Physics Letters B 771 (2017) 199–205

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Embedding cosmological inflation, axion dark matter and seesaw mechanism in a 3-3-1 gauge model

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ARTICLE INFO

Article history:
Received 25 March 2017
Received in revised form 9 May 2017
Accepted 10 May 2017
Available online 19 May 2017
Editor: M. Trodden

ABSTRACT

The Peccei–Quinn symmetry is an intrinsic global symmetry of the 3-3-1 gauge models. Its spontaneous breaking mechanism engendering an invisible KSVZ-like axion links the 3-3-1 models with new physics at $\sim 10^{10}$ GeV scale. The axion that results from this mechanism is an interesting candidate for the dark matter of the universe, while its real partner may drive inflation if radiative corrections are taken into account. This is obtained by connecting the type I seesaw mechanism with the spontaneous breaking of the Peccei–Quinn symmetry. In the end of the day we have a scenario providing a common answer to the strong-CP problem, inflation, dark matter and neutrino mass.

1. Introduction

The $SU(3)_C \times SU(3)_L \times U(1)_N$ (3-3-1) gauge models for the electroweak interactions are interesting in their own right. For example, in these models generation cannot replicate unrestrictedly as in the standard model (SM), since it is not exact replica of one another and each is separately anomalous. However, when three generations are taken into account, gauge anomaly is automatically canceled [1,2], providing a reason for the existence of three families of fermions.

Moreover, the set of constraints provided by the gauge invariance of the Yukawa interactions together with those coming from the anomaly cancellation conditions are enough to fix the electric charges of the particles in the 3-3-1 model, thus providing an understanding of the pattern of electric charge quantization [3,4].

In what concerns the Peccei–Quinn (PQ) symmetry, it is an automatic symmetry of these models, thus elegantly solving the strong CP-problem [5]. However, the original versions of the 3-3-1 gauge models furnish an unrealistic axion because of its sizable couplings with the standard particles [6,7]. In order to have an invisible axion a neutral scalar singlet must be added to the conventional scalar sector [8–11].

Regarding neutrino masses, canonical seesaw mechanisms, as type I and type II, as well as the inverse seesaw mechanism are easily implemented in the framework of the 3-3-1 models [12–16].

From the phenomenological point of view, a remarkable aspect of the 3-3-1 models relies on flavor physics. Rare decays, lepton number violation and flavor changing neutral current are natural outcome of the model [17–24]. Recent collider phenomenology of these models are performed in Refs. [25–27].

Last in the sequence but not least in importance, we remember that conventional particle content of some 3-3-1 models includes a stable and neutral particle that may play the role of cold dark matter in the WIMP form [28–31]. These interesting features turn the 3-3-1 models an appealing candidates for physics beyond the SM. In this point we call the attention to the fact that the physics of the early universe, particularly inflation, has been poorly explored within these models [32,33]. Thus, in view of the recent experimental advances in probing inflation observables, it turns imperative to search for mechanisms that allow implementation of inflation in the framework of the 3-3-1 gauge models.

Concerning implementation of cosmological issues within phenomenological gauge models, as the SM, we remark that there are two distinct ways of providing a common solution to cosmological inflation, cold dark mater and neutrino masses within the SM. The first arises within the type I seesaw mechanism for small neutrino masses. By adding right handed neutrinos and at least one neutral scalar in the singlet form to the SM, besides considering spontaneous breaking of global lepton number within type I seesaw mechanism, one has that the real part of the neutral singlet may drive inflation while the imaginary part may be the dark matter of the universe [34,35].

On the other hand, the implementation of the PQ symmetry in the standard model may be accomplished by adding exotic vector

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http://dx.doi.org/10.1016/j.physletb.2017.05.034

like quarks, right-handed neutrinos and neutral scalar singlet to its particle content. This scenario is called SMASH [36–40]. The PQ symmetry is spontaneously broken when the neutral scalar singlet develops vacuum expectation value (VEV) different from zero. In this circumstance, the imaginary part of this scalar singlet will be the invisible axion, which may play the role of dark matter, while the real part may drive inflation. Moreover, on coupling the neutral scalar singlet to the right-handed neutrino, through an Yukawa interaction, the VEV of the neutral scalar, that must lie in the range $(10^{10}$–$10^{11})$ GeV, will generate mass to heavy neutrinos that may trigger the type I seesaw mechanism yielding small masses for the standard neutrinos. The problem with this scenario is that it generates an inflaton potential of the type $\lambda\phi^4$ which is practically excluded by the current bounds from PLANCK15 [41]. A way of circumventing such a problem is, either to consider that the inflaton couples non-minimally with the scalar curvature R, or take into account radiative corrections to the inflaton potential.

The main difference among these two proposals relies on the dark matter sector. In the former case the dark matter candidate, a Majoron, gains mass from quantum gravity effect and then is classified as warm dark matter, while in the latter case the dark matter candidate is the axion and is classified as cold dark matter candidate. The axion gains mass from QCD and quantum gravity effects. Care must be taken because quantum gravity effect may destabilize the axion as dark matter candidate. We take care of this by means of large discrete symmetry.

Since the PQ symmetry is an automatic symmetry of the 3-3-1 gauge models, consequently its version involving right-handed neutrinos realizes automatically the SMASH proposal. In this case it turns imperative to check if the real partner of the axion will drive inflation. We show that this is possible when radiative corrections are taken into account. We examine also reheating phase. It is engendered by the decay of the inflaton into the conventional scalars. The model easily provides a reheating temperature of 10^9 GeV for typical values of the parameters required by canonical inflation models. In addition, standard neutrinos will gain mass through the type I seesaw mechanism and the axion is the natural dark matter candidate of the model.

The paper is divided in the following way: In Sec. 2 we revisit the 3-3-1 model that contains an invisible axion in its spectrum. Next, in Sec. 3, we develop the inflationary paradigm in such model. We finally conclude in Sec. 4.

2. The 3-3-1 model, the Peccei–Quinn symmetry and the invisible axion

The model developed here is one proposed in Ref. [42] which is a modification of the original one [1,43,44]. To realize our proposal, heavy neutrinos in the singlet form must be added to the leptonic sector of the model

$$f_L^a = \begin{pmatrix} \nu_L^a \\ e_L^a \\ (\nu_R^c)^a \end{pmatrix} \sim (1\,,\,3\,,\,-1/3)\,,\;\; e_{aR} \sim (1, 1, -1)\,,$$
$$N_{aR} \sim (1, 1, 0) \tag{1}$$

with $a = 1, 2, 3$ representing the three known generations. We are indicating the transformation under 3-3-1 after the similarity sign, "$\sim$".

The quark sector is kept intact with one generation of left-handed fields coming in the triplet fundamental representation of $SU(3)_L$ and the other two composing an anti-triplet representation with the content

$$Q_{iL} = \begin{pmatrix} d_{iL} \\ -u_{iL} \\ d'_{iL} \end{pmatrix} \sim (3\,,\,\bar{3}\,,\,0)\,,\quad Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ u'_{3L} \end{pmatrix} \sim (3\,,\,3\,,\,1/3)\,, \tag{2}$$

and the right-handed fields

$$u_{iR} \sim (3, 1, 2/3)\,,\; d_{iR} \sim (3, 1, -1/3)\,,\; d'_{iR} \sim (3, 1, -1/3)$$
$$u_{3R} \sim (3, 1, 2/3)\,,\; d_{3R} \sim (3, 1, -1/3)\,,\; u'_{3R} \sim (3, 1, 2/3), \tag{3}$$

where $j = 1, 2$ represent different generations. The primed quarks are the exotic ones but with the usual electric charges.

In order to generate the masses for the gauge bosons and fermions, the model requires only three Higgs scalar triplets. For our proposal here we add a neutral scalar in the singlet form such that the scalar content is composed by

$$\chi = \begin{pmatrix} \chi^0 \\ \chi^- \\ \chi'^0 \end{pmatrix} \sim (1\,,\,3\,,\,-1/3)\,,\;\; \eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta'^0 \end{pmatrix} \sim (1\,,\,3\,,\,-1/3),$$
$$\rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho'^+ \end{pmatrix} \sim (1\,,\,3\,,\,2/3)\,,\qquad \phi \sim (1, 1, 0). \tag{4}$$

Thus the particle content of the model in Ref. [42] is extended by the fields N_{a_R} and ϕ.

In order to keep intact the physics results of the Ref. [42], the Lagrangian of the model must be invariant by the following set of discrete symmetries $Z_{11} \otimes Z_2$ but now with Z_{11} acting as

$$\begin{aligned}
&\phi \to \omega_1^{-1}\phi\,, && f_{aL} \to \omega_1 f_{aL}\,,\\
&\rho \to \omega_2^{-1}\rho\,, && d_{aR} \to \omega_2 d_{aR}\,,\\
&\chi \to \omega_3^{-1}\chi\,, && u'_{3R} \to \omega_3 u'_{3R}\,,\\
&Q_{iL} \to \omega_4^{-1} Q_{iL}\,, && d'_{iR} \to \omega_4 d'_{iR}\,,\\
&\eta \to \omega_5^{-1}\eta\,, && u_{aR} \to \omega_5 u_{aR}\,,\\
&Q_{3L} \to \omega_0 Q_{3L}\,, && N_R \to \omega_5^{-1} N_R\,,\\
&e_{aR} \to \omega_3 e_{aR}\,,
\end{aligned} \tag{5}$$

where $\omega_k \equiv e^{2\pi i \frac{k}{11}}$, $\{k = 0, \pm1, ..., \pm5\}$.

The Z_2 symmetry must act as

$$(\rho\,,\chi\,,d'_R\,,u'_{3R}\,,u_R\,,d_R\,,e_R) \;\to\; -(\rho\,,\chi\,,d'_R\,,u'_{3R}\,,u_R\,,d_R\,,e_R)\,. \tag{6}$$

These discrete symmetries yield the following Yukawa couplings

$$\begin{aligned}
\mathcal{L}^Y = {} & G_1 \bar{Q}_{3L} u'_{3R}\chi + G_2^{ij}\bar{Q}_{iL} d'_{jR}\chi^* + G_3^{3a}\bar{Q}_{3L} u_{aR}\eta + G_4^{ia}\bar{Q}_{iL} d_{aR}\eta^* \\
& + G_5^{3a}\bar{Q}_{3L} d_{aR}\rho + G_6^{ia}\bar{Q}_{iL} u_{aR}\rho^* + g_{ab}\bar{f}_{aL} e_{bR}\rho \\
& + h_{ab}\bar{f}_{aL}\eta N_{bR} + h'_{ab}\phi \bar{N}_{aR}^C N_{bR} + \text{H.c.}
\end{aligned} \tag{7}$$

The transformations displayed in Eqs. (5) and (6) are a little different from the original case [42]. The reason for this is to accommodate the last two terms in the Lagrangian above which are crucial for our proposal, as we will see later. The physics of the original case remains the same because the new terms involve heavy neutrinos that are standard model singlets.

The potential does not change. It is exactly the same as in the original case, i.e.,

$$
\begin{aligned}
V_H = {} & \mu_\phi^2\phi^2+\mu_\chi^2\chi^2+\mu_\eta^2\eta^2+\mu_\rho^2\rho^2+\lambda_1\chi^4+\lambda_2\eta^4+\lambda_3\rho^4 \\
& +\lambda_4(\chi^\dagger\chi)(\eta^\dagger\eta)+\lambda_5(\chi^\dagger\chi)(\rho^\dagger\rho)+\lambda_6(\eta^\dagger\eta)(\rho^\dagger\rho) \\
& +\lambda_7(\chi^\dagger\eta)(\eta^\dagger\chi)+\lambda_8(\chi^\dagger\rho)(\rho^\dagger\chi)+\lambda_9(\eta^\dagger\rho)(\rho^\dagger\eta) \\
& +\lambda_{10}(\phi\phi^*)^2+\lambda_{11}(\phi\phi^*)(\chi^\dagger\chi)+\lambda_{12}(\phi\phi^*)(\rho^\dagger\rho) \\
& +\lambda_{13}(\phi\phi^*)(\eta^\dagger\eta)+\lambda_\phi\epsilon^{ijk}\eta_i\rho_j\chi_k\phi+\text{H.c.} \qquad (8)
\end{aligned}
$$

Other tiny change arises in the definition of the PQ charges. In order to have chiral quarks under $U(1)_{PQ}$, we need the following transformation

$$
\begin{aligned}
& u_{aL}\to e^{-i\alpha X_u}u_{aL}\,,\quad u_{aR}\ \to\ e^{i\alpha X_u}u_{aR}\,,\\
& u'_{3L}\to e^{-i\alpha X'_u}u'_{3L}\,,\quad u'_{3R}\ \to\ e^{i\alpha X'_u}u'_{3R}\,,\\
& d_{aL}\to e^{-i\alpha X_d}d_{aL}\,,\quad d_{aR}\ \to\ e^{i\alpha X_d}d_{aR}\,,\\
& d'_{iL}\to e^{-i\alpha X'_d}d'_{iL}\,,\quad d'_{iR}\ \to\ e^{i\alpha X'_d}d'_{iR}\,. \qquad (9)
\end{aligned}
$$

For the leptons we can define their PQ charges by

$$
\begin{aligned}
& e_{aL}\to e^{i\alpha X_e}e_{aL}\,,\quad e_{aR}\ \to\ e^{i\alpha X_{eR}}e_{aR}\,,\quad N_{aL}\to e^{i\alpha X_N}N_{aL}\\
& \nu_{aL}\to e^{i\alpha X_\nu}\nu_{aL}\,,\quad \nu_{aR}\ \to\ e^{i\alpha X_{\nu R}}\nu_{aR}\,. \qquad (10)
\end{aligned}
$$

With these assignments and taking the Yukawa interactions in Eq. (7) into account, as well as the non-Hermitean terms $\eta\rho\chi\phi$, we easily see that the PQ charges for the scalars are constrained and imply the following relations:

$$
X_d=-X_u\,,\quad X_{d'}=-X_{u'}\,,\quad X_\nu=X_{eR}\,,\quad X_e=X_{\nu R}\,. \qquad (11)
$$

We can make the further choice $X_d=X_{d'}$, leading to

$$
X_d=X_{d'}=-X_u=-X_{u'}=-X_e=X_{eR}=X_\nu=-X_{\nu R}=X_N\,, \qquad (12)
$$

implying that the PQ symmetry is chiral for the leptons, too. The scalars transform as

$$
\begin{aligned}
& \phi\to e^{-2i\alpha X_d}\phi\,, && \eta^0\ \to\ e^{2i\alpha X_d}\eta^0\\
& \eta^-\to\eta^-\,, && \eta'^0\ \to\ e^{2i\alpha X_d}\eta'^0\\
& \rho^+\to\rho^+\,, && \rho^0\ \to\ e^{-2i\alpha X_d}\rho^0\\
& \rho'^+\to\rho'^+\,, && \chi^0\ \to\ e^{2i\alpha X_d}\chi^0\\
& \chi^-\to\chi^-\,, && \chi'^0\ \to\ e^{2i\alpha X_d}\chi'^0.
\end{aligned}
\qquad (13)
$$

It is now clear that the entire Lagrangian of the model is $U(1)_{PQ}$ invariant, providing a natural solution to the strong-CP problem.

To accomplish our proposal, let us consider that only χ'^0, ρ^0, η^0 and ϕ develop VEV and expand such fields in the standard way,

$$
\begin{aligned}
& \chi'^0=\frac{1}{\sqrt{2}}(v_{\chi'}+R_{\chi'}+iI_{\chi'})\,,\qquad \eta^0=\frac{1}{\sqrt{2}}(v_\eta+R_\eta+iI_\eta)\,,\\
& \rho^0=\frac{1}{\sqrt{2}}(v_\rho+R_\rho+iI_\rho)\,,\qquad \phi=\frac{1}{\sqrt{2}}(v_\phi+R_\phi+iI_\phi)\,. \qquad (14)
\end{aligned}
$$

With such expansion, we obtain the set of constraint equations that guarantee that the potential has a minimum

$$
\begin{aligned}
& \mu_\chi^2+\lambda_1 v_{\chi'}^2+\frac{\lambda_4}{2}v_\eta^2+\frac{\lambda_5}{2}v_\rho^2+\frac{\lambda_{11}}{2}v_\phi^2+\frac{A}{v_{\chi'}^2}=0,\\
& \mu_\eta^2+\lambda_2 v_\eta^2+\frac{\lambda_4}{2}v_{\chi'}^2+\frac{\lambda_6}{2}v_\rho^2+\frac{\lambda_{13}}{2}v_\phi^2+\frac{A}{v_\eta^2}=0,\\
& \mu_\rho^2+\lambda_3 v_\rho^2+\frac{\lambda_5}{2}v_{\chi'}^2+\frac{\lambda_6}{2}v_\eta^2+\frac{\lambda_{12}}{2}v_\phi^2+\frac{A}{v_\rho^2}=0,\\
& \mu_\phi^2+\lambda_{10} v_\phi^2+\frac{\lambda_{11}}{2}v_{\chi'}^2+\frac{\lambda_{12}}{2}v_\rho^2+\frac{\lambda_{13}}{2}v_\eta^2+\frac{A}{v_\phi^2}=0\,, \qquad (15)
\end{aligned}
$$

where we have defined $A\equiv\lambda_\phi v_\eta v_\rho v_{\chi'} v_\phi$. The physical scalars are obtained by substituting these constraints into the mass matrices given by the second derivative of the potential. The axion arises from the mass matrix M_I^2 given by

$$
-\frac{A}{2}\begin{pmatrix}
\frac{1}{v_{\chi'}^2} & \frac{1}{v_\eta v_{\chi'}} & \frac{1}{v_\rho v_{\chi'}} & \frac{1}{v_{\chi'}v_\phi}\\
\frac{1}{v_\eta v_{\chi'}} & \frac{1}{v_\eta^2} & \frac{1}{v_\eta v_\rho} & \frac{1}{v_\eta v_\phi}\\
\frac{1}{v_\rho v_{\chi'}} & \frac{1}{v_\eta v_\rho} & \frac{1}{v_\rho^2} & \frac{1}{v_\eta v_\rho}\\
\frac{1}{v_{\chi'}v_\phi} & \frac{1}{v_\eta v_\phi} & \frac{1}{v_\eta v_\rho} & \frac{1}{v_\phi^2}
\end{pmatrix} \qquad (16)
$$

in the basis $(I_{\chi'}, I_\eta, I_\rho, I_\phi)$. Its diagonalization furnishes an axion given by, $a=\frac{1}{\sqrt{1+\frac{v_{\chi'}^2}{v_\phi^2}}}\left(I_\phi-\frac{v_{\chi'}}{v_\phi}I_{\chi'}\right)$. As $v_\phi\gg v_{\chi'}$ we have that $a\sim I_\phi$.

Now let us focus on the CP-even component of ϕ. It will be our inflaton candidate. It composes the following mass matrix M_R^2 given by

$$
\begin{pmatrix}
\lambda_1 v_{\chi'}^2-\frac{A}{2v_{\chi'}^2} & \frac{\lambda_4 v_{\chi'}v_\eta}{2}+\frac{A}{2v_\eta v_{\chi'}} & \frac{\lambda_5 v_{\chi'}v_\rho}{2}+\frac{A}{2v_\rho v_{\chi'}} & \frac{A}{2v_\phi v_{\chi'}}\\
\frac{\lambda_4 v_{\chi'}v_\eta}{2}+\frac{A}{2v_\eta v_{\chi'}} & \lambda_2 v_\eta^2-\frac{A}{2v_\eta^2} & \frac{\lambda_6 v_\eta v_\rho}{2}+\frac{A}{2v_\rho v_\eta} & \frac{A}{2v_\eta v_\phi}\\
\frac{\lambda_5 v_{\chi'}v_\rho}{2}+\frac{A}{2v_\rho v_{\chi'}} & \frac{\lambda_6 v_\eta v_\rho}{2}+\frac{A}{2v_\rho v_\eta} & \lambda_3 v_\rho^2-\frac{A}{2v_\rho^2} & \frac{A}{2v_\rho v_\phi}\\
\frac{A}{2v_\phi v_{\chi'}} & \frac{A}{2v_\eta v_\phi} & \frac{A}{2v_\rho v_\phi} & \lambda_{10}v_\phi^2-\frac{A}{2v_\phi^2}
\end{pmatrix} \qquad (17)
$$

in the basis $(R_{\chi'}, R_\eta, R_\rho, R_\phi)$. As $v_\phi\gg v_\rho, v_\eta, v_{\chi'}$, we have that R_ϕ decouples and its mass is predicted to be $m_{R_\phi}^2\sim\lambda_{10}v_\phi^2$.

In this point we call the attention to the fact that the presence of large discrete symmetries has the function of stabilizing the axion against quantum gravity effects. In order to see this, perceive that the effective operators responsible for the gravitational contribution to the axion mass is of the form ϕ^n/M_{Pl}^{n-4}. A Z_N symmetry automatically suppress terms of this kind till some $n=N-1$. The main surviving term contributing to the axion mass is the one with $n=N$. It is true that with Z_{11} the axion is protected only for energy scales not bigger than $\langle\phi\rangle\simeq 10^{10}$ GeV. Nevertheless, this is not a threat for the model since we still have values for the θ angle and axion mass (gravitationally induced) [45],

$$
\begin{aligned}
& M_a^{Grav}\simeq\sqrt{\frac{\langle\phi\rangle^{N-2}}{M_{Pl}^{N-4}}}\simeq 10^{-12}\ \text{eV}\simeq 10^{-7}m_a\,,\\
& \theta_{eff}\simeq\frac{\langle\phi\rangle^N}{M_{Pl}^{N-4}\Lambda_{QCD}^4}\simeq 10^{-19}\,, \qquad (18)
\end{aligned}
$$

where we have used $M_{Pl}\simeq 10^{19}$ GeV, $\Lambda_{QCD}\simeq 300$ MeV, and $m_a\simeq 10^{-5}$ eV is the instanton induced axion mass. These values are consistent with astrophysical and cosmological bounds (see PDG [46]). If we had taken $\langle\phi\rangle\simeq 10^{11}$ GeV, the axion would still be protected under gravitation, but the θ value would be on the threshold of its bound $\theta_{eff}\lesssim 10^{-9}$. So we can have a valid solution to the strong-CP problem for Z_{11} for scales $\langle\phi\rangle\lesssim 10^{10}$ GeV in this version of 3-3-1. We finish this section remarking that the incorporation of PQ symmetry in 3-3-1 model as done here has as main purpose the explanation of the strong CP-problem and, as a byproduct, the invisible axion fulfilling the conditions to be a viable dark matter candidate.

3. Implementing inflation

Here we consider inflation in the specific framework of the 3-3-1 model presented in the previous section. Our aim is to show that the real component of the ϕ field will drive inflation with its potential satisfying the slow roll conditions while providing the current prediction for the scalar spectral index, n_s, and obeying the current bound on the scalar to tensor ratio, r.

First thing to note is that the ϕ potential involves the terms,

$$V_\phi = \mu_\phi^2 \phi^* \phi + \lambda_{10}(\phi^*\phi)^2 + \lambda_{11}(\phi^*\phi)(\chi^\dagger\chi) + \lambda_{12}(\phi^*\phi)(\rho^\dagger\rho) + \lambda_{13}(\phi^*\phi)(\eta^\dagger\eta) + \lambda_\phi \epsilon^{ijk}\eta_i\rho_j\chi_k\phi + \text{H.c.} \tag{19}$$

However, as $v_\phi >> v_{\eta,\rho,\chi}$, the terms in this potential that really matter during inflation are

$$V_\phi = \mu_\phi^2 \phi^*\phi + \lambda_{10}(\phi^*\phi)^2. \tag{20}$$

This is the chaotic inflation scenario with the inflaton being the real part of ϕ. From now on we use the notation $R_\phi \equiv \Phi$

A VEV around 10^{10} GeV for ϕ implies that the dominant term in the above potential is $\lambda_{10}\Phi^4$. However, as we know, the $\lambda_{10}\Phi^4$ chaotic inflation is not favored by recent values of r measured by PLANCK2015 [41]. Thus, in order to circumvent this problem, we take into account radiative corrections to the potential which now reads

$$V(\Phi) = V_{tree} + V_{eff}, \tag{21}$$

with $V_{tree} = \lambda_{10}\Phi^4$ and V_{eff} being the radiative corrections due to the coupling of Φ to the particle content of the 3-3-1 model. The radiative corrections are engendered by the couplings of our inflaton with the right-handed neutrinos and the scalars whose intensities are determined by the parameters λ_{11}, λ_{12}, λ_{13}, λ_ϕ and h'. As we will see below, successful reheating after inflation requires $\lambda_{\phi,11,12,13}$ very small. Thus the intensity of the radiative corrections is practically determined by h' which is the coupling of the inflaton, Φ, to the heavy neutrino, N_R, and is given by the last term of the Lagrangian in Eq. (7).

According to Coleman–Weinberg approach[47], we obtain the following expression to the effective potential

$$V_{eff} = \frac{1}{64\pi^2}\sum_i \left[(-1)^{2J}(2J+1)m_i^4 \ln\frac{m_i^2}{\Delta^2}\right], \tag{22}$$

where m_i is the ϕ-field dependent mass where $i = \eta, \rho, \chi, \phi, N_R$. J is the spin of the respective contribution. The intensities of the scalar contributions are dictated by the couplings $\lambda_{\phi,11,12,13}$ while the intensities of the heavy neutrino contributions are dictated by h''s. As we will see in the end of this section, efficient reheating implies $\lambda_{\phi,11,12,13} \ll h'$. Thus, for reasons of simplicity, we just consider contributions due to N_R. This means to take in Eq. (22) $m_{i=N_R} = -\sqrt{2}h'\Phi$. In this circumstance, for our proposal here, and for simplicity reasons, it is just sufficient to consider one family of heavy neutrinos. After all this the potential that really matters during the inflationary period is given by

$$V(\Phi) \approx \lambda'\left(\Phi^4 + a'\Phi^4 \ln\frac{\Phi}{\Delta}\right), \tag{23}$$

where $\lambda' = \frac{\lambda_{10}}{4}$ and $a' = \frac{a+160\lambda'^2}{32\pi^2\lambda'} \approx \frac{a}{32\pi^2\lambda'}$. Δ is a renormalization scale. This approximation is justified because the amplitude of curvature perturbation demands a small λ_{10}. The term a carries the radiative contribution and in our case it is given by $a = -16h'^4$. The negative sign is a characteristic feature of fermion contributions. We would like to point out that, according to our assumptions, the loop dominant contribution that generates the expression above comes from box diagram composed by four heavy neutrinos running in the box. This is the reason why the loop correction given above gets proportional to h'^4. If we go further with the calculation, the two loop diagram is a correction of the box diagram and get proportional to h'^6 which is smaller than one loop contribution for $h' < 1$. Thus we do not need to worry about higher order corrections than one-loop.

We can now treat the issue of inflation, which occurs as long as the slow roll approximation is satisfied ($\epsilon \ll 1$, $\eta \ll 1$, $\zeta^2 \ll 1$). Throughout this section we follow the approach given in Refs. [34,35]. The slow roll parameters are given by [48]

$$\epsilon(\Phi) = \frac{m_P^2}{16\pi}\left(\frac{V'}{V}\right)^2, \qquad \eta(\Phi) = \frac{m_P^2}{8\pi}\left(\frac{V''}{V}\right),$$
$$\zeta^2(\Phi) = \frac{m_P^4}{64\pi^2}\left(\frac{V'''V''}{V^2}\right), \tag{24}$$

where $m_P = 1.22 \times 10^{19}$ GeV.

The spectral index n_S, the scalar to tensor ratio r and the running of spectral index $\alpha \equiv \frac{dn_S}{d\ln k}$ are defined as [49]

$$n_S = 1 - 6\epsilon + 2\eta, \qquad r = 16\epsilon,$$
$$\alpha = 16\epsilon\eta - 24\epsilon^2 - 2\zeta^2. \tag{25}$$

For a wave number $k = 0.05$ Mpc^{-1}, the Planck results indicate $n_S = 0.9644 \pm 0.0049$ and $r < 0.149$ [41].

The number of e-folds is given by

$$N = \frac{-8\pi}{m_P^2}\int_{\Phi_i}^{\Phi_f}\frac{V}{V'}d\Phi, \tag{26}$$

where Φ_f marks the end of inflation and is defined by $(\epsilon, \eta, \zeta^2) = 1$. To find Φ_i we set $N = 50, 60$ and 70 and solve Eq. (26) for Φ_i.

Another important parameter is the amplitude of curvature perturbation

$$\Delta_R^2 = \frac{8V}{3m_P^4\epsilon}. \tag{27}$$

Planck measurement of this parameter gives $\Delta_R^2 = 2.215 \times 10^{-9}$ for a wave number $k = 0.05$ Mpc^{-1}. We use this experimental value of Δ_R^2 to fix λ'.

Let us discuss our results beginning with Fig. 1. There we show the behavior of the scalar to tensor ratio, r, related to a' for some values of Δ. First of all, so as to have an idea of the values of Φ_i and Φ_f, for the case of $\Delta = 3m_P$, and considering the setup presented above, we have that inflation ends with $\Phi_f \sim 10^{18}$ GeV and, for the particular case of 60 e-folds, we get the initial value $\Phi_i \sim 4 \times 10^{19}$ GeV. Note that as a' goes to zero all the curves converge to a point around $r = 0.26$. This is the expected value for r provided by Φ^4 chaotic inflation. Thus, in our case, the current bounds on r requires $a' \neq 0$. This means that radiative corrections turn to be absolutely necessary in our analysis. We also stress that the scalar to tensor ratio demands trans-Planckian regime for Δ because the sub-Planckian case faces problems in the integration on the e-fold number to reach the value 60 unless a' goes to zero, again recovering the Φ^4 chaotic inflation. Even for the trans-Planckian case, on assuming $a' < 0$, the current values of r do not allow Δ to exceed the regime of $\sim 6m_P$. In other words, our inflation model requires sizable radiative corrections in order to obey

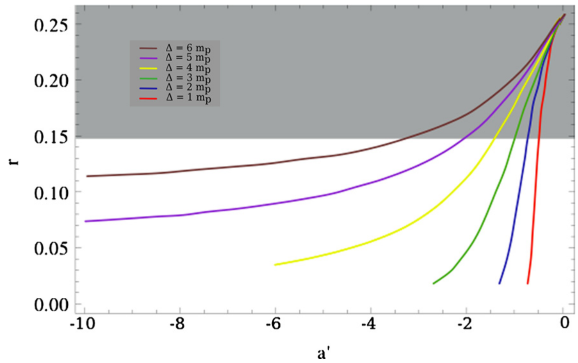

Fig. 1. r vs a' for several values of Δ. The region in gray is excluded by Planck.

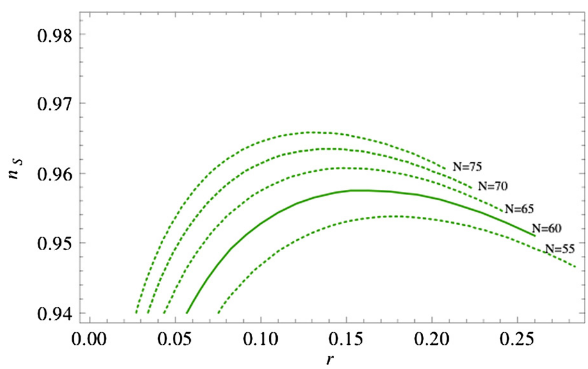

Fig. 2. n_S vs r for $\Delta = 3m_P$.

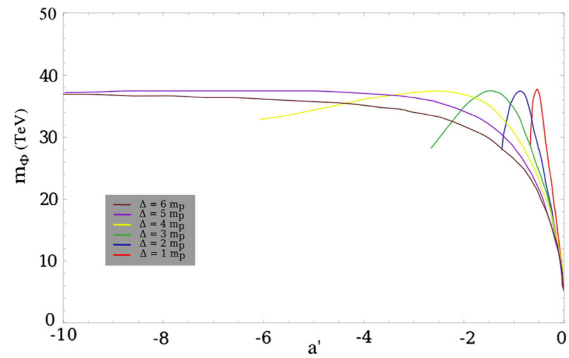

Fig. 3. m_ϕ vs a' for several values of Δ.

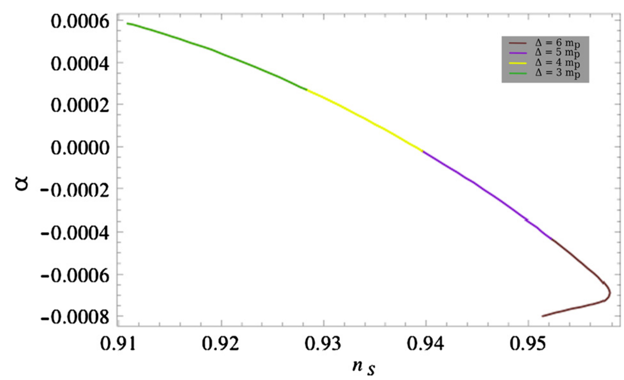

Fig. 4. α vs n_S for several values of Δ.

the current value of n_S and the bound on r. All this run into $a' \neq 0$ and Δ around few m_P.

In Fig. 2 we present our results for n_S and r in a plot confronting n_s with r for $\Delta = 3m_P$ and a' obeying the values corresponding to the green curve in Fig. 1 for several e-fold values. As we can see in that plot, the model predictions for n_S and r are in perfect agreement with the experimental bounds provided by PLANCK2015. This result is valid for any other choice of the values for the parameter Δ presented in Fig. 1.

Another interesting outcome we have obtained concerns the inflaton mass. Its expression at tree level is extracted from the diagonalization of the mass matrix M^2_R in Eq. (17). As reheating demands very tiny λ_ϕ and $v_\phi \gg v_\rho, v_\eta, v_{\chi'}$, then the $(M^2_R)_{44}$ element of that matrix decouples incurring into the following expression for the inflaton mass at tree level, $m_\Phi \sim \sqrt{2\lambda_{10}}v_\phi$. When radiative corrections are plugged in, this expression receives a correction that depends on the parameters a' and Δ. In Fig. 3 we plot the behavior of the inflaton mass m_Φ with a' for some values of Δ. Even if v_ϕ is around 10^{10} GeV, but as the coupling λ_{10} is very small, as required by reheating phase, the inflaton gains a small mass when compared to the conventional chaotic inflation case. According to the prediction of our model, the inflaton may develop mass until few tens of TeV. This has implications to the reheating phase, as discussed below.

For sake of completeness, in Fig. 4 we plot the running index α versus n_S for some values of Δ. There we have a relatively small α value for all points as it has to be in chaotic inflation.

We finish this section by discussing reheating [50]. First of all notice that our inflaton couples to the heavy neutrinos through the Yukawa coupling in Eq. (7), and to scalars according to the last four terms in the potential in Eq. (19). Because $v_\phi \sim 10^{10}$ GeV, the inflaton develops mass around tens of TeV, as shown in Fig. 3. This order of magnitude for the inflaton mass forbids that it decays into a pair of heavy neutrinos because, as we will see below, in the model developed here heavy neutrinos gain mass around $\sim 10^7$ GeV. Thus reheating will be solely due to the decay of the inflaton into a pair of scalars.

Perceive that the 3-3-1 model in question has a complex scalar sector that turns quite impossible to obtain a well-behaved mixing matrix that connects the symmetrical scalars to the physical ones. This is particularly sure with the CP-even neutral scalars. This is because its mass matrix, given in Eq. (17), is a 4×4 one. Its diagonalization requires numerical approach. For our proposal here it is just enough to parametrizes the couplings among the inflaton and a pair of Higgs, provided by those last four terms in the potential in Eq. (19), by the general form: $\frac{\lambda}{8} v_\phi \Phi hh$ where λ represents any combination of the couplings $\lambda_{\phi,11,12,13}$ while h represents any combination of the eigenstates R_η, R_ρ and $R_{\chi'}$, or simply the heavier one, that is practically $R_{\chi'}$. This does not matter too much because the reheating phase puts constraint into the couplings λ's. We assume that one of the couplings $\lambda_{\phi,11,12,13}$ will be the dominant one and proceed with calculation.

In view of this, the expression for the decay width of the channel $\Phi \to hh$ is

$$\Gamma(\Phi \to hh) \sim \frac{\lambda^2 v_\phi^2}{32\pi m_\Phi}. \tag{28}$$

In this case reheating temperature is estimated to be [51]

$$T_R \sim 0.1\sqrt{\Gamma(\Phi \to hh) m_P}. \tag{29}$$

For $v_\phi = 10^{10}$ GeV and $m_\Phi \sim 10$ TeV, a reheating temperature around 10^9 GeV requires $\lambda \sim 10^{-6}$. This means that the couplings $\lambda_{\phi,11,12,13}$ in the potential must develop values at most around this order of magnitude which are typical values in chaotic inflation models [51]. In summary, although the inflaton has an unusual small mass, the model is efficient in reheating the universe.

4. Some remarks and conclusions

When η^0 and ϕ develop VEVs, the last two terms in the Lagrangian in Eq. (7) yields Dirac and Majorana mass terms for ν_L and N_R,

$$\mathcal{L} \supset M_D \bar{\nu}_L N_R + M \bar{N}_R^C N_R + \text{H.c.}, \tag{30}$$

where $M_D = h\frac{v_\eta}{\sqrt{2}}$ and $M = \frac{h' v_\phi}{\sqrt{2}}$. These terms provide the following mass matrix for the six massive neutrinos,

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M \end{pmatrix}. \tag{31}$$

This is the mass matrix for the type I seesaw mechanism whose diagonalization, for $M \gg M_D$, leads to [52–56]

$$m_{\nu_L} \simeq \frac{M_D^2}{M} \quad \text{and} \quad M_R \simeq M. \tag{32}$$

We are interested in estimating the order of magnitude of the masses, only. As $a' \approx \frac{a}{32\pi^2\lambda'}$ and $a = -16h'^4$, for a' around 10 and $\lambda' \sim 10^{-14}$, as required by $\Delta_R^2 = 2.215 \times 10^{-9}$, we get $h' \sim 10^{-3}$, which results in $M_R \sim 10^7$ GeV. So as to obtain standard neutrinos at eV scale in agreement with solar and atmospheric neutrino oscillation, we just need $M_D \sim (10^{-1} - 10^{-2})$ GeV. This is obtained for h in the range $\sim (10^{-3} - 10^{-4})$ for $v_\eta \sim 10^2$ GeV. Such range of values for h are of the same order of the Yukawa couplings in the standard model.

Axion dark matter is considered as an attractive alternative to thermal WIMP dark matter. Our axion is invisible and receives mass through chiral anomaly, $m_a^2 \sim \frac{\Lambda_{QCD}^4}{f_{pq}^2}$, which is about 10^{-3} eV for $f_{pq} \sim 10^{10}$ GeV and $\Lambda_{QCD} \sim 10^{-1}$ GeV, turning our axion a natural candidate for cold dark matter. As PQ symmetry is broken during inflation, our axion will be produced in the early universe through the misalignment mechanism and its relic abundance is cast in Refs. [57,58].

Just few words about heavy neutrinos with masses around 10^7 GeV. These neutrinos interact with charged scalars, as allowed by the Yukawa coupling $h\bar{f}_L\eta N_R$, and may give rise to baryogenesis through leptogenesis. Because of the complexity and importance of such subject, we treat it separately elsewhere. However, for a previous treatment of this issue in a similar situation, but different scenario, we refer the reader to the Refs. [59,60].

In summary, several papers have proposed extensions of the standard model that provide a common origin to the understanding of the strong CP-problem, dark matter, inflation, and small neutrino masses. In this paper we argued that such proposal is elegantly realized in the framework of a 3-3-1 gauge model. In it the strong CP-problem is solved with the PQ symmetry whose associated axion is invisible and may constitute the dark matter of the universe. Inflation is driven by the real part of the neutral scalar singlet that contains the axion. Successful inflation was obtained by considering radiative corrections to the inflaton potential. The model has an unusual inflaton with mass of tens of TeV. Reheating is achieved through the decay of the inflaton into scalars, and neutrinos gain small mass through the type I seesaw mechanism.

We end by saying that we do not expect that the 3-3-1 model be the final theory valid in the range from TeV up to Planck scale. It is more probable that it is an effective theory of a more fundamental one that prevails in the high energy scale, as for example grand unification theories. If this is the case, it is reasonable to expect that the predictions done here be preserved in the context of such a final theory.

Acknowledgements

This work was supported by Conselho Nacional de Pesquisa e Desenvolvimento Científico – CNPq (C.A.S.P, P.S.R.S.) and Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – CAPES (J.G.R.).

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