Description of processes passing at finite space-time intervals in the framework of quantum field theory

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Abstract. We consider a novel quantum field-theoretical approach to the description of processes passing at finite space-time intervals based on the Feynman diagram technique in the coordinate representation. The most know phenomena of this type are neutrino and neutral kaon oscillations. The experimental setting of these processes demands that the rules of passing to the momentum representation in the Feynman diagram technique be adjusted in accordance with it, which leads to a modification of the Feynman propagator in the momentum representation. The approach does not make use of wave packets: both initial and final particle states are described by plane waves, which simplifies the calculations considerably. We demonstrate the validity of the formalism by applying it to describing the decay of an unstable particle and neutrino oscillations. It is shown that the considered approach correctly reproduces the known results.

1. Introduction

The Standard Model allows one to describe a great amount of different elementary particle interaction processes with a high accuracy in the framework of perturbative S-matrix formalism and Feynman diagram technique. However there is a number of phenomena which cannot be described in the framework of the standard perturbation theory. In particular, such phenomena are oscillations of the strange neutral mesons and neutrino oscillations, which take place at macroscopic space and time intervals. These processes are described either in the quantum mechanical approach in terms of plane waves [1, 2, 3, 4, 5] or in the QM or QFT approaches in terms of wave packets [6, 7, 8]. The first one is inconsistent, since the production of states without definite mass violates the energy-momentum conservation. The descriptions in terms of wave packets circumvent this problem, but the corresponding calculations are rather complicated. The standard S-matrix perturbation theory is not convenient for describing processes passing at finite distances and finite time intervals.

The idea of the novel approach is to adjust the standard S-matrix formalism for the description of such processes. We consider the processes of production and detection as a whole, use the Feynman diagram technique in the coordinate representation to write down the amplitude and then pass to the momentum representation in a way, which corresponds to the experimental setting. Effectively it leads to a modification of the Feynman propagator in the momentum representation, while all the other Feynman rules in the momentum representation are kept intact. The approach is based on the paper by R. Feynman [9] and was developed in the papers

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[10, 11, 12]. In the present paper, in the framework of the proposed approach, the processes of the decay of an unstable particle and neutrino oscillations are considered.

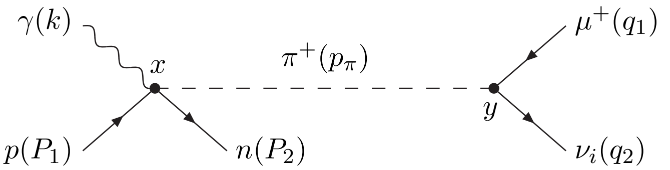
2. Decay of an unstable particle

We work in the framework of the minimal extension of the Standard Model by the right neutrino singlets. The charged-current interaction Lagrangian of the leptons takes the form

$$L_{cc} = -\frac{g}{2\sqrt{2}} \left(\sum_{i,k=1}^{3} \bar{l}_i \gamma^\mu (1-\gamma^5) U_{ik} \nu_k W_\mu^- + h.c. \right), \tag{1}$$

where l_i is the field of the charged lepton of the i-th generation, U_{ik} denotes the PMNS-matrix, and ν_k stands for the field of the neutrino state with definite mass.

Let us take the positively charged pion as an example of an unstable particle and consider the process of pion photoproduction with its subsequent decay to a lepton pair. The process corresponds to the following diagram:



(2)

The four-momenta of the particles are designated as it is shown in the diagram. The pion is a virtual particle and is described by a propagator in the coordinate representation. The points of production x and decay y are supposed to be separated by a fixed macroscopic interval. In this case the virtual particle is almost on the mass shell, i.e. the relation $\left|p^2 - m_i^2\right| \big/ \vec{p}^2 \ll 1$ holds [7].

Following the prescription formulated in [10, 11, 12], we construct the so-called time-dependent propagator of an unstable scalar particle in the momentum representation as the Fourier transform of the exact renormalized propagator $D_e(x)$ of the scalar field in the coordinate representation, multiplied by the additional delta function, which fixes the time interval T between the events:

$$D(p,T) = \int d^4x e^{ipx} D_e(x)\, \delta\left(x^0 - T\right) \approx \frac{i}{2p^0} e^{-i\frac{m^2 - p^2 - im\Gamma}{2p^0}T}. \tag{3}$$

Here m is the physical mass of the pion, Γ is its width, and we have used the condition that the particle is close the mass shell.

Now we can use this expression to write out the amplitude of the process in the momentum representation for the case, where the time interval between the points x and y equals T:

$$M^{(i)} = i\frac{G_F}{2\sqrt{2}p_\pi^0} \cos\theta_c f_\pi m_{(\mu)} U_{2i}^* e^{-i\frac{m_\pi^2 - p_\pi^2 - im_\pi\Gamma}{2p_\pi^0}T} \bar{\nu}_i(q_2) \left(1 - \gamma^5\right) v(q_1)\, M_P(k, P_1, p_\pi, P_2). \tag{4}$$

Here θ_c is the Cabibbo angle, f_π is the pion decay constant of the dimension massof , m_π is the pion mass, Γ is its width, $m_{(\mu)}$ is the muon mass; $M_P(k, P_1, p_\pi, P_2)$ stands for the amplitude of the π-meson photoproduction, whose explicit form is unimportant for us.

Next we find the squared modulus of the amplitude, averaged with respect to the polarizations of the initial particles and summed over the polarizations of the final particles. Due to the simple structure of the scalar propagator, the squared amplitude factorizes as follows:

$$\left\langle \left|M^{(i)}\right|^2 \right\rangle = \left\langle |M_1|^2 \right\rangle \left\langle \left|M_2^{(i)}\right|^2 \right\rangle \frac{1}{4\left(p_\pi^0\right)^2} e^{-\frac{m_\pi\Gamma}{p_\pi^0}T}. \tag{5}$$

We have only one term in the amplitude, thus there is no interference.

Now we are in a position to find the differential probability of the process. Let us introduce a 4-momentum p, which satisfies $p^2 = m_\pi^2$ and the vector $\vec{p}$ is directed from the source to the detector and defined by the energy-momentum conservation in the production vertex. According to the prescription of [10, 11, 12] we multiply the squared modulus of the amplitude (5) by the delta-function of energy-momentum conservation $(2\pi)^4\delta(k + P_1 - P_2 - q_1 - q_2)$, substitute p instead of p_π everywhere in the ampltude, multiply the result by $2\pi\delta(k + P_1 - P_2 - p)$ and integrate it with respect to the phase volume of the final particles. Besides this, when the intermediate virtual particle momentum is fixed, one can pass from the time interval T to the distance L traveled by the pion according to the formula $T = Lp^0/|\vec{p}|$. Summing over the final neutrino type i, which is unobservable, we arrive at the differential probability:

$$\frac{dW}{d\vec{p}} = \frac{dW_1}{d\vec{p}} W_2 e^{-\frac{m_\pi \Gamma}{|\vec{p}|}L}. \tag{6}$$

Thus, we find that the differential probability of the whole process is the product of the differential probability $\frac{dW_1}{d\vec{p}}$ of the production of a π-meson with a definite momentum, the probability W_2 of its decay and the standard exponential factor, describing the probability of the pion reaching the detector.

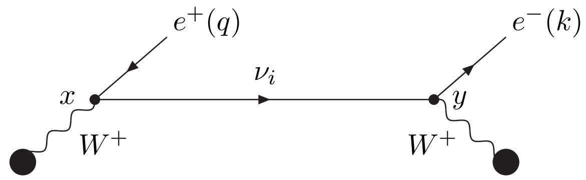
Finally we observe that the experimental situation fixes only the direction of the intermediate pion momentum, but not its length. It means that one has to integrate the probability (6) with respect to $|\vec{p}|$, which gives us the probability of the whole process as follows:

$$\frac{dW}{d\Omega} = \int \frac{dW}{d\vec{p}} |\vec{p}|^2 \, d|\vec{p}| = \frac{dW_1}{d\Omega} \, W_2|_{|\vec{p}|=|\vec{p}|^*} \, e^{-\frac{m_\pi \Gamma}{|\vec{p}|^*}L}, \tag{7}$$

where $|\vec{p}|^*$ is the root of the equation $k^0 + P_1^0 - P_2^0|_{\vec{P}_2=\vec{k}+\vec{P}_1-\vec{p}} - p^0 = 0$ with respect to $|\vec{p}|$.

3. Neutrino oscillations

Let us consider a process, where a neutrino is emitted and detected in the charged-current interaction with nuclei:



(8)

The filled circles represent weak hadron currents $j_\mu^{(1,2)}$.

As in the previous case, we introduce the extra delta-function into the integral and construct the time-dependent propagator of i-th neutrino mass eigenstate in the momentum representation. When the neutrino is almost on the mass shell, the result takes the form:

$$S_i^c(p, T) = \int d^4x e^{ipx} S_i^c(x) \, \delta\left(x^0 - T\right) \approx i\frac{\hat{p} + m_i}{2p^0} e^{-i\frac{m_i^2 - p^2}{2p^0}T}. \tag{9}$$

Due to the smallness of the neutrino masses we can neglect them everywhere except in the exponential. The amplitude of the process in the momentum representation for the case $y^0 - x^0 = T$ reads:

$$M = -i\frac{G_F^2}{4p_n^0} \sum_{i=1}^{3} |U_{1i}|^2 \, e^{-i\frac{m_i^2 - p_n^2}{2p_n^0}T} j_\rho^{(2)} \bar{u}(k) \gamma^\rho \left(1 - \gamma^5\right) \hat{p}_n \gamma^\mu \left(1 - \gamma^5\right) v(q) \, j_\mu^{(1)}. \tag{10}$$

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In our approximation, the squared modulus of the amplitude, averaged and summed over particles' polarizations, factorizes:

$$\left\langle |M|^2 \right\rangle = \left\langle |M_1|^2 \right\rangle \left\langle |M_2|^2 \right\rangle \frac{1}{4\left(p_n^0\right)^2} \left[1 - 4 \sum_{\substack{i,k=1 \\ i<k}}^{3} |U_{1i}|^2 |U_{1k}|^2 \sin^2 \left(\frac{m_i^2 - m_k^2}{4p_n^0} T \right) \right]. \tag{11}$$

Performing calculations, which are analogous to those in the case of the pion, we obtain the differential probability:

$$\frac{dW}{d\vec{p}} = \frac{dW_1}{d\vec{p}} W_2 P_{ee}\left(L\right). \tag{12}$$

Here we introduced a special notation

$$P_{ee}\left(L\right) = \left[1 - 4 \sum_{\substack{i,k=1 \\ i<k}}^{3} |U_{1i}|^2 |U_{1k}|^2 \sin^2 \left(\frac{m_i^2 - m_k^2}{4\,|\vec{p}|} L \right) \right] \tag{13}$$

for the expression, which, in the standard approach, is called the distance-dependent electron neutrino survival probability.

Integrating expression (12) with respect to $|\vec{p}|$, one arrives at the total probability of detecting an electron in the process under consideration:

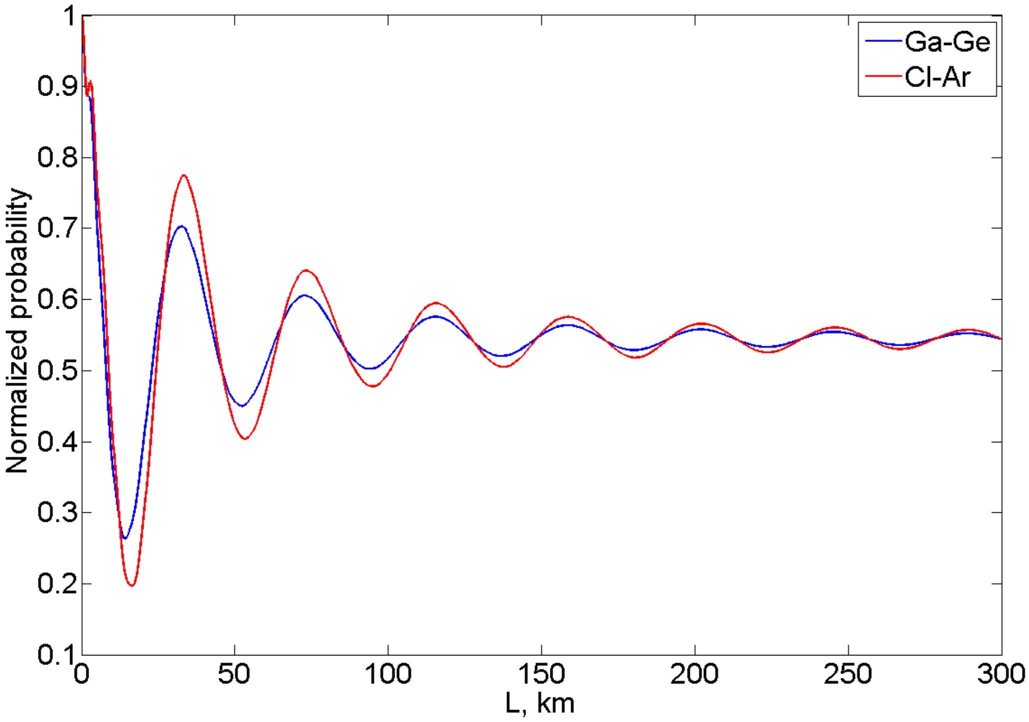
$$\frac{dW}{d\Omega} = \int\limits_{|\vec{p}|_{\min}}^{|\vec{p}|_{\max}} \frac{dW}{d\vec{p}} \, |\vec{p}|^2 \, d\,|\vec{p}| = \int\limits_{|\vec{p}|_{\min}}^{|\vec{p}|_{\max}} \frac{dW_1}{d\vec{p}} W_2 P_{ee}\left(L\right) |\vec{p}|^2 \, d\,|\vec{p}|, \tag{14}$$

where $\frac{dW_1}{d\vec{p}}$ is the differential probability of the production of a neutrino with a fixed momentum and W_2 is the neutrino interaction probability in the detector. The lower limit of integration is determined by the registration process, and the upper one by the production process.

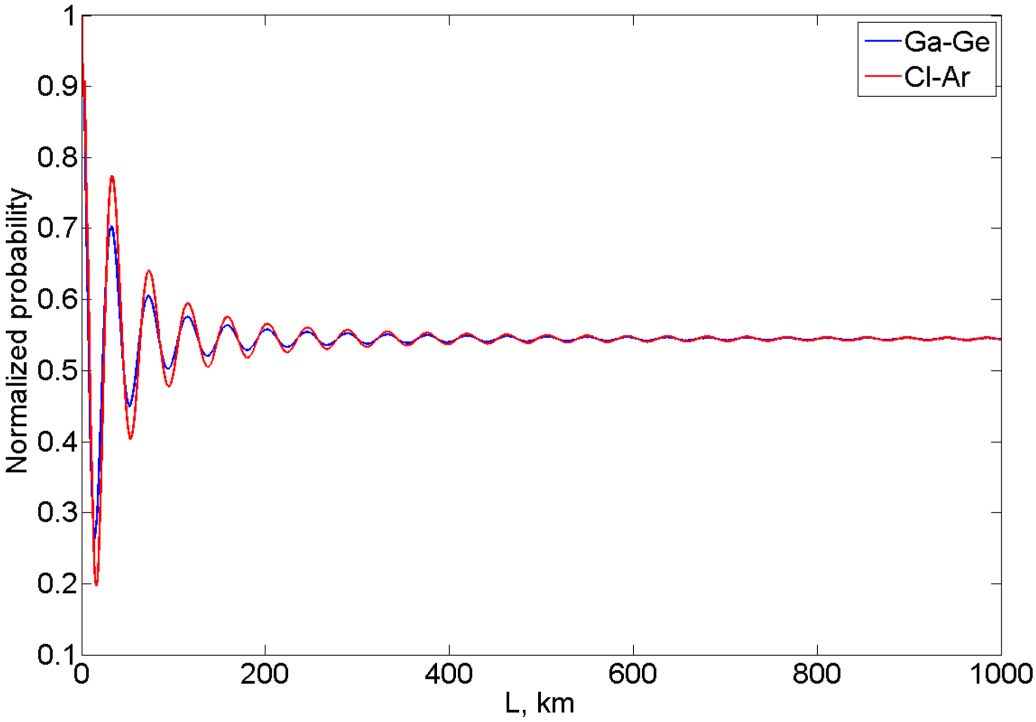
Let us take as an example the production reaction of the solar carbon cycle $^{15}\mathrm{O} \to {}^{15}\mathrm{N} + e^+ + \nu_i$ and the detection by the chlorine-argon and gallium-germanium detectors. If one neglects the nuclear form-factors and considers only the ground state of the final nucleus, the integrand without the oscillating factor can be approximated by the function:

$$\begin{aligned} \frac{dW_1}{d\vec{p}} W_2 \, |\vec{p}|^2 \quad &= \quad C\sqrt{\left(|\vec{p}|_{\max} - |\vec{p}|\right)\left(|\vec{p}|_{\max} - |\vec{p}| + 2m\right)} \left(|\vec{p}|_{\max} - |\vec{p}| + m\right) |\vec{p}|^2 \times \\ &\times \sqrt{\left(|\vec{p}| - |\vec{p}|_{\min}\right)\left(|\vec{p}| - |\vec{p}|_{\min} + 2m\right)} \left(|\vec{p}| - |\vec{p}|_{\min} + m\right). \end{aligned} \tag{15}$$

The results of numerical integration with the parameters $m_2^2 - m_1^2 = 10^{-4}$ eV^2, $m_3^2 - m_1^2 = m_3^2 - m_2^2 = 10^{-3}$ eV^2, $\theta_{12} = 0.59$, $\theta_{13} = 0.16$, $\theta_{23} = 0.70$, $|\vec{p}|_{\min}^{\mathrm{Ga-Ge}} = 232$ keV, $|\vec{p}|_{\min}^{\mathrm{Cl-Ar}} = 814$ keV, $|\vec{p}|_{\max} = 1732$ keV are presented in figure 1 (the probability is normalized to its value at the point $L = 0$). Since the intermediate neutrinos have a momentum distribution, the oscillations fade out with distance. This fact gives rise to the coherence length. If, similar to the rule adopted in optics, we take the condition of oscillations' visibility to be $(I_{\max} - I_{\min})/(I_{\max} + I_{\min}) > 0.1$, where $I_{\max\,/\,\min}$ denotes the neutrino registration probability in the adjacent maximum and minimum of the oscillation pattern, we arrive at the coherence lengths $L_{\mathrm{coh}}^{\mathrm{Cl-Ar}} \approx 103$ km, $L_{\mathrm{coh}}^{\mathrm{Ga-Ge}} \approx 71$ km.



a) Distance L from 0 to 300 km.



b) Distance L from 0 to 1000 km.

Figure 1. Normalized probability of the process depending on the distance.

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The standard quantum-mechanical treatment in terms of wave packets gives [5]:

$$L_{kj}^{\text{coh}} = \frac{4\sqrt{2}\,|\vec{p}|^2}{\left|m_k^2 - m_j^2\right|}\sigma_x \propto 10 \div 100 \text{ km}, \tag{16}$$

where σ_x is the half-width at half-height of the neutrino wave packet in the coordinate representation, which is inverse to the half-width at half-height of the neutrino momentum length distribution. This expression is in agreement with the result above. But unlike the standard approach, where there is a coherence lengths for each pair of neutrino mass eigenstates, here we have only one length. It is specific for the whole experiment and cannot be decomposed into coherence lengths for pairs of neutrino mass eigenstates.

4. Conclusion

A novel quantum field-theoretical approach to the description of processes passing at finite space-time intervals is discussed. It is based on the Feynman diagram technique in the coordinate representation supplemented by modified rules of passing to the momentum representation, which reflect the experimental situation at hand. Wave packets are not employed, we use only the description in terms of plane waves, which considerably simplifies the calculations. The neutrino flavour states turn out to be unnecessary and only the mass eigenstates are used.

We have explicitly shown that the approach allows to consistently describe such processes as decay of an unstable particle and neutrino oscillations. The predictions for the processes are found to completely coincide with the results given by the standard formalism. It is shown that the neutrino oscillation pattern depends on the detection process, and neutrino oscillations fade out with distance due to the presence of neutrino momentum distribution. The coherence lengths for two specific examples are found based on the analogy with interference in optics with the help of the visibility function.

The advantages of the approach are technical simplicity and physical transparency.

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