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$\Lambda_b \to \Lambda \ell^+ \ell^-$ Transition in Universal Extra Dimension Model

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Abstract. - The flavor changing neutral current transition of $\Lambda_b \to \Lambda \ell^+ \ell^-$ is investigated in universal extra dimension approach with a single extra dimension using the related form factors calculated by QCD sum rules both in full theory, which recently are available as well as heavy quark effective theory. The dependency of the Wilson coefficients on the compactification scale $1/R$ up to 1000 GeV is presented and compared with results obtained in the SM. The total decay rate and branching ratio of this transition is also calculated both in universal extra dimension model and the SM. Our results show that when $1/R$ is increased, the results of universal extra dimension become close to the SM predictions. However, in low $1/R$'s, there is a discrepancy between two predictions. This can be considered as a signal for existing extra dimensions.

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1. Introduction

The Standard Model (SM) is in perfect consistency with all confirmed collider data. However, there are some problems which can not be explained by the SM. Some of these problems are: origin of the matter in the universe, gauge and fermion mass hierarchy, number of generations, matter-antimatter asymmetry, unification, and so on. Hence, we need to have a more fundamental theory beyond the SM.

Extra dimension (ED) model [1] with small compactification radius is one of the most reasonable candidates to have a solution for aforesaid problems. A part of ED which allows the SM fields (both gauge bosons and fermions) to propagate in the extra dimensions is universal extra dimension (UED) approach. An example of the UED model also, where just a single universal extra dimension compactified on a circle of radius R is considered is so called the Appelquist, Cheng and Dobrescu (ACD) model [2]. The radius R is the extra parameter that causes the difference between SM and its beyond. The particles propagating in extra dimension are called Kaluza-Klein (KK) particles.

The aim of the paper is to find the effects of the KK modes on branching ratio related to the $\Lambda_b \to \Lambda \ell^+ \ell^-$ transition. We will use the form factors obtained both using QCD sum rules in full theory, which they recently are available [3] and also those obtained in heavy quark effective theory (HQET) [4]. Using the values of the form factors, we present the sensitivity of the branching ratio to the compactification parameter, $1/R$. The low energy experiments show that the lower bound on $1/R$ is 400 GeV. So we discuss the dependence of physical observables in the interval 400 $\leq$1/R$\leq$ 1000.

The paper is organized as follows: In section 2, we introduce the responsible Hamiltonian for the considered transition and present the values of the Wilson coefficient obtained at different $1/R$ and also the SM. In this section, we also introduce the form factors obtained in full QCD and HQET. Section 3 is devoted to the numerical analysis of the branching ratio and our discussion. Note that the complete version of this work has been published in [5].

2. Theoretical Framework

2.1 Effective Hamiltonian Responsible for $\Lambda_b \to \Lambda l^+ l^-$ Transition

At quark level, this transition is proceed via the flavor changing neutral current(FCNC) transition of $b \to s \ell^+ \ell^-$ via electroweak penguin and weak box diagrams see Fig. 1.

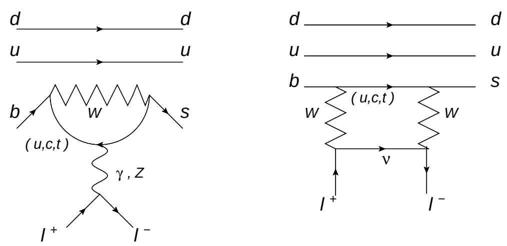

Figure 1: Penguin and box diagram responsible for the $\Lambda_b \to \Lambda \ell^+ \ell^-$ transition

The effective hamiltonian corresponding to these diagrams can be written as:

$$\begin{aligned} \mathcal{H}_{eff} &= \frac{G_F}{4\sqrt{2}} V_{tb} V_{ts}^* \Big\{ C_7^{eff} \bar{s} i \sigma_{\mu\nu} (1+\gamma_5) q^\nu b \bar{\ell} \gamma^\mu \ell + C_9^{eff} \bar{s} \gamma_\mu (1-\gamma_5) b \bar{\ell} \gamma^\mu \ell \\ &+ C_{10} \bar{s} \gamma_\mu (1-\gamma_5) b \bar{\ell} \gamma^\mu \gamma_5 \ell \Big\} \ . \end{aligned} \tag{1}$$

$1/R$ [GeV]	C_7^{eff}	C_{10}	$C_9^{eff}(0.4)$
400	-0.266419	-4.65118	$4.62718 + 0.298476i$
500	-0.277351	-4.51581	$4.61719 + 0.298476i$
600	-0.283593	-4.43995	$4.61159 + 0.298476i$
700	-0.287468	-4.39337	$4.60815 + 0.298476i$
800	-0.29003	-4.36279	$4.60589 + 0.298476i$
900	-0.291808	-4.34166	$4.60433 + 0.298476i$
1000	-0.293092	-4.32646	$4.60321 + 0.298476i$
SM	-0.298672	-4.26087	$4.59837 + 0.298476i$

Table 1: Values of C_7^{eff}, C_{10}, and $C_9^{eff}(0.4)$ in both UED and SM.

where C_7^{eff}, C_{10}, and C_9^{eff} are Wilson coefficients. The explicit expressions of these coefficients calculated both in ACD and SM models are given in [8, 6, 7, 9]. Using the expressions for Wilson coefficients, we obtain the numerical values presented in Table 1. The C_9^{eff} depends on $s = \frac{q^2}{m^2_{\Lambda_b}}$ but the others have no such dependency. In Table 1, as an example, we only present the C_9^{eff} values at $s = 0.4$, which lies in the physical region.

2.2 Transition Matrix Elements

To calculate the amplitude for $\Lambda_b \to \Lambda l^+ l^-$ transition, we need to sandwich the effective hamiltonian between the initial and final baryonic states. To proceed, we should define the following matrix elements in terms of form factors.

$$\begin{aligned} &\langle \Lambda \,|\bar{s}\gamma_\mu(1-\gamma_5)b|\, \Lambda_b\rangle \ , \\ &\langle \Lambda \,|\bar{s}\sigma_{\mu\nu}(1+\gamma_5)q^\nu b|\, \Lambda_b\rangle \ . \end{aligned} \tag{2}$$

In full QCD, these transition matrix elements are parameterized in terms of twelve form factors, f_i, g_i, f_i^T and g_i^T with $i = 1 \to 3$, as [3];

$$\begin{aligned} \langle \Lambda \,|\bar{s}\gamma_\mu b|\, \Lambda_b\rangle &= \bar{u}_\Lambda\Big[f_1\gamma_\mu + if_2\sigma_{\mu\nu}q^\nu + f_3 q_\mu\Big]u_{\Lambda_b} \ , \\ \langle \Lambda \,|\bar{s}\gamma_\mu\gamma_5 b|\, \Lambda_b\rangle &= \bar{u}_\Lambda\Big[g_1\gamma_\mu\gamma_5 + ig_2\sigma_{\mu\nu}\gamma_5 q^\nu + g_3 q_\mu\gamma_5\Big]u_{\Lambda_b} \ , \\ \langle \Lambda \,|\bar{s}i\sigma_{\mu\nu}q^\nu b|\, \Lambda_b\rangle &= \bar{u}_\Lambda\Big[f_1^T\gamma_\mu + if_2^T\sigma_{\mu\nu}q^\nu + f_3^T q_\mu\Big]u_{\Lambda_b} \ , \\ \langle \Lambda \,|\bar{s}i\sigma_{\mu\nu}\gamma_5 q^\nu b|\, \Lambda_b\rangle &= \bar{u}_\Lambda\Big[g_1^T\gamma_\mu\gamma_5 + ig_2^T\sigma_{\mu\nu}\gamma_5 q^\nu + g_3^T q_\mu\gamma_5\Big]u_{\Lambda_b} \ . \end{aligned} \tag{3}$$

where $q = p_{\Lambda_b} - p_\Lambda$ and u_{Λ_b} and u_Λ are spinors of the Λ_b and Λ baryons, respectively.

In HQET, twelve form factors are reduced to two, namely F_1 and F_2 [10, 11], i.e.,

$$\langle \Lambda(p) \mid \bar{s}\Gamma b \mid \Lambda_b(p+q)\rangle = \bar{u}_\Lambda(p)[F_1(Q^2) + \not{v}F_2(Q^2)]\Gamma u_{\Lambda_b}(p+q), \tag{4}$$

where Γ refers to any Dirac matrices and $\not{v} = (\not{p} + \not{q})/m_{\Lambda_b}$. Comparing the matrix elements in full theory and $HQET$, we immediately obtain the following relations among the form factors at HQET (see also [12])

$$\begin{aligned}
f_1 &= g_1 = f_2^T = g_2^T = F_1 + \frac{m_\Lambda}{m_{\Lambda_b}} F_2 \ , \\
f_2 &= g_2 = f_3 = g_3 = \frac{F_2}{m_{\Lambda_b}} \ , \\
f_1^T &= g_1^T = \frac{F_2}{m_{\Lambda_b}} q^2 \ , \\
f_3^T &= -\frac{F_2}{m_{\Lambda_b}} (m_{\Lambda_b} - m_\Lambda) \ , \\
g_3^T &= \frac{F_2}{m_{\Lambda_b}} (m_{\Lambda_b} + m_\Lambda) \ .
\end{aligned} \tag{5}$$

Form Factors in Full QCD

In Full theory, all twelve form factors have been calculated and in the context of QCD sum rules in [3]. The form factors $f_1,\ f_2,\ f_3,\ g_1,\ g_2,\ g_3,\ f_2^T,\ f_3^T,\ g_2^T$ and g_3^T can be extrapolated in terms of q^2 as [3]:

$$f_i(q^2)[g_i(q^2)] = \frac{a}{\left(1 - \dfrac{q^2}{m_{fit}^2}\right)} + \frac{b}{\left(1 - \dfrac{q^2}{m_{fit}^2}\right)^2} \ , \tag{6}$$

where the fit parameters $a,\ b$ and m_{fit}^2 in full theory are given in Table 2.

On the other hand, f_1^T and g_1^T is well extrapolate to [3]:

$$f_1^T(q^2)[g_1^T(q^2)] = \frac{c}{\left(1 - \dfrac{q^2}{m_{fit}^{'2}}\right)} - \frac{c}{\left(1 - \dfrac{q^2}{m_{fit}^{''2}}\right)^2} \ , \tag{7}$$

where, the values for the parameters c, $m_{fit}^{'2}$ and $m_{fit}^{''2}$ are presented in Table 3.

Form Factors in Heavy Quark Effective Theory(HQET)

The form factors, F_1 and F_2 are calculated in Table [4]. The parametrization of these form factors is given as: [12, 13]:

$$F(s) = \frac{F(0)}{1 - a_F s + b_F s^2} \ .$$

where, the values of the parameters $F(0),\ a_F$ and b_F are given in Table 4

	QCD sum rules		
	a	b	m^2_{fit}
f_1	-0.046	0.368	39.10
f_2	0.0046	-0.017	26.37
f_3	0.006	-0.021	22.99
g_1	-0.220	0.538	48.70
g_2	0.005	-0.018	26.93
g_3	0.035	-0.050	24.26
f_2^T	-0.131	0.426	45.70
f_3^T	-0.046	0.102	28.31
g_2^T	-0.369	0.664	59.37
g_3^T	-0.026	-0.075	23.73

Table 2: Parameters appearing in the fit function of the form factors, f_1, f_2, f_3, g_1, g_2, g_3, f_2^T, f_3^T, g_2^T and g_3^T in full theory for $\Lambda_b \to \Lambda \ell^+ \ell^-$. In this Table only central values of the parameters are presented.

3. Branching Ratio

Using the amplitude and definition of the transition matrix elements in terms of form factors, the differential decay rate responsible for the considered transition is obtained as:

$$\frac{d\Gamma}{ds} = \frac{G^2 \alpha_{em}^2 m_{\Lambda_b}}{8192\pi^5} |V_{tb} V_{ts}^*|^2 v \sqrt{\lambda} \left[\mathcal{T}_0(s) + \frac{1}{3} \mathcal{T}_2(s) \right] , \tag{8}$$

where the explicit expressions for the functions $\mathcal{T}_0(s)$ and $\mathcal{T}_2(s)$ can be found in [3]. Here, $s = q^2/m^2_{\Lambda_b}$, $\lambda = \lambda(1, r, s) = 1 + r^2 + s^2 - 2r - 2s - 2rs$, $r = m^2_\Lambda/m^2_{\Lambda_b}$, and $v = \sqrt{1 - \frac{4m_\ell^2}{q^2}}$. Integrating the differential decay rate over s in the region $4m_\ell^2 \leq q^2 \leq (m_{\Lambda_b} - m_\Lambda)^2$, we obtain the total decay rate. Using the lifetime of the Λ_b baryon, $\tau_{\Lambda_b} = 1.383 \times 10^{-12}\ s$ [14], and the values, $m_\Lambda = 1.1156\ GeV$, $m_{\Lambda_b} = 5.620\ MeV$, $m_b = 4.8 GeV$, $m_\tau = 1.77\ GeV$, $m_\mu = 0.106\ GeV$ [15], $m_t = 167\ GeV$, $m_c = 1.46$ [16], $\mid V_{tb} V_{ts}^* \mid= 0.041$ and $G_F = 1.17 \times 10^{-5}$ GeV^{-2} [14], we can plot the branching ratios in terms of compactification factor, $1/R$. The branching ratio of $\Lambda_b \to \Lambda l^+ l^-$ for different leptons are shown in Figs **??**, **??**, and **??** both in full theory and HQET. From these figures, we see that the order of branching ratio reduces with increasing

	QCD sum rules		
	c	$m_{fit}^{'2}$	$m_{fit}^{''2}$
f_1^T	-1.191	23.81	59.96
g_1^T	-0.653	24.15	48.52

Table 3: Parameters appearing in the fit function of the form factors f_1^T and g_1^T in full theory for $\Lambda_b \to \Lambda \ell^+ \ell^-$.

	$F(0)$	a_F	b_F
F_1	0.462	-0.0182	-0.000176
F_2	-0.077	-0.0685	0.00146

Table 4: Form factors for $\Lambda_b \to \Lambda \ell^+ \ell^-$ decay in a three parameter fit.

the mass of leptons which is in a consistency with our expectations. In low $1/R$, we we see overall a considerable discrepancy between the predictions of the ACD and SM models both in full and HQET theories. This can be considered as a sign of existing the extra dimensions. However, when $1/R$ approaches to 1000 GeV, the predictions of the ACD and SM models become very close to each other. These figures, also depict that the $\Lambda_b \to \Lambda l^+ l^-$ transition is more probable in full theory in comparison with HQET. The order of branching ratios show a possibility to study this channel at LHCb.

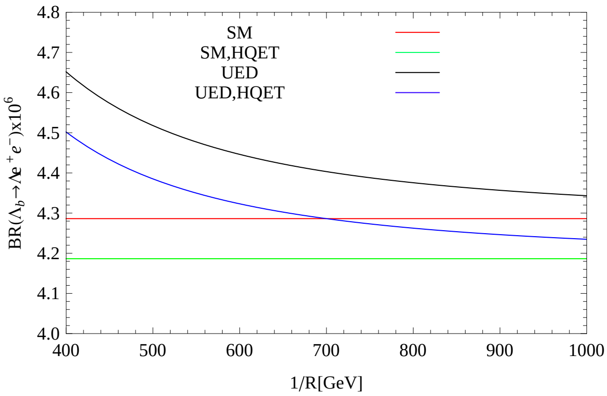

Figure 1: Dependency of the branching ratio on $1/R$ for electron

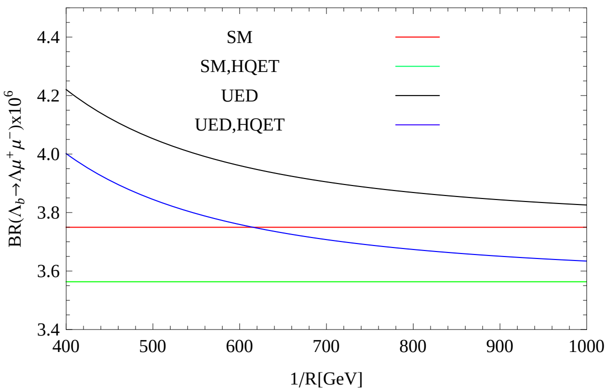

Figure 2: Dependency of the branching ratio on $1/R$ for muon

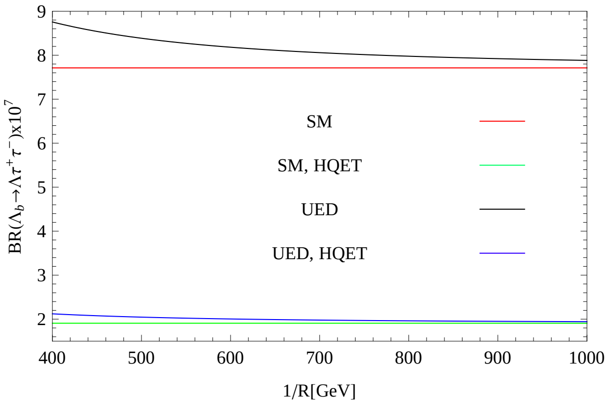

Figure 3: Dependency of the branching ratio on $1/R$ for tau

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