Quintessential Inflation at the Maxima of the Potential

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Abstract. There is great interest in understanding a possible late accelerated expansion of the universe. Data suggest that the universe was still decelerating around redshift 1 and started to accelerate more recently at redshift 0.5. In models where the expansion is driven by a cosmological constant the acceleration should become increasingly greater with time, thus inflation never ends. This could also be the case with most models of quintessence or quintessential inflation where the late accelerated expansion is produced by a monotonically decreasing scalar potential. Here we would like to explore the possibility that a recent inflation has already or is about to end. This possibility is not ruled out by existing data and could be testable with far more, higher accuracy, supernovae on the Hubble diagram. We construct a two-stage inflationary model which can accommodate early inflation as well as a second stage of inflation (quintessence) with a single scalar field ϕ. Using an analogy from a mechanical problem we propose an inflaton field solution to the equations of motion which can account for two inflationary epochs. Inflation occurs close to the maxima of the potential. As a consequence both inflations are necesarilly finite. A first inflation is produced when fluctuations displace the inflaton field from its higher maximum rolling down the potential as in new inflation. Instead of rolling towards a global minimum the inflaton approaches a lower maximum where a second inflation takes place. The model is not realistic, however, because matter has not been taken into account at the end of the first inflation where particle production should occur as in non-oscillatory models. This is a delicate problem which will be treated elsewhere.

1 Introduction

The idea that the universe underwent an early inflationary expansion is now widely accepted [1]. This era of inflation makes plausible certain initial conditions for standard cosmology and provides a mechanism for structure formation. More speculatively the idea that the universe is at present undergoing inflation (usually denoted by the term quintessence) is the subject of much current interest [2]. Several models have been proposed where typically the potential energy of a scalar field, in general different from the one producing early inflation, is dominating the dynamics of the universe. Usually the potential is an inverse power of the field decreasing monotonically towards

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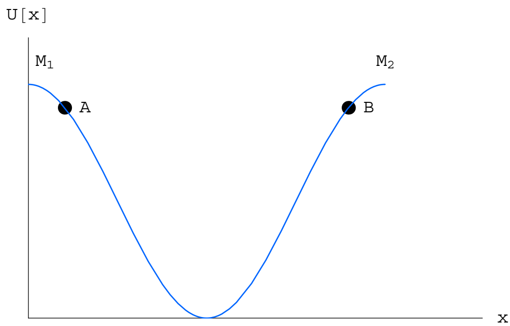

Fig. 1. The potential energy $U(x)$ of a particle in classical mechanics. In the absence of friction if we leave the particle at the point A with vanishing velocity it will eventually reach B, with $U(A) = U(B)$, also with vanishing velocity. In the limit when $A \to M_1$ it will take an infinite amount of time for the particle to reach $B \to M_2$.

zero. In the present work we are interested in studying a model which accommodates two stages of inflation by the evolution of a single scalar field [3]. Here, however, we look at the possibility that both inflations are produced when the inflaton is close to the maxima of the potential. The fact that both inflations occur at the maxima implies that they are necesarilly finite. This opens the interesting possibility where the second inflation has already or is about to end. This possibility is not ruled out by existing data and could be testable with far more, higher accuracy, supernovae on the Hubble diagram [4]. In what follows we construct a two-stage inflationary model by using an analogy with a problem from classical mechanics. The resulting potential could be obtained from supergravity (see Appendix).

Let us consider a potential $U(x)$ as shown in Fig. 1. When there is no friction the equation of motion for a particle of mass $m = 1$ is given by

$$\ddot{x} + U'(x) = 0. \tag{1}$$

We study the problem of a particle that leaves with vanishing velocity somewhere from the left of the minimum, let us say A and reaches B some time later. If we fix the origin of time at the minimum of $U(x)$ then the particle leaves A in the past reaching B sometime in the future. As A becomes close to the maximum at M_1 the particle spends longer close to the maxima. In the limit when $A \to M_1$ it takes an infinite amount of time for the particle to reach M_2. The particle would spend most of the time leaving M_1 and trying to reach M_2. As a result the kinetic energy is negligible close to the maxima; the potential energy dominates. We call this the limiting solution. The maximum at M_1 is located at $x = 0$ thus we require $x(t = -\infty) = 0$ and $x(t = +\infty)$ locates the maximum at M_2.

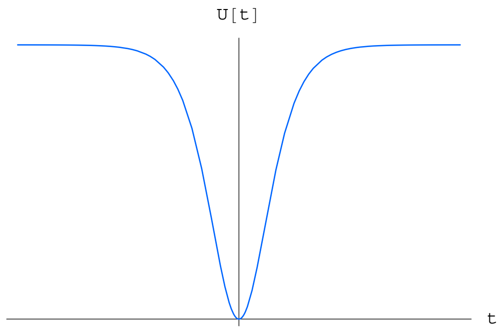

Fig. 2. The potential energy of the particle of Fig. 1 now as a function of time. The particle spends most of its time close to the maxima M_1, M_2 where the kinetic energy is negligible.

As a concrete example let us consider the potential

$$U(x) = \cos^2(x). \tag{2}$$

It is easy to check that the limiting solution is

$$x(t) = 2\arctan[\tanh(\frac{t}{\sqrt{2}})] + \frac{\pi}{2}, \tag{3}$$

where $x(t = -\infty) = 0$, $x(t = +\infty) = \pi$ and $\dot{x}(t = -\infty) = \dot{x}(t = +\infty) = 0$. The potential $U(x)$ is already illustrated in Fig. 1. As a function of time the potential is shown in Fig. 2. If we could lower the r.h.s. branch of this potential we could use this mechanical problem as an analogy to construct a model with two stages of inflation. Actually this can be done as follows. Instead of the potential $U(x)$ let us consider a new potential $\bar{U}(x)$ illustrated in Fig. 3. Now the maximum at M_2 is much smaller than the maximum at M_1. If we impose a solution of the type given by (3) it is clear that we need a friction term in the corresponding (1) to stop the particle precisely at M_2. Thus imposing a limiting solution to the potential $\bar{U}(x)$ determines the friction term and, as before the particle will spend most of the time close to M_1 and close to M_2 with negligible kinetic energy. As a function of time the potential of Fig. 4 shows the two plateaus at $t \to -\infty$ and $t \to +\infty$ corresponding to the maxima at M_1 and M_2 respectively.

In inflationary models of the "new" type one typically starts with a very flat potential and inflation occurs close to the maximum at $\phi = 0$, where ϕ is the inflaton field. There could be a previous "primordial" stage of inflation probably of the chaotic type setting the initial conditions for new inflation. For simplicity in what follows we will call this new inflationary epoch a first stage or simply first inflation (although probably there was inflation before) characterized by a scale Λ_1. This scenario is illustrated in Fig. 5. Here we study the possibility of a second stage of inflation at a scale Λ_2, where $\Lambda_2 \ll$

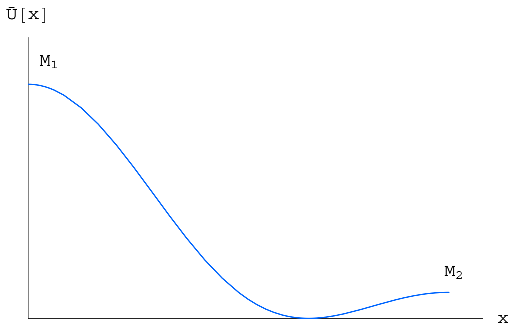

Fig. 3. A particle leaves the maximum at M_1 with vanishing velocity. It will just reach M_2 also with vanishing velocity if there is a friction term which stops the particle precisely at M_2.

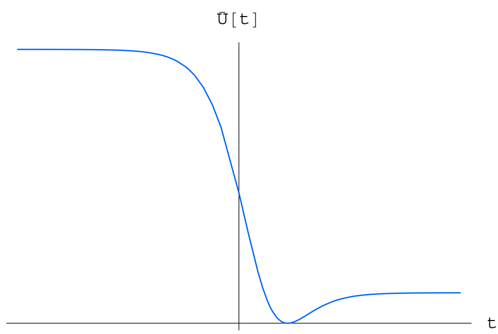

Fig. 4. The potential $\bar{U}(x)$ of Fig. 3 as a function of time. With an appropriate friction term the particle which leaves M_1 with vanishing velocity will just manage to reach M_2 in an infinite time. The particle spends most of its time close to the maxima with vanishing kinetic energy. Two plateaus appear at different energy scales.

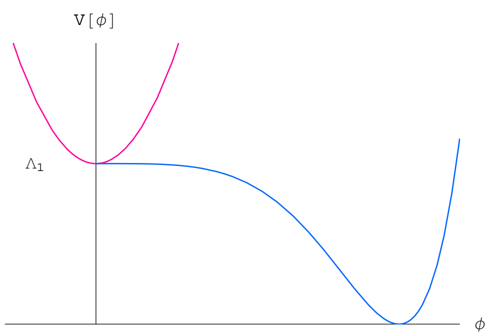

Fig. 5. A typical new inflationary potential gives rise to an early epoch of inflation (which in this work we call first inflation). Note, however, that there could have been a previous "primordial" stage of inflation providing initial conditions for new inflation to occur.

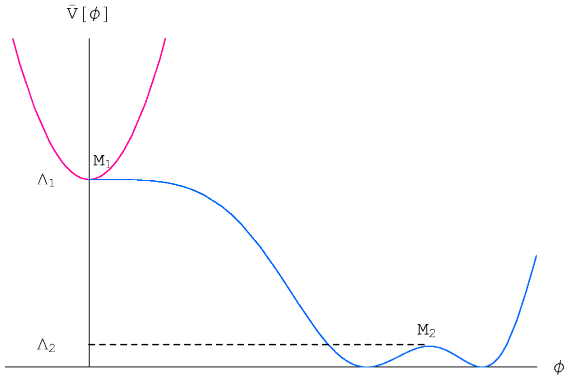

Fig. 6. The scenario studied here requires the presence of a second maximum at a scale Λ_2. There is a solution to the field equations wich makes the inflaton (due to the expansion of the universe) to evolve very slowly close to both maxima thus providing an inflationary model with two stages of inflation. In the limiting solution it takes an infinite amount of time for the scalar field to reach M_2 starting to roll from M_1. In a more realistic situation fluctuations displace ϕ from the maximum at M_1 thus allowing for the possibility of a finite second inflation. The scalar field ending in oscillations around one of its minima (see Fig. 9).

Λ_1. The mechanical analogy indicates that the second inflation will occur also close to a maximum, we then expect something like Fig. 6.

In Sect. 2 we construct a two-stage inflationary model. Section 3 deals with the problem of initial conditions and conclude in Sect. 4 with a discussion of the main results.

2 The Model

The inflaton field and Friedmann's equations are, as usual, given by

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \tag{4}$$

$$3H^2 = V + \frac{1}{2}\dot{\phi}^2, \tag{5}$$

where we have set the reduced Planck mass $M = 2.44 \times 10^{18} GeV$ to unity. The equations above can be rewritten as

$$3H^2 + \dot{H} = V, \tag{6}$$

$$\dot{H} = -\frac{1}{2}\dot{\phi}^2, \tag{7}$$

where the dot means derivative w.r.t. cosmic time and a prime denotes a field derivative. The limiting solution is conveniently written as

$$\phi(t) = \frac{2b}{\sqrt{6(1-b^2)}}\left(2\arctan[\tanh[\frac{\sqrt{3}a(1-b)}{2b}(t+t_o)]] + \frac{\pi}{2}\right), \qquad (8)$$

where the parameters a, b, t_o are determined by imposing physical conditions on the potential. The peculiar way in which these parameters are introduced above simplifies the analysis on the resulting potential. The main difference w.r.t. the mechanical problem is that the limiting solution determines (up to an integration constant) the "friction" term $3H\dot{\phi}$ through (7) *and* the potential through (6). Equivalently by providing $H(t)$ we could get $\phi(t)$ and V. The derivative of the Hubble function is

$$\dot{H} = -a^2\frac{1-b}{1+b}sech^2[\frac{\sqrt{3}a(1-b)}{b}(t+t_o)]. \qquad (9)$$

Integrating this expression we get the Hubble function

$$H(t) = \frac{a}{\sqrt{3}(1+b)}\left(1 - b\tanh[\frac{\sqrt{3}a(1-b)}{b}(t+t_o)]\right), \qquad (10)$$

the arbitray integration constant $a/(\sqrt{3}(1+b))$ has been chosen this way to get an overall scale for $H(t)$ and to guarantee a positive Hubble function for $b < 1$. This choice also makes the potential very simple when $t \to -\infty$. From (6) the potential follows

$$V(t) = \frac{a^2}{(1+b)^2}\left(b - \tanh[\frac{\sqrt{3}a(1-b)}{b}(t+t_o)]\right)^2. \qquad (11)$$

We can invert (8) and from (11) obtain the potential as a function of ϕ, this is given by

$$V(\phi) = \frac{a^2}{(1+b)^2}\left(b + \cos[\sqrt{\frac{3(1-b^2)}{2b^2}}\phi]\right)^2. \qquad (12)$$

A potential of this type could follow from supergravity (see Appendix). The parameters are determined from the following conditions. The potential at $t \to -\infty$ should reach the scale Λ_1 of the first inflation, this fixes a

$$V(t \to -\infty) = a^2 \equiv \Lambda_1^4, \qquad \Rightarrow a = \Lambda_1^2. \qquad (13)$$

At $t \to +\infty$ the potential reaches the second scale Λ_2 of inflation, this fixes b

$$V(t \to +\infty) = a^2\frac{(1-b)^2}{(1+b)^2} \equiv \Lambda_2^4, \qquad \Rightarrow b = \frac{1-d}{1+d}; \qquad d \equiv (\frac{\Lambda_2}{\Lambda_1})^2. \qquad (14)$$

Actually, (14) has two solutions for b but the second solution leads eventually to a negative Hubble function and thus to a contracting universe. This case is not studied here. Inflation ends (or starts) at points where the acceleration of the scale factor $a(t)$ vanishes

$$\frac{\ddot{a}(t)}{a(t)} = H^2 + \dot{H} = 0. \tag{15}$$

This equation has two solutions. The end of the first inflation is taken at $t = 0$ thus fixing t_o

$$t_o = -\frac{b}{\sqrt{3}a(1-b)} arctanh[\frac{\sqrt{6}(1-b^2)-b}{3-2b^2}]. \tag{16}$$

The beginning of the second inflation is given by

$$t_{s2} = -\frac{\ln(49-20\sqrt{6})}{2\sqrt{3}}\frac{b}{a(1-b)}, \tag{17}$$

which, by virtue of (14), can be written as

$$t_{s2} = -\frac{\ln(49-20\sqrt{6})}{4\sqrt{3}\Lambda_2^2}(1-(\frac{\Lambda_2}{\Lambda_1})^2) \approx \frac{0.662}{\Lambda_2^2}, \tag{18}$$

the last result follows because $\Lambda_2/\Lambda_1 \ll 1$.

Corresponding to $\rho_\phi \approx 0.7\rho_c$ with $\rho_\phi \approx V(\phi)$, we take $\Lambda_2 = 2.744 \times 10^{-12}\sqrt{h}GeV$, where h is somewhere between 0.68 and 0.75. Thus, (18) gives the time when the second inflation starts with respect to the end of the first inflation at $t = 0$, which for any practical purpose could be taken as the Big-Bang. Using the reduced Planck time $T = 2.7 \times 10^{-43} sec$, (18) gives

$$t_{s2} \approx \frac{4.5 \times 10^9}{h} years. \tag{19}$$

This is not, however, a realistic model because particle production at the end of the first inflation t_{end1} has not been considered. Shortly after t_{end1} the radiation, followed by matter energy density should dominate the inflaton energy density. Because we want a second stage of inflation to occur at a second maximum then reheating after the first inflation should be produced as in non-oscillatory models [5], a delicate problem which will be dealt with elsewhere.

In Fig. 7 we show the total, potential and kinetic energies, as well as the acceleration of the scale factor of the universe as functions of time for the limiting solution given by (8). Finally Fig. 8 shows the equation of state parameter $\omega = p/\rho$. The example above was developed using the limiting solution. This is only an approximation to a more general situation where the scalar field is initially displaced from its maximum at M_1.

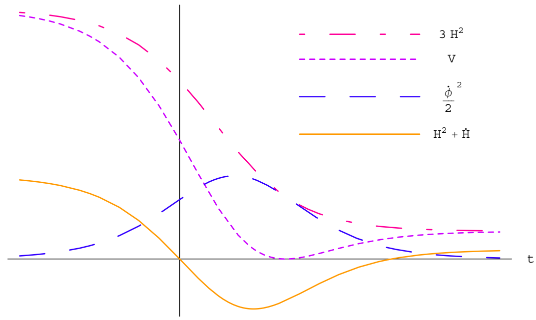

Fig. 7. We show the total, potential and kinetic energies, as well as the acceleration of the scale factor of the universe, $\ddot{a}(t)/a(t)$, as functions of time for the limiting solution given by (8). The origin of time has been chosen so that the end of the first inflation occurs at $t = 0$. Thus we find that the start of the second inflation denoted by t_{s2} is given by $t_{s2} \approx 4.5 \times 10^9/h$ years.

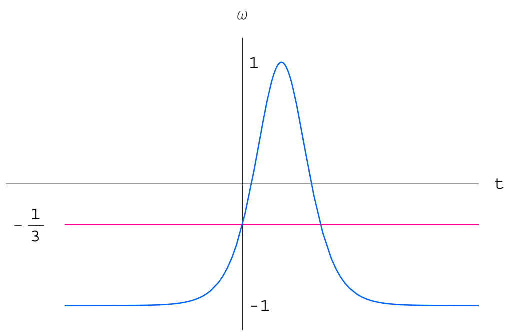

Fig. 8. The equation of state parameter $\omega = p/\rho$ as a function of time. For $t \to \pm\infty$ ω takes the cosmological constant value of -1. Inflation occurs for $\omega \leq -1/3$.

3 Initial Conditions

In a more realistic situation the inflaton leaves not from the maximum at M_1 but from a slightly displaced position. The potential is shown in Fig. 6. A mechanism setting the field away from M_1 is provided by its fluctuations. We have that

$$\delta\phi \approx \frac{H(t \to -\infty)}{2\pi} \approx \frac{\Lambda_1^2}{2\pi\sqrt{3}}. \tag{20}$$

Depending on the initial conditions the scalar field approaches M_2 ending in oscillations around one of the minima. The time evolution of ϕ is illustrated in Fig. 9. Figure 9a corresponds to a field which is unable to reach the maximum at M_2 ending in oscillations around the first minimum while Fig. 9b shows the time evolution of the field when this is able to overcome the maximum M_2 ending at the second minimum. In both cases the flat part of the figure

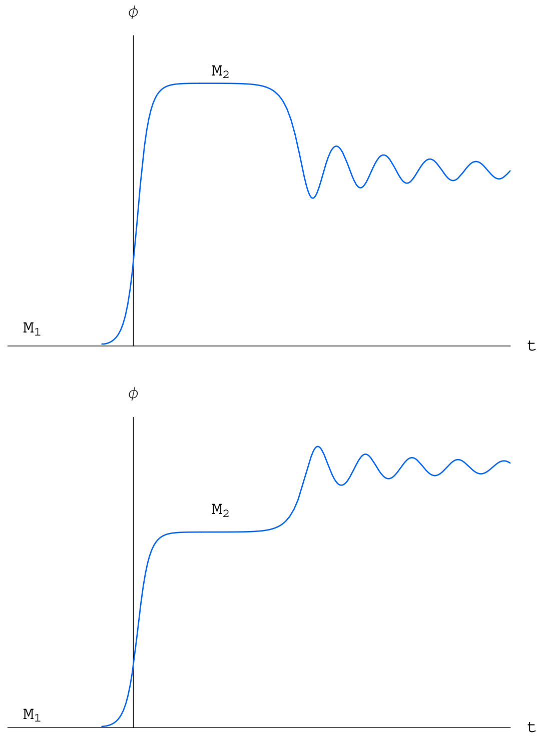

Fig. 9. The inflaton leaves from close to M_1 where it has been displaced due to its fluctuations $\delta\phi \approx H(t \to -\infty)/2\pi \approx \Lambda_1^2/2\pi\sqrt{3}$. After some time it approaches the second maximum at M_2 (see Fig. 6) ending in oscillations around the first minimum Fig. 9a or the second Fig. 9b.

is where the second inflation occurs and its duration clearly depends on the initial conditions with which the universe was prepared.

The equivalent to Fig. 7 for this case is shown in Fig. 10 where we plot the total, potential and kinetic energies as well as the acceleration of the scale factor of the universe. Finally Fig. 11 shows the behaviour of the equation of state parameter $\omega = p/\rho$. This should be compared with Fig. 8 for the case of the limiting solution.

4 Conclusions

We have studied a model of inflation which can accommodate two inflationary eras. Both stages of inflation are drived by the potential energy of a single scalar field. The new feature is that inflation occurs close to the maxima of the potential where the kinetic energy is negligible. As a consequence both inflations are of finite duration. It is then possible that the second inflation

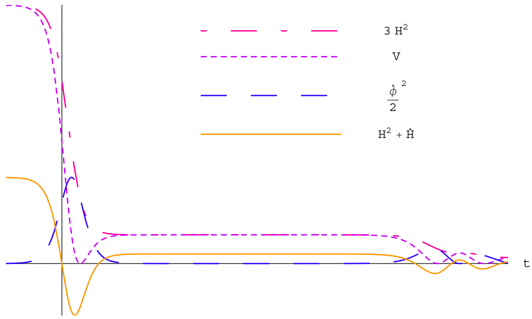

Fig. 10. Corresponding to Fig. 7 where now the scalar field starts its rolling displaced from the maximum at M_1 due to its fluctuations. All the curves are flat to the left of the figure with the inflaton close to M_1. It finally approaches M_2 (second plateau). After some time close to the second maximum at M_2 the inflaton rolls to a minimum (see Fig. 6) with the oscillatory behaviour shown at the far right of the figure.

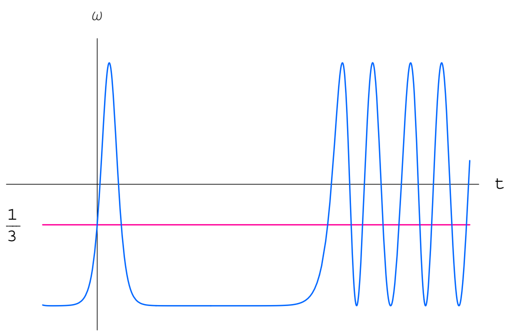

Fig. 11. The equation of state parameter $\omega = p/\rho$ as a function of time when the scalar field has been displaced from the maximum at M_1. Note how during the oscillations of the inflaton there are short periods of inflation.

has already or is about to end which should be testable by substantially increasing the number and accuracy of supernovae on the Hubble diagram. In the ideal case, which we call the limiting solution, the scalar field takes an infinite amount of time to reach the second, smaller, maximum. In a more realistic case the scalar field is displaced from the higher maximum by its fluctuations ending in oscillations in one of the minima of the potential. The origin of time is fixed by the requirement that $H^2 + \dot{H}$ vanishes at $t = 0$. Thus the end of the first inflation defines the origin of time which for any practical purpose could be taken as the Big-Bang. A realistic model should incorporate an era of radiation followed by matter domination after the end of the first inflation. Potentially problematic is that the initial conditions are fine-tuned to avoid the scalar field undershoot or overshoot the second maximum of the potential. On the other hand we have been able to show that a potential of the type (12) could be derived from supergravity. In supergravity the only natural scale is the Planck scale and we can find arguments to explain the possible origin of the first scale of inflation [6] while the second could be understood in terms of the first one by considering friction terms due to the expansion of the universe and possible interactions of the inflaton with matter fields.

Appendix

Let us consider the supergravity potential for one chiral superfield with scalar component z and without D-terms [7]

$$V = e^K \left[F^* (K_{zz^*})^{-1} F - 3|W|^2\right], \tag{21}$$

where

$$F \equiv \frac{\partial W}{\partial z} + \left(\frac{\partial K}{\partial z}\right) W, \qquad K_{zz^*} \equiv \frac{\partial^2 K}{\partial z \partial z^*} \ . \tag{22}$$

The reduced Planck mass $M \sim 2.4 \times 10^{18}$ GeV has been set equal to one. The superpotential and Kähler potential denoted W and K respectively. Here we are interested in models where W and K are given by polynomial expressions such as

$$W = \sum_{n=0}^{\infty} a_n z^n, \tag{23}$$

and

$$K = \sum_{n=1}^{\infty} b_n (zz^*)^n, \tag{24}$$

where a_n and b_n are real coefficients. In general this structure leads to expressions that contain cos-form potentials for the angular field ϕ which is a real field defined from z in the following way

$$z = \chi e^{i\phi} \ . \tag{25}$$

By using the superpotential and Kähler potential as given by (23) and (24), it is straightforward to show that the supergravity potential can be written in the form

$$V = e^K \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \left[\frac{(n+K_1)(m+K_1)}{K_2} - 3\right] a_n a_m z^n z^{*m}, \tag{26}$$

where K_i denote the sums

$$K_1 = \sum_{n=1}^{\infty} n b_n (zz^*)^n \ , \tag{27}$$

$$K_2 = \sum_{n=1}^{\infty} n^2 b_n (zz^*)^n \ . \tag{28}$$

Notice that for superpotentials and Kähler potentials of the form (23) and (24), respectively, (26) is entirely equivalent to the supergravity potential given by (21). Let us now insert the radial and angular fields by writing z in the way expressed by (25), $z = \chi e^{i\phi}$. The potential is then given by [8]

$$V = e^K \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \left[\frac{(n+K_1)(m+K_1)}{K_2} - 3\right] a_n a_m \chi^{n+m} \cos[(n-m)\phi], \tag{29}$$

It is easy to show that (29) can give rise to potentials of the type (12). Let us write the superpotential and Käler potential in the form

$$W = a_0 + a_1 z + a_2 z^2, \tag{30}$$

and

$$K = zz^* = \chi^2, \tag{31}$$

Assuming that the χ field has relaxed to its v.e.v., χ_0 and eliminating e.g., a_1 we get

$$V(\phi) = c_1 (c_2 + \cos[\phi])^2, \tag{32}$$

where

$$c_1 = 2e^{\chi_0^2}\chi_0\sqrt{(\chi_0^2-1)a_0a_2}, \tag{33}$$

and

$$c_2 = \frac{((\chi_0^2-2)a_0 + (\chi_0^4+2)a_2)\sqrt{(\chi_0^2-3)a_0^2 - 2\chi_0^2(\chi_0^2-1)a_0a_2 + \chi_0^2(\chi_0^4+\chi_0^2+4)a_2^2}}{\sqrt{(\chi_0^2-2)^2a_0^2 - 2(\chi_0^6-2\chi_0^4+2\chi_0^2+2)a_0a_2 + (\chi_0^4+2)^2a_2^2}}. \tag{34}$$

It is then not untinkable that a model of the type (12) could arise from a particle physics model.

Acknowledgements

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