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BEAM EMITTANCE GROWTH CAUSED BY TRANSVERSE DEFLECTING FIELDS IN A LINEAR ACCELERATOR*

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ABSTRACT

The effect of the beam-generated transverse deflecting fields on the emittance of an intense bunch of particles in a high-energy linear accelerator is analyzed in this paper. The equation of motion is solved by a perturbation method for cases of a coasting beam and a uniformly accelerated beam. The results are applied to obtain some design tolerance specifications for the recently proposed SLAC Single Pass Collider.

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1. Introduction

In this paper we examine the effect of transverse deflecting fields, generated by the passage of an intense bunch of particles through a conducting pipe, on the emittance of the same bunch of particles which generated these fields. This problem is important in determining the limits to the luminosity of linear colliding beam devices which are now receiving much attention[1) as possible successors to colliding beam storage rings for achieving very high energy in the electron-positron center-of-mass system. These linear colliding beam devices require beam transverse dimensions of the order of microns at their collision point to achieve useful reaction rates and the minimum beam transverse dimension is directly related to the emittance of the beam. Significant emittance growth during the acceleration of the intense bunch required for linear colliders can degrade the performance of these devices.

When a point charge travels off-axis down a pipe, it interacts with the walls of the pipe and leaves behind a transverse wake field which will deflect particles traveling behind the point charge. If an intense bunch of particles travels through a structure whose transverse dimensions are large compared to the length of the bunch, the transverse wake field will be such that all particles behind the head of the bunch are deflected further away from the axis of the structure. Thus, as the bunch travels down along the pipe, the total area in transverse phase space occupied by all the particles in the bunch will increase.[2)

We examine the effects of these wake fields with particular application to linear electron accelerators. We first find the equations of motion for particles in the bunch, and then solve these equations by a perturbation method for the cases of a coasting beam and a uniformly accelerated beam. For the case of a very strong wake, an asymptotic analytic solution is found.[3)] Finally, the results of this analysis are applied to the SLAC linac for bunches of an intensity required for linear colliding beam applications.

2. Equations of Motion

In what follows we treat the bunch as relativistically and as if its transverse dimensions were zero. We calculate the displacement of a point on the bunch, $x(z,s)$, as a function of z, the longitudinal position relative to the center of the bunch (z is positive toward the head of the bunch), and s, the distance from the beginning of the accelerator. The approximation of zero transverse dimensions for the bunch is a good one in most cases of interest where the transverse dimension of the bunch is very much less than the size of the pipe. Thus the transverse wake field is uniform across the bunch. In effect, $x(z,s)$ is the displacement of the center of a slice through the bunch at the position z.

The transverse force at z depends on the displacement of all charges with $z' > z$ and is given by

$$F_x(z,s) = e^2 \int_z^\infty dz'\rho(z')W(z'-z)x(z',s') \qquad (1)$$

where ρ is the line density of the particles in the bunch ($\int\rho dz$ is normalized to the total number of particles in the bunch, N), $e\cdot W\cdot x$ is the transverse field produced by a point charge displaced from the axis by x at a distance $z' - z$ behind the point charge, and $s' = s - z' + z$ refers to the retarded time for the field. We have assumed that the displacement of a particle changes sufficiently slowly with s so that the average W of the structure can be used (certainly the case in electron linac).

The equation of motion for x(z,s) can be written as

$$\frac{d}{ds}\left[\gamma(s)\frac{d}{ds}x(z,s)\right] + \left(\frac{2\pi}{\lambda(s)}\right)^2\gamma(s)x(z,s) = r_o\int_z^\infty dz'\rho(z')W(z'-z)x(z',s') \tag{2}$$

where $\gamma(s)$ is the energy of the beam at position s in units of mc^2, m being the rest mass of the particle; $\lambda(s)$ is the instantaneous wavelength of betatron focusing at position s; and $r_o = e^2/mc^2$ is the classical radius of the particle.

We assume that W, γ, λ, and ρ are known functions, and that the betatron focusing is provided by a smooth function rather than coming from a series of widely spaced quadrupoles. We also assume that the bunch length is much shorter than the betatron wavelength so that the retardation can be ignored, i.e., $x(z',s')$ on the right-hand side of eq. (2) can be replaced by $x(z',s)$.

We will first solve eq. (2) using a perturbation method for a coasting beam in which the bunch has constant energy as a function of s.

3. Coasting Beam Solution

For a coasting beam we have $\gamma(s) = \gamma_o$ and $\lambda(s) = \lambda_o$ both independent of s. Consider a bunch injected into the linac with a displacement of $x = x_o$ and a slope $x' = 0$. In the limit of no wake field, the zeroth order solution for the bunch motion is simply

$$x^{(o)}(z,s) = x_o \cos k_o s \tag{3}$$

where we have defined $k_o = 2\pi/\lambda_o$.

We expand x(z,s) in a series of powers of the wake field

$$x(z,s) = \sum_{n=o}^{\infty} x^{(n)}(z,s) \tag{4}$$

and obtain the n^{th} order term from the $(n-1)^{th}$ order term by solving

$$\frac{d^2}{ds^2} x^{(n)}(z,s) + k_o^2 x^{(n)}(z,s) = \frac{r_o}{\gamma_o} \int_z^{\infty} dz' \rho(z') W(z'-z) x^{(n-1)}(z',s) \tag{5}$$

which is a direct consequence of eq. (2). The solution of eq. (5) is

$$x^{(n)}(z,s) = \int_o^s ds' G(s,s') \frac{r_o}{\gamma_o} \int_z^{\infty} dz' \rho(z') W(z'-z) x^{(n-1)}(z',s') \tag{6}$$

with G(s,s') the Green's function:

$$G(s,s') = \frac{1}{k_o} \sin k_o (s-s') \tag{7}$$

Inserting $x^{(o)}$ from eq. (3), we obtain by iteration

$$x^{(1)}(z,L) = x_o\left(\frac{r_o}{\gamma_o k_o}\right)\left(\frac{L}{2}\sin k_o L\right)\cdot\int_z^\infty dz_1\rho(z_1)W(z_1 - z)$$

$$x^{(2)}(z,L) = x_o\left(\frac{r_o}{\gamma_o k_o}\right)^2\left(-\frac{L^2}{8}\cos k_o L + \frac{L}{8k_o}\sin k_o\right)$$

$$\cdot\int_z^\infty dz_1\rho(z_1)W(z_1 - z)\int_{z_1}^\infty dz_2\rho(z_2)W(z_2 - z_1) \qquad (8)$$

...

where we have taken values at the end of linac $s = L$. For a weak wake field, one can ignore all the terms higher than the first order and the deviation of a particle from its zeroth order trajectory is given by $x^{(1)}$. It is sufficient to keep only the first-order term if $|x^{(1)}| << |x_o|$ which requires

$$\left|\frac{Lr_o}{2\gamma_o k_o}\int_z^\infty dz_1\rho(z_1)W(z_1 - z)\right| << 1 \qquad (9)$$

The fact that $|x^{(1)}|$ is proportional to s is a direct consequence of the resonant driving situation. Since $x^{(1)} \propto s\sin(k_o s)$ and $x^{(o)} \propto \cos(k_o s)$, a series of snapshot pictures of the bunch looks like the sketch in fig. 1. The value of $x^{(1)}$ depends on the position z along the bunch, and at the head of the bunch $x^{(1)}$ vanishes as it should. If the wake field is not weak, one has to carry the calculation to higher orders.

In many practical situations, the pipe is long enough so that the bunch has completed many betatron oscillations before reaching $s = L$, i.e., $k_o L >> 1$. In this case, we find after some albegra that eq. (8) reduces to

$$x^{(n)}(z,L) \approx x_o \left(\frac{r_o}{\gamma_o k_o}\right)^n \frac{1}{n!} \left(\frac{L}{2i}\right)^n e^{ik_o L} \cdot R_n(z) \tag{10}$$

where R_n is defined by

$$R_n(z) = \int_z^\infty dz_1 \rho(z_1) W(z_1 - z) \int_{z_1}^\infty dz_2 \rho(z_2) W(z_2 - z_1) \ldots \int_{z_{n-1}}^\infty dz_n \rho(z_n) W(z_n - z_{n-1}) \tag{11}$$

It is understood that only the real part of eq. (10) is meaningful. Note that $x^{(n)}(z,s)$ has the useful property that it is factorizable in the variables s and z, although $x(z,s)$ is not.

A closed form can be obtained for R_n if we approximate ρ by a rectangular distribution and W by a linear function, i.e.,

$$\rho = \begin{cases} N/\ell & \text{for } |z| < \ell/2 \\ 0 & \text{for } |z| \geq \ell/2 \end{cases} \tag{12a}$$

$$W = W_o z/\ell \tag{12b}$$

Both approximations are close enough to reality in many applications to allow a good assessment to be made of the importance of higher order terms. In particular the linear wake is a quite good approximation for the SLAC linac. We find

$$R_n(z) = \frac{1}{(2n)!}\left[NW_o\left(\frac{1}{2} - \frac{z}{\ell}\right)^2\right]^n \tag{13}$$

and

$$x(z,L) = x_o e^{ik_oL} \sum_{n=o}^{\infty} \frac{1}{n!(2n)!}\left(\frac{\eta}{2i}\right)^n \tag{14}$$

where we have defined

$$\eta = \frac{r_oLNW_o}{\gamma_ok_o}\left(\frac{1}{2} - \frac{z}{\ell}\right)^2 \tag{15}$$

The validity condition eq. (9) for the first-order approximation becomes $|\eta/4| << 1$. In the limit $|\eta| >> 1$, one can find an asymptotic expression for eq. (14):

$$x(z,L) \approx \frac{x_o}{\sqrt{6\pi}} |\eta|^{-1/6}\left(\exp \frac{3\sqrt{3}}{4} |\eta|^{1/3}\right)\cos\left[k_oL - \frac{\eta}{|\eta|}\left(\frac{3}{4}|\eta|^{1/3} - \frac{\pi}{12}\right)\right] \tag{16}$$

For intermediate values of $|\eta|$, the power series expression (14) is more accurate.

4. Accelerated Beam Solution

We assume that the energy of the beam increases linearly with s as a result of acceleration in such a way that $\gamma(s) = \gamma_o(1 + Gs)$, with $\gamma_o mc^2$ the beam energy at injection, and G the acceleration gradient. We assume that the strength of the focusing force in the linac scales with beam energy so that the instantaneous betatron wavelength remains constant, $\lambda(s) = \lambda_o$.

We first make a change of variable in the equation of motion (2) from s to a new variable $u = 1 + Gs$. Equation (2) then becomes

$$\frac{d^2x}{du^2} + \frac{1}{u}\frac{dx}{du} + \left(\frac{k_o}{G}\right)^2 x = \frac{r_o}{\gamma_o G^2 u} \int_z^\infty dz'\rho(z')W(z'-z)x(z',u) \tag{17}$$

where we have defined $k_o = 2\pi/\lambda_o$. This equation will again be solved by an iteration procedure as in the coasting beam case. The zeroth order solution is obtained by setting the right-hand side of (17) to zero and demanding the initial conditions $x(z,u) = x_o$ and $dx(z,u)/du = 0$ at $s = 0$ or $u = 1$:

$$x^{(o)}(z,u) = x_o \cdot \frac{N_1\left(\frac{k_o}{G}\right)J_o\left(\frac{k_o}{G}u\right) - J_1\left(\frac{k_o}{G}\right)N_o\left(\frac{k_o}{G}u\right)}{N_1\left(\frac{k_o}{G}\right)J_o\left(\frac{k_o}{G}\right) - J_1\left(\frac{k_o}{G}\right)N_o\left(\frac{k_o}{G}\right)} \tag{18}$$

where J_o, J_1, N_o, and N_1 are the usual Bessel functions.

If we expand $x(z,u)$ as in eq. (4), the n^{th} order term $x^{(n)}$ can be obtained from the $(n-1)^{th}$ order term $x^{(n-1)}$ by solving eq. (17) with the x's on the left-hand side replaced by $x^{(n)}$ and the x on the right-hand side replaced by $x^{(n-1)}$. The solution can again be obtained using a Green's function:

$$x^{(n)}(z,u) = \int_1^u du'G(u,u') \frac{r_o}{\gamma_o G^2 u'} \int_z^\infty dz'\rho(z')W(z'-z)x^{(n-1)}(z',u') \tag{19}$$

with the Green's function

$$G(u,u') = \frac{\pi u'}{2}\left[N_o\left(\frac{k_o}{G}u\right)J_o\left(\frac{k_o}{G}u'\right) - J_o\left(\frac{k_o}{G}u\right)N_o\left(\frac{k_o}{G}u'\right)\right] \qquad (20)$$

In practice, over most of the length of a linear accelerator, the betatron oscillation wavelength is much shorter than the distance required to double the energy, i.e., $k_o \gg G$. The acceleration is then adiabatic and the arguments in the Bessel functions that appear in eqs. (18) and (20) are much larger than unity. We can use the large argument expressions for the Bessel functions to obtain

$$x^{(o)}(z,s) \approx \frac{x_o}{\sqrt{u}}\cos\left[\frac{k_o}{G}(u-1)\right] = \frac{x_o}{\sqrt{1+Gs}}\cos k_o s \qquad (21)$$

$$G(u,u') \approx \frac{G}{k_o}\sqrt{\frac{u'}{u}}\sin\left[\frac{k_o}{G}(u-u')\right] \qquad (22)$$

The factor $\sqrt{1+Gs}$ in eq. (21) is the usual adiabatic damping factor.

The terms in the series solution are

$$x^{(n)}(z,s) = \frac{x_o}{\sqrt{1+Gs}}\left(\frac{r_o}{\gamma_o k_o}\right)^n I_n(s)R_n(z) \qquad (23)$$

where $R_n(z)$ has been defined in eq. (11) and $I_n(s)$ is given by the recursion relation

$$I_n(s) = \int_o^s \frac{ds'}{1+Gs'}\sin\left[k_o(s-s')\right] I_{n-1}(s')$$

$$I_o(s) = \cos k_o s \qquad (24)$$

In general, the solution to eq. (24) for I_n is rather complicated. However, if the beam energy at the end of acceleration is much higher than the beam energy at injection, i.e., $(1+GL) \gg 1$, eq. (24) can be solved to yield

$$I_n(L) \approx \frac{1}{n!} e^{ik_oL} \left[\frac{1}{2iG} \ell n(1+GL)\right]^n \tag{25}$$

where taking the real part is understood.

Comparing the accelerated beam results, eqs. (23) and (25), with the coasting beam results, eq. (10), we note that the accelerated beam results can be obtained from the coasting beam result to all orders by the simple substitution rule

Coasting Beam		Accelerated Beam
constant energy γ_o	$\longrightarrow$	injection energy γ_o
injection displacement x_o	$\longrightarrow$	$\sqrt{\frac{\gamma_o}{\gamma_f}}\, x_o$
length of linac L	$\longrightarrow$	$\frac{1}{G} \ell n \frac{\gamma_f}{\gamma_o}$

(26)

where the last rule $L \to \frac{1}{G} \ell n(\gamma_f/\gamma_o)$ does <u>not</u> apply to the betatron oscillation phase $\exp(ik_oL)$. The particle energy at the end of acceleration is $\gamma_f mc^2$. The accelerated beam result reduces to the coasting beam result in the limit $G \to 0$.

For a bunch with a rectangular charge distribution and in the linear wake field approximation, eqs. (14) and (16) still hold if we replace x_o by $(\gamma_o/\gamma_f)^{1/2} x_o$ and η of eq. (15) by

$$\eta = \frac{Lr_o N W_o}{k_o(\gamma_f - \gamma_o)} \ln\frac{\gamma_f}{\gamma_o} \cdot \left(\frac{1}{2} - \frac{z}{\ell}\right)^2 \qquad (27)$$

In fig. 2 we have plotted x(z,L) along the bunch for values of $k_o L = 0$, $\pi/2$, π, and $3\pi/2$ (modulus 2π). The wake field strength is such that the value of η at the very tail of the bunch is equal to 150. It is clear from fig. 2 that the distortion of the bunch can be very large.

5. Misalignment Effects

In the previous analysis, we have assumed that the accelerator structure is perfectly aligned and the wake field is produced as a consequence of beam injection with a displacement error. In this section, we will study the effect caused by misalignment of the accelerator pipe. We assume the beam is injected into the linac with perfect precision and it travels down the linac in a straight line in the limit of low beam intensity. We will study the case with acceleration under the approximation that the acceleration is adiabatic.

The equation of motion can be written as

$$\frac{d^2x}{du^2} + \frac{1}{u}\frac{dx}{du} + \left(\frac{k_o}{G}\right)^2 x = \frac{r_o}{\gamma_o G^2 u}\int_z^\infty dz'\rho(z')W(z'-z)\left[x(z',u) - d(s)\right] \qquad (28)$$

where d(s) is the transverse position error of the pipe structure at position s. The accelerator will be treated as N_c structures, with the i^{th} structure misaligned by a distance d_i. Compared with eq. (17), eq. (28) contains an additional force term on the right-hand side that comes from the pipe misalignment.

The zero-th order solution to eq. (28) is $x^{(o)} = 0$ since the beam is assumed to be injected without error. The trajectory of the head of the bunch strictly follows $x^{(o)}$ and is therefore a perfect straight line. The first-order perturbation term comes solely from the misalignments d_i:

$$x^{(1)}(z,s) = -\sum_{\substack{i \\ (s>s_i)}} \frac{r_o d_i \ell_i}{\gamma_o k_o} \cdot \frac{1}{(1+Gs)^{1/2}(1+Gs_i)^{1/2}} \sin\left[k_o(s-s_i)\right] \cdot R_1(z) \tag{29}$$

where the factor $\sin\left[k_o(s-s_i)\right]$ is characteristic of the response to an angular kick, and the quantity $R_n(z)$ has been defined in eq. (11).

Note that unlike the case for an injection error, the first-order term in this case is not driven by a force with the natural frequency of this system and the resonant driving condition does not apply. As a consequence we do not expect a large value of $x^{(1)}$. On the other hand, the second-order term $x^{(2)}$ is driven by $x^{(1)}$ which does oscillate with the natural frequency of the system and may acquire large amplitudes. For this reason, it is necessary to carry out the perturbation calculation up to the second order in wake field. The second-order term can be obtained by substituting eq. (29) into eq. (19).

$$x^{(2)}(z,s) = \sum_{\substack{i \\ (s>s_i)}} \frac{r_o^2 d_i \ell_i}{2\gamma_o^2 k_o^2 G} \cdot \frac{\left[\ell n\ (1+Gs)/(1+Gs_i)\right]}{(1+Gs)^{1/2}(1+Gs_i)^{1/2}} \cos\left[k_o(s-s_i)\right] \cdot R_2(z) \tag{30}$$

If we assume the misalignment errors d_i are uncorrelated from one pipe structure to the next,

$$\langle x^{(1)2}\rangle = \frac{1}{2N_c}\langle d^2\rangle\left(\frac{r_oL}{\gamma_f k_o}\right)^2 \ell n\,\frac{\gamma_f}{\gamma_o}\cdot R_1^2(z)$$

$$\langle x^{(2)2}\rangle = \frac{1}{24N_c}\langle d^2\rangle\left(\frac{r_oL}{\gamma_f k_o}\right)^4 \ell n^3\left(\frac{\gamma_f}{\gamma_o}\right)\cdot R_2^2(z) \qquad (31)$$

where $\langle d^2\rangle^{1/2}$ is the rms value of the misalignment and we have assumed that all structures have the same length $\ell_i = L/N_c$. In eq. (31), we have approximated the sum over i by an integral over the length of the linac and we have also made the approximation that $\gamma_f >> \gamma_o$.

The factor $1/N_c$ in the expression for $\langle x^{(1)2}\rangle$ is a consequence of the fact that the first-order perturbation is not driven resonantly. That $x^{(2)}$ is driven resonantly by $x^{(1)}$ is shown by the fact that $\langle x^{(2)2}\rangle$ does not acquire an additional factor of $1/N_c$.

If we assume a rectangular distribution (12a) and a linear wake field (12b), we can use eq. (13) to obtain the quantities $R_n(z)$. The ratio of $\langle x^{(2)2}\rangle$ to $\langle x^{(1)2}\rangle$ under these assumptions is $\eta^2/1728$.

The emittance growth due to misalignment can be substantially reduced by empirically controlling the injection offset x_o and angle x_o' at the beginning of the linac. The corresponding first- and second-order contributions have been obtained in eq. (23):

$$x^{(1)}(z,L) = \frac{1}{\sqrt{1+GL}}\left(\frac{r_o}{\gamma_o k_o}\right)R_1(z)\,\frac{\ell n(1+GL)}{2G}\left(x_o \sin k_oL - \frac{x_o'}{k_o}\cos k_oL\right)$$

$$x^{(2)}(z,L) = \frac{-1}{\sqrt{1+GL}}\left(\frac{r_o}{\gamma_o k_o}\right)^2 R_2(z)\,\frac{\ell n^2(1+GL)}{8G^2}\left(x_o \cos k_oL + \frac{x_o'}{k_o}\sin k_oL\right) \qquad (32)$$

By choosing proper values of x_o and x'_o it is possible to cancel either the first-order misalignment contribution, eq. (29), or the second-order misalignment contribution, eq. (30), by a corresponding contribution from (32). For example, if the second order misalignment term dominates, one might choose

$$\begin{pmatrix} x_o \\ \\ x'_o \end{pmatrix} = \frac{4G}{\ln^2(1+GL)} \sum_i d_i \ell_i \frac{\ln[(1+GL)/(1+Gs_i)]}{(1+Gs_i)^{1/2}} \cdot \begin{pmatrix} \cos k_o s_i \\ \\ k_o \sin k_o s_i \end{pmatrix} \tag{33}$$

so that the second-order contribution from the injection offset and angle concels the second-order contribution from the misalignments. The required x_o and x'_o have rms values given by

$$\langle x_o^2 \rangle = \frac{8}{3N_c} \frac{\gamma_f/\gamma_o}{\ln(\gamma_f/\gamma_o)} \langle d^2 \rangle = \frac{1}{k_o^2} \langle x_o'^2 \rangle \tag{34}$$

With x_o and x'_o given by eq. (33), the first-order term obtained by the sum of misalignment and injection contributions is

$$x^{(1)} = \frac{1}{\sqrt{1+GL}} \left(\frac{r_o}{\gamma_o k_o} \right) R_1(z) \sum_i \frac{d_i \ell_i}{\sqrt{1+Gs_i}} \left[1 - 2\,\frac{\ln(1+Gs_i)}{\ln(1+GL)} \right] \sin[k_o(L-s_i)] \tag{35}$$

The rms value of this $x^{(1)}$ is given by

$$\langle x^{(1)2} \rangle = \frac{1}{6N_c} \langle d^2 \rangle \left(\frac{r_o L}{\gamma_f k_o} \right)^2 \ln \frac{\gamma_f}{\gamma_o} \cdot R_1^2(z) \tag{36}$$

which is 1/3 of that for the case with no injection cancellation effort. Thus this scheme of minimizing the emittance growth due to misalignment by controlling the injection conditions not only cancels the second-order misalignment contribution but also significantly reduces the first-order contribution.

6. Application to SLAC Linac

The wake field for the SLAC linac has been calculated by K. Bane and P. Wilson.[4] In the regions of interest to us, the wake field W(z) is linear in z. The relevant parameters for the proposed single pass collider operation are[1]

$$
\begin{array}{ll}
N = 5 \times 10^{10} & \sigma_z = 1 \text{ mm} \\
\gamma_o = 2.4 \times 10^3 \ (1.2 \text{ GeV}) & W_o = 5.9 \times 10^5 \text{m}^{-3} \\
\gamma_f = 10^5 \ (50 \text{ GeV}) & \sigma_x = 70 \ \mu\text{m} \\
L = 3 \times 10^3 \text{ m} & N_c = 240 \\
\lambda_o = 100 \text{ m} &
\end{array}
\tag{37}
$$

Under these conditions, the asymptotic expression must be used in order to find the proper tolerance criterion for not having a significant emittance growth. The value of η, according to eq. (27), is 37 at $z=0$, 94 at $z=-\sigma_z=-\frac{1}{2\sqrt{3}}\ell$, and 150 at $z=-\ell/2$. The bunch shape for this case has been shown in fig. 2. The corresponding values of the maximum magnitude of $x(z,L)$ are $1.5x_o$ for $z=0$ and $6.1x_o$ for $z=-\sigma_z$. If we require a wake field displacement at $z=-\sigma_z$ of $\leq \sigma_x$, we obtain a tolerance on the injection displacement of $|x_o| \leq 11\ \mu$m.

This tolerance on the injection error, x_o, sets the requirement for injection stability since the injection error can always be canceled by a set of static magnets. The corresponding tolerance on the jitter-inc of the injection kicker magnet is ± 1 μrad.

Misalignment effects are cominated by the second-order perturbation term rather than the first-order term. For a particle at $z = -\sigma_z$, for example, the ratio $x^{(2)}_{rms}$ to $x^{(1)}_{rms}$ is about 2.2.

The misalignment effect can be minimized by injecting the beam with empirically determined offset x_o and angle x'_o. Since the second-order contribution dominates, the optimum choise of x_o and x'_o is given by eq. (33). The expected rms value of the required injection offset, given by eq. (34), is $\langle x_o^2\rangle^{1/2} = 0.35\ \langle d^2\rangle^{1/2}$. After optimizing by controlling the injection conditions, the resultant beam size growth, $\langle x^{(1)2}\rangle^{1/2}$, is given by eq. (36), which is found to be 0.25 $\langle d^2\rangle^{1/2}$ at the bunch center and 0.62 $\langle d^2\rangle^{1/2}$ at σ_z behind the bunch center. For this beam size growth at $z = -\sigma_z$ to be less than the transverse beam size σ_x at the end of the linac, we demand a misalignment tolerance of $\langle d^2\rangle^{1/2} = 0.11$ mm.

The effect of the accelerator misalignment is determined by examining the reduction in luminosity arising from the emittance growth. Since the luminosity is inversely proportional to the emittance, the reduction factor R is approximately given by

$$R = \int_{-\ell/2}^{\ell/2} \frac{dz/\ell}{1 + \left[\langle x^{(1)2}\rangle/\sigma_x^2\right]} \tag{38}$$

where $\langle x^{(1)2} \rangle$ is given by eq. (36). In fig. 3, we have plotted the luminosity reduction factor versus the rms orbit distortion, $\langle d^2 \rangle^{1/2}$. For $\langle d^2 \rangle^{1/2} = 0.1$ mm, the reduction in luminosity is about 20%. The third-order term is appreciable only at the very tail of the bunch and thus does not affect the luminosity noticeably.

We have not taken account of the spread in betatron frequencies in the beam which comes from the energy spread in the bunch. Since different particles have slightly different energies and thus different betatron frequencies, there will be a Landau damping effect which will become significant if $\Delta k_o L > \pi$ (Δk_o is the spread in the betatron wave number in the bunch). A numerical tracking program is being prepared to investigate this effect.

7. Summary

We have studied the effect of transverse wake fields on the emittance growth of an intense bunch of particles in a linear accelerator. We first set up the equation of motion for particles in the bunch and then solve it by a perturbation method for cases of a coasting beam and a uniformly accelerated beam. The coasting beam result is given by eqs. (4), (10), and (11). For the case of an accelerated beam, we have found a substitution rule, eq. (26), that allows one to obtain the result from the coasting beam result.

For practical applications, we simplify the calculation by assuming a rectangular charge distribution and a linear wake function. Under these assumptions, an asymptotic expression of the bunch shape, eqs. (15), (16), and (27), can be obtained for cases with strong wake fields.

We have looked at the effects caused by misalignments of the linac acceleration sections. The perturbation method is again applied. The effect of misalignment on beam emittance growth can be minimized by controlling the injection conditions of the bunch. After the minimization scheme, the expected rms perturbation to beam emittance is given by eq. (36).

These results are applied to the recently proposed SLAC single pass collider. We find that the emittance growth and the associated reduction in luminosity will be tolerable provided the jittering in the injection angle is within ± 1 μrad and the accelerator misalignment is less than ±0.1 mm.

Acknowledgments

We would like to thank Dr. Rae Stiening for many useful discussions concerning this work.

REFERENCES

1) Design Report for the SLAC Single Pass Collider Project (1980).

2) R. Helm and G. Loew, The Stanford Two-Mile Accelerator, edited by R. B. Neal, New York, W. A. Benjamin, Inc. (1968), p. 173. See also references quoted therein.

3) W. K. H. Panofsky and M. Bander, Rev. Sci, Instr. 39, No. 2, 206 (1968).

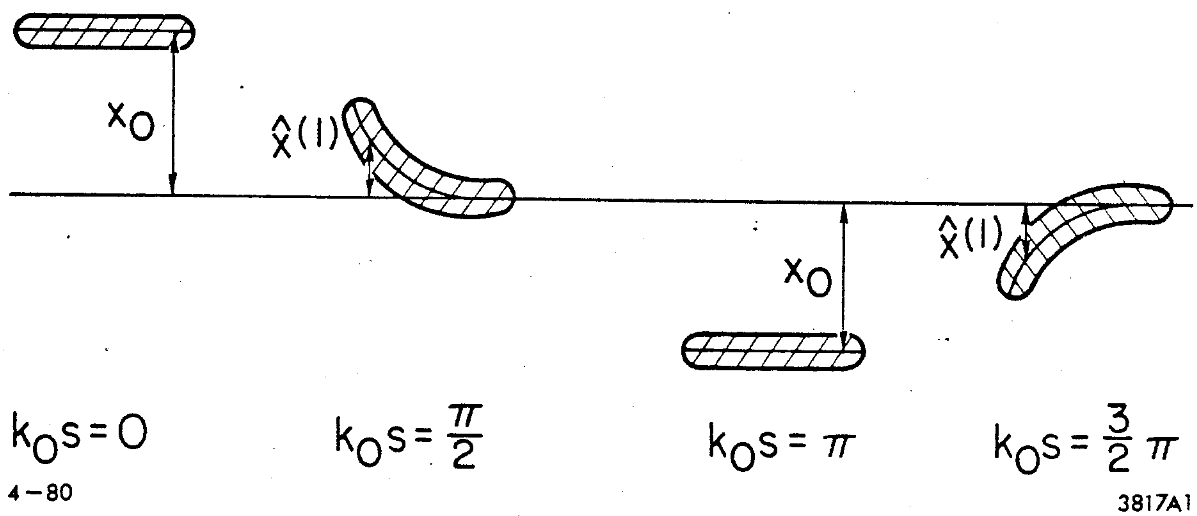
4) K. Bane and P. B. Wilson, "Wake Function for the SLAC linac," unpublished CN-16 (1980).

FIGURE CAPTIONS

Fig. 1. Sketches of bunch shape for a case with weak wake field at four instances of time.

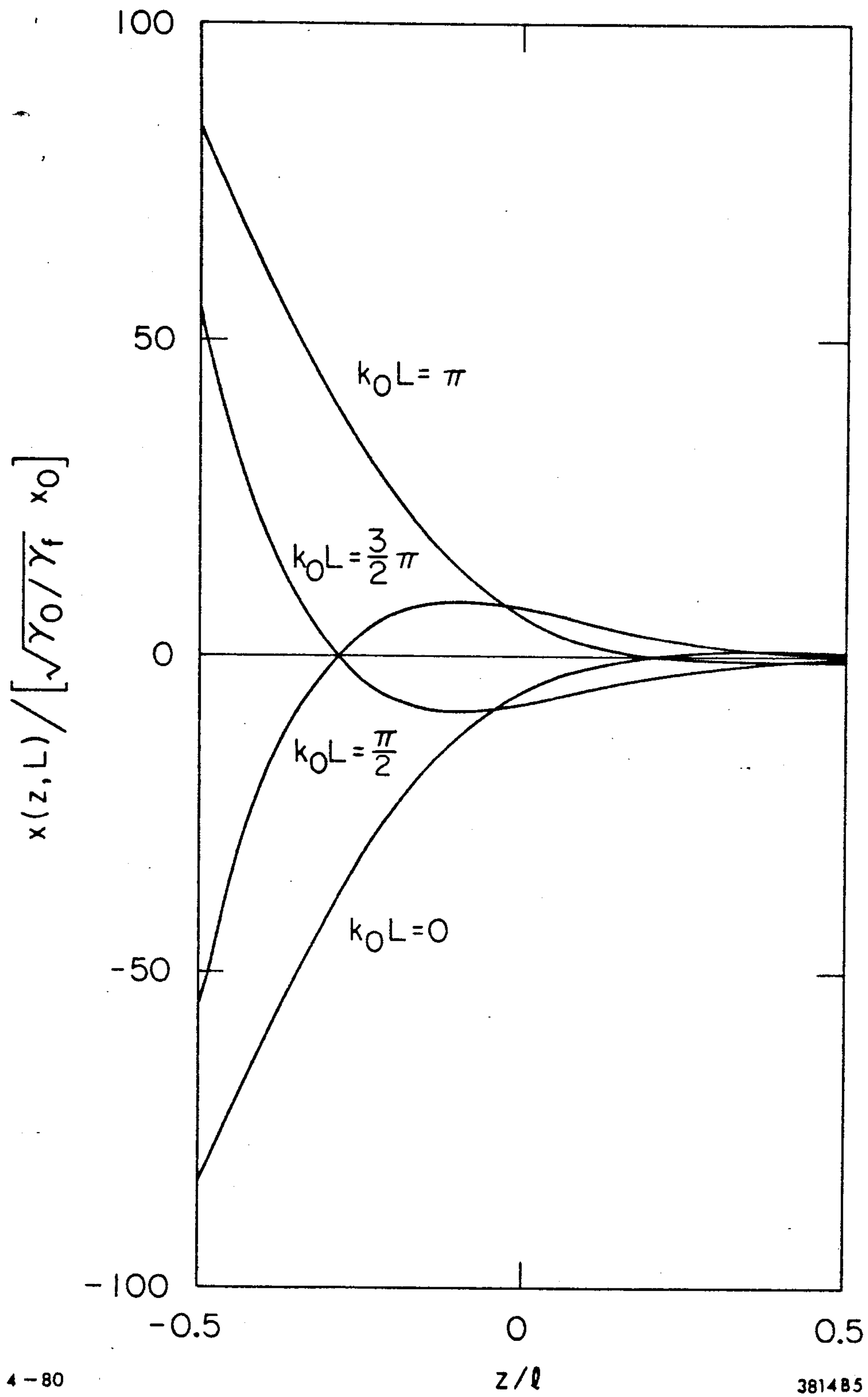
Fig. 2. Bunch distortion at the end of accelerator for four different values of total betatron phase k_oL (modulus 2π). The wake field strength parameter η is taken to be 150 at the bunch tail.

Fig. 3. Luminosity reduction factor R versus accelerator misalignment tolerance $\langle d^2\rangle^{1/2}$ for the case of the SLAC Single Pass Collider.



x_0
$\hat{x}(1)$
x_0
$\hat{x}(1)$
$k_0 s = 0$
$k_0 s = \frac{\pi}{2}$
$k_0 s = \pi$
$k_0 s = \frac{3}{2}\pi$
4–80
3817A1

Fig. 1



100
50
0
-50
-100
-0.5
0
0.5
z/ℓ
$x(z,L)/\left[\sqrt{\gamma_0/\gamma_f}\ x_0\right]$
$k_0L = \pi$
$k_0L = \frac{3}{2}\pi$
$k_0L = \frac{\pi}{2}$
$k_0L = 0$
4-80
3814B5

Fig. 2

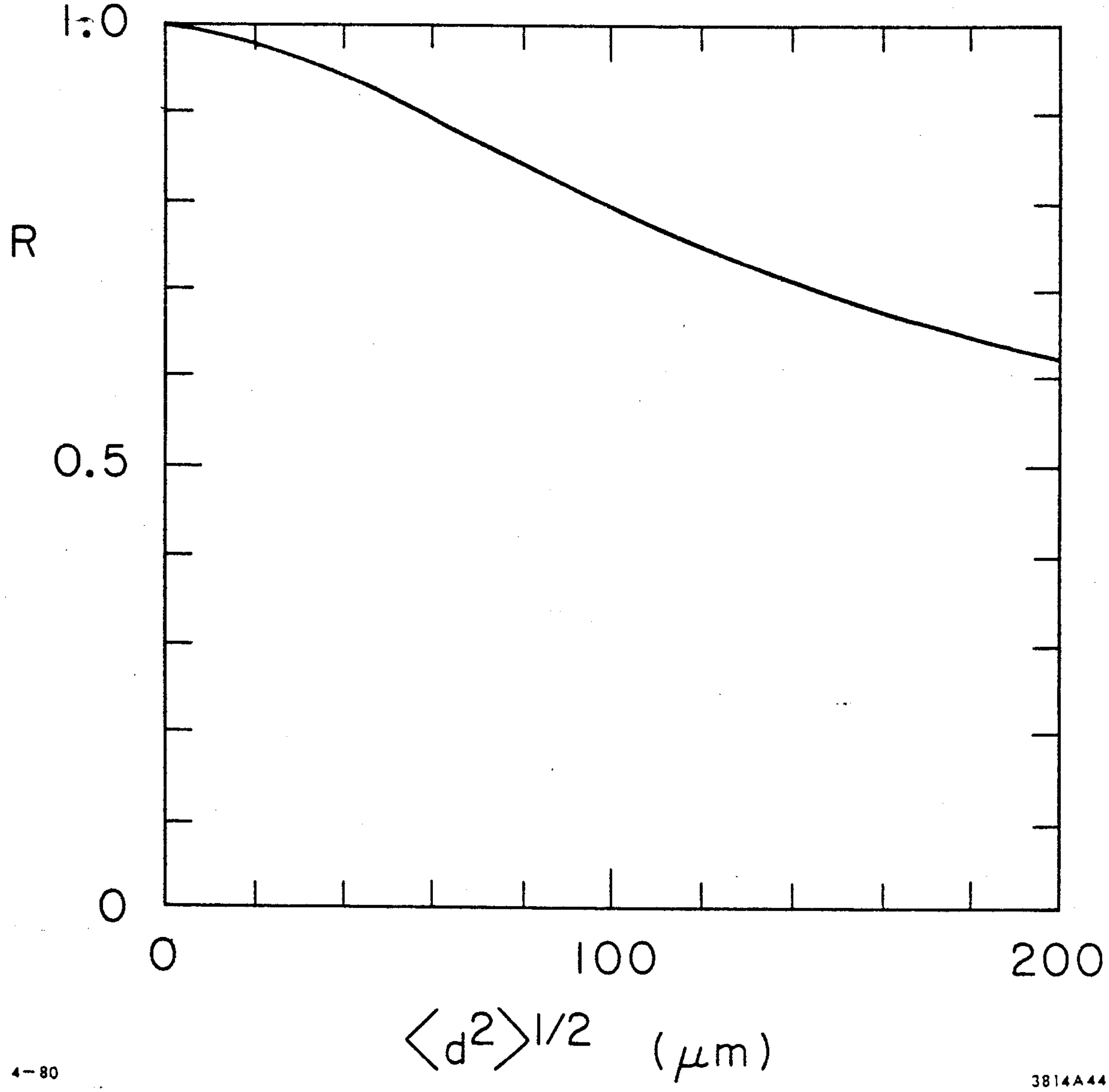


Fig. 3