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One-Loop β Functions for Yukawa Couplings in the Electroweak-Scale Right-Handed Neutrino Model

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Abstract. Fermions in the model of electroweak-scale right-handed neutrinos (EWRH) with masses of the order of 300 GeV or more could result in dynamical electroweak symmetry breaking by forming condensates through the exchange of a fundamental Higgs scalar doublet or triplet. These condensates are dynamically studied within the framework of the Schwinger-Dyson equation. With the electroweak symmetry broken by condensates, the fully worked-out model of EWRH in which there are two doublets and two triplets, one of which is composite and the others being the original fundamental scalar doublet and triplet could be suitable for recent LHC discovery of the 125 GeV scalar particle.

1. Introduction

The SM has been established for a long time and has shown its abilities to address many problems in High Energy Physics. Nevertheless, some problems can not be explained by SM. One of which is SM Higgs mechanism where spontaneous symmetry breaking (SSB) is put in by hand. With Higgs potential $V(\phi) = \mu^2\phi^+\phi + \lambda(\phi^+\phi)^2$, this mechanism does not explain why μ^2 is negative when it could, a priori, have been positive. μ^2 and hence $m_H^2 = -2\mu^2 = 2\lambda v^2$ are unstable to large radiative corrections from much higher enery scales (gauge hierarchy problem, fine-tuning needed to keep Higgs light, etc.). Moreover, the SM Yukawa mechanism for generating fermion masses, with $m_f = y_f v/\sqrt{2}$, does not give insight into these masses, since it just puts in the value of Yukawa couplings y_f by hand, and some y_f's range down to 10^{-5} with no explanation. Might it be a possible way to give a dymanical reason for fermion masses? Consider two major previous cases of superconductivity and Gell-Mann Levy σ model for spontaneous chiral symmetry breaking (SχSB) where the fundamental scalar fields were used to model SSB. For these cases, the actual underlying physics did not involve fundamental scalar fields but instead a bilinear fermion condensate. For instance, the physical properties of superconductor materials can be explained by dynamically forming a condensate state of Cooper pairs (ee) in BCS theory; since these are charged, they give mass to photon, resulting in Meissener effect. In the second case, the actual origin of SχSB in QCD is the dynamical formation of a $(\bar{q}q)$ condensate. These previous models of symmetry breaking, therefore, bring about the underlying physics responsible for this symmetry breaking may well be a dynamically induced fermion condensate.
The condition for condensation mentioned in [1] is that the Yukawa couplings are larger than critical Yukawa couplings α^c. For instance, when the energy scale is of the order of $O(1\text{TeV})$, the Yukawa coupling of a heavy fourth generation in [2] is large enough, exceeding α^c, to form

condensates through the exchange of a fundamental Higgs scalar doublet. In this paper, a similar mechanism has been discussed. We have also investigated the one-loop β functions in the model of [3] to find the energy scale forming condensate states of fermions. For the fact that condensates of mirror fermions can be a part of DEWSB, the fully worked-out model of EWν_R in which there are one singlet, two doublets and two triplets could be compatible with recent LHC discovery of Higgs particle [4], [5], while the SM with fourth generations has been ruled out.
The plan of the paper is as follows. Firstly, we summarize the essential elements of the EWν_R model of [3]. Secondly, the analytical formula and numerical calculations related to the one-loop beta functions of Yukawa coulings are presented. We conclude with some remarks concerning the DEWSB in the model of [3].

2. The EWν_R model

The essential elements of the model presented in [3] can be reviewed as below
The $SU(2)_L \times U(1)_Y$ fermion content of the EWν_R model of [3]
- $SU(2)_L$ lepton doublets:

$$\begin{aligned} SM: l_L &= \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}; \\ Mirror: l_R^M &= \begin{pmatrix} \nu_R \\ e_R^M \end{pmatrix}, \end{aligned} \tag{1}$$

for the SM left-handed lepton doublet and for the right-handed mirror lepton doublet respectively.
- $SU(2)_L$ lepton singlets:

$$SM: e_R; \tag{2}$$

$$Mirror: e_L^M \tag{3}$$

for the SM right-handed lepton singlet left-handed mirror lepton singlet respectively.
Similarly, for the quarks, we have
- $SU(2)_L$ quark doublets:

$$\begin{aligned} SM: q_L &= \begin{pmatrix} u_L \\ d_L \end{pmatrix}; \\ Mirror: q_R^M &= \begin{pmatrix} u_R \\ d_R^M \end{pmatrix}, \end{aligned} \tag{4}$$

for the SM left-handed quark doublet and for the right-handed mirror quark doublet respectively.
- $SU(2)_L$ quark singlets:

$$SM: u_R, d_R; Mirror: u_L^M, d_L^M, \tag{5}$$

for the SM right-handed lepton singlet left-handed mirror lepton singlet respectively.
Higgs content in the model of EWν_R
-Higgs doublets

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = (1, 2, Y/2 = 1/2). \tag{6}$$

-Higgs triplets

$$\widetilde{\chi} = \frac{1}{\sqrt{2}}\vec{\tau}\cdot\vec{\chi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\chi^+ & \chi^{++} \\ \chi^0 & -\frac{1}{\sqrt{2}}\chi^+ \end{pmatrix} = (1, 3, Y/2 = 1), \tag{7}$$

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$$\xi \;=\left(\begin{array}{c}\xi^+\\ \xi^0\\ \xi^-\end{array}\right)\;=(1,3,Y/2=0)\,. \tag{8}$$

-Higgs singlet

$$\phi_S=(1,1,Y/2=0). \tag{9}$$

Higgs triplets such as in (7) were considered earlier in various contexts [6],[7],[8].
In the search for the beta functions discussed in this work, it is important to know Yukawa Lagrangian. The Yukawa couplings can be found in [9]. In the case of SM fermions, we have the usual Yukawa interactions

$$\mathcal{L}_{SM}=-g_{SM}\bar{\Psi}_L\Phi\Psi_R+\text{h.c.}. \tag{10}$$

For mirror fermions we consider the terms

$$\mathcal{L}_{e^M}=-g_l^M\bar{l}_R^M\Phi e_L^M+\text{h.c.}, \tag{11}$$

$$\mathcal{L}_{d^M}=-g_d^M\bar{q}_R^M\Phi d_L-g_d^M\bar{q}_R^M\widetilde{\Phi}d_L^M+\text{h.c.}, \tag{12}$$

$$\mathcal{L}_{\nu_R}=g_Ml_R^{M,T}\sigma_2\tau_2\widetilde{\chi}l_R^M, \tag{13}$$

$$\mathcal{L}_S=-g_{Sl}\bar{l}_Ll_R^M\phi_S+\text{h.c.}. \tag{14}$$

As energy increases, the Yukawa interactions become strong, bound states of fermions can be formed. The top quark, however, can not form a bound state by exchanging the Higgs boson [2] because of a severely fine-tuned picture of DEWSB. Moreover, the vacuum expectation values of Higgs doublets and Higgs triplets break the global $SU(2)_L\times SU(2)_R$ down to the custodial $SU(2)_D$. Thus, we are interested in the running of Yukawa couplings: g_l^M, g_d^M and g_M.

3. Oneloop beta functions for Yukawa couplings in the EWν_R model

3.1. Analytical formula

In the EWν_R model, right-handed neutrino interacts with Higgs triplet $\widetilde{\chi}$. Hence, the running of Yukawa coupling g_M is firstly investigated in this section. The one loop contributions to the Yukawa coupling renormalization constant are composed of four terms: vertex corrections, fermion self-energy, scalar self-energy and gauge interactions. At high enery scale, gauge couplings, however, become too small compared to Yukawa couplings then gauge contributions to the one-loop β functions can be neglected. For this reason the first three terms are computed. Consider the vertex corrections which contribution diagrams modifying the proper one-loop β function for g_M are shown in fig. 1

$$\Gamma_1=-(i)^3g_M^3\left(1+\frac{1}{2}\right)\int\frac{d^4k}{(2\pi)^4}\left(\frac{i}{\gamma^\mu p_\mu-\gamma^\mu k_\mu}\right)^2\frac{i}{k^2}. \tag{15}$$

The contribution coming from equation (15) is computed by using dimensional regularization. We have

$$\beta_1=3\times\frac{g_M^3}{16\pi^2}. \tag{16}$$

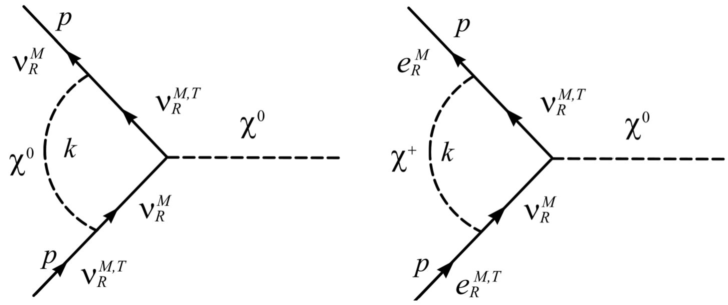

Figure 1. Contribution diagrams of vertex corrections modifying the proper one-loop β function for g_M

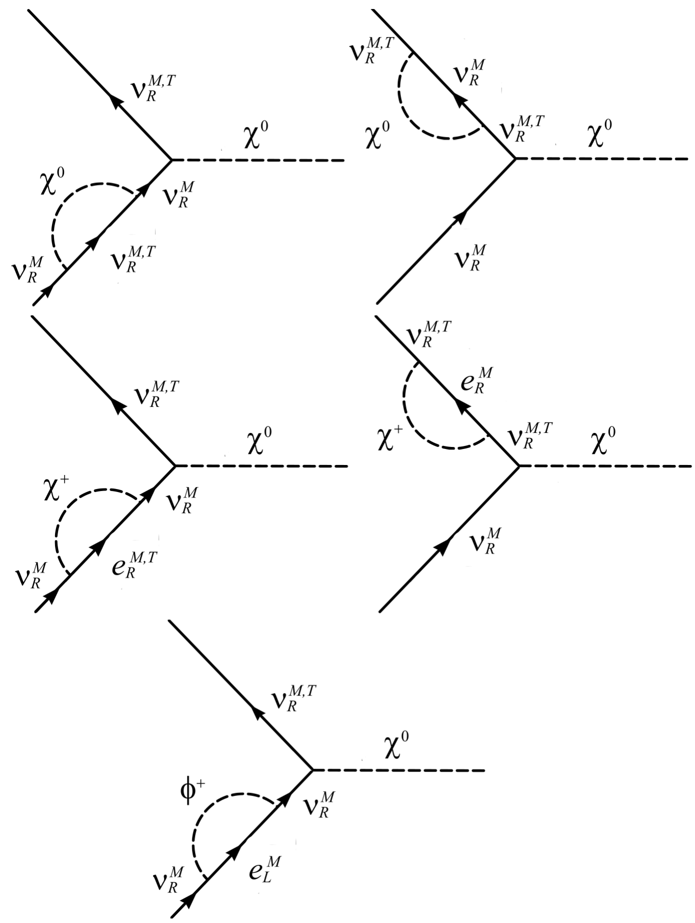

Figure 2. Contribution diagrams of femion self-energy modifying the proper one-loop β function for g_M

The fermion self-energy term, which diagrams shown in fig. 2, is given by

$$\Sigma_f(p) = \left[2(-ig_M)^2 + 2\left(-\frac{i}{\sqrt{2}}g_M\right)^2 + \left(-ig_l^M\right)^2\right] \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2} \frac{i(\gamma^\mu p_\mu \gamma^\mu k_\mu)}{(p-k)^2}, \tag{17}$$

and its contribution to the beta function is

$$\beta_2 = \left(\frac{3}{2}g_M^2 + \frac{1}{2}g_l^{M2}\right)\frac{g_M}{16\pi^2}. \tag{18}$$

Finally, the Feynman diagram for the scalar self-energy is shown in fig. 3 and

$$\Sigma_s(p) = (-ig_M)^2(-)\int\frac{d^4k}{(2\pi)^4}Tr\left(\frac{i}{\gamma^\mu k_\mu}\frac{i}{\gamma^\mu p_\mu - \gamma^\mu k_\mu}\right), \tag{19}$$

$$\beta_3 = 2\times\frac{g_M^3}{16\pi^2}. \tag{20}$$

The total contribution to the beta function is then

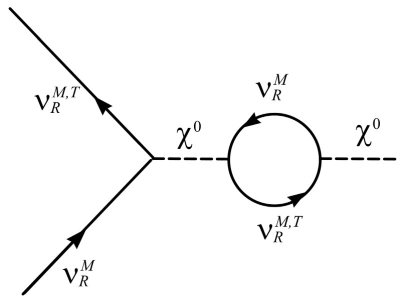

Figure 3. Contribution diagram of scalar self-energy modifying the proper one-loop β function for g_M

$$\beta_{g_M} = \frac{dg_M}{dt} = \beta_1 + \beta_2 + \beta_3 = \frac{\left(13g_M^3 + g_L^{M2}g_M\right)}{32\pi^2}. \tag{21}$$

Similarly, by using dimensional regularization, we found the the evolution of Yukawa couplings of both mirror leptons and mirror quarks

$$\beta_{g_l^M} = \frac{dg_l^M}{dt} = \frac{22g_l^{M3} + 3g_M^2 g_l^M}{64\pi^2}, \tag{22}$$

$$\beta_{g_d^M} = \frac{dg_d^M}{dt} = \frac{3g_d^{M3}}{8\pi^2}. \tag{23}$$

3.2. Numerical calculations

As shown in fig.4, with initial values of g_M, g_l^M, and g_d^M being $2.50, 2.46$ and 2.73 respectively or the coresponding induced masses being about 300 GeV, the Yukawa couplings of mirror fermions increase dramatically as energy increases and Ladau pole singularrities appear at the order of $O(1\text{TeV})$. Moreover, by using the Schwinger-Dyson equation in the rainbow approximation to compute the self-energy of fermions and subsequently their condensates, we found the critical Yukawa couplings of g_M, g_l^M, and g_d^M are π, π and $\pi/2$ respectively [10]. Above which condensates carrying the electroweak quantum numbers can be formed which could spontaneously break electroweak symmetry. It is amazing that from fig.4 both mirror fermions form condensates at the order of $O(1\text{TeV})$ while top quark which initial Yukawa coupling is approximately 1 hardly to do so. For this reason, mirror fermions are responsible for DEWSB in the EWν_R model.

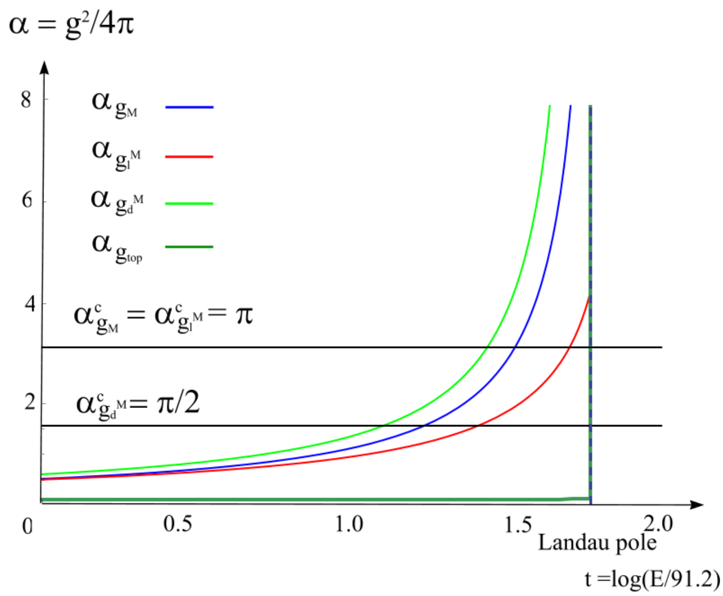

Figure 4. The evolution of Yukawa couplings

4. Conclusion

The central results of our paper, on one hand, is eqs.(21), (22) and (23) which give beta functions for the Yukawa couplings in the EWν_R model. For suitble initial values of Yukawa couplings, the Landau singularities appear at the order of O(1TeV). As critical Yukawa couplings are known, electroweak symmetry can be broken dynamically by condensates of mirror quarks and right-handed neutrinos. On the orther hand, because DEWSB scale is of the order of O(1TeV) with forbidden case that mirror leptons get condensate states later than the others do (badly symmetry broken), the mass distribution of fermions in the EWν_R model and constraints on form factors by using conditions assuring proper alignment of the two VEVs can be investigated in near future.

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