reaction center-of-mass energy E_{cm}. This belief was based on the observation that the lowest order production amplitudes (in the electromagnetic coupling constant) are the "single photon annihilation processes;" $e^+ + e^- \rightarrow \gamma \rightarrow X$ (see Fig. 1), which contain a factor k^{-2} in the photon propagator, so that the cross section due to this amplitude alone would be proportional to $(k^2)^{-2}$. When the incoming e^- and e^+ momenta in the laboratory are equal in magnitude but have opposite sign, such as when the e^+ and e^- are both stored in rings with equal energy, then in the laboratory k has no space-like component, and its time-like component is equal to E_{cm}. Therefore it is clear that, in such experiments, the one-photon annihilation cross section contains a factor due to the photon propagator of $(E_{cm})^{-4}$. This is partly compensated by a factor $\leq E_{cm}^2$ coming from the vertices, so that, indeed, the production cross section contains a factor which decreases at least as fast as $(E_{cm})^{-2}$.

On the other hand, it was suspected for some time[3] that, for high values of E_{cm}, the production cross sections will be dominated by the "photon-photon scattering" amplitude $e^+ + e^- \rightarrow e^- + e^+ + \gamma + \gamma \rightarrow e^- + e^+ + X$ (see Fig. 2). The reason for this dominance is that the two γ's can have large space-like momentum components, so that they need not be far off their mass shells even if the final state has high center-of-mass energy. Therefore, this amplitude does <u>not</u> contain the factor $(E_{cm})^{-2}$.

F. E. Low calculated the cross section for $X = \pi^0$ and found it to vary approximately as the square of the logarithm of E_{cm}.[4] P. C. DeCelles, and J. F. Goehl[5] pointed out that the two-pion system in relative S state can be studied by observing two-pion production by two virtual photons. N. Arteaga-Romero, A. Jaccarini and

P. Kessler[6] have calculated the cross sections for muon, pion and kaon production due to the photon-photon scattering amplitude. Subsequently, S. Brodsky, T. Kinoshita and H. Terazawa[7] evaluated the cross sections for muon pair, pion pair, π^0 and η production through the same mechanism. These four important references demonstrated that at high E_{cm} values photon-photon scattering dominates single-photon annihilation, that valuable information could be gained by observing these processes, and that the cross section actually increases with the electron (positron) energy. The results just mentioned give added impetus to the search for means to realize collisions at high E_{cm}.

The purpose of this paper is to describe a device with which e + e collisions at high E_{cm} can be achieved. These collisions are induced by colliding a bunch of stored electrons (or positrons) circulating in a storage ring with synchronized bunches of electrons (possibly positrons) produced by a high energy linac. To illustrate the possibilities offered by such a device, we consider as an example the case when particles with energy $E_s = 2$ GeV, stored in the storage ring now under construction at SLAC,[8] collide with electrons with energy $E_\ell = 20$ GeV from the Stanford Linear Accelerator. With these parameters, to a good approximation,

$$E_{cm} = 2\sqrt{E_s E_\ell} \qquad (1)$$

which gives $E_{cm} \approx 12.6$ GeV. In the next section, we estimate the luminosity achievable in the proposed device, and, in the last section, we estimate the expected counting rates for various production processes.[9]

II. LUMINOSITY

The general arrangement of the proposed system is shown in Fig. 3. The linac beam collides with the stored beam in a straight section of the storage ring, and can either be dumped or led on to another experiment. Periodically, the linac supplies particles for filling the storage ring via another beam line not shown. In the remainder of this section, we shall refer to the particles in the linac beam as electrons and to the stored particles as positrons. Some number N_s of positrons is stored in the ring and circulates there. Then, f times a second, a burst of N_ℓ electrons from the linac is conducted through the interaction region where it collides with the stored particles. Both linac and stored beams are focused down to small "waists" at the interaction region to increase the particle density and consequently the interaction rate.

The region over which the products of interactions can be observed is limited by practical detector considerations to a fraction of a meter. In order to maximize the useful interaction rate, both the linac beam and the stored beam should be concentrated in short pulses so that each linac electron encounters the entire swarm of stored positrons in the interaction region. This can be done by concentrating the stored beam into a single radiofrequency bunch and delivering the linac burst in a series of short pulses at intervals of the orbital period of the stored bunch.

The interaction rate in a colliding-beam system is described in terms of the luminosity, the interaction rate per unit cross section. The luminosity of the proposed system is

$$L = \left(\frac{fN_\ell}{A_{eff}}\right) N_s \qquad (2)$$

where A_{eff} is the effective area of the beam collision.

It is obvious that the smallest value of A_{eff} which is feasible should be used. Roughly speaking, for colliding beams of differing widths and heights, the effective area is determined by the product of the greater height with the greater width, and there is no advantage in making one beam smaller in either dimension than the other. Thus, we may limit our considerations to colliding beams of the same size and shape. Various factors must be considered in choosing the size and shape, but the most important turns out to be the disruptive influence of the linac-beam space-charge forces on the stored beam. This effect depends only on the factors in the parentheses in Eq. (2), and we treat it in the following paragraphs.

Consider a positron traveling around the storage ring. Every (1/f) seconds it encounters a pulse of N_ℓ electrons from the linac in the interaction region from which it receives a transverse impulse which depends on its lateral position. Between impulses it circulates in the ring, undergoing radiation damping and fluctuations as well as Landau damping due to nonlinear forces, and we assume that these influences disturb its motion sufficiently that its lateral positions on successive encounters with the linac pulse are statistically independent. We treat the case that the beams are distributed transversely as round Gaussians. A positron passing through the linac pulse at radial distance r from its axis receives an increment of radial momentum

$$\Delta p_r(r) = -2r_e mcN_\ell \left[\frac{1 - \exp\left(\frac{r^2}{2\sigma^2}\right)}{r} \right] \tag{3}$$

where r_e is the classical radius of the electron and m its rest mass, c is the speed of light and σ is the standard deviation of the Gaussian distribution.

$$\psi(r) = \frac{N_\ell}{2\pi\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) \tag{4}$$

[$2\pi r\psi(r)dr$ gives the number of electrons between r and r + dr.] The polar scattering angle corresponding to Δp_r is $\theta = (\Delta p_r/\gamma mc)$ where γ is the relativistic energy parameter of the stored positrons. Since we are assuming that both beams are the same size, the positions at which the positron passes through the linac pulse are distributed according to Eq. (4) with $N_\ell = 1$, so the mean square polar scattering angle is given,

$$\langle\theta^2\rangle = \left(\frac{2r_eN_\ell}{\gamma}\right)^2 2\pi \int_0^\infty r dr \psi(r) \left[\frac{1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)}{r}\right]^2$$

$$= \left(\frac{r_eN_\ell}{\gamma\sigma}\right)^2 2\int_0^\infty \frac{du}{u} e^{-u}(1 - e^{-u})^2$$

$$\langle\theta^2\rangle = \left(\frac{r_eN_\ell}{\gamma\sigma}\right)^2 2\ln(4/3). \tag{5}$$

The transverse motion of the positron in the storage ring is described in terms of two orthogonal normal modes of betatron oscillation, which, in an ideal ring with no horizontal-to-vertical couplings, are the horizontal and vertical oscillations, but, in general, have other directions. The mean square projected scattering angle on either direction is $\frac{1}{2}\langle\theta^2\rangle$. Now, random increments in the projected angles are equivalent to random increments in the corresponding normal-mode oscillation amplitudes A,

$$\Delta A^2 = \left(\frac{r_e\beta N_\ell}{\gamma\sigma}\right)^2 \ln(4/3). \tag{6}$$

The parameter β denotes the local reduced betatron wavelength for the mode at the interaction region,[10] assumed to be the same for both modes. These random increments may be regarded, in the same way synchrotron-radiation fluctuations are,[11] as producing a rate of oscillation-amplitude growth of the form,

$$\left.\frac{d\langle A^2\rangle}{dt}\right)_{scatt} = f\langle\Delta A^2\rangle .$$

The corresponding growth rate due to radiation is

$$\left.\frac{d\langle A^2\rangle}{dt}\right)_{rad} = \frac{2A_r^2}{\tau} = \frac{4\sigma_r}{\tau}$$

where τ is the normal-mode radiation damping time, assumed to be the same for both modes, A_r is the rms modal amplitude which would obtain if there were only radiation effects present and σ_r is the corresponding rms size of the beam. Combining the rates of growth with the rate of decay due to radiation damping, we get

$$\frac{d\langle A^2\rangle}{dt} = f\langle\Delta A^2\rangle + \frac{4}{\tau}\left(\sigma_r^2 - \sigma^2\right) , \tag{7}$$

where σ is the rms size of the beam in the presence of the beam-beam interaction.

For statistical equilibrium, the rate of growth is zero, and we obtain the following equation for the beam size:

$$\left(\frac{\sigma}{\sigma_r}\right)^4 - \left(\frac{\sigma}{\sigma_r}\right)^2 - f\tau\left(\frac{r_e\beta N_\ell}{2\gamma\sigma_r^2}\right)^2 \ln(4/3) = 0. \tag{8}$$

If the last term is small compared to one, the stored beam is little disturbed by the linac beam and $\sigma \approx \sigma_r$. If the last term is large compared to one, beam size is determined by the beam-beam interaction and $\sigma^2 \sim N_\ell$.

For the round Gaussian beams of equal size under consideration, the effective area is $A_{eff} = 4\pi\sigma^2$, so the luminosity is

$$L = \frac{fN_s}{4\pi}\left(\frac{N_\ell}{\sigma^2}\right) . \tag{9}$$

Solving Eq. (8) for the quantity (N_ℓ/σ^2), we find that it is a monotonically increasing function of N_ℓ, but one which approaches a constant value as N_ℓ grows large in comparison to a critical number,

$$N_c = \frac{2\gamma\sigma_r^2}{r_e\beta}\left[f\tau \ln(4/3)\right]^{-1/2} . \tag{10}$$

The constant value approached by the quantity (N_ℓ/σ^2) is (N_c/σ_r^2), so there is a limiting luminosity given by

$$L_{max} = \frac{2f\gamma N_s}{4\pi r_e\beta\left[f\tau \ln(4/3)\right]^{1/2}} , \tag{11}$$

which applies for $N_\ell \gg N_c$. For other than asymptotic values of N_ℓ, the luminosity must be obtained from Eq. (8) and Eq. (9). In terms of the critical number,

$$L = L_{max}\left(\frac{N_c}{2N}\right)\left\{\left[1+\left(\frac{2N}{N_c}\right)^2\right]^{1/2} - 1\right\} \quad . \tag{12}$$

To sum up, we have found that the impulses due to the linac beam pulses cause the stored beam bunch to spread transversely in such a fashion that, when $N_\ell \gg N_c$, the effect on luminosity of an increase in linac beam-pulse population (N_ℓ) is just counteracted by that of the consequent increase in beam area ($4\pi\sigma^2$), and the luminosity remains constant. Thus, linac pulse populations greatly in excess of N_c are not effective to increase the luminosity, and, if the linac can produce pulse populations of the order of N_c, a luminosity of the order of L_{max} should be achievable.

Now we apply these considerations to the SLAC storage ring now under construction. Turning first to the factor β, it should be made small. However, small values of β imply that the length of the waist at the interaction region is small; because the length of high-particle-density region is of the order of 2β. This is turn implies that the linac pulses as well as the stored bunch must be similarly short which will reduce the bunch populations that the linac can produce. A compromise is in order. The bunch length in the SLAC storage ring, as designed, will be about 30 cm at 2 GeV, so we choose $\beta = 15$ cm.

Next we turn to the factor N_s. A definite limitation on the number of stored positrons is set by the radiofrequency accelerating power available to the beam in the storage ring. The synchrotron-radiation energy loss per revolution for a 2-GeV positron is 110 KeV. We assume an available rf power of 220 kilowatts so that a circulating current of 2 amperes can be stored. There are, of course, other potential limitations on N_s. The rate of positron filling from the linac must be great enough to overcome the rate of loss of stored particles due to bremsstrahlung in the residual gas in the vacuum chamber and to the Touschek Effect.[12] On the basis of present SLAC linac performance, we estimate a filling rate into one bunch to be about 9 amperes per hour, while we estimate the stored-beam lifetime to be an hour or more, so the filling rate is quite sufficient. Another requirement is that the stored-particle motions be stable in their own space-charge fields. Either incoherent or coherent instability may arise. The incoherent space-charge limit on the intensity of a single stored beam arises when the space-charge forces shift particle oscillation frequencies to the extent that individual particle motions become unstable. Using a frequency-shift formula due to Laslett,[13] we estimate the frequency shift to be less than 10^{-2} for 2 amperes stored in the ring, a very safe value. Coherent instabilities of stored beams have been observed in most existing storage rings, and various mechanisms of origin have been studied. Storage in a single circulating bunch is especially favorable for controlling one class of such instabilities which can be suppressed by a proper choice of betatron frequencies.[14] Furthermore, the SLAC storage ring will be equipped with feedback systems and other devices to control all of the presently understood coherent instabilities. For estimating the luminosity, we assume that these devices will permit the storage of 2 amperes without deleterious coherent instability.

A circulating current of 2 amperes corresponds to

$$N_s = 10^{13}.$$

Other relevant parameters for 2 GeV storage ring operation with a round beam are: $\gamma = 4 \times 10^4$, $\sigma_r = 0.013$ cm (corresponding to $\beta = 15$ cm), $\tau = 0.028$ sec, and $f = 360 \text{ sec}^{-1}$. We obtain with these parameters

$$L_{max} = 3.2 \times 10^{29} \text{ cm}^{-2} \text{ sec}^{-1}$$

$$N_c = 1.9 \times 10^{11}.$$

We propose a mode of operation in which the linac delivers three one-nanosecond (30-cm) pulses spaced at 700-nanosecond intervals within the rf accelerating burst. These three pulses in a burst strike the circulating bunch on three successive turns around the ring. For purposes of estimating the luminosity, we lump their scattering effects as though they all came at once. We assume 10^{11} electrons per one-nanosecond pulse, so $N_\ell = 3 \times 10^{11}$. From Eq. (12) we obtain the estimated luminosity,

$$L = 2.4 \times 10^{29} \text{ cm}^{-2} \text{ sec}^{-1} \tag{13}$$

and this is the figure we shall use.

III. COUNTING RATES

We denote the number of $e + e \rightarrow e + e + \mathcal{X}$ events per hour (taking place in the interaction region) by $N(\mathcal{X})$, and the cross section of this process by $\sigma(\mathcal{X})$. The $N(\mathcal{X})$ is given by the well known formula

$$N(\mathcal{X}) = \left[L \{\text{cm}^{-2} \text{ sec}^{-1}\} \sigma(\mathcal{X}) \{\mu\text{barn}\} \, 3.6 \times 10^{-27} \right] \{\text{hr}^{-1}\} \, . \tag{14}$$

Here the expression in curly brackets gives the dimensions of the factor immediately preceding the bracket. Substituting the value of L from Eq. (13) into Eq. (14), one obtains

$$N(\mathcal{X}) = \left[0.86 \times 10^{3} \, \sigma(\mathcal{X}) \{\mu\text{barn}\} \right] \{\text{hr}^{-1}\} \, . \tag{15}$$

Expressions for $\sigma(\mathcal{X})$ are given in Ref. 7 for 1 GeV $\leq E_{cm} \leq$ 6 GeV, when $\mathcal{X}$ is one of the following: $\mu^+\mu^-$, $\pi^+\pi^-$, π^0 or η. Extrapolating these $\sigma(\mathcal{X})$ to the case under discussion, i.e., when E_{cm} = 12.6 GeV, we find the approximate results: $\sigma(\mu^+\mu^-) \approx 7\times10^{-2}$ μbarns, $\sigma(\pi^+\pi^-) \approx 1\times10^{-2}$ μbarns and $\sigma(\eta) \approx \sigma(\pi^0) \approx 1\times10^{-3}$ μbarns. With these values Eq. (15) gives the approximate values listed in the second column of Table I. These cross sections may be compared to the single-photon cross section for $\mathcal{X} = \mu\mu$ (Fig. 1) of about 0.5×10^{-3} μbarn.

Not all reactions which take place in the interaction region can be observed, because some of the final particles escape detection. Let us denote by $\overline{N}(X)$ the number per hour of those reactions, $e + e \rightarrow e + e + X$, in which at least one of the final electrons (either an e^+ or an e^-) is detected. Let us denote by θ_L^f the angle in the laboratory between the momentum $\overline{p}^f$ of particle "f" in the final state, and the direction of the incoming electron, and by θ_{cm}^f the same angle measured in the reaction center-of-mass frame. To estimate $\overline{N}(X)$, we assume that particle f is detected always, except when it scatters into the walls of the beam channel through one of the two magnets immediately adjacent to the interaction region. We assume that those particles which leave the beam before they pass through those magnets can be detected by the interaction region apparatus. On the other hand, we assume that those particles which scatter almost parallel or antiparallel to the incoming electron beam, and travel with one of the beams within the bore through these two magnets, can be detected leaving a bending magnet. (The scattered particles have less energy than the unscattered electrons and positrons.) We shall not discuss here in detail the experimental techniques of detection, nor the problem of scattered electron background produced by bremsstrahlung. The parameters of the storage ring under construction at the Stanford Linear Accelerator Center are such that particles scattered with an angle

$$9.0 \text{ mr} \lesssim \theta_L^f \lesssim 17.5 \text{ mr}, \tag{16a}$$

and

$$\pi \text{ radians} - 17.5 \text{ mr} \lesssim \theta_L^f \lesssim \pi \text{ radians} - 9.0 \text{ mr} \tag{16b}$$

hit the walls of the magnets while passing through the bore. Therefore, according to our assumptions, all final particles, except those which scatter into the laboratory angle intervals given in Eq. (16), can be detected. Performing a Lorentz transformation from the laboratory to the reaction center-of-mass system, we find that the angles θ_L^f and θ_{cm}^f are related by[6]

$$\mathrm{tg}\,\theta_L^f = \sqrt{1-u^2}\;\frac{(p^f/m^f)\sin\theta_{cm}^f}{(p^f/m^f)\cos\theta_{cm}^f + u\left[1+(p^f/m^f)^2\right]^{1/2}} \tag{17a}$$

$$\xrightarrow[p^f/m^f\to\infty]{} \sqrt{1-u^2}\;\frac{\sin\theta_{cm}^f}{\cos\theta_{cm}^f + u}\,, \tag{17b}$$

where u is the velocity of the reaction center-of-mass as measured in the laboratory, m^f is the rest mass of particle f, and $p^f \equiv |\vec{p}_{cm}^{\,f}|$. According to Eq. (17), the laboratory angle intervals given in Eq. (16) are transformed into the reaction center-of-mass angle intervals

$$28.5\ \mathrm{mr} \lesssim \theta_{cm}^f \lesssim 56.0\ \mathrm{mr}, \tag{18a}$$

and

$$\pi\ \mathrm{radians} - 56.0\ \mathrm{mr} \lesssim \theta_{cm}^f \lesssim \pi\ \mathrm{radians} - 28.5\ \mathrm{mr}, \tag{18b}$$

so that, according to our assumptions concerning the geometry of the experimental region, all final particles can be detected, except those for which θ_{cm}^f lies in the intervals given by Eq. (18).

We intend to calculate that fraction of all $e + e \to e + e + X$ events, for which either $\theta_{cm}^{e^+}$ or $\theta_{cm}^{e^-}$ lies in one of the intervals given by Eq. (18). To do this, we use the Weizsäcker-Williams approximation[6,7] which leads to the expression

$$d\sigma\left(\theta_{cm,M}^{e^+};\,\theta_{cm,M}^{e^-}\right)/dM = \mathrm{const}\times\sigma_{\gamma\gamma}(M)\;M^3\int\frac{d\omega}{\omega^5}\,f\left(\theta_{cm,M}^{e^+}\right)f\left(\theta_{cm,M}^{e^-}\right) \tag{19}$$

given by Eq. (10) in the Appendix of the third of Ref. 6. In Eq. (19), M is the rest mass of X; $\sigma_{\gamma\gamma}(M)$ the cross section for production of X with such a mass by two photons, and $d\sigma\left(\theta_{cm,M}^{e^-};\,\theta_{cm,M}^{e^+}\right)/dM$ is the differential cross section (with respect to M) for those $e + e \to e + e + X$ processes, for which $\theta_{cm}^{e^-} \leq \theta_{cm,M}^{e^-}$ and $\theta_{cm}^{e^+} \leq \theta_{cm,M}^{e^+}$. For the range of parameters of interest to us, we can use the approximation[6]

$$f\left(\theta^{f}_{cm, M}\right) \approx \ln\left(\frac{E^2_{cm}}{4\omega m}\ \theta^{f}_{cm, M}\ \{\text{radians}\}\right) , \tag{20}$$

where $f = e^+$ or e^-. Again, the curly bracket indicates that $\theta^{f}_{cm, M}$ is measured in radians. The electron mass is m. The logarithm in Eq. (20) is a slowly changing function of ω, and, in the following approximation, we will consider it as a constant (function of ω). With this approximation

$$\sigma\left(\theta^{e^+}_{cm, M}, \theta^{e^-}_{cm, M}\right) \approx D \times \ln\left(\frac{E^2_{cm}}{m}\ \theta^{e^+}_{cm, M}\right) \ln\left(\frac{E^2_{cm}}{m}\ \theta^{e^-}_{cm, M}\right), \tag{21}$$

where D depends neither on $\theta^{e^+}_{cm, M}$ nor on $\theta^{e^-}_{cm, M}$.

The cross section for those events for which $\theta^{e^-}_{cm}$ lies in the interval (18b) (electron scattered backwards), is very small, and we simply neglect these events. Similarly, we neglect those events for which $\theta^{e^+}_{cm, M}$ lies in the interval (18a) (positron scattered backwards). With this approximation, the cross section for those events for which $\theta^{e^-}_{cm}$ lies in any of the two intervals (18), is the same as the cross section for those events for which $\theta^{e^-}_{cm}$ lies in the interval (18a). We denote this cross section by $\sigma\left(28.5 \text{ mr} \le \theta^{e^-}_{cm} \le 56.0 \text{ mr}\right)$. It can be calculated from the formula

$$\sigma\left(28.5 \text{ mr} \le \theta^{e^-}_{cm} \le 56.0 \text{ mr}\right) = \sigma\left(\theta^{e^+}_{cm, M} = \pi \text{ radians},\ \theta^{e^-}_{cm, M} = 56.0 \text{ mr}\right)$$
$$- \sigma\left(\theta^{e^+}_{cm, M} = \pi \text{ radians},\ \theta^{e^-}_{cm, M} = 28.5 \text{ mr}\right). \tag{22}$$

A similar expression gives $\sigma\left(\pi \text{ radians} - 28.5 \text{ mr} \le \theta^{e^+}_{cm} \le \pi \text{ radians} - 56.0 \text{ mr}\right)$, which, within our approximations, is the cross section for all those events for which $\theta^{e^+}_{cm}$ lies in any of the two intervals (18).

Substituting Eq. (21) into Eq. (22), we obtain

$$\sigma\left(28.5 \text{ mr} \le \theta^{e^-}_{cm} \le 56.0 \text{ mr}\right) = D \times \left[\ln\left(\frac{E_{cm}}{m}\ 56.0 \times 10^{-3}\right) - \ln\left(\frac{E_{cm}}{m}\ 28.5 \times 10^{-3}\right)\right]$$
$$\times f\left(\theta^{e^+}_{cm, M} = \pi \text{ radians}\right) \tag{23}$$

To obtain D, we evaluate $\sigma\left(\theta^{e^+}_{cm,M} = 4 \text{ mr}, \theta^{e^-}_{cm,M} = 4 \text{ mr}\right)$ from Eq. (21), and compare the result with the published values of this cross section.[6] Substituting the D so obtained into Eq. (23), we find

$$\sigma\left(28.5 \text{ mr} \le \theta^{e^-}_{cm} \le 56.0 \text{mr}\right) \lesssim 0.06\, \sigma(X) \qquad (24)$$

We conclude that, under our assumptions, in only 6% of all $e + e \longrightarrow e + e + X$ events does the final e^- escape detection. Similarly, we find that about 94% of all final e^+ can be detected. We conclude that

$$\overline{N}(X) \approx N(X) . \qquad (25)$$

The angular distribution of the particles which make up the system X is roughly isotropic, although peaked in the forward and backward directions, in the center-of-mass system, when X is either $\mu^+ + \mu^-$ or $\pi^+ + \pi^-$, for $E_{cm} = 4$ GeV or 6 GeV.[6,7] The same is true for the angular distribution of γ rays coming from $X \longrightarrow 2\gamma$, when $X = \pi^o$ or η. Equation (17) shows that particles which are extremely nonrelativistic in the reaction center-of-mass system all have very small θ^f_L. Howevery, already when $(p^f/m^f)^2 = 1/4$ or higher, and assuming that the production of particle f is isotropic as a function of θ^f_{cm}, one finds that, with our assumption concerning the geometry of the interaction region, more than 85% of all particles f can be detected. On the other hand, particles with $(p^f/m^f)^2 < 1/4$ constitute only a small fraction of all the final particles because of the small phase space which is available to them. If $u \longrightarrow 1$, then θ^f_L would become small, and many particles would escape detection. However, in the experiment we are considering, $u \approx 0.8$, which is not high enough for this to happen. We conclude that most particles which belong to the final system X, can be detected.

These are encouraging results, and there is hope that the Stanford Linear Accelerator and the storage ring under construction there will be combined in the future in the manner suggested in this paper. The present orientation of the storage ring is such that the axis of its interaction region is roughly perpendicular to the 20-GeV linac beam; however, with the addition of a bent beam line and a new beam dump the linac beam could be brought to the storage ring.

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9. The history of this paper is the following: John Rees had estimated the luminosity obtainable by colliding a linac beam with a stored beam,[1] but found that the expected counting rates, although acceptable, were rather low. At the Frascati Informal Meeting on $e^+ - e^-$ on colliding beam rings in September 1970, during P. Kessler's report, P. L. Csonka estimated the counting rates obtainable at SLAC on the basis of cross section values given by Kessler and found them high. He was referred to John Rees and his work. The discrepancy was later

traced to the differences in cross sections assumed by the two of them, and the present paper was then written.

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TABLE I

X	$N(X)$
$\mu^+\mu^-$	60 hr^{-1}
$\pi^+\pi^-$	10 hr^{-1}
π^o or η	1 hr^{-1}

The number of $e + e \rightarrow e + e + X$ events taking place in the interaction region at E_{cm} = 12.6 GeV is $N(X)$. The number of those events in which at least either the final electron or the final positron can be detected under the assumptions discussed in the text is close to $N(X)$. The number of those events in which all particles contained in X can be detected is also close to $N(X)$.

FIGURE CAPTIONS

1. "Single-photon annihilation" of an electron-positron pair, producing the final particles X.

2. Each of the two photons is produced by an e (either electron or positron). The colliding photons produce the outgoing particles X in a "photon-photon scattering."

3. An outline of the proposed arrangement: 20 GeV linac electrons collide with 2 GeV stored positrons in the interaction region.

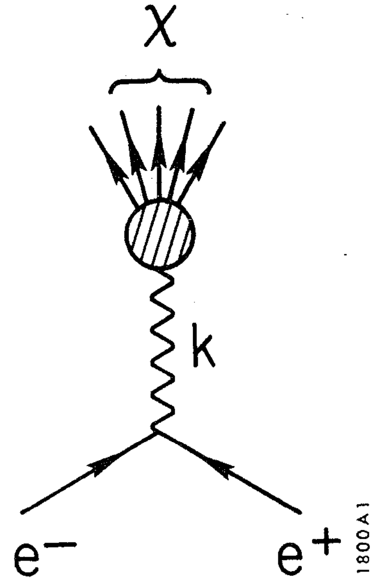

Fig. 1

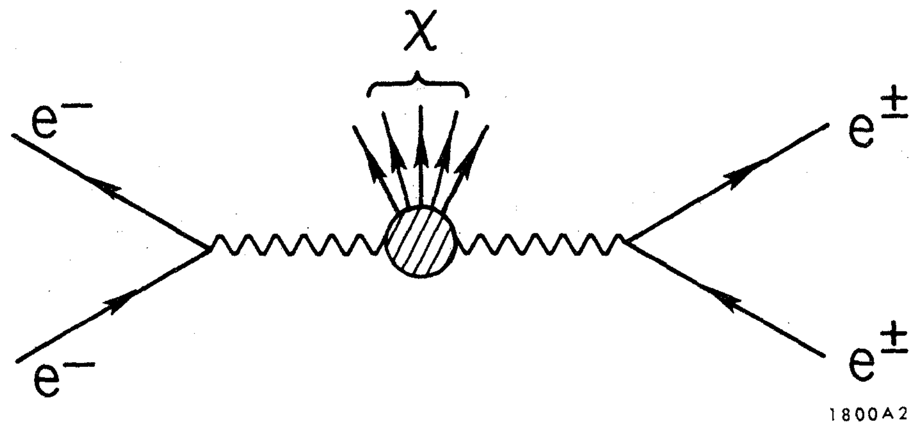

Fig. 2

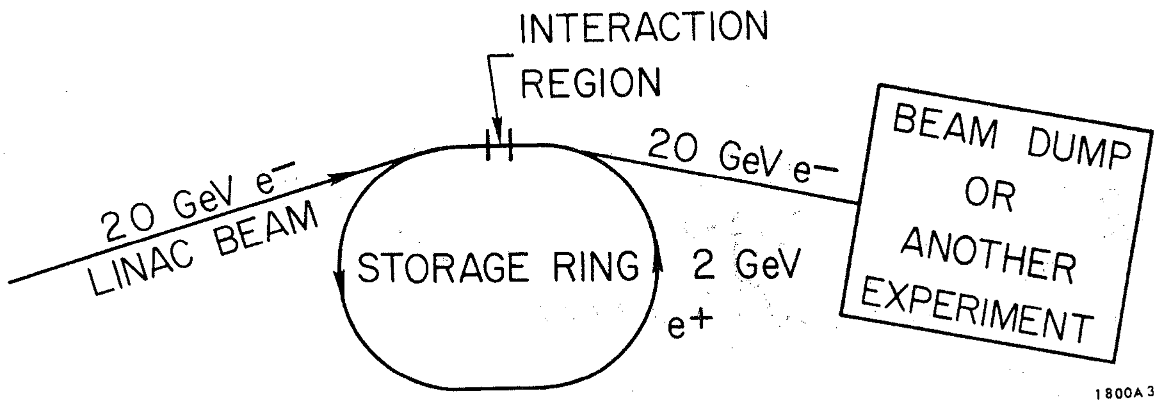
INTERACTION
REGION
20 GeV e−
LINAC BEAM
STORAGE RING
20 GeV e−
2 GeV
e+
BEAM DUMP
OR
ANOTHER
EXPERIMENT
1800A3

Fig. 3