Torsion, a multivalued gauge degree of freedom in Einstein's gravity

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Abstract. We show that there is an entire gauge symmetry of a novel kind which interpolates between an infinity of formulations of the laws of gravity, ranging from Einsteins pure curvature formulation to a pure torsion formulation in a teleparallel geometry. As a consequence, torsion and curvature are not independent and that torsion is an alternative description of curvature in gravity.

1. It is well known that the solutions of Einstein equations can equally well be obtained in a Riemann-Cartan spacetime with torsion, in which the Cartan curvature vanishes identically. This *teleparallel formulation* of gravity has been proposed by Einstein as an alternative to his famous theory of relativity in Riemannian spacetime, and its properties have been studied in many publications [1, 2]. In this formulation, the Einstein-Hilbert action is equal to a combination of scalars formed from torsion tensors [2]. In this contribution to the special volume on analog models of gravitational theories I would like to point out that there is an entire family of theories in which a combination of curvature and torsion provide us with an alternative description of the same gravitational forces [3, 4, 5]. This result, obtained first in Ref. [5], was reached on the basis of the close analogy of spaces with gravity and torsion to crystals with defects [6, 7].

2. A nontrivial extension of Einsteins theory in curved spacetime to Riemann-Cartan spacetime was advanced since 1959 [7, 8, 9]. It has the appealing feature that it can be rewritten as a gauge theory that is invariant under local Poincaré transformations, i.e., under both local translations and local Lorentz transformations. This brings the geometric theory of gravity to a similar form as the gauge theories of weak, electromagnetic, and strong interactions. In the gauge formulation, torsion is treated as an independent field which couples only to the intrinsic spin of the elementary particles.

The extension has, however, several unsatisfactory features. First, it has the same problem as Einstein's theory that it cannot be quantized and is necessarily classical. But spin carries a power of $\hbar$, and this is zero in the classical limit. So there exists, strictly speaking, no classical source of torsion. Moreover, since torsion is assumed to couple to spin with the universal gravitational coupling strength, its smallness implies that spin cannot play any sizable role in the forces between celestial bodies. For example, even if the earth consisted only of polarized atoms, its total intrinsic spin would be 10^{15} times smaller than the rotational spin around the axis.

A second argument against a gravitational theory with torsion was brought forward by Weinberg [1]. Since torsion is a tensor there is really no need for it to ensure general coordinate invariance.

A third argument is that the coupling of spinning particles to torsion would make the photon massive. To avoid this, the authors of [8] forbid the direct coupling to photons. They ignore the fact that by the allowing spin-1 ρ mesons to couple to torsion, their electromagnetic coupling will make photons massive after all.

Until the recent results of the satellite experiment Gravity Probe B [11] some people had hoped that the observed Lense-Thirring effect would deviate from Einstein's prediction. But it did not. Thus a simple possibility of discovering torsion experimentally has faded [12].

These problems make it doubtful that the generalization of gravity to Riemann-Cartan spacetime proposed in the papers [7, 8, 9] has a chance of being true. In this note we want to give a symmetry argument for this. It is inspired by a simple analog model of gravity, a *world crystal* with defects [9, 13] [2] whose lattice constant is of the order of a Planck length. Some consequences of such a world crystal were pointed out in a recent study of black holes in such a scenario [19].

In the world crystal, there exists a new type of extra gauge symmetry in which zero torsion is merely a special gauge. A completely equivalent gauge is the absence of Cartan curvature, which is found in Einstein's teleparallel universe. And there exists an infinite number of intermediate gauges.

3. To prepare the grounds for the argument, recall that a crystal can have two different types of topological line-like defects [7, 9], which in a four-dimensional world crystal are world surfaces (which may be the objects of string theory).

First, there are translational defects called *dislocations* (Fig. 1). These are produced by a

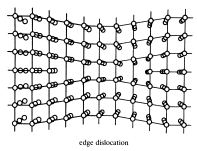

Figure 1. Formation of a dislocation line (of the edge type) by a Volterra process. The Burgers vector **b** characterizes the missing layer. There exist two more types where **b** points in orthogonal directions.

cutting process due to Volterra: a single-atom layer is removed from the crystal, allowing the remaining atoms to relax to equilibrium under the elastic forces. A second type of topological defects is of the rotation type, the so-called *disclinations* (Fig. 2). They arise by removing an entire wedge from the crystal and re-gluing the free surfaces.

The defects cause a failure of derivatives to commute in front of the displacement field $u_i(\mathbf{x})$. In three dimensions, the dislocation density is given by the tensor

$$\alpha_{ij}(\mathbf{x}) = \epsilon_{ikl}\nabla_k\nabla_l u_j(\mathbf{x}). \tag{1}$$

If $\omega_i \equiv \frac{1}{2}\epsilon_{ijk}[\nabla_j u_k(\mathbf{x}) - \nabla_k u_j(\mathbf{x})]$ denotes the local rotation field, the disclination density is defined by

[1] See the letter exchanges in the 2006 issue *Physics Today* **60**, 10, 16 following Weinberg's article [10].

[2] The gravitational action in this model is obtained by integrating out the lattice vibrations. It is thus a consequence of their *entropy*. This mechanism has recently been emphasized by Verlinde [14]. The same mechanism had been used a long time ago to generate stiffness of strings [15, 16] This is of course just reformulation of good-old Sakharov's idea [17, 18].

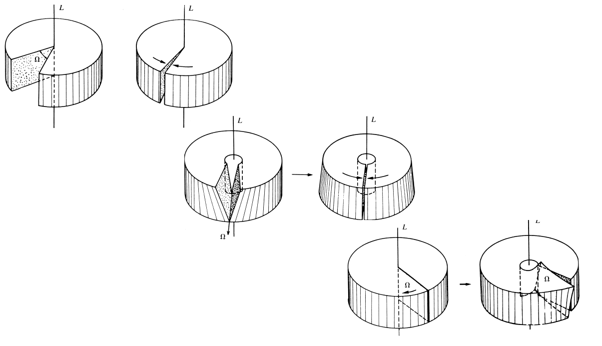

Figure 2. Three different possibilities of constructing disclinations: wedge, splay, and twist disclinations. They are characterized by the Frank vector $\mathbf{\Omega}$.

$$\theta_{ij}(\mathbf{x}) = \epsilon_{ikl}\nabla_k\nabla_l\omega_j(\mathbf{x}). \tag{2}$$

The defect densities satisfy the conservation laws

$$\nabla_i\theta_{ij} = 0, \quad \nabla_i\alpha_{ij} = -\epsilon_{jkl}\theta_{kl}. \tag{3}$$

These are fulfilled as Bianchi identities if we express $\theta_{ij}(\mathbf{x})$, $\alpha_{ij}(\mathbf{x})$ in terms of plastic gauge fields $\beta^p_{kl}, \phi^p_{lj}$, setting $\theta_{ij} = \epsilon_{ikl}\nabla_k\phi^p_{lj}$, $\alpha_{il} = \epsilon_{ijk}\nabla_j\beta^p_{kl} + \delta_{il}\phi^p_{kk} - \phi^p_{li}$. The defect densities are invariant under the gauge transformations $\beta^p_{kl} \to \beta^p_{kl} + \nabla_k u^p_l - \epsilon_{klr}\omega^p_r$, $\phi^p_{li} \to \phi^p_{li} + \partial_l\omega^p_i$, where $\omega^p_i \equiv \frac{1}{2}\epsilon_{ijk}\nabla_j u^p_k$. Thus $h_{ij} \equiv \beta^p_{ij} + \epsilon_{ijk}\omega^p_k$ and $A_{ijk} \equiv \phi^p_{ij}\epsilon_{jkl}$ are *translational* and *rotational* defect gauge fields in the crystal.

4. In order to appreciate the strategy of the paper, let us discuss a simple analog field theoretic model. Instead of Einstein's theory, we consider a model of a simple real field ρ with an Euclidean Lagrangian $\mathcal{L} = (\partial_\mu\rho)^2 - \rho^2 + \rho^4$ and a partition function $Z = \int \mathcal{D}\rho\, e^{-\int dx\mathcal{L}}$. The field ρ plays the role of the real metric tensor field $g_{\mu\nu}$. We may extend the above model trivially by an extra gauge field A_μ that has no physical consequences. This is done by re-expressing the Lagrangian in terms of a complex field $\psi = e^{i\theta}\rho$ and the gauge field A_μ as $\bar{\mathcal{L}} = |(\partial_\mu - iA_\mu)\psi|^2 - |\psi|^2 + |\psi|^4$. Now we form the partition function $\bar{Z} = \int \mathcal{D}\psi\mathcal{D}\psi^*\mathcal{D}A_\mu\Phi\, e^{-\int dx\bar{\mathcal{L}}}$, where Φ is an arbitrary gauge-fixing functional multiplied by the associated Faddeev-Popov determinant.

It is easy to verify that the classical field equations following from the new $\bar{Z}$ are exactly the same as those of the original Z.

The same thing holds for the flcutuating theory. The gauge field A_μ possesses two different spin contents. One is of spin zero, a pure gauge field $A_\mu = \partial_\mu\Lambda$ which possesses no electromagnetic field strength and leaves the physical content of the field system completely invariant. The second content has spin 1, where A^μ is orthogonal to the spin-zero part. This does possess an electromagnetic field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. . Since the action is independent of $F_{\mu\nu}$ this spin-1 content does not contribute to the partition function $\bar{Z}$ either. If an action term of the Maxwell type $F_{\mu\nu}F^{\mu\nu}$ were present in the exponent of $\bar{Z}$, the theory would be only invariant under ordinary gauge transformations with single-valued scalar fields $\Lambda(x)$. Without such a Maxwell term, it is invariant under *all* gauge transformations with single and multi-valued [9] scalar fields $\Lambda(x)$, and thus to fluctuation of electromagnetic fields.

There is, obviously, no way of observing A_μ. In the gravitational theory to be discussed, the partition function Z of the ρ field plays the role of Einstein's theory formulated in terms of the metric field $g_{\mu\nu}$. The reformulation $\bar{Z}$ in terms of a gauge field will be the gauge field reformulation of Einstein's theory. The decomposition

$$\rho = \psi^*\psi = (\rho e^{-i\theta})(e^{i\theta}\rho) \tag{4}$$

will be the analog of the analog of this decomposition made for the metric rather than the real field ρ. The local phase θ may be any arbitrary function, single- as well as multivalued.

5. The Volterra processes can be represented mathematically by multivalued transformations from an Euclidean crystal with coordinates $\bar{x}^a$ to a crystal with defects and coordinates x^μ, as illustrated in Figs. 3 and 4 for two-dimensional crystals.

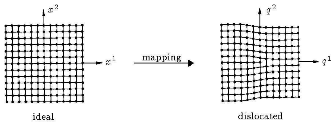

Figure 3. Multivalued mapping of the perfect crystal to an edge dislocation with a Burgers vector **b** pointing in the 2-direction.

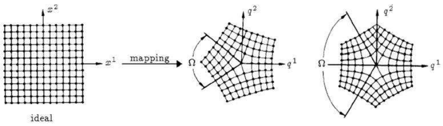

Figure 4. Multivalued mapping of the perfect crystal to a wedge disclination of Frank vector Ω in the third direction.

For an edge dislocation the mapping is $\bar{x}^1 = x^1$, $\bar{x}^2 = x^2 + (b/2\pi)\phi(x)$, where $\phi(x) \equiv (1/2\pi)\arctan(x^2/x^1)$. Initially, this function has a cut from the origin towards left infinity. In a second step, the cut is removed and the multivalued version of the arctan is taken. This makes $\phi(\mathbf{x})$ the Green function of the commutator $[\partial_1, \partial_2]$: $(\partial_1\partial_2 - \partial_2\partial_1)\phi(x) = \delta^{(2)}(\mathbf{x})$. For a wedge disclination, the mapping is $d\bar{x}^i = \delta^i{}_\mu\,[x^\mu + (\Omega/2\pi)\varepsilon^\mu{}_\nu x^\nu \phi(x)]$.

A combination of the two

$$\eta_{ij}(\mathbf{x}) \equiv \theta_{ij}(\mathbf{x}) - \tfrac{1}{2}\nabla_m[\epsilon_{min}\alpha_{jn}(\mathbf{x}) + \{ij\} + \epsilon_{ijn}\alpha_{mn}] \tag{5}$$

forms the *defect tensor*

$$\eta_{ij}(\mathbf{x}) \equiv \epsilon_{ikl}\epsilon_{jmn}\nabla_k\nabla_m u^p_{ln}(\mathbf{x}), \quad u^p_{ln} \equiv \tfrac{1}{2}(\beta^p_{ln} + \beta^p_{nl}). \tag{6}$$

It is a symmetric tensor due to the conservation laws (3), and represents the Einstein tensor $\bar{G}_{ij} \equiv \bar{R}_{ij} - \frac{1}{2}g_{ij}\bar{R}_k{}^k$ of the geometry of the world crystal [3].

The expressions can easily be defined on a simple-cubic world crystal if we replace ∇_i by lattice derivatives, as shown in [7, 9]. There it is also shown that, in three spacetime dimensions, the disclination density $\theta_{ij}(\mathbf{x})$ represents the Einstein tensor G^{C}_{ij} associated with the *Cartan curvature tensor* $R^{\mathrm{C}}_{ijk}{}^l$ of the Riemann-Cartan geometry of the world crystal. The relation is

$$G^{\mathrm{C}}_{ji}(\mathbf{x}) = \epsilon_{ikl}\nabla_k\nabla_l\omega_j(\mathbf{x}) = \theta_{ij}(\mathbf{x}). \tag{7}$$

The dislocation density $\alpha_{ij}(\mathbf{x})$ represents the torsion $S_{lkj} = \frac{1}{2}(\Gamma_{lkj} - \Gamma_{klj})$ of the Riemann-Cartan geometry. Here the relation is

$$\alpha_{ij} = \epsilon_{ikl}S_{lkj}. \tag{8}$$

6. The standard form of a defect with Burgers vector b_l and Frank vector Ω_q has a displacement field

$$u_l(\mathbf{x}) = -\delta(\mathbf{x}, V)[b_l + \epsilon_{lqr}\Omega_q(x_r - \bar{x}_r)], \tag{9}$$

[3] Geometric objects formed from Christoffel symbols are denoted by a bar. Otherwise the full affine connection is used.

where ϵ_{lqr} is the antisymmetric unit tensor, $\bar{x}_r$ the axis of rotation of the disclination part, and $\delta(\mathbf{x}; V)$ is the delta function on the volume V, i.e., in three dimensions:

$$\delta(\mathbf{x}; V) = \int_V d^3x' \, \delta^{(3)}(\mathbf{x} - \mathbf{x}'). \tag{10}$$

Its derivative is the delta function on the Volterra surface S of V:

$$-\nabla\delta(\mathbf{x}; V) = \delta(\mathbf{x}; S) = \int_S d\mathbf{S}' \, \delta^{(3)}(\mathbf{x} - \mathbf{x}'). \tag{11}$$

For the new gauge symmetry, the crucial observation is that as a simple consequence of (9), a dislocation line in the world crystal can either be obtained by a Volterra process of cutting out a thin slice of material of thickness $\mathbf{b}$, or alternatively by cutting out a wedge of Frank vector $\mathbf{\Omega}$, and reinserting it at distance $\mathbf{b}$ from the cut. Thus the dislocation line is indistinguishable from a pair of disclination lines with opposite Frank vector Ω whose axes of rotation are separated by a distance $\mathbf{b}$ (Fig. 5a). Conversely, a disclination line is equivalent to a stack of dislocation lines with fixed Burgers vector $\mathbf{b}$ (Fig. 5b).

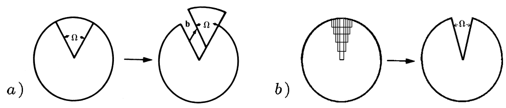

Figure 5. Equivalence between a) dislocation and pair of disclination lines, b) disclination and stack of dislocation lines.

Analytically, this is most easily seen in the two-dimensional version of the relation (5):

$$\eta_{33} = \theta_{33} + \epsilon_{3mn}\nabla_m\alpha_{3n}. \tag{12}$$

Each term is invariant under the plastic gauge transformations $\beta^p_{kl} \to \beta^p_{kl} + \nabla_k u^p_l - \epsilon_{kl}\omega^p_3$, $\phi^p_l \to \phi^p_l + \partial_l\omega^p_3$. The general defect has a displacement field

$$u_l = -\delta(V_2)[b_l - \Omega\epsilon_{3lr}(x_r - \bar{x}_r)]. \tag{13}$$

The first term is a dislocation, the second term a disclination. According to Fig. 5, the latter can be read as a superposition of dislocations with the same Burgers vector $\tilde{b}_l = -\int_{\bar{x}}^{x} dx'_r \Omega\epsilon_{3lr}$. The former may be viewed as a dipole of disclinations: $-\bar{\nabla}_l[-\frac{1}{2}b_m\epsilon_{3km}]\epsilon_{3kr}(x_r - \bar{x}_r)$.

7. Generalizing the defect relations (5) and (12) to $D \geq 4$ spacetime dimensions and allowing for large deviations from Euclidean space, we find the defect relation

$$\bar{G}_{\mu\nu} = G_{\mu\nu} - \tfrac{1}{2}D^{*\lambda}\left(S_{\mu\nu,\lambda} - S_{\nu\lambda,\mu} + S_{\lambda\mu,\nu}\right) \tag{14}$$

where $\bar{G}_{\mu\nu}$ is the Einstein tensor and $G_{\mu\nu}$ its Cartan version, while $S_{\mu\kappa}{}^{,\tau}$ is the Palatini tensor related to the torsion field $S_{\mu\kappa}{}^{\tau}$ by

$$\tfrac{1}{2}S_{\mu\kappa}{}^{,\tau} \equiv S_{\mu\kappa}{}^{\tau} + \delta_\mu{}^\tau S_{\kappa\lambda}{}^{\lambda} - \delta_\kappa{}^\tau S_{\mu\lambda}{}^{\lambda}. \tag{15}$$

The symbol D_μ denotes the covariant derivative defined by $D_\mu v_\nu \equiv \partial_\mu v_\nu - \Gamma_{\mu\nu}{}^\lambda v_\lambda$, $D_\mu v^\lambda \equiv \partial_\mu v^\lambda + \Gamma_{\mu\nu}{}^\lambda v^\nu$, and $D^*_\mu \equiv D_\mu + 2S_{\mu\kappa}{}^\kappa$. The defect conservation laws (3) read

$$D^*_\mu G\,{}_\lambda{}^\mu + 2S^{\nu\lambda\kappa}G\,{}_{\kappa\nu} - \tfrac{1}{2}S^{\nu\kappa,\mu}R\,{}_{\lambda\mu\nu\kappa} = 0\,, \tag{16}$$

$$D^{*\mu}S_{\lambda\kappa,\mu} = G\,{}_{\lambda\kappa} - G\,{}_{\kappa\lambda}. \tag{17}$$

They are Bianchi identities ensuring the single-valuedness of observables, connection $\Gamma_{\mu\nu}{}^\lambda$ and metric $g_{\mu\nu}$, via the integrability conditions $[\partial_\sigma, \partial_\tau]\Gamma_{\mu\nu}{}^\lambda = 0$ and $[\partial_\sigma, \partial_\tau]g_{\mu\nu} = 0$.

In a four-dimensional Riemann-Cartan spacetime, the geometry is described by the direct generalizations of *translational* and *rotational* defect gauge fields h_{ij} and A_{ijk}, which are here the vierbein field $h^\alpha{}_\mu$, and the spin connection $A_{\mu\alpha}{}^\beta$. The square of the former is the metric $g_{\mu\nu} = h^\alpha{}_\mu h_{\alpha\nu}$. The latter is defined by the covariant derivative $D_\lambda h_\beta{}^\mu = \partial_\lambda h_\beta{}^\mu - A_{\lambda\beta}{}^\gamma h_\gamma{}^\mu + \Gamma_{\lambda\nu}{}^\mu h_\beta{}^\nu \equiv D^L_\lambda h_\beta{}^\mu + \Gamma_{\lambda\nu}{}^\mu h_\beta{}^\nu$. The field strength of $A_{\mu\alpha}{}^\beta \equiv (A_\mu)_\alpha{}^\beta$

$$F_{\mu\nu\beta}{}^{\gamma} \equiv \{\partial_\mu A_\nu - \partial_\nu A_\nu - [A_\mu, A_\nu]\}_\lambda{}^\kappa, \tag{18}$$

determines the Cartan curvature $R_{\mu\nu\lambda}{}^{\kappa} \equiv h^{\beta}{}_{\lambda} h_{\gamma}{}^{\kappa} F_{\mu\nu\beta}{}^{\gamma}$. The field strength of $h^{\gamma}{}_{\nu}$ is the torsion:

$$S_{\alpha\beta}{}^{\gamma} \equiv \tfrac{1}{2} h_{\alpha}{}^{\mu} h_{\beta}{}^{\nu} [D^L_\mu h^{\gamma}{}_{\nu} - (\mu \leftrightarrow \nu)]. \tag{19}$$

The relations (14), (16), and (17) follow from this.

8. The theory is gauge invariant under local Lorentz transformations as a direct consequence of the fact that the metric can alternatively be written as [compare (4)]

$$g_{\mu\nu} = h^{\gamma}{}_{\mu} \Lambda^{a}{}_{\gamma} \Lambda_{a}{}^{\beta} h_{\beta\nu}, \tag{20}$$

where $\Lambda_a{}^\beta$ is an *arbitrary local* Lorentz transformation (single- as well as multi-valued), and that the Hilbert-Einstein Lagrangian $\mathcal{L}_{\rm EH} = -(1/2\kappa)R$ is *independent* of $\Lambda^a{}_\alpha$. The extra $\Lambda_a{}^\beta$ transforms the gauge field $A_{\mu\alpha}{}^\beta$ as

$$A_{\mu\alpha}{}^{\beta} \to A_{\mu\alpha}{}^{\beta} + \Delta A_{\mu\alpha}{}^{\beta}, \quad \Delta A_{\mu\alpha}{}^{\beta} \equiv \Lambda_a{}^{\beta} \partial_\mu \Lambda^a{}_\alpha. \tag{21}$$

At this point are ready to introduce the *new* gauge invariance announced in the title: we allow $\Lambda_a{}^\beta$ in Eq. (20) to be a *multivalued Lorentz transformation*. This is *not* integrable, so that $\Delta A_{\mu\alpha}{}^\beta$ is a *nontrivial* gauge field. Indeed, the rotational field strength $F_{\mu\nu\alpha}{}^\gamma$ can be expressed as $F_{\mu\nu\alpha}{}^\gamma \equiv \Lambda_a{}^\gamma[\partial_\mu, \partial_\nu]\Lambda^a{}_\alpha \neq 0$ and yields a nonzero Cartan curvature $R_{\mu\nu\lambda}{}^\kappa \neq 0$. The important observation is that a *multivalued* $\Lambda^a{}_\alpha$ is able to *change the geometry* [4]. The right-hand side of (14) is *independent* of the vector field $A_{\mu\alpha}{}^\beta$. This allows us to move torsion into Cartan curvature and back, fully or partially, by complete analogy with the defect transformations in two-dimensional crystals in Fig. 21. We can choose for $A_{\mu\alpha}{}^\beta$ any function we like. For example we may choose it to make the torsion vanish, and $A_{\mu\alpha}{}^\beta$ reduces to the usual spin connection of Einstein's gravity, the well-known combination of the objects of anholonomity $\Omega_{\mu\nu}{}^\lambda = \frac{1}{2}[h_\alpha{}^\lambda \partial_\mu h^\alpha{}_\nu - (\mu \leftrightarrow \nu)]$. In the opposite extreme $A_{\mu\alpha}{}^\beta = 0$, the Cartan curvature is zero, spacetime is teleparallel, and the Lagrangian is equal to the combination of torsion tensors: $\mathcal{L}_S = -(1/2\kappa)(-4D_\mu S^\mu + S_{\mu\nu\lambda}S^{\mu\nu\lambda} + 2S_{\mu\nu\lambda}S^{\mu\lambda\nu} - 4S^\mu S_\mu)$, where $S_\mu \equiv S_{\mu\nu}{}^\nu$. In any of the new gauges, the field equations follow from the Hilbert-Einstein Lagrangian

$$\mathcal{L}_{\rm EH} = -(1/2\kappa)R + \mathcal{L}_S, \tag{22}$$

where $A_{\mu\alpha}{}^\beta$ is fixed by any some convenient gauge-fixing functional. It does not follow any field equation.

9. Adding matter fields of masses m to the Einstein Lagrangian, and varying with respect to $h^\alpha{}_\mu$, we find in the zero-torsion gauge the Einstein equation

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \tag{23}$$

where $T_{\mu\nu}$ is the sum over the symmetric energy-momentum tensors of all matter fields. Each contains the canonical energy-momentum tensor ${}^{\rm m}\Theta_{\mu\nu}$ and the spin current densities ${}^{\rm m}\Sigma_{\mu\nu}{}^\lambda$ in the combination due to Belinfante [20] [5],

$${}^{\rm m}T_{\kappa\nu} = {}^{\rm m}\Theta_{\kappa\nu} - \tfrac{1}{2} D^{*\mu} \left({}^{\rm m}\Sigma_{\kappa\nu,\mu} - {}^{\rm m}\Sigma_{\nu\mu,\kappa} + {}^{\rm m}\Sigma_{\mu\kappa,\,\nu} \right), \tag{24}$$

which is the matter analog of the defect relation (14).

[4] The new freedom brought about by multivalued gauge transformations in many areas of physics is explained in the textbook [9]. For instance, we can *derive* the physical laws in a magnetic field from those in field-free space, thus finding the minimal coupling rule of the magnetic vector potential. Similarly, we can *derive* the physical laws in curved space from those in flat space.

[5] For more details see Sect. 17.7 in the textbook [9].

The new gauge invariance of (14) has the physical consequence that the external gravitational field in the far-zone of a celestial body does not care whether angular momentum comes from rotation of matter or from internal spins. The off-diagonal elements of the metric in the far-zone, and thus the Lense-Thirring effect measured in [11], depend only on the total angular momentum $J^{\lambda\mu} = \int d^3x(x^\lambda T^{\mu 0} - x^\mu T^{\lambda 0})$, which by the Belinfante relation (24) is the sum of orbital angular momentum $L^{\lambda\mu} = \int d^3x(x^\lambda \Theta^{\mu 0} - x^\mu \Theta^{\lambda 0})$ and spin $S^{\lambda\mu} = \int d^3x \Sigma^{\lambda\mu,0}$. A star consisting of polarized matter has the same external gravitational field in the far-zone as a star rotating with the corresponding orbital angular momentum. This is the *universality of orbital momentum and intrinsic angular momentum* in gravitational physics observed in Ref. [21].

Since torsion is merely a *new-gauge degree* of freedom in describing a gravitational field, it cannot be detected experimentally, not even by spinning particles. A field with arbitrary spin may be coupled to gravity via the covariant derivative $D_\mu \equiv \partial_\mu \mathbf{1} + \frac{i}{2} A_{\mu\alpha}{}^\beta \Sigma^\alpha{}_\beta$, where $\Sigma^\alpha{}_\beta$ are the generators of the Lorentz group, in the Dirac case $\Sigma_{\alpha\beta} = \frac{i}{4}[\gamma^\alpha, \gamma_\beta]$. But since the torsion is a tensor, we may equally well use an infinity of alternative covariant derivatives $D^q_\mu \equiv \partial_\mu \mathbf{1} + \frac{i}{2} A^q_{\mu\alpha}{}^\beta \Sigma^\alpha{}_\beta$, where $A^q_{\mu\alpha}{}^\beta \equiv A_{\mu\alpha}{}^\beta - qK_{\mu\alpha}{}^\beta$, and $K_{\mu\alpha\beta} = h_\alpha{}^\nu h_\beta{}^\lambda K_{\mu\nu\lambda} \equiv h_\alpha{}^\nu h_\beta{}^\lambda (S_{\mu\nu\lambda} - S_{\nu\lambda\mu} + S_{\lambda\mu\nu})$. Any coupling constant q is permitted by covariance. In order to see which q is physically correct we come back to the above-discussed photon mass problem, and consider the covariant electromagnetic field tensor $F^q_{\mu\nu} \equiv D^q_\mu A_\nu - D^q_\nu A_\mu$. Working out the covariant derivative we find $\partial_\mu A_\nu - \partial_\nu A_\mu - 2(1-q) S_{\mu\nu}{}^\lambda A_\lambda$, which shows that Maxwell Lagrangian $-\frac{1}{4} F^q_{\mu\nu} F^{q\,\mu\nu}$ acquires a mass term, unless we fix the coupling strength to the value $q = 1$.

For this value of q, a little algebra [7, 9] shows that the torsion drops out from the gauge field $A^q_{\mu\alpha}{}^\beta$. This reduces to the good-old Fock-Ivanenko spin connection that has been used in Einstein gravity without torsion:

$$A^1_{\mu\alpha}{}^\beta = \bar{A}_{\mu\alpha}{}^\beta = h_\alpha{}^\nu h^{\beta\lambda}(\Omega_{\mu\nu\lambda} - \Omega_{\nu\lambda\mu} + \Omega_{\lambda\mu\nu}). \tag{25}$$

Having ensured that the photon does not couple to torsion, this choice also prevents all all other spinning baryonic matter to do so, thus avoiding that a photon mass arises via virtual processes.

10. In summary, we have shown that if the Hilbert-Einstein Lagrangian is expressed in terms of the translational and rotational gauge fields $h^\alpha{}_\mu$ and $A^1_{\mu\alpha}{}^\beta$, the Cartan curvature can be converted to torsion and back, totally or partially, by a *new type of multivalued gauge transformation* in Riemann-Cartan spacetime. In this general formulation, Einstein's original theory is obtained by going to the zero-torsion gauge, while his teleparallel theory is in the gauge in which the Cartan curvature tensor vanishes. But any intermediate choice of the field $A_{\mu\alpha}{}^\beta$ is allowed.

Higher gradient terms in the elastic energy of the world crystal are capable of generating an action of the gauge field [6] $A_{\mu\alpha}{}^\beta$. These would break the above gauge symmetry and give the gauge field a life of its own. The coupling to spin, however, must go only via the torsion-free soin connection $A^1_{\mu\alpha}{}^\beta$ to avoid a photon mass.

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