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Accidental composite dark matter

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Abstract: We build models where Dark Matter candidates arise as composite states of a new confining gauge force, stable thanks to accidental symmetries. Restricting to renormalizable theories compatible with SU(5) unification, we find 13 models based on SU(N) gauge theories and 9 based on SO(N). We also describe other models that require non-renormalizable interactions. The two gauge groups lead to distinctive phenomenologies: SU(N) theories give complex DM, with potentially observable electric and magnetic dipole moments that lead to peculiar spin-independent cross sections; SO(N) theories give real DM, with challenging spin-dependent cross sections or inelastic scatterings. Models with Yukawa couplings also give rise to spin-independent direct detection mediated by the Higgs boson and to electric dipole moments for the electron. In some models DM has higher spin. Each model predicts a specific set of lighter composite scalars, possibly observable at colliders.

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Contents

1 Introduction

A striking success of the Standard Model is that all observed global symmetries are understood as accidental symmetries of the renormalizable Lagrangian. This explains in particular the stability of the proton as a consequence of baryon number conservation.

In nature, besides the proton, at least another particle should be stable to provide the necessary Dark Matter (DM) abundance required by cosmological observations. It is natural to imagine that dark matter too is stable because of accidental symmetries. This idea can be minimally realized by adding to the SM one extra multiplet that cannot have any Yukawa interaction with SM particles, and that contains a DM candidate ([1, 2]; a different proposal to get accidentally stable DM was presented by [3]).

The fact that bounds from DM searches require a successful weak-scale DM candidate to have no electric charge, no color, and almost no coupling to the Z (the vectorial coupling to the Z must be a few orders of magnitude smaller than a typical weak coupling) calls for an explanation. A simple way of explaining why DM is so dark and stable is to add to the SM (with its elementary Higgs) new fermions Ψ charged under a new technicolor interaction that confines at a scale $\Lambda_{\rm TC}$. Techni-quarks are assumed to lie in (possibly reducible) real representations under the SM gauge group, such that their condensates do not break the electro-weak symmetry, realising the framework dubbed 'vector-like confinement' in [4]. The renormalizable Lagrangian of the theory is

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \bar\Psi_i (i\not{D} - m_i)\Psi_i - \frac{\mathcal{G}^{A2}_{\mu\nu}}{4g^2_{\rm TC}} + \frac{\theta_{\rm TC}}{32\pi^2}\mathcal{G}^A_{\mu\nu}\tilde{\mathcal{G}}^A_{\mu\nu} + [H\bar\Psi_i(y^L_{ij}P_L + y^R_{ij}P_R)\Psi_j + \text{h.c.}] \quad (1.1)$$

where the latter term, Yukawa interactions with the Higgs doublet H, can be allowed by quantum numbers. The topological term for technicolor gauge fields is physical for non-vanishing techni-quark masses m_i.

We assume that when technicolor interactions confine at a scale $\Lambda_{\rm TC}$, the approximate global techni-flavor symmetry is broken by condensates producing light techni-pions (TCπ) and other heavier composite particles, such as techni-baryons (TCb). All these particles are splitted in mass by SM gauge interactions in such a way that the lightest stable techni-particle (charged under an accidental symmetry that keeps it stable) tends to be the 'most neutral' one.

Composite Dark Matter has been rarely considered in the literature, and mostly in models with different goals, e.g. with supersymmetry [5], with composite [6–9] or partially composite Higgs [10–12], with a mirror-SM sector [13] or quirks [14] or a fourth generation [15] as well as from a phenomenological point of view, in order to realise special situations (such as inelastic DM, asymmetric DM, strongly interacting DM, magnetic DM, etc.) often motivated by anomalies [16–20]. An approach similar to the present study was considered in [21, 22, 24]. In [21, 24] bosonic techni-baryon DM in SU(4) gauge theories was studied. In [22] we began a general study of composite DM adopting a specific point of view with respect to the naturalness problem, according to which the Lagrangian does not contain any massive parameter, power divergences are unphysical, all masses arise via dimensional transmutation. The resulting assumption $m_i = 0$ lead to very predictive models [22]. Allowing for techni-quark masses (if lighter than about 1 TeV, they do not induce unnaturally large corrections to the Higgs mass [23]) and for an order one $\theta_{\rm TC}$ modifies the mass spectrum of the theory, inducing electric dipole moments (EDMs) for TCb that leads to a sizeable Dark Matter direct detection signal with characteristic dependence on velocity and transferred momentum.

The issue of composite dark matter is logically independent from the point of view in [22] on naturalness. We here revisit the DM issue remaining agnostic about the explanation of smallness of the electro-weak scale: we just assume that for some reason the SM is

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much lighter than other unspecified new physics, such that accidental symmetries appear at low energy. We make the following simplifying assumptions:

1. We study both $\mathrm{SU}(N)_{\mathrm{TC}}$ and $\mathrm{SO}(N)_{\mathrm{TC}}$ techni-color gauge groups, but we restrict to techni-quarks in the fundamental representations of the TC group.

2. We consider techni-quark representations that can be embedded in SU(5)-unified models.

3. We do not consider techni-scalars, that would generate a different set of TCb, and would allow to realise partial compositeness in a fundamental theory.

The accidentally stable Dark Matter candidates. This scenario has the following accidental symmetries that lead to automatically stable composite DM candidates:

- *Techni-baryon number.* The Lagrangian is accidentally symmetric under a $\mathrm{U}(1)_{\mathrm{TB}}$ global symmetry (sometimes broken by anomalies down to $\mathbb{Z}_2$) that rotates the techni-quarks Ψ with the same phase. This guarantees the *stability of the lightest techni-baryon.*

- *Species number.* When the techni-quarks are in a reducible representation of the SM, each phase rotation acting individually on a Ψ_i is an accidental techni-flavor symmetry of the renormalizable Lagrangian. This leads to *stable techni-pions made of different species* $\bar{\Psi}_i\Psi_j$. TCb made of different species can also be stable if their decay to TCπ is kinematically forbidden.

- *G-parity.* In models with electro-weak representations the Lagrangian can be invariant under a discrete symmetry known as G-parity [25], that acts on techni-quarks as $\Psi \to \exp(i\pi T^2)\Psi^c$. In $\mathrm{SU}(N)_{\mathrm{TC}}$ theories G-parity acts on TCπ so that even (odd) isospin TCπ are even (odd) under G-parity. Standard Model states are G-parity even, so that the lightest G-parity odd TCπ is stable. This symmetry is broken by non-vanishing hypercharge.

We assume that, in a successful model, all stable particles must be good DM candidates.

Breaking of accidental symmetries. The symmetries above can be violated by various effects.

First, when the quantum numbers allow for Yukawa interactions with the Higgs, this breaks both species number and G-parity while preserving techni-baryon number. States whose stability was insured by these broken symmetries will then decay with specific patterns. We assume that all allowed couplings are present and that decays are fast enough that unstable particles are not relevant for dark matter.

Second, species number and $G-$parity can also be broken by dimension 5 operators,

$$\frac{1}{M}\bar{\Psi}\Psi HH\,, \qquad \frac{1}{M}\bar{\Psi}\sigma^{\mu\nu}\Psi B_{\mu\nu}\,. \tag{1.2}$$

The lifetime of TCπ is shorter than the age of the universe for $M < \bar{M}_{\mathrm{Pl}} \equiv 2.4\times 10^{18}$GeV.

SU(5)	SU(3)$_c$	SU(2)$_L$	U(1)$_Y$	charge	name	Δb_3	Δb_2	Δb_Y
1	1	1	0	0	N	0	0	0
$\bar{5}$	$\bar{3}$	1	1/3	1/3	D	1/3	0	2/9
	1	2	$-1/2$	$0, -1$	L	0	1/3	1/3
10	$\bar{3}$	1	$-2/3$	$-2/3$	U	1/3	0	8/9
	1	1	1	1	E	0	0	2/3
	3	2	1/6	$2/3, -1/3$	Q	2/3	1	1/9
15	3	2	1/6	$2/3, -1/3$	Q	2/3	1	1/9
	1	3	1	$0, 1, 2$	T	0	4/3	2
	6	1	$-2/3$	$-2/3$	S	5/3	0	8/9
24	1	3	0	-1, 0, 1	V	0	4/3	0
	8	1	0	0	G	2	0	0
	$\bar{3}$	2	5/6	$4/3, 1/3$	X	2/3	1	25/9
	1	1	0	0	N	0	0	0

Table 1. Techni-quarks are assumed to belong to fragments of SU(5) representations (plus their conjugates for complex representations). We give the SM decomposition, assign standard names used throughout the paper, and list the contributions Δb_i to the SM β-function coefficients (to be multiplied by the multiplicity of the techni-color representation).

Third, techni-baryon and species number can be broken by operators of dimension 6 or higher depending on the quantum numbers. In the first case the lifetime is consistent with the present experimental bound from indirect searches [26, 27]

$$\tau \sim \frac{8\pi\, M^4}{M_{\rm DM}^5} \sim 10^{26}\,\text{sec} \times \left(\frac{M}{\bar{\rm M}_{\rm Pl}}\right)^4 \left(\frac{100\,\text{TeV}}{M_{\rm DM}}\right)^5 \gtrsim 10^{25}\,\text{sec}\frac{100\,\text{TeV}}{M_{\rm DM}} \tag{1.3}$$

if M is comparable to $\bar{M}_{\rm Pl}$ and $M_{\rm DM} \approx 100\,\text{TeV}$. For dimension 7 operators the scale M must be larger than $\approx 10^{14}\,\text{GeV}$.

Any species number symmetry can also be broken by adding e.g. ad-hoc scalars with quantum numbers such that desired extra Yukawa couplings arise.

The upshot is that techni-baryon number is more robust than species number or G-parity in the framework of vector-like confinement, at least working within the standard assumptions of effective field theory. TCb are then the most promising dark matter candidate. We will focus mostly on TCb dark matter in what follows.

The paper is structured as follows. We identify successful DM models based on SU$(N)_{\rm TC}$ in section 2 and models based on SO$(N)_{\rm TC}$ in section 3. In section 4 we discuss the effect of techni-quark masses and of the $\theta_{\rm TC}$ on the spectrum and the generation of Electric Dipole moments. In section 5 we discuss the resulting phenomenology. Conclusions are given in section 6. In the appendices we provide technical details of the techni-baryon classification and we collect models that require higher dimensional operators.

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2 $\mathrm{SU}(N)_{\mathrm{TC}}$ Composite Dark Matter models

In this section we consider an $\mathrm{SU}(N)_{\mathrm{TC}}$ techni-color group with N_{TF} techni-quarks in its fundamental representation. We assume that the dynamics is as in QCD: when techni-color interactions become strong, confinement takes place and the global flavor symmetry $\mathrm{SU}(N_{\mathrm{TF}})_L \otimes \mathrm{SU}(N_{\mathrm{TF}})_R$ is spontaneously broken to the diagonal sub-group $\mathrm{SU}(N_{\mathrm{TF}})$ producing $N_{\mathrm{TF}}^2 - 1$ Goldstone bosons in the adjoint representation of the unbroken group. We assume the standard large N scaling

$$\Lambda_{\mathrm{TC}} \sim \frac{4\pi}{\sqrt{N}} f\,, \qquad m_B \sim N \Lambda_{\mathrm{TC}} \tag{2.1}$$

where, to be definite, we denote with Λ_{TC} the mass of the lightest vector meson, with f the Goldstone bosons decay constant, and with m_B the techni-baryon mass.

We consider a model as viable from the point of view of Dark Matter phenomenology, provided that all its stable states have no color, no charge and no hypercharge. This implies that dark matter should belong to a multiplet with integer isospin. As in weakly coupled theories, the neutral component within an electroweak multiplet becomes the lightest component, with a calculable splitting, of order 100 MeV, induced by electro-weak symmetry breaking [1–3].

We analyzed these requirements using the tools in appendix A and the package LieArt [28]. We assume an SU(5) unification scheme, so we select techni-quarks from components of the simpler SU(5) representations listed in table 1. In general for a SM representation there are two inequivalent assignments of techni-quark quantum numbers:

$$R \equiv R_N \oplus \bar{R}_{\bar{N}}, \quad \text{and} \qquad \tilde{R} \equiv \bar{R}_N \oplus R_{\bar{N}} \tag{2.2}$$

where R_N and $\bar{R}_N$ transform in the fundamental of $\mathrm{SU}(N)_{\mathrm{TC}}$, while $R_{\bar{N}}$ and $\bar{R}_{\bar{N}}$ in anti-fundamentals. Since the V, N and G representations are real under the SM gauge group, one has $V = \tilde{V}$, $N = \tilde{N}$ and $G = \tilde{G}$. For each SM representation, an unbroken species symmetry exists corresponding to a U(1) that rotates the (anti)fundamental of $\mathrm{SU}(N)_{\mathrm{TC}}$ with charge $+1$ (-1). Because of this accidental symmetry, TCπ made by different species are stable unless the symmetry is broken e.g. by Yukawa couplings. Techni-baryon number, that guarantees the stability of the lightest TCb, is the sum of all species numbers.

It is convenient to classify models in the following way:

1. *Golden-class* models, such that all stable states are acceptable DM candidates with just renormalizable interactions.[1] Yukawa couplings are often needed in order to break accidental symmetries, avoiding unwanted stable TCπ. All possible Yukawa couplings among the SU(5) fragments are:

$$HL(\tilde{E} \text{ or } \tilde{T} \text{ or } N \text{ or } V), \qquad HQ(\tilde{D} \text{ or } \tilde{U}), \qquad HDX, \tag{2.3}$$

as well as similar interactions with $H \leftrightarrow H^\dagger$ or $x \leftrightarrow \tilde{x}$ where x denotes all techni-quarks.

[1] The dimension-less models considered in [22] are a sub-set of these models, with the extra assumption of vanishing techni-quark masses.

2. *Silver-class* models where non-renormalizable interactions or ad-hoc extra particles are introduced in order to break accidental symmetries that lead to unwanted stable particles.[2]

3. Models with no DM candidates.

An important restriction on the techni-quark content arises from the requirement that $\mathrm{SU}(N)_{\mathrm{TC}}$ with N_{TF} flavors of techni-quarks (e.g. a singlet N contributes as $N_{\mathrm{TF}}=1$) is asymptotically free. Defining the gauge β-function coefficients as $d\alpha_i^{-1}/d\log Q=-b_i/2\pi$ we have

$$b_{\mathrm{TC}}=-\frac{11}{3}N+\frac{2}{3}N_{\mathrm{TF}}<0. \tag{2.4}$$

Furthermore we demand that the SM gauge couplings do not develop Landau poles below the Planck scale:

$$b_3=-7+\Delta b_3\lesssim 3, \qquad b_2=-\frac{19}{6}+\Delta b_2\lesssim 6.5, \qquad b_Y=\frac{41}{6}+\Delta b_Y\lesssim 18. \tag{2.5}$$

where the numerical factors have been computed assuming $\Lambda_{\mathrm{TC}}\sim 100\,\mathrm{TeV}$, motivated by DM as a thermal relic, see section 5. Colored techni-quarks such as U or D contribute as $\Delta b_3=2N/3$, while a G state gives $\Delta b_3=4N$. The weak doublet L contributes as $\Delta b_2=2N/3$, while for the weak triplet V we have $\Delta b_2=8N/3$. Finally $\Delta b_Y=\frac{2}{3}\sum_R \dim(R)Y_R^2$ (e.g. a singlet E contributes as $\Delta b_Y=4N/3$). The contributions $\Delta b_{2,3,Y}$ are summed over techni-quarks, and the constant terms in the β-function coefficients $b_{2,3,Y}$ are the SM contributions.

Summarising, the constraints on the techni-quark content are:

$$N_{\mathrm{TF}}<\frac{11}{2}N, \qquad \Delta b_3\lesssim 10, \qquad \Delta b_2\lesssim 10, \qquad \Delta b_Y\lesssim 11. \tag{2.6}$$

This implies that one weak triplet V is allowed by the constraint on Δb_2 for $N=3$ technicolors but not for $N\geq 4$. Models that contain the techni-quark G, S, X are not allowed, not even for $N=3$, because of Δb_3 or Δb_Y.

2.1 Techni-pions and techni-baryons of $\mathrm{SU}(N)_{\mathrm{TC}}$

Techni-pions are $\Psi\bar\Psi$ states in the adjoint representations of $\mathrm{SU}(N_{\mathrm{TF}})$ under the unbroken techni-flavor symmetry. Their decomposition under the SM group is given by

$$\mathrm{Adj}_{\,\mathrm{SU}(N_{\mathrm{TF}})}=\left[\sum_{i=1}^{N_S}R_i\right]\otimes\left[\sum_{i=1}^{N_S}\bar R_i\right]\ominus 1 \tag{2.7}$$

where the sum runs over the N_S species (e.g. a model with $\Psi=L\oplus N$ techni-quarks has $N_S=2$ species and $N_{\mathrm{TF}}=2+1$ techni-flavors). SM gauge interaction generate a positive contribution to $\mathrm{TC}\pi$ masses that can be estimated as

$$\Delta_{\mathrm{gauge}}m^2_{\mathrm{TC}\pi}\sim\frac{g^2}{(4\pi)^2}\Lambda^2_{\mathrm{TC}}. \tag{2.8}$$

[2]Higher dimensional operators violate flavour in general. Assuming that the scale suppressing these operators is around the GUT or Planck scale, as required for baryon violating operators, this does not lead to phenomenological problems.

The $N_S - 1$ singlets under the SM gauge group do not acquire mass from gauge interactions. In our previous study [22] we assumed vanishing techni-quark masses, such that these singlet TCπ were massless in absence of Yukawa interactions, and thereby experimentally excluded because of their axion-like coupling to SM vectors. Here we allow for techni-quark masses, such that the singlets become massive avoiding phenomenological problems. The contribution from techni-quark masses to TCπ masses scales as

$$\Delta_{\text{mass}} m^2_{\text{TC}\pi} \sim m_\Psi \Lambda_{\text{TC}} \tag{2.9}$$

and can be described using chiral Lagrangian techniques.

Techni-pions can be stable because of G-parity or species number if they are made by different species. For example in QCD, the charged pion π^+ decays because species number is broken by weak interactions, while $G-$parity is broken by hypercharge allowing π_0 to decay through the anomaly. Among our representations, only the weak triplet V is symmetric under G-parity leading to stable TCπ.

TCb are techni-color singlets constructed with N techni-quarks. They are fermions for N odd and bosons for N even, leading to vastly different dark matter phenomenology. The SM quantum numbers of TCb multiplets are determined by group theory: the TCb fill representations of the unbroken $\text{SU}(N_{\text{TF}})$ global techni-flavor symmetry that can be decomposed under the SM. TCb wave-function is totally antisymmetric in techni-color. Furthermore, one can argue that the lighter TCb have the smallest possible spin, and the lowest possible angular momentum (fully symmetric s-wave function in space). Due to Fermi statistics, this implies that TCb must be fully symmetric in spin and techni-flavour. This determines the representation of the lighter TCb under the unbroken global techni-flavor symmetry corresponding to a Young tableau with two rows with $N/2$ boxes (N even) or two rows with $(N+1)/2$ and $(N-1)/2$ boxes (N odd) and also the spin. Explicitly for $N = 3, 4, 5$ they are,

$$\text{lighter TCb} = \begin{cases} \yng(2,1) & \text{for } N = 3 \\ \yng(2,2) & \text{for } N = 4 \\ \yng(3,2) & \text{for } N = 5\,. \end{cases} \tag{2.10}$$

A fully symmetric representation is obtained by a tensor product of each techni-flavor representation with an identical spin representation: for even (odd) N we obtain spin-0 (spin 1/2) DM. The case $N_{\text{TF}} = 1$ is special because flavour cannot be anti-symmetrized, TCb have spin $N/2$. The heavier TCb (analog of the decuplet in QCD) transform instead in the following representations

$$\text{heavier TCb} = \begin{cases} \yng(3) & \text{for } N = 3 \\ \yng(3,1) \oplus \yng(4) & \text{for } N = 4 \\ \yng(4,1) \oplus \yng(5) & \text{for } N = 5 \end{cases} \tag{2.11}$$

and have higher spin described by an identical spin representation. The mass difference between the heavier and the lighter TCb is expected of order Λ_{TC}.

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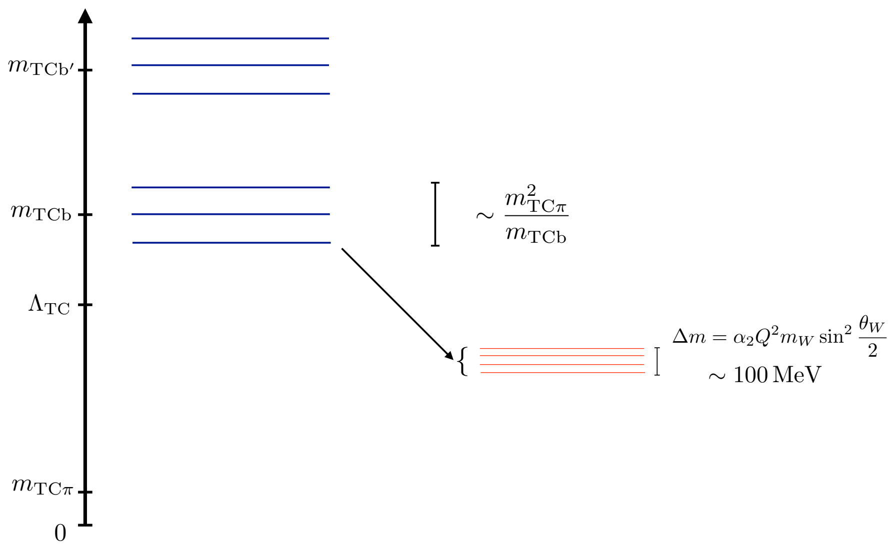

Figure 1. Spectrum of techni-color DM models. Splitting between techni-flavor multiplets is of order the dynamical scale Λ_{TC} splitting between different SM representation $\Lambda_{\mathrm{TC}}/100$ or larger and hyperfine splitting from electro-weak symmetry breaking of order 100 MeV.

Heavier TCb usually decay into a lighter TCb and TCπ; however some heavier TCb could be accidentally stable due to species number if they are the lightest states with TCb and species number. This can happen for techni-quark masses comparable to Λ_{TC}. An analog exists in QCD where, in absence of the weak interactions, the lightest strange baryon (Λ, with quark content uds) would be stable because its decay to kaons and nucleons is not kinematically allowed. Furthermore, the spin 3/2 baryon $\Omega^-(1672)$ (quark content sss) cannot decay to $\Xi^0 K^-$ through strong interactions: its decay is allowed only by strangeness-violating weak interactions.

TCb flavour multiplets are split by SM gauge interactions, by techni-quark masses and possibly by techni-quark Yukawa interactions and by higher dimensional operators (that we neglect). While for the TCπ one can argue that in the limit of zero techni-quark masses the lightest multiplets are those with the smallest charge under the SM gauge group, the same sentence is not rigorously proved for TCb. Indeed, while the long distance gauge contribution to the energy of charged fields is proportional to their total charge, the short distance contribution is difficult to estimate. Experience with electromagnetic splitting of baryons in QCD hints however to the fact that the lightest states are indeed the ones with smaller charge. This is what we will assume in the following. We estimate,

$$\Delta_{\mathrm{mass}} m_B \sim m_\Psi\,, \qquad \Delta_{\mathrm{gauge}} m_B \sim \frac{g^2}{(4\pi)^2}\Lambda_{\mathrm{TC}}. \tag{2.12}$$

Finally, the breaking of the electro-weak symmetry induces calculable splittings within the components of each electro-weak multiplet (of order 100 MeV), with the result that the component with smallest electric charge is the lightest state. The spectrum of the theory is illustrated in figure 1.

2.2 $\mathrm{SU}(N)_{\mathrm{TC}}$ golden-class models

In this section we present the golden-class models for $\mathrm{SU}(N)_{\mathrm{TC}}$ strong interactions. The models are obtained scanning over techni-quarks made by combinations of the SU(5) fragments of table 1. excluding models that lead to sub-Planckian Landau poles for g_Y, g_2 or g_3. We require that the lightest stable TCb has no color, no hypercharge, and integer isospin. For example, for $N = 3$, the possible DM candidates are made of the following techni-quarks:

$$LLE,\ DDU,\ EU\tilde{D},\ QQ\tilde{D}\ DLQ,\ UQ\tilde{L},\ Vx\tilde{x}, \tag{2.13}$$

where x denotes any techni-quark, any E can be substituted by a T, any V can be substituted by a N. By replacing all techni-quarks with their tilded counterparts one obtains equivalent descriptions of the same models.

However, if species number is conserved, most of the models that can give rise to such TCb DM candidates also lead to extra stable TCπ with $Y \neq 0$ or color, that are thereby excluded by DM direct searches (unless their thermal abundance is small enough). In the context of renormalizable golden-class models, Yukawa couplings to the Higgs doublet determine the accidental symmetries. For example, a Yukawa coupling to the Higgs boson is allowed by gauge quantum numbers in a model containing the techni-quarks $\Psi = L \oplus \tilde{E}$. The Yukawa coupling $HL\tilde{E}$ breaks the unwanted species number. On the contrary, no Yukawa coupling is allowed in a model with $\Psi = L \oplus E$ that would lead to the first TCb in eq. (2.13). In appendix B we present a list of silver-class models (limited for simplicity to $N = 3, 4$ and to the case of 1 or 2 species) where extra effects (non-renormalizable interactions or other particles) are needed to break unwanted symmetries.

The list of $\mathrm{SU}(N)_{\mathrm{TC}}$ golden-class models presented below is summarized in table 2.[3] We start the description of golden-class models from models that only involve color-less techni-quarks.

The simplest model contains the singlet N as the only techni-quark, such that the lightest DM TCb has spin $N/2$. Interactions with SM particles arise only adding extra states, as described below.

a) $\mathrm{SU}(N)_{\mathrm{TC}}$ model $\Psi = V$. The model has a single specie of techni-quarks: a triplet with zero hypercharge in the adjoint of $\mathrm{SU}(2)_L$, such that $N_{\mathrm{TF}} = 3$. No Yukawa coupling is allowed. If $N \geq 4$ the g_2 gauge coupling becomes non-perturbative below the Planck scale. Thereby this model is only allowed for $N = 3$. Both TCb and TCπ lie in the 8 of $\mathrm{SU}(3)_{\mathrm{TF}}$, that decomposes as

$$8 = 3_0 \oplus 5_0 \qquad \text{under } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y. \tag{2.14}$$

The TCπ triplet is stable because of G-parity, and the TCb triplet is stable because of techni-baryon number. These are good DM candidates. This model has been already presented in [22].

[3] We do not consider models that contain SM representations with multiplicity as these do not lead to new DM candidates. In some cases however this might change the spin of the lightest TCb.

SU(N) techni-color. Techni-quarks	Yukawa couplings	Allowed N	Techni-pions	Techni-baryons	under
$N_{\rm TF} = 3$			8	$8, \bar{6}, \dots$ for $N = 3, 4, \dots$	$\mathrm{SU}(3)_{\rm TF}$
$\Psi = V$	0	3	3	$VVV = 3$	$\mathrm{SU}(2)_L$
$\Psi = N \oplus L$	1	$3, \dots, 14$	unstable	$N^{N*} = 1$	$\mathrm{SU}(2)_L$
$N_{\rm TF} = 4$			15	$\overline{20}, 20', \dots$	$\mathrm{SU}(4)_{\rm TF}$
$\Psi = V \oplus N$	0	3	3×3	$VVV, VNN = 3,\ VVN = 1$	$\mathrm{SU}(2)_L$
$\Psi = N \oplus L \oplus \tilde{E}$	2	$3, 4, 5$	unstable	$N^{N*} = 1$	$\mathrm{SU}(2)_L$
$N_{\rm TF} = 5$			24	$\overline{40}, \overline{50}$	$\mathrm{SU}(5)_{\rm TF}$
$\Psi = V \oplus L$	1	3	unstable	$VVV = 3$	$\mathrm{SU}(2)_L$
$\Psi = N \oplus L \oplus \tilde{L}$	2	3	unstable	$NL\tilde{L} = 1$	$\mathrm{SU}(2)_L$
$=$	2	4	unstable	$NNL\tilde{L}, L\tilde{L}L\tilde{L} = 1$	$\mathrm{SU}(2)_L$
$N_{\rm TF} = 6$			35	$70, \overline{105'}$	$\mathrm{SU}(6)_{\rm TF}$
$\Psi = V \oplus L \oplus N$	2	3	unstable	$VVV, VNN = 3, VVN = 1$	$\mathrm{SU}(2)_L$
$\Psi = V \oplus L \oplus \tilde{E}$	2	3	unstable	$VVV = 3$	$\mathrm{SU}(2)_L$
$\Psi = N \oplus L \oplus \tilde{L} \oplus \tilde{E}$	3	3	unstable	$NL\tilde{L}, \tilde{L}\tilde{L}\tilde{E} = 1$	$\mathrm{SU}(2)_L$
$=$	3	4	unstable	$NNL\tilde{L}, L\tilde{L}L\tilde{L}, N\tilde{E}\tilde{L}\tilde{L} = 1$	$\mathrm{SU}(2)_L$
$N_{\rm TF} = 7$			48	112	$\mathrm{SU}(7)_{\rm TF}$
$\Psi = L \oplus \tilde{L} \oplus E \oplus \tilde{E} \oplus N$	4	3	unstable	$LLE, \tilde{L}\tilde{L}\tilde{E}, L\tilde{L}N, E\tilde{E}N = 1$	$\mathrm{SU}(2)_L$
$\Psi = N \oplus L \oplus \tilde{E} \oplus V$	3	3	unstable	$VVV, VNN = 3,\ VVN = 1$	$\mathrm{SU}(2)_L$
$N_{\rm TF} = 9$			80	240	$\mathrm{SU}(9)_{\rm TF}$
$\Psi = Q \oplus \tilde{D}$	1	3	unstable	$QQ\tilde{D} = 1$	$\mathrm{SU}(2)_L$
$N_{\rm TF} = 12$			143	572	$\mathrm{SU}(12)_{\rm TF}$
$\Psi = Q \oplus \tilde{D} \oplus \tilde{U}$	2	3	unstable	$QQ\tilde{D}, \tilde{D}\tilde{D}\tilde{U} = 1$	$\mathrm{SU}(2)_L$

Table 2. Golden-class models with $\mathrm{SU}(N)_{\rm TC}$ techni-color that give viable TCb and/or TCπ Dark Matter candidates with $Q = Y = 0$, starting from techni-quarks coming from SU(5) fragments listed in table 1. The darker rows give the techni-flavour content of the lightest TCb and TCπ considering only masses induced by techni-color interactions. The lighter rows show the viable models, the number of Yukawa interactions, and the $\mathrm{SU}(2)_L$ content of the stable TCπ and the stable TCb, assuming that the lighter component is the one with the least SM charge. A * denotes a higher spin TCb. .

b) $\mathrm{SU}(N)_{\rm TC}$ model $\Psi = V \oplus N$. The previous model can be simply extended to $N_S = 2$ techni-quarks by adding an N (SM gauge singlet) such that $N_{\rm TF} = 4$. Again, no Yukawa coupling is allowed and the model can be considered only for $N = 3$ because of sub-Planckian Landau poles. TCπ lie in the 15 of $\mathrm{SU}(4)_{\rm TF}$ that decomposes as

$$\mathrm{TC}\pi\,:\ 15 = 1_0 \oplus 3 \times 3_0 \oplus 5_0 \qquad \text{under } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y. \tag{2.15}$$

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The three triplets are stable because of species number and because of G-parity. The lighter TCb lie in the $\overline{20}$ representation of $\mathrm{SU}(4)_{\mathrm{TF}}$ that decomposes as

$$\mathrm{TCb}: \overline{20} = 1_0 \oplus 3 \times 3_0 \oplus 2 \times 5_0 \qquad \text{under } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y. \tag{2.16}$$

The lightest TCb is a stable DM candidate, and its identity depends on the techni-quark masses. For $m_V \ll m_N$, the triplet 3_0 (VVV) is expected to be the lightest. For $m_N \ll m_V \gtrsim \Lambda_{\mathrm{TC}}$ the extra TCb NNN^* (denoted with a * and not included in the list above because it has spin 3/2) could become the stable DM candidate; at the same time the $\mathrm{SU}(4)_{\mathrm{TF}}$ classification breaks down.

c) $\mathbf{SU}(\boldsymbol{N})_{\mathbf{TC}}$ models $\boldsymbol{\Psi = N \oplus L}$ and $\boldsymbol{\Psi = N \oplus L \oplus \tilde{E}}$. In both models, enough Yukawa couplings are allowed such that only techni-baryon number is conserved and all TCπ are unstable. For the $N \oplus L$ ($N_{\mathrm{TF}} = 3$) and the $N \oplus L \oplus \tilde{E}$ ($N_{\mathrm{TF}} = 4$) models respectively, these are:

$$\begin{matrix} \mathrm{TC}\pi: 8 = 1_0 \oplus 2_{\pm 1/2} \oplus 3_0 \\ \mathrm{TC}\pi: 15 = 1_{2\times 0,\pm 1} \oplus 2 \times 2_{\pm 1/2} \oplus 3_0 \end{matrix} \qquad \text{under } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y. \tag{2.17}$$

For $N = 3$, the spin 1/2 TCb do not contain any DM candidate, for example in the the $N \oplus L$ model they are

$$\mathrm{TCb}: 8 = 1_{-1} \oplus 2_{-1/2,-3/2} \oplus 3_{-1} \qquad \text{under } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y. \tag{2.18}$$

The DM candidate is the singlet NNN^*, which only exists with spin 3/2. Thereby these models are viable only as long as the techni-quark masses m_L and $m_{\tilde{E}}$ are of order Λ_{TC} and large enough that NNN^* is the lightest TCb. This state lies in the 10 of $\mathrm{SU}(3)_{\mathrm{TF}}$ in the $N \oplus L$ model

$$\mathrm{TCb}^*: 10 = 1_0 \oplus 2_{-1/2} \oplus 3_{-1} \oplus 4_{-3/2} \qquad \text{under } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \tag{2.19}$$

and in the $\overline{20}''$ of $\mathrm{SU}(4)_{\mathrm{TF}}$ in the $N \oplus L \oplus \tilde{E}$ model.

The same is true for $N = 4$, where the only DM candidate is the singlet $NNNN^*$ that lies in the completely symmetric spin 2 representation $\square\square\square\square$. In the $N \oplus L$ model, this representation decomposes as

$$\mathrm{TCb}^*: 15' = 1_0 \oplus 2_{-1/2} \oplus 3_{-1} \oplus 4_{-3/2} \oplus 5_{-2} \qquad \text{under } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y. \tag{2.20}$$

The $N \oplus L$ model is allowed by perturbativity constraints up to $N = 14$, while the $N \oplus L \oplus \tilde{E}$ is allowed up to $N = 5$ (with increasing spin of the DM candidate).

d) $\mathbf{SU}(\boldsymbol{N})_{\mathbf{TC}}$ models $\boldsymbol{\Psi = V \oplus L}$ and $\boldsymbol{V \oplus L \oplus \tilde{E}}$. Other possible extensions of the first model are $\Psi = V \oplus L$ and $V \oplus L \oplus \tilde{E}$. A possible problem of these models is that, even for $N = 3$, the $\mathrm{SU}(2)_L$ gauge coupling becomes non perturbative around 10^{17} GeV. In view of the Yukawa couplings VLH, $\tilde{E}LH$, all TCπ are unstable and given by

$$\mathrm{TC}\pi: 24 = 1_0 \oplus 2_{\pm 1/2} \oplus 2 \times 3_0 \oplus 4_{\pm 1/2} \oplus 5_0 \qquad \text{under } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y, \tag{2.21}$$

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in the $V \oplus L$ model, and by

$$\text{TC}\pi : 35 = 2 \times 1_0 \oplus 2 \times 2_{\pm 1/2} \oplus 3_{2\times 0,\pm 1} \oplus 4_{\pm 1/2} \oplus 5_0 \qquad \text{under } \text{SU}(2)_L \otimes \text{U}(1)_Y \,, \tag{2.22}$$

in the $V \oplus L \oplus \tilde{E}$ model.

In both models the TCb DM candidate is the VVV state that forms a weak triplet as in the $\Psi = V$ model: the extra techni-quarks L (and possibly $\tilde{E}$) does not lead to any extra DM candidates and play a minor role provided that they are heavy enough. In the $V \oplus L$ model, the lightest TCb multiplet is a $\overline{40}$ of $\text{SU}(5)_{\text{TF}}$ that decomposes as:

$$\text{TCb} : \overline{40} = 1_{-1} \oplus 2_{2\times(-1/2),-3/2} \oplus 3_{0,2\times(-1)} \oplus 2 \times 4_{-1/2} \oplus 5_{0,-1} \oplus 6_{-1/2} \tag{2.23}$$

under $\text{SU}(2)_L \otimes \text{U}(1)_Y$.

e) $\mathbf{SU}(\boldsymbol{N})_{\mathbf{TC}}$ models $\boldsymbol{\Psi = V \oplus N \oplus L}$ and $\boldsymbol{\Psi = V \oplus N \oplus L \oplus \tilde{E}}$. As in the previous models, sub-Planckian Landau poles are avoided only for $N = 3$ (where g_2 becomes non perturbative around 10^{17} GeV). Since L and $\tilde{E}$ cannot enter in an hypercharge-less TCb, the DM candidates are the same of the $V \oplus N$ model. Unlike in the $V \oplus N$ model, the Yukawa couplings VLH, NLH and $L\tilde{E}H$ break all species number symmetries, such that all TCπ are unstable. In the $V \oplus N \oplus L$ model ($N_{\text{TF}} = 6$), the TCπ are

$$\text{TC}\pi : 35 = 2 \times 1_0 \oplus 2 \times 2_{\pm 1/2} \oplus 4 \times 3_0 \oplus 4_{\pm 1/2} \oplus 5_0 \qquad \text{under } \text{SU}(2)_L \otimes \text{U}(1)_Y \,. \tag{2.24}$$

In the $V \oplus N \oplus L \oplus \tilde{E}$ model ($N_{\text{TF}} = 7$), the list extends to

$$\text{TC}\pi : 48 = 1_{3\times 0,\pm 1} \oplus 3 \times 2_{\pm 1/2} \oplus 3_{4\times 0,\pm 1} \oplus 4_{\pm 1/2} \oplus 5_0 \qquad \text{under } \text{SU}(2)_L \otimes \text{U}(1)_Y \,. \tag{2.25}$$

f) $\mathbf{SU}(\boldsymbol{N})_{\mathbf{TC}}$ model $\boldsymbol{\Psi = N \oplus L \oplus \tilde{L}}$. The model allows two Yukawa couplings (NLH, $N\tilde{L}H$) such that there are no stable TCπ and allows for DM TCb candidates not present in the previous models. The unstable TCπ are:

$$\text{TC}\pi : 24 = 1_{2\times 0,\pm 1} \oplus 2 \times 2_{\pm 1/2} \oplus 3_{2\times 0,\pm 1} \qquad \text{under } \text{SU}(2)_L \otimes \text{U}(1)_Y \,. \tag{2.26}$$

Sub-Planckian Landau poles are avoided for $N \leq 7$. Here we discuss the TCb DM candidates for $N = 3, 4$.

For $N = 3$, the lighter TCb fill a $\overline{40}$ of $\text{SU}(5)_{\text{TF}}$ that decomposes as

$$\text{TCb} : \overline{40} = 1_{2\times 0,\pm 1} \oplus 2_{3\times(\pm 1/2),\pm 3/2} \oplus 3_{2\times 0,\pm 1} \oplus 4_{\pm 1/2} \tag{2.27}$$

under $\text{SU}(2)_L \otimes \text{U}(1)_Y$, so that the TCb DM candidates are singlets made of $N\tilde{L}L$.

For $N = 4$, the lighter TCb are

$$\text{TCb} : \overline{50} = 1_{3\times 0,\pm 1,\pm 2} \oplus 2_{2\times(\pm 1/2),\pm 3/2} \oplus 3_{2\times 0,2\times(\pm 1)} \oplus 4_{\pm 1/2} \oplus 5_0 \,, \tag{2.28}$$

under $\text{SU}(2)_L \otimes \text{U}(1)_Y$. The TCb DM candidates are singlets made of $LL\tilde{L}\tilde{L}$ and $L\tilde{L}NN$.

g) $\mathrm{SU}(N)_{\mathrm{TC}}$ model $\Psi = N \oplus L \oplus \tilde{L} \oplus \tilde{E}$. This is a non trivial extension of the previous model, with one more Yukawa coupling allowed ($L\tilde{E}H$), so that there are no stable TCπ. The model is allowed only for $N = 3, 4$, since for greater values of N the coupling g_Y develops a sub-Planckian Landau pole. The unstable TCπ can be listed as:

$$\mathrm{TC}\pi : 35 = 1_{3\times 0, 2\times(\pm 1)} \oplus 2_{3\times(\pm 1/2), \pm 3/2} \oplus 3_{2\times 0, \pm 1} \qquad \text{under } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y . \tag{2.29}$$

This model gives a TCb DM candidate not present in the previous models: $\tilde{L}\tilde{L}\tilde{E}$ and $N\tilde{L}\tilde{L}\tilde{E}$ for $N = 3$ and $N = 4$ respectively.

For $N = 3$, the lightest multiplet of TCb decomposes under $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ as

$$\mathrm{TCb} : 70 = \mathrm{TCb}_{N\oplus L\oplus\tilde{L}} \oplus 1_{0,3\times(-1),2\times(-2)} \oplus 2_{2\times(-1/2),3\times(-3/2),(-5/2)} \oplus 3_{0,2\times(-1),-2} \tag{2.30}$$

where $\mathrm{TCb}_{N\oplus L\oplus\tilde{L}}$ is defined in eq. (2.27). For $N = 4$ we get

$$\begin{aligned}\mathrm{TCb} : \overline{105'} = \mathrm{TCb}_{N\oplus L\oplus\tilde{L}} &\oplus 1_{0,2\times(-1),3\times(-2)} \oplus 2_{1/2,3\times(-1/2),4\times(-3/2),2\times(-5/2)} \\ &\oplus 3_{0,3\times(-1),2\times(-2),-3} \oplus 4_{-1/2,-3/2}\end{aligned} \tag{2.31}$$

where now $\mathrm{TCb}_{N\oplus L\oplus\tilde{L}}$ refers to eq. (2.28). In each case, besides the TCb DM candidates of the $N \oplus L \oplus \tilde{L}$ model, there a singlet DM candidate made of $\tilde{L}\tilde{L}\tilde{E}$ or $N\tilde{L}\tilde{L}\tilde{E}$.

h) $\mathrm{SU}(N)_{\mathrm{TC}}$ model $\Psi = L \oplus \tilde{L} \oplus E \oplus \tilde{E} \oplus N$. The model has $N_{\mathrm{TF}} = 7$ and for $N = 3$ gives $\Delta b_Y = 12$ so that hypercharge has a Landau pole around the Planck scale, so that it cannot be extended to $N > 3$. Thanks to the presence of N, it allows for 4 Yukawa couplings ($L\tilde{E}H$, $\tilde{L}EH$, LNH, $\tilde{L}NH$) that break all species number symmetries. The unstable TCπ are:

$$\mathrm{TC}\pi : 48 = 1_{4\times 0, 3\times(\pm 1), \pm 2} \oplus 2_{4\times(\pm 1/2), 2\times(\pm 3/2)} \oplus 3_{2\times 0, \pm 1} \tag{2.32}$$

under $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. The lightest TCb fill a 112 of $\mathrm{SU}(7)_{\mathrm{TF}}$, that decomposes as

$$\mathrm{TCb} : 112 = \mathrm{TCb}_{N\oplus L\oplus\tilde{L}} \oplus 1_{4\times(0,\pm 1),2\times(\pm 2)} \oplus 2_{4\times(\pm 1/2),3\times(\pm 3/2),\pm 5/2} \oplus 3_{2\times(0,\pm 1),\pm 2} \tag{2.33}$$

under $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. The TCb DM candidates are those of the $N \oplus L \oplus \tilde{L}$ model, defined in eq. (2.27), plus the singlets LLE, $\tilde{L}\tilde{L}\tilde{E}$ and $E\tilde{E}N$.

We next consider models with coloured techni-quarks.

i) $\mathrm{SU}(N)_{\mathrm{TC}}$ model $\Psi = Q \oplus \tilde{D}$. The simplest golden-class model with colored techni-quarks is $\Psi = Q \oplus \tilde{D}$, that is allowed for $N = 3, 4$ and gives a DM candidate only for $N = 3$. The model has $N_{\mathrm{TF}} = 6$ and does not lead to unwanted stable states because species number is broken by the Yukawa coupling $Q\tilde{D}H$.

The model predicts a set of unstable TCπ in the 80 representation of $\mathrm{SU}(9)_{\mathrm{TF}}$, that decomposes under the SM gauge group $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ as

$$\mathrm{TC}\pi : 80 = (1,1)_0 \oplus (1,2)_{\pm 1/2} \oplus (1,3)_0 \oplus 2(8,1)_0 \oplus (8,2)_{\pm 1/2} \oplus (8,3)_0 . \tag{2.34}$$

For $N = 3$ the multiplet of lighter TCb fills a 240 of $\mathrm{SU}(9)_{\mathrm{TF}}$, that decomposes as

$$\begin{aligned}\mathrm{TCb}:\ 240 = {} & (1,1)_0 \oplus (1,2)_{\pm 1/2} \oplus (1,3)_0 \oplus (8,1)_{2\times 0,-1} \oplus (10,1)_0 \oplus (8,2)_{1/2,2\times(-1/2)} \\ & \oplus (10,2)_{\pm 1/2} \oplus 2(8,3)_0 \oplus (10,3)_0 \oplus (8,4)_{1/2}\end{aligned} \tag{2.35}$$

under the SM gauge group $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. The DM candidate is the neutral singlet $QQ\tilde{D}$, which can be the lightest TCb.

l) $\mathbf{SU}(\boldsymbol{N})_{\mathbf{TC}}$ model $\boldsymbol{\Psi = Q \oplus \tilde{D} \oplus \tilde{U}}$. This extension of the previous model allows for two Yukawa couplings, $QH\tilde{D}$ and $QH\tilde{U}$, so that there are no stable TCπ. This model has $N_{\mathrm{TF}} = 12$ and is allowed only for $N = 3$, where $\Delta b_3 = 8$. It predicts an extended set of unstable TCπ, that fills a 143 of $\mathrm{SU}(12)_{\mathrm{TF}}$:

$$\mathrm{TC}\pi:\ 143 = \mathrm{TC}\pi_{Q\oplus\tilde{D}} \oplus (1,1)_{0,\pm 1} \oplus (1,2)_{\pm 1/2} \oplus (8,1)_{0,\pm 1} \oplus (8,2)_{\pm 1/2} \tag{2.36}$$

under $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. The model contains two TCb DM candidates: $QQ\tilde{D}$ and $\tilde{D}\tilde{D}\tilde{U}$. The lighter TCb fill a 572 of $\mathrm{SU}(12)_{TF}$, that decomposes as

$$\begin{aligned}\mathrm{TCb}:\ 572 = {} & \mathrm{TCb}_{Q\oplus\tilde{D}} \oplus (1,1)_{0,2\times 1} \oplus (1,2)_{2\times 1/2,3/2} \oplus (1,3)_1 \oplus (8,1)_{2\times 0,4\times 1,2} \oplus (10,1)_{0,2\times 1} \\ & \oplus (8,2)_{4\times 1/2,2\times 3/2} \oplus (10,2)_{2\times 1/2,3/2} \oplus 2\times(8,3)_1 \oplus (10,3)_1\end{aligned} \tag{2.37}$$

under $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. The TCb DM candidates are those of the $\Psi = Q \oplus \tilde{D}$ model plus a singlet made of $\tilde{D}\tilde{D}\tilde{U}$.

Notice that colored techni-quarks never provide golden-class models for $N \geq 4$. For example the model $\Psi = G$ leads, for $N = 4$, to an acceptable TCb DM candidate; but g_3 develops a sub-Planckian Landau pole.[4] Landau poles also exclude the model $\Psi = Q \oplus \tilde{U} \oplus \tilde{D}$ (two Yukawa couplings allowed, no stable TCπ) that for $N = 6$ provides a TCb DM candidate, $QQ\tilde{D}\tilde{D}\tilde{D}\tilde{U}$.

3 $\mathrm{SO}(N)_{\mathrm{TC}}$ Composite Dark Matter models

In this section we consider models based on $\mathrm{SO}(N)$ techni-color interactions with techni-quarks in the vector representation of $\mathrm{SO}(N)$.[5] The techni-quark content is restricted by demanding that $g_{Y,2,3}$ do not develop sub-Planckian Landau poles, and that $\mathrm{SO}(N)_{\mathrm{TC}}$ is asymptotically free. Normalizing the generators in the fundamental as $\mathrm{Tr}(T^a T^b) = \delta^{ab}$, the $\mathrm{SO}(N)_{\mathrm{TC}}$ β-function coefficient reads

$$b_{\mathrm{TC}} = -\frac{11}{3}(N-2) + \frac{2}{3}N_{\mathrm{TF}} < 0 \qquad \text{so that } N_{\mathrm{TF}} < \frac{11}{2}(N-2). \tag{3.1}$$

[4]Detailed group-theoretical computations show that the simplest model $\Psi = G$ leads, for $N = 3$ only to coloured lighter TCb; this can be cured by adding extra techni-quarks (e.g. $\Psi = G \oplus N$) but their addition lead to stable coloured TCπ or TCb. Furthermore g_3 develops a Landau pole below the Planck scale. So these are not golden-class models.

[5]We do not consider $\mathrm{Sp}(N)$ techni-color interactions, since there are no stable techno-baryons: the anti-symmetric combination of N techni-quarks decays into N techni-mesons. We also ignore models with chiral representations of the gauge group, which lead to more complicated patterns of symmetry breaking that are not under good theoretical control. Our results partly hold also for fermions in more general real representations, but TCb may have different properties [29].

Considering again techni-quarks in fragments of the simplest SU(5) representations in table 1, vectorial techni-quarks Ψ are defined as:

$$\Psi \equiv \begin{cases} C_N \oplus \bar{C}_N & \text{for complex SM representations } C \in \{E, L, D, U, Q, S, T, X\} \\ R_N & \text{for real SM representations } R \in \{N, V, G\} \end{cases} . \tag{3.2}$$

The dynamics of the theory is as follows. In the limit of negligible techni-quarks masses, the anomaly free global symmetry is $\mathrm{SU}(N_{\rm TF}) \otimes \mathbb{Z}_{(3+(-1)^N)N_{\rm TF}}$ which is spontaneously broken to $\mathrm{SO}(N_{\rm TF}) \otimes \mathbb{Z}_2$ by the condensates

$$\langle C_N \bar{C}_N \rangle = 2\langle R_N R_N \rangle \sim 4\pi\Lambda^3_{\rm TC} . \tag{3.3}$$

The spontaneous breaking produces $N_{\rm TF}(N_{\rm TF}+1)/2 - 1$ pseudo-Goldstone bosons that transform in the two-index symmetric representation of the unbroken $\mathrm{SO}(N_{\rm TF})$ group. The condensate preserves the accidental U(1) symmetry rotating C_N and $\bar{C}_N$ with opposite phases, that generalises the species symmetry defined for $\mathrm{SU}(N)_{\rm TC}$ theories.

The important novelty of this class of models is that the technicolor representation is real. This has various consequences: TCπ are $\Psi\Psi$ states and there is no distinction between TCb and anti-TCb. Moreover N, V, G techni-quarks lie in real representations under both $G_{\rm SM}$ and $\mathrm{SO}(N)_{\rm TC}$ and can have Majorana masses that do not arise in $\mathrm{SU}(N)_{\rm TC}$ models.

3.1 Techni-pions and techni-baryons of $\mathrm{SO}(N)_{\rm TC}$

There are important differences with respect to $\mathrm{SU}(N)_{\rm TC}$ models.

Techni-pions are now $\Psi\Psi$ states, such that, if species number is conserved, TCπ made of $C_N C_N$ are stable because they have species number 2. Furthermore they have quantum numbers under the SM gauge group not compatible with DM phenomenology. Real techni-quarks R_N instead do not produce stable TCπ since the techni-quark condensate and masses break their species number.

The presence of at least one techni-quark in a real representation is a necessary ingredient to build viable models without unwanted stable TCπ. In fact, Yukawa couplings of the form HR_NC_N can break the unwanted species symmetries allowing all TCπ to decay. The allowed Yukawa interactions with the Higgs are (analogously to eq. (2.3)):

$$HL(E \text{ or } T \text{ or } N \text{ or } V), \qquad HQ(D \text{ or } U), \qquad HDX. \tag{3.4}$$

G-parity can still be defined as in $\mathrm{SU}(N)_{\rm TC}$ theories. However, with our choice of representations, G-parity is only conserved by the SM multiplet V that in $\mathrm{SO}(N)_{\rm TC}$ theories only gives rise to (unstable) G-even TCπ.

Techni-baryons (TCb) are, as in $\mathrm{SU}(N)_{\rm TC}$ theories, antisymmetric combinations of N techni-quarks. Techni-baryon number is not conserved, such that TCb cannot have an asymmetry, two TCb can annihilate and TCb can now be real particles, e.g. Majorana fermions. The lightest TCb is stable and can be a DM candidate. For N odd stability simply follows from the accidental $\Psi \to -\Psi$ symmetry. For generic N stability follows because the $\mathrm{SO}(N)$ gauge theory actually has an accidental $\mathrm{O}(N)$ symmetry; the quotient

$\mathbb{Z}_2 = \mathrm{O}(N)/\,\mathrm{SO}(N)$ (that distinguishes orthogonal matrices according to the sign of their determinant) acts as a global symmetry group. All TCb built with the N-index anti-symmetric tensor are odd under this $\mathbb{Z}_2$ symmetry, and the lightest odd state is stable.

Since the same anti-symmetric tensor with N-indices is invariant under both $\mathrm{SU}(N)_{\rm TC}$ and $\mathrm{SO}(N)_{\rm TC}$, the TCb following from a given set of techni-quarks are the same. They must however be decomposed under different techni-flavor groups conserved by technicolor interactions: $\mathrm{SU}(N_{\rm TF})$ for $\mathrm{SU}(N)_{\rm TC}$, and $\mathrm{SO}(N_{\rm TF})$ for $\mathrm{SO}(N)_{\rm TC}$. Since $\mathrm{SO}(N_{\rm TF}) \subset \mathrm{SU}(N_{\rm TF})$, one can start from the TCb of $\mathrm{SU}(N_{\rm TF})$ and split them into $\mathrm{SO}(N_{\rm TF})$ multiplets. The group-theoretic decomposition rules that connect the TCb representations of $\mathrm{SU}(N_{\rm TF})$ and $\mathrm{SO}(N_{\rm TF})$ are the following:[6]

$$\begin{aligned}
N=3: &\qquad \left(\yng(2,1)\right)_{\mathrm{SU}(N_{\rm TF})} = \left(\yng(2,1) \oplus \yng(1)\right)_{\mathrm{SO}(N_{\rm TF})} \\
N=4: &\qquad \left(\yng(2,2)\right)_{\mathrm{SU}(N_{\rm TF})} = \left(\yng(2,2) \oplus \yng(2) \oplus 1\right)_{\mathrm{SO}(N_{\rm TF})} \\
N=5: &\qquad \left(\yng(3,2)\right)_{\mathrm{SU}(N_{\rm TF})} = \left(\yng(3,2) \oplus \yng(2,1) \oplus \yng(3) \oplus \yng(1)\right)_{\mathrm{SO}(N_{\rm TF})} .
\end{aligned} \tag{3.5}$$

This leads to a novel physical phenomenon: $\mathrm{SO}(N)_{\rm TC}$ gives different masses to the TCb multiplets that were degenerate in $\mathrm{SU}(N)_{\rm TC}$ models. For example, in ordinary QCD, if color $\mathrm{SU}(3)$ were replaced by $\mathrm{SO}(3)$ (with 3 quarks in its real fundamental representation), the 'eightfold way' would split into 'threefold way' and 'pentafold' way:

$$8 = \left(\yng(2,1)\right)_{\mathrm{SU}(3)} = \left(\yng(2,1) \oplus \yng(1)\right)_{\mathrm{SO}(3)} = 5 \oplus 3\,, \tag{3.6}$$

with a similar decomposition for the heavier decuplet of spin-3/2 baryons:

$$10 = \left(\yng(3)\right)_{\mathrm{SU}(3)} = \left(\yng(3) \oplus \yng(1)\right)_{\mathrm{SO}(3)} = 7 \oplus 3\,. \tag{3.7}$$

Unfortunately, QCD gives us no guidance in understanding a crucial question for composite DM phenomenology: *which* $\mathrm{SO}(N_{\rm TF})$ *multiplet contains the lighter TCb, given that more representations have the same spin?*

Given that composite spin-1 resonances behave as gauge vectors of the techni-flavor symmetries, and that gauging of global symmetries likely generates positive contributions to TCb masses, a plausible answer is that the lightest TCb multiplet is the one in the smallest representation of $\mathrm{SO}(N_{\rm TF})$ among those with lowest spin. We will make this assumption in what follows (dedicated lattice simulations could check this, present results do not allow to settle the issue [31]). This means that for N odd the lightest TCb will be in the vectorial representation of $\mathrm{SO}(N_{\rm TF})$ (denoted by $\square$) with the same quantum numbers as techni-quarks Ψ itself, while for even N it will be a singlet of $\mathrm{SO}(N_{\rm TF})$.

Even within the assumption above, if techni-quark masses are comparable to $\Lambda_{\rm TC}$, it becomes possible that the lightest TCb belongs to a higher $\mathrm{SO}(N_{\rm TF})$ representation. For

[6]The information contained in these $\mathrm{SO}(N_{\rm TF})$ Young diagrams is redundant for small $N_{\rm TF}$. Only diagrams with as many rows as the rank of the corresponding $\mathrm{SO}(N_{\rm TF})$ group are independent. The rank of $\mathrm{SO}(N_{\rm TF})$ is $N_{\rm TF}/2$ for $N_{\rm TF}$ even and $(N_{\rm TF}-1)/2$ for $N_{\rm TF}$ odd.

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SO(N) techni-color. Techni-quarks	Yukawa couplings	Allowed N	Techni-pions	Techni-baryons	under
$N_{\rm TF}=3$			5	$3,1,\ldots$ for $N=3,4,\ldots$	$\mathrm{SO}(3)_{\rm TF}$
$\Psi=V$	0	$3,4,\ldots,7$	unstable	$V^N=3,1,\ldots$	$\mathrm{SU}(2)_L$
$N_{\rm TF}=4$			9	$4,1,\ldots$	$\mathrm{SO}(4)_{\rm TF}$
$\Psi=N\oplus V$	0	$3,4,\ldots,7$	3	$VVN=1, V(VV+NN)=3,$ $VV(VV+NN)=1,\ldots$	$\mathrm{SU}(2)_L$ $\mathrm{SU}(2)_L$
$N_{\rm TF}=5$			14	$5,1\ldots$	$\mathrm{SO}(5)_{\rm TF}$
$\Psi=L\oplus N$	1	$3,4,\ldots,14$	unstable	$L\bar{L}N=1,$ $L\bar{L}(L\bar{L}+NN)=1,\ldots$	$\mathrm{SU}(2)_L$ $\mathrm{SU}(2)_L$
$N_{\rm TF}=7$			27	$1,\ldots$	$\mathrm{SO}(7)_{\rm TF}$
$\Psi=L\oplus V$	1	4	unstable	$(L\bar{L}+VV)^2=1$	$\mathrm{SU}(2)_L$
$\Psi=L\oplus E\oplus N$	2	$4,5$	unstable	$(E\bar{E}+L\bar{L})^2+NN(L\bar{L}+E\bar{E})=1$	$\mathrm{SU}(2)_L$
$N_{\rm TF}=8$			35	1	$\mathrm{SO}(8)_{\rm TF}$
$\Psi=G$	0	4	unstable	$GGGG=1$	$\mathrm{SU}(2)_L$
$\Psi=L\oplus N\oplus V$	2	4	unstable	$(L\bar{L}+VV)^2+NN(L\bar{L}+VV)=1$	$\mathrm{SU}(2)_L$
$N_{\rm TF}=9$			44	1	$\mathrm{SO}(9)_{\rm TF}$
$\Psi=L\oplus E\oplus V$	2	4	unstable	$(E\bar{E}+L\bar{L}+VV)^2=1$	$\mathrm{SU}(2)_L$
$N_{\rm TF}=10$			54	1	$\mathrm{SO}(10)_{\rm TF}$
$\Psi=L\oplus E\oplus V\oplus N$	3	4	unstable	as $L\oplus E\oplus V+NN(L\bar{L}+E\bar{E}+VV)=1$	$\mathrm{SU}(2)_L$

Table 3. Golden-class models with $\mathrm{SO}(N)_{\rm TC}$ techni-color. Notations are as in table 2. In various models the DM candidate is a linear combination of states.

completeness, we therefore also specify the SM decomposition of the higher $\mathrm{SO}(N_{\rm TF})$ representations appearing in eq. (3.5). Notice that for $N=4$, the $\yng(2)$ representation coincides with the representation of the TCπ, so we only need to specify the $\yng(2,2)$ representation. Analogously, for $N=5$ we only need to decompose $\yng(3,2)$ and $\yng(3)$.

Finally, the members of the lightest TCb $\mathrm{SO}(N_{\rm TF})$ multiplet are further split by SM gauge interactions and the lightest TCb is the one with the smallest SM charge.

3.2 SO(N)$_{\rm TC}$ golden-class models

As discussed above, avoiding unwanted stable TCπ implies that the model must contain at least one real V, N, G state with Majorana mass. This leads to real DM states, with important consequences for DM phenomenology discussed in section 5.2. With the assumption that the lightest TCb multiplet is the one in the smallest representation of $\mathrm{SO}(N_{\rm TF})$ among those with lowest spin, table 3 lists the golden-class models discussed below. These are the models that give a DM candidate without unwanted stable particles. In appendix B we will present the silver-class models that need extra assumptions to break accidental symmetries in order to avoid unwanted stable states.

a) SO(N)$_{\rm TC}$ model $\Psi=V$. This model has $N_{\rm TF}=3$; TCπ are unstable, as they lie in the G-even representation 5_0 under $\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y$. Landau poles are avoided for

$N \leq 7$ and TCb DM candidates V^N exist for any N. For $N = 3$, the lightest DM candidate has spin 1/2 and lies in the 3_0 representation while the heavier TCb lie in the $\ydiagram{2,1} = 5_0$ multiplet. For $N = 4$ the TCb DM candidate is a scalar singlet. Also, we have heavier TCb in the $\ydiagram{2} = 5_0$ representation of $\mathrm{SU}(2)_L$, while the representation $\ydiagram{2,2}$ is absent. Finally, for $N = 5$, the lightest DM candidate is a 3_0 multiplet with spin 1/2, the heavier TCb are a $\ydiagram{2,1} = 5_0$ multiplet and a $\ydiagram{3} = 7_0$ multiplet while the $\ydiagram{3,2}$ representation is absent.

b) $\mathrm{SO}(N)_{\mathrm{TC}}$ model $\Psi = N \oplus V$. This extension of the previous model has $N_{\mathrm{TF}} = 4$ and it is allowed up to $N = 7$. The model gives an extended list of TCπ

$$\mathrm{TC}\pi: \; 9 = 1_0 \oplus 3_0 \oplus 5_0 \qquad \text{under } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y. \tag{3.8}$$

The 3_0 is stable because of species number, giving a TCπ DM candidate. For $N = 3$ the lightest TCb DM candidate lives in the 4-dimensional $\ydiagram{1}$ representation of $\mathrm{SO}(4)_{\mathrm{TF}}$ that is composed by a singlet NVV and a triplet made by a linear combination of VNN and VVV. For $N = 4$ the TCb DM candidate is a singlet linear combination of $VVVV$, $VVNN$. The remaining heavier TCb for $N = 3$ are

$$\mathrm{TCb}: \; \ydiagram{2,1} = 16 = 2 \times 3_0 \oplus 2 \times 5_0 \quad \text{under } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y\,. \tag{3.9}$$

As explained before, for $N = 4$ it is enough to specify the following decomposition

$$\mathrm{TCb}: \; \ydiagram{2,2} = 10 = 2 \times 5_0 \quad \text{under } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y\,, \tag{3.10}$$

to describe all possible TCb.

c) $\mathrm{SO}(N)_{\mathrm{TC}}$ model $\Psi = G$. For $N = 4$ this model with $N_{\mathrm{TF}} = 8$ avoids a sub-Planckian Landau pole for g_3 and, at the same time, techni-color is asymptotically free, $b_{\mathrm{TC}} = -2$. The model leads to the following colored TCπ, that undergo anomalous decays to gluons:

$$\mathrm{TC}\pi: \; 35 = 8_0 \oplus 27_0 \qquad \text{under } \mathrm{SU}(3)_c \otimes \mathrm{U}(1)_Y\,. \tag{3.11}$$

The TCb DM candidate is the SM singlet $GGGG$ and the remaining heavier TCb are:

$$\mathrm{TCb}: \; \ydiagram{2,2} = 300 = 1_0 \oplus 8_0 \oplus 3 \times 27_0 \oplus 64_0 \oplus \Big(10_0 \oplus 28_0 \oplus 35_0 \oplus \text{h.c.}\Big) \tag{3.12}$$

under $\mathrm{SU}(3)_c \otimes \mathrm{U}(1)_Y$, plus a set of TCb living in the same representations as the TCπ above.

d) $\mathrm{SO}(N)_{\mathrm{TC}}$ model $\Psi = L \oplus N$. This model with $N_{\mathrm{TF}} = 5$ allows for a Yukawa coupling that involve the neutral state N, such that all TCπ decay. They fill a 14 of $\mathrm{SO}(5)_{\mathrm{TF}}$ that decomposes as

$$\mathrm{TC}\pi: \; 14 = 3_{\pm 1,0} \oplus 2_{\pm 1/2} \oplus 1_0 \qquad \text{under } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y. \tag{3.13}$$

This model exists for $3 \leq N \leq 14$. Let us consider $N = \{3, 4, 5\}$, for which the lightest TCb are all SM singlets. For example, for $N = 3, 4$ they are $NL\bar{L}$ and $L\bar{L}(L\bar{L} + NN)$

respectively. To specify the complete set of TCb, we need the following decompositions

$$
\text{TCb}:\begin{cases}
\yng(2,1) = 35 = 1_{0,\pm1}\oplus 2_{\pm1/2,\pm1/2,\pm3/2}\oplus 3_{2\times0,\pm1}\oplus 4_{\pm1/2} & \text{for } N=3\\
\yng(2,2) = 35 = 1_{0,\pm1,\pm2}\oplus 2_{\pm1/2,\pm3/2}\oplus 3_{0,\pm1}\oplus 4_{\pm1/2}\oplus 5_0 & \text{for } N=4\\
\yng(3,1)\oplus\yng(3) = 105\oplus 30 = (6_{\pm1/2}\oplus 5_{\pm1,2\times0}\oplus 4_{\pm3/2,2\times(\pm1/2)} & \\
\quad\oplus\ 3_{2\times(\pm1,0),\pm2}\oplus 2_{2\times(\pm3/2,\pm1/2),\pm\frac{5}{2}}\oplus 1_{0,\pm(1,2)})\oplus & \\
\quad(4_{\pm3/2,\pm1/2}\oplus 3_{\pm1,0}\oplus 2_{\pm1/2}+1_0) & \text{for } N=5
\end{cases}
\tag{3.14}
$$

under $\mathrm{SU}(2)_L\otimes \mathrm{U}(1)_Y$. Taking into account the Yukawa couplings, in this model and in the following models the TCb mix giving real eigenstates which are all good DM candidates, with a peculiar phenomenology discussed in section 5.2.

In the limit $m_N \gg \Lambda_{\rm TC}$ the N state can be integrated out realizing nicely the silver-class model $\Psi = L$ presented in appendix B.

e) $\mathbf{SO}(\boldsymbol{N})_{\mathbf{TC}}$ model $\boldsymbol{L}\oplus\boldsymbol{V}$. This model with $N_{\rm TF}=7$ is similar to the $\Psi = L\oplus N$ but with a more complex set of TCπ

$$
\text{TC}\pi:\ 27 = 5_0\oplus 4_{\pm1/2}\oplus 3_{\pm1,0}\oplus 2_{\pm1/2}\oplus 1_0 \qquad \text{under } \mathrm{SU}(2)_L\otimes \mathrm{U}(1)_Y. \tag{3.15}
$$

The strong coupling $g_{\rm TC}$ is asymptotically free for $N\geq 4$ and g_2 avoids a sub-Planckian Landau pole for $N\leq 4$ (with $N=5$ slightly excluded). For $N=4$ the TCb DM candidate is the SM singlet $(L\bar{L}+VV)^2$ and the remaining heavier TCb decompose under $\mathrm{SU}(2)_L\otimes \mathrm{U}(1)_Y$ as:

$$
\begin{aligned}
\text{TCb}:\ \yng(2,2) = 168 = {} & 1_{3\times0,\pm1,\pm2}\oplus 2_{3\times(\pm1/2),\pm3/2}\oplus 3_{4\times0,3\times(\pm1)}\oplus 4_{4\times(\pm1/2),\pm3/2}\\
& \oplus 5_{4\times0,\pm1}\oplus 6_{2\times1/2}\oplus 7_{0,\pm1} \qquad \text{under } \mathrm{SU}(2)_L\otimes \mathrm{U}(1)_Y\ .
\end{aligned}\tag{3.16}
$$

f) $\mathbf{SO}(\boldsymbol{N})_{\mathbf{TC}}$ model $\boldsymbol{\Psi} = \boldsymbol{L}\oplus\boldsymbol{E}\oplus\boldsymbol{N}$. This model with $N_{\rm TF}=7$ and 2 Yukawa couplings HLN and HLE predicts the following unstable TCπ

$$
\text{TC}\pi:\ 27 = 3_{\pm1,0}\oplus 2_{\pm3/2}\oplus 2\times 2_{\pm1/2}\oplus 1_{\pm2,\pm1}\oplus 2\times 1_0 \qquad \text{under } \mathrm{SU}(2)_L\otimes \mathrm{U}(1)_Y. \tag{3.17}
$$

The model exists for $N=4,5$. For $N=4$ the DM candidate is a singlet, then to fully specify the complete set of TCb we need to decompose the multiplet:

$$
\begin{aligned}
\text{TCb}:\ \yng(2,2) = 168 = {} & 5_0\oplus 4_{\pm3/2,2\times(\pm1/2)}\oplus 3_{\pm3,2\times(\pm2),5\times(\pm1),5\times0}\\
& \oplus 2_{2\times(\pm5/2),5\times(\pm3/2),7\times(\pm1/2)}\oplus 1_{3\times(\pm2),4\times(\pm1),6\times0}
\end{aligned}\tag{3.18}
$$

under $\mathrm{SU}(2)_L\otimes \mathrm{U}(1)_Y$.

g) $\mathbf{SO}(\boldsymbol{N})_{\mathbf{TC}}$ model $\boldsymbol{\Psi} = \boldsymbol{L}\oplus\boldsymbol{V}\oplus\boldsymbol{E}$. This model with $N_{\rm TF}=9$ and 2 Yukawa couplings HLV and HLE, gives rise to the set of unstable TCπ:

$$
\text{TC}\pi:\ 44 = 5_0\oplus 4_{\pm1/2}\oplus 3_{2\times(\pm1),0}\oplus 2_{\pm3/2,2\times(\pm1/2)}\oplus 1_{\pm2}\oplus 2\times 1_0 \qquad \text{under } \mathrm{SU}(2)_L\otimes \mathrm{U}(1)_Y. \tag{3.19}
$$

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The model exists and gives a singlet TCb DM candidate for $N = 4$. The multiplet of the remaining heavier TCb is:

$$\begin{aligned}\text{TCb}: \begin{array}{|c|c|}\hline \; & \; \\ \hline \; & \; \\ \hline\end{array} = 495 = {}& 1_{8\times 0,4\times(\pm 1),3\times(\pm 2)} \oplus 2_{10\times(\pm 1/2),6\times(\pm 3/2),2\times(\pm 5/2)} \\ &\oplus 3_{11\times 0,10\times(\pm 1),3\times(\pm 2),\pm 3} \oplus 4_{9\times(\pm 1/2),5\times(\pm 3/2),\pm 5/2} \\ &\oplus 5_{7\times 0,4\times(\pm 1),2\times(\pm 2)} \oplus 6_{3\times(\pm 1/2),\pm 3/2} \oplus 7_{0,\pm 1}\end{aligned} \tag{3.20}$$

under $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$.

h) $\mathbf{SO}(N)_{\mathbf{TC}}$ model $\Psi = L \oplus V \oplus N$. This model has $N_{\mathrm{TF}} = 8$ and 2 Yukawa couplings (HLV, HLN) are allowed, so that all TCπ decay:

$$\mathrm{TC}\pi : 35 = 5_0 \oplus 4_{\pm 1/2} \oplus 3_{\pm 1,0,0} \oplus 2 \times 2_{\pm 1/2} \oplus 2 \times 1_0 \qquad \text{under } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y. \tag{3.21}$$

The model is allowed only for $N = 4$ and gives a singlet TCb DM candidate. The complete set of TCb contains the multiplet:

$$\begin{aligned}\text{TCb}: \begin{array}{|c|c|}\hline \; & \; \\ \hline \; & \; \\ \hline\end{array} = 300 = {}& 7_{\pm 1,0} \oplus 6_{3\times(\pm 1/2)} \oplus 5_{2\times(\pm 1),8\times 0} \oplus 4_{\pm 3/2,8\times(\pm 1/2)} \oplus 3_{6\times(\pm 1),9\times 0} \\ &\oplus 2_{2\times(\pm 3/2),7\times(\pm 1/2)} \oplus 1_{\pm 2,2\times(\pm 1),6\times 0} \qquad \text{under } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y.\end{aligned} \tag{3.22}$$

i) $\mathbf{SO}(N)_{\mathbf{TC}}$ model $\Psi = L \oplus E \oplus V \oplus N$. This model with $N_{\mathrm{TF}} = 10$ and 3 Yukawa couplings HLV, HLN, HLE predicts the following unstable TCπ:

$$\mathrm{TC}\pi : 54 = 5_0 \oplus 4_{\pm 1/2} \oplus 2 \times 3_{\pm 1,0} \oplus 2_{\pm 3/2,3\times(\pm 1/2)} \oplus 1_{\pm 2,\pm 1} \oplus 3 \times 1_0 \quad \text{under } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y. \tag{3.23}$$

For $N = 4$, the model gives the singlet TCb DM candidates, while the multiplet of the remaining heavier TCb is:

$$\begin{aligned}\text{TCb}: \begin{array}{|c|c|}\hline \; & \; \\ \hline \; & \; \\ \hline\end{array} = 770 = {}& 1_{5\times(\pm 2),8\times(\pm 1),14\times 0} \oplus 2_{3\times(\pm 5/2),11\times(\pm 3/2),19\times(\pm 1/2)} \\ &\oplus 3_{\pm 3,5\times(\pm 2),17\times(\pm 1),20\times 0} \oplus 4_{\pm 5/2,7\times(\pm 3/2),15\times(\pm 1/2)} \\ &\oplus 5_{2\times(\pm 2),6\times(\pm 1),11\times 0} \oplus 6_{\pm 3/2,4\times(\pm 1/2)} \oplus 7_{\pm 1,0}\end{aligned} \tag{3.24}$$

under $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$.

4 Techni-quark masses and the θ_{TC} angle

In [22] we considered composite dark matter theories in the limit of massless techni-quarks. With masses (such that also the CP-violating θ_{TC} angle becomes physical) the theory has a few more free parameters, that significantly affect its phenomenology. From a phenomenological point of view, we are mostly interested in checking that a successful TCb DM candidate is indeed the lightest TCb and in computing its interactions. The main new feature relevant for DM direct detection is that DM TCb fermion has magnetic and electric dipoles with moments

$$\bar{\Psi}\gamma_{\mu\nu}(\mu_M + i d_E \gamma_5)\Psi \; F_{\mu\nu}/2. \tag{4.1}$$

We estimate

$$\mu_M \sim \frac{e}{M_{\rm DM}}, \qquad d_E \sim \frac{e\,\theta_{\rm TC}}{16\pi^2}\frac{\min[m_\Psi]}{f^2} \sim \frac{e\,\theta_{\rm TC}\min[m_\Psi]}{M_{\rm DM}^2}. \tag{4.2}$$

A magnetic moment with order 1 gyro-magnetic ratio is typical of composite states. The smaller electric dipole is generated when CP is violated by a non-zero $\theta_{\rm TC}$. For $\theta_{\rm TC} \sim \mathcal{O}(1)$ EDM could give striking effects in direct detection as we will see in section 5. Chromo-dipoles are generated in models with colored constituents.

4.1 A QCD-like example

To illustrate the effects of the $\theta_{\rm TC}$ angle, assumed to be large unlike the QCD θ-angle, we work out in detail the silver-class model with $\mathrm{SU}(3)_{\rm TC}$ and $\Psi = L \oplus E$ techni-quarks, described in section B.1. In this scenario the techni-strong dynamics is identical to QCD with three flavors and therefore we can rescale QCD data to make definite predictions. For this choice of quantum numbers no Yukawa couplings are allowed, such that charged TCπ are stable at renormalizable level. We assume that non-renormalizable operators break species number symmetry leading to unstable TCπ, and that DM is the singlet neutral TCb.

The TCπ in the adjoint of $\mathrm{SU}(3)_{\rm TF}$ and the anomalous $\mathrm{U}(1)$ singlet are described by the hermitian matrix

$$\Pi = \begin{pmatrix} \pi_3^0/\sqrt{2} + \pi_1^0/\sqrt{6} & \pi_3^+ & \pi_2^- \\ \pi_3^- & -\pi_3^0/\sqrt{2} + \pi_1^0/\sqrt{6} & \pi_2^{--} \\ \pi_2^+ & \pi_2^{++} & -2\pi_1^0/\sqrt{6} \end{pmatrix} + \frac{\eta'}{\sqrt{3}}\mathbb{1}_3. \tag{4.3}$$

This is as in QCD, but with different charges for the isospin doublets. The effective TCπ Lagrangian described in [30] reads

$$\mathscr{L}_{\rm TC\pi} \approx \frac{f^2}{4}\left\{ \mathrm{Tr}[D_\mu U D^\mu U^\dagger] + 2B_0 \mathrm{Tr}[M(U+U^\dagger)] - \frac{a}{3}\left[\theta_{\rm TC} - \frac{i}{2}(\ln\det U - \ln\det U^\dagger)\right]^2 \right\}, \tag{4.4}$$

where $U = \langle U\rangle e^{i\sqrt{2}\Pi/f}$ is the TCπ matrix. The second term in the lagrangian describes the effect of techni-quark masses, where $M = \mathrm{diag}(m_L, m_L, m_E)$, and B_0 is the chiral condensate. The last term encodes the effect of the $\theta_{\rm TC}$ angle and $\mathrm{U}(1)$ axial anomaly that gives mass to the techni-η', $m_{\eta'}^2 \sim a + \mathcal{O}(m)$.

The VEV $\langle U\rangle$ is determined dynamically by minimising the potential. One can conveniently look for a solution of the form

$$\langle U\rangle = \mathrm{diag}\,(e^{-i\phi_L}, e^{-i\phi_L}, e^{-i\phi_E}). \tag{4.5}$$

The extrema of the potential are determined by the Dashen's equations:

$$\chi_L^2 \sin\phi_L = \frac{a}{3}(\theta_{\rm TC} - 2\phi_L - \phi_E), \qquad \chi_E^2 \sin\phi_E = \frac{a}{3}(\theta_{\rm TC} - 2\phi_L - \phi_E), \tag{4.6}$$

where we defined $\chi_{E,L}^2 \equiv -2m_{E,L}B_0$. It is easy to check that $\langle U\rangle \neq \mathbb{1}$ when $\theta_{\rm TC} \neq 0$ and techni-quark masses are different from zero. A non-vanishing $\theta_{\rm TC}$ modifies the TCπ

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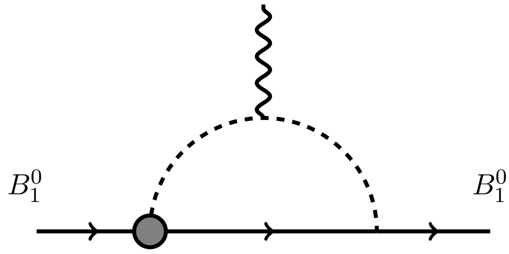

Figure 2. Leading logarithmic contribution to the EDM of the DM candidate B_1^0. Internal dashed and continuous lines represent respectively π_2, B_2 states. The gray dot correspond to the CP violating πBB vertex proportional to b_1, b_2, while the other πBB vertex is proportional to the D and F derivative couplings.

spectrum such that $m_{E,L} \to m_{E,L} \cos\phi_{E,L}$ in the mass formulæ and generates CP violating interactions among the TCπ. In the limit $m'_\eta \gg m_\pi$ (corresponding to $\chi^2_{L,E} \ll a$) and neglecting gauge contributions one finds

$$m^2_{\pi_3} = \chi^2_L \cos\phi_L\,, \qquad m^2_{\pi_2} = \frac{\chi^2_L \cos\phi_L + \chi^2_E \cos\phi_E}{2}\,, \qquad m^2_{\pi_1} = \frac{1}{3}(\chi^2_L \cos\phi_L + 2\chi^2_E \cos\phi_E)\,. \tag{4.7}$$

Since $\cos\phi_{E,L}$ can be negative the effect of $\theta_{\rm TC}$ cannot be entirely reabsorbed by redefining the techni-quark masses (for example, in real world QCD, the measured pion spectrum is compatible with $\theta = 0$ but not with $\theta = \pi$ [32]).

The spectrum of TCb can be computed with similar techniques. The octet contains

$$B = \begin{pmatrix} B_3^0/\sqrt{2} + B_1^0/\sqrt{6} & B_3^+ & B_2^- \\ B_3^- & -B_3^0/\sqrt{2} + B_1^0/\sqrt{6} & B_2^{--} \\ B_{2'}^+ & B_{2'}^{++} & -2B_1^0/\sqrt{6} \end{pmatrix}\,. \tag{4.8}$$

where B_2 and B_2' are the analog of the nucleon and the Ξ doublet respectively, B_3 of the triplet Σ and B_1^0 of the singlet Λ. The effective lagrangian for the TCb can be found in [30]. It contains the following terms relevant to the present discussion:

$$\begin{aligned} \mathscr{L}_{kin} &= {\rm Tr}[\bar{B}(i\not{D} - m_B)B] - 2\left(b_1\ {\rm Tr}[\bar{B}M_\theta B] + b_2\ {\rm Tr}[\bar{B}BM_\theta]\right)\,, \\ \mathscr{L}_{BB\Pi,\theta} &= -\frac{2\sqrt{2}a}{3f}\left(\theta_{\rm TC} - 2\phi_L - \phi_E\right)\left(b_1 {\rm Tr}[\bar{B}\Pi B] + b_2 {\rm Tr}[\bar{B}B\Pi]\right) + \dots\,, \\ \mathscr{L}_{BB\Pi} &= -\frac{D+F}{\sqrt{2}f}{\rm Tr}[\bar{B}\gamma^\mu\gamma_5(D_\mu\Pi)B] - \frac{D-F}{\sqrt{2}f}{\rm Tr}[\bar{B}\gamma^\mu\gamma_5 B(D_\mu\Pi)] + \dots\,, \end{aligned} \tag{4.9}$$

where m_B is the common TCb mass generated by the strong interactions and M_θ is the techni-quark mass matrix that depends on $\theta_{\rm TC}$ angle through eq. (4.5)

$$M_\theta = {\rm diag}(\chi^2_L \cos\phi_L,\, \chi^2_L \cos\phi_L,\, \chi^2_E \cos\phi_E). \tag{4.10}$$

The second line of eq. (4.9) describes the CP violating interactions induced by $\theta_{\rm TC}$ relevant for the computation of electric dipoles and the third line contains derivative interactions with the TCπ. Dots stand for non-linear terms irrelevant for the present discussion. All

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the parameters of the effective lagrangian are determined by rescaling QCD data in terms of the dynamical scale,

$$\begin{array}{llll} \dfrac{m_\rho}{f}\sim 8\,, & \dfrac{B_0}{m_\rho}\sim -2\,, & \dfrac{m_B}{m_\rho}\sim 1.3\,, & \Delta^g m_\pi^2\sim \dfrac{3\alpha_2}{4\pi}(J(J+1))\,m_\rho^2 \\ m_B\,b_1\sim 0.15\,, & m_B\,b_2\sim -0.3\,, & D\sim 0.6\,, & F\sim 0.4 \end{array} \tag{4.11}$$

where J is the isospin of the TCπ multiplet. From the first line of eq. (4.9) the mass splittings between TCb due to techni-quark masses reads

$$\begin{aligned} \Delta m_{B_2} &= 2(b_1\chi_L^2\cos\phi_L + b_2\chi_E^2\cos\phi_E)\,, & \Delta m_{B_{2'}} &= 2(b_2\chi_L^2\cos\phi_L + b_1\chi_E^2\cos\phi_E)\,, \\ \Delta m_{B_3} &= 2(b_1+b_2)\chi_L^2\cos\phi_L\,, & \Delta m_{B_1} &= 2/3(\chi_L^2\cos\phi_L + 2\chi_E^2\cos\phi_E)(b_1+b_2)\,. \end{aligned} \tag{4.12}$$

The LLE states, corresponding to the triplet B_3 and the singlet B_1 have zero hypercharge. Therefore they can be viable DM candidates if they are the lightest TCb. Using the QCD values of b_1 and b_2,we find that techni-quark masses always favor B_2 or $B_{2'}$ to be the lightest TCb. The neutral LLE state can be the lightest TCb when the mass splitting due to SM gauge interactions is more important than the mass splitting due to techni-quark masses. This can be realised in the symmetric limit $\chi_L = \chi_E \equiv \chi$ where techni-quark masses respect the techni-flavor symmetry and the singlet B_1^0 (analog of the Λ) is most likely the lightest TCb.

In the limit $\chi_L = \chi_E \ll a$ we can solve Dashen's equations analytically. The solution has multiple branches labelled by the integer n [53],

$$\phi_L = \phi_E - 2\pi n \simeq \frac{\theta_{\rm TC} - 2\pi n}{3}\,. \tag{4.13}$$

The solution with minimum energy has a discontinuity at $\theta_{\rm TC} = \pi$ where it jumps from $n = 0$ to $n = 1$. This is necessary to restore the periodicity in $\theta_{\rm TC}$.

4.2 Electric dipole of the DM candidates

We parameterize dipole moments in terms of gyromagnetic factors $g_{M,E}$ as

$$\mu_M = \frac{e g_M}{2M_{\rm DM}}, \qquad d_E = \frac{e g_E}{2M_{\rm DM}}. \tag{4.14}$$

Following [33], to leading order the dipole moments are proportional to the electric charge,

$$\mu(B) = \alpha\,{\rm Tr}[BB^\dagger Q] + \beta\,{\rm Tr}[BQB^\dagger] \tag{4.15}$$

where α and β are properties of the strong dynamics that, for the QCD-like model, can be extracted from the measured magnetic moments of baryons in QCD. What is different in our context is the charge matrix $Q = {\rm diag}(0\,, -1\,, 1)$. Plugging in the equation above we estimate $g_M^{B_1} \sim 2.8$.

The same argument applies to the EDMs. To estimate the coefficient we proceed as in [30] for the computation of the neutron EDM. The CP violating vertices from the mass terms in eq. (4.9) generate one-loop graphs that contribute to the EDM. The dominant contributions are given by the logarithmically divergent diagrams represented in figure 2. Similarly to the computation of the neutron EDM we obtain the estimate,

$$g_E^{B_1} \simeq -\frac{3\chi^2}{4\pi^2 f^2} M_{\rm DM}\,[b_1(D+F) - b_2(D-F)]\ln\frac{m_{B_2}^2}{m_{\pi_2}^2} \times \sin\frac{\theta_{\rm TC}}{3} \qquad \text{for } \theta_{\rm TC} < \pi. \quad (4.16)$$

For $\theta_{\rm TC} \lesssim 1$ using the numerical values in (4.11) we obtain

$$g_E^{B_1} \simeq -0.15\,\frac{m_{\pi_2}^2}{f^2}\log\frac{m_B^2}{m_\pi^2} \times \theta_{\rm TC}. \quad (4.17)$$

The discussion above can be easily generalised to other models. For example the model $\Psi = V$ for $N = 3$ has again the same dynamics as QCD. From eq. (4.15) one can see that the magnetic and electric dipole moments of the TCb dark matter candidate (the neutral component of an isospin triplet) are zero.

For different N and $N_{\rm TF}$ the relevant dynamics can be parametrized in terms of few unknown parameters that could in principle be extracted from lattice simulations. For TCπ the discussion is identical to eq. (4.4) with a number of Dashen's equation equal to the number of SM representations of the model. TCb are in general described by a tensor of SU($N_{\rm TF}$) $B_{i_1\, i_2 \ldots i_N}$ with the symmetry of Young tableaux as in (2.10). Their effective lagrangian is constructed writing all possible techni-flavor invariant combinations of the techni-baryon fields B and $\bar{B}$ with the techni-quark mass matrix M transforming in the adjoint representation of SU($N_{\rm TF}$).

For N odd there are two non-trivial invariants:

$${\rm Tr}[\bar{B}MB]\,,\ {\rm Tr}[\bar{B}BM]\,. \quad (4.18)$$

Since the TCπ are in the adjoint representation, other two invariants can be written with derivative interactions that do not break the global symmetries.

For N even, a single invariant can be written down: group theory uniquely fixes the mass splitting among TCb up to its overall coefficient. For example, in the model with $N = N_{\rm TF} = 4$ we predict equal mass differences between the TCb.

5 Phenomenology of Composite Dark Matter

We here briefly outline the phenomenology of the scenarios with TCb dark matter.[7]

This crucially depends on the TCb mass. Cosmology singles out two special values:

$$M_{\rm DM} \approx \begin{cases} 100\,{\rm TeV} & \text{if DM is a thermal relic,} \\ 3\,{\rm TeV} & \text{if DM is a complex state with a TCb asymmetry [26, 27].} \end{cases} \quad (5.1)$$

In the first case, the cosmological relic abundance is determined by the non-relativistic annihilation cross-section of TCb, that annihilate into TCπ through strong interactions

[7] If TCπ are stable due to accidental symmetries their mass should not exceed few TeV not to overclose the universe. The TCπ DM in this case likely dominates and behaves as the minimal dark matter candidates studied in [1–3].

and to SM states through gauge interactions. We can neglect the second sub-dominant effect. Rescaling the measured $p\bar{p}$ annihilation cross-section one finds [22] that the thermal DM abundance is reproduced for $M_{\rm DM} \sim 200\,\text{TeV}$.

5.1 Direct detection of complex Dark Matter

In various models, the DM candidate is a complex state with $Y = 0$ in the triplet or quintuplet representation of $\text{SU}(2)_L$. Its weak interactions lead to a direct-detection cross section characteristic of Minimal Dark Matter, which is too small to be observed in the present context where the DM mass is around 100 TeV, if DM is a thermal relic. Moreover in various models DM is a SM singlet, such that even this cross section is absent.

The main hope for direct detection of thermal TCb DM relies on the fact that composite DM made of charged constituents can have special interactions with the photon, leading to significant rates of low-energy scatterings. Scalar DM S can only have the dimension 6 interaction $(S^* i\partial_\mu S)\partial_\nu F_{\mu\nu}$ or higher, which does not lead to interesting rates. Fermionic DM Ψ instead can have dipole interactions as in eq. (4.1) leading to the following cross section for direct detection [34, 35]:

$$\frac{d\sigma}{dE_R} \approx \frac{e^2 Z^2}{4\pi E_R}\left(\mu_M^2 + \frac{d_E^2}{v^2}\right) \tag{5.2}$$

where v is the relative DM/nucleus velocity and E_R is the nucleus recoil energy. For simplicity, we here assumed a nucleus $\mathcal{N}$ with $A, Z \gg 1$, mass $M_{\mathcal{N}} \approx A m_N$, a recoil energy $E_R \ll M_{\mathcal{N}} v^2$, and approximated nuclear form factors with their unit value that holds at small enough E_R. In the same approximation, this cross section can be compared to the standard approximation used in searches for spin-independent DM interactions:

$$\frac{d\sigma}{dE_R} = \frac{M_{\mathcal{N}}\sigma_{\rm SI}A^2}{2\mu^2 v^2}, \qquad \mu = \frac{M_{\mathcal{N}} M_{\rm DM}}{M_{\mathcal{N}} + M_{\rm DM}}. \tag{5.3}$$

We see that the dipole cross sections has a characteristic testable enhancement at low recoil-energy E_R, arising because the DM/matter scattering is mediated by the massless photon. Furthermore, the magnetic-dipole cross section has a characteristic suppression at small $v > v_{\rm min} = \sqrt{M_{\mathcal{N}} E_R/2\mu^2}$, which could be tested relying on the seasonal variation in the average v^2.

We parameterize the dipole moments in terms of their gyro-magnetic and gyro-electric constant g_M and g_E as in eq. (4.14). Composite DM generically predicts an order one gyromagnetic factor g_M, and a possibly sizeable gyro-electric factor $g_E \sim \theta_{\rm TC}\text{Min}[m_\Psi]/M_{\rm DM}$ as discussed in section 4.

This means that for $M_{\rm DM} \approx 100\,\text{TeV}$ and $g_M \sim 1$ the magnetic effect is 3 orders of magnitude below the experimental limit,

$$\sigma_{\rm SI} < 10^{-44}\,\text{cm}^2 \frac{M_{\rm DM}}{\text{TeV}} \qquad \text{for } M_{\rm DM} \gg M_{\mathcal{N}} \tag{5.4}$$

and at the level of the neutrino background, see also [36]. The electric effect is comparable to the present LUX bound for $g_E \approx 0.01$ and $M_{\rm DM} \approx 100\,\text{TeV}$, as illustrated in figure 3a.

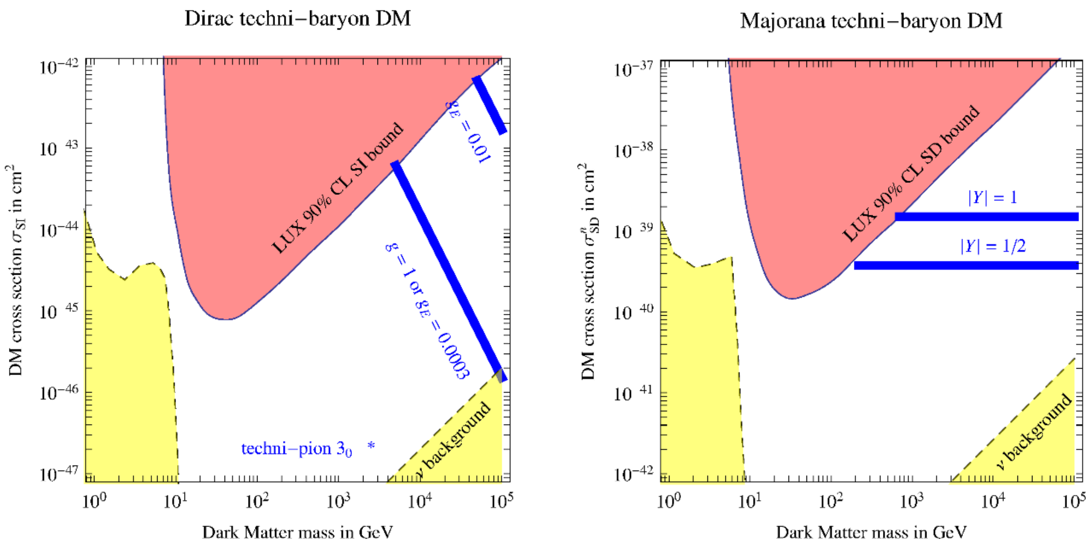

Figure 3. Predictions for direct detection of Dark Matter. Left: Dirac TCb predicted by $\mathrm{SU}(N)_{\mathrm{TC}}$ models to have magnetic and/or electric dipole moments, giving spin-independent cross section. Right: Majorana techni-baryons predicted by $\mathrm{SO}(N)_{\mathrm{TC}}$ models to have hypercharge giving spin-dependent cross section.

In some models DM has chromo-dipoles, that lead to a similar scattering rate with e^4Z^2/E_R replaced by $g_3^4/\Lambda_{\mathrm{QCD}}$ times a nuclear form factor, which is strongly suppressed at energies below Λ_{QCD}. Thereby chromo-dipoles do not compete with electric dipoles.

Some composite DM models predict that DM is a TCb with higher-spin. Spin 1 DM can have characteristic spin-dependent interactions which are, however, suppressed by the transferred momentum [37]. More interestingly, a composite spin-1 TCb B_μ could have a dimension-4 interaction $B_\mu B_\nu^* F^{\mu\nu}$ with a photon. Even when the lighter TCb is mostly composed of neutral SM singlets N, it also contains a small component of charged heavier techni-quarks with a momentum asymmetry (an effect analogous to the strange momentum asymmetry in nucleons [38]).

5.2 Direct detection of real Dark Matter

Techni-baryon DM in $\mathrm{SO}(N)_{\mathrm{TC}}$ gauge theories has novel interesting features compared to $\mathrm{SU}(N)_{\mathrm{TC}}$ models: there is no techni-baryon number conservation, so DM is a real state with no techni-baryon asymmetry. In most golden-class models, the techni-quarks have Yukawa couplings to the Higgs. As a consequence the DM candidates TCb with $Y = 0$ mix with TCb with $Y \neq 0$ after electro-weak symmetry breaking. The resulting lightest TCb is a Majorana fermion for N odd, a real scalar for N even. To illustrate this point, let us consider for example the $L \oplus N$ model with 3 techni-colors. The multiplet of lighter TCb in eq. (3.14) contains a Majorana singlet 1_0 and a Dirac weak doublet $2_{\pm 1/2}$. In view of the Yukawa couplings among the techni-quarks, the mass matrix for the neutral TCb components has the form

$$\begin{array}{c} \\ 1_0 \\ 2_{1/2} \\ 2_{-1/2} \\ \vdots \end{array} \begin{array}{c} \begin{array}{cccc} 1_0 & 2_{1/2} & 2_{-1/2} & \cdots \end{array} \\ \begin{pmatrix} m_{1_0} & y_L v & y_R v & \cdots \\ y_L^* v & 0 & m_{2_{1/2}} & \cdots \\ y_R^* v & m_{2_{1/2}} & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \end{array} \tag{5.5}$$

where the TCb Higgs couplings y can, in principle, be derived from the Yukawa couplings among techni-quarks. The dots refers to other TCb states that are expected to be heavier but could still be relevant if they mix significantly.

The mass matrix is analogous to one of the bino and higgsino in supersymmetry. Furthermore, in our scenario TCb have a common mass m_B generated by strong dynamics and are mildly split by techni-quark masses and gauge interactions: thereby the spectrum resembles the case known as 'well tempered neutralinos' [39]. Due to the mixing with Majorana states, the lightest DM TCb is a Majorana fermion. This significantly changes the phenomenology of direct detection: a Majorana fermion can neither have vector couplings to the Z, avoiding the severe constraints from spin independent cross section, nor dipole moments, removing the signals discussed in section 5.1. However, Majorana technibaryon DM χ can have an axial coupling to the Z,

$$-\, g_A Z_\mu \frac{g_2}{\cos\theta_{\rm W}} \frac{\bar{\chi}\gamma_\mu\gamma_5\chi}{2}\,, \tag{5.6}$$

that leads to a spin dependent cross-section with the nuclei. Using the present LUX bound [40, 41] $\sigma^n_{\rm SD} < 1.7\ 10^{-39} M_{\rm DM}/\,{\rm TeV}$, one finds

$$|g_A| < 1.2 \frac{M_{\rm DM}}{\rm TeV}. \tag{5.7}$$

The situation is illustrated in figure 3b.

This is a significant constraint only if the mixing angle among states of different hyper charge is large so that $g_A \sim \mathcal{O}(1)$. This situation is achieved for

$$\Delta m \equiv |m_{2_{1/2}} - m_{1_0}| \lesssim yv. \tag{5.8}$$

Even assuming negligible techni-quark masses, SM gauge interactions split singlets and doublets by a few per cent:

$$\Delta m \approx \frac{\alpha_2}{4\pi} m_B \sim 0.03 \times m_B. \tag{5.9}$$

For a TCb mass around 100 TeV the condition (5.8) is unlikely to be realised: in the opposite regime $\Delta m \gg yv$ the lightest TCb has suppressed coupling to the Z,

$$g_A \sim \frac{y^2 v^2}{\Delta m^2} \ll 1. \tag{5.10}$$

Another effect of phenomenological relevance can arise if $m_{2_{1/2}} \ll m_{1_0}$. In this case the lighter complex doublet splits into two real states, with a mass difference $\Delta m_{2_{1/2}} \approx y^2v^2/\Delta m$. The Z gives a tree level coupling between the real mass eigenstates, becoming irrelevant for direct DM searches if $\Delta m_{2_{1/2}} \gtrsim 100\,{\rm keV}$. A smaller mass difference can be obtained for $y \sim 10^{-3}$ and gives rise to inelastic DM phenomenology [42].

gauge group	Techni-quark content	Techni-pion content under $SU(2)_L \otimes U(1)_Y$								
		1_0	$1_{\pm 1}$	$1_{\pm 2}$	$2_{\pm 1/2}$	$2_{\pm 3/2}$	3_0	$3_{\pm 1}$	$4_{\pm 1/2}$	5_0
$SU(N)_{TC}$	V						$1_{\rm stable}$			1
	$N \oplus V$	1					$3_{\rm stable}$			1
	$N \oplus L$	1			1		1			
	$N \oplus L \oplus \tilde{E}$	2	1		2		1			
	$V \oplus L$	1			1		2		1	1
	$V \oplus L \oplus \tilde{E}$	2			2		2	1	1	1
	$V \oplus L \oplus N$	2			2		4		1	1
	$N \oplus L \oplus \tilde{L}$	2	1		2		2	1		
	$N \oplus L \oplus \tilde{L} \oplus \tilde{E}$	3	2		3	1	2	1		
	$N \oplus L \oplus \tilde{E} \oplus V$	3	1		3		4	1	1	1
	$N \oplus L \oplus \tilde{L} \oplus E \oplus \tilde{E}$	4	3	1	4	2	2	1		
$SO(N)_{TC}$	V									1
	$L \oplus N$	1			1		1	1		
	$N \oplus V$	1					$1_{\rm stable}$			1
	$L \oplus V$	1			1		1	1	1	1
	$L \oplus N \oplus E$	2	1	1	2	1	1	1		
	$L \oplus E \oplus V$	2		1	2	1	1	2	1	1
	$L \oplus N \oplus V$	2			2		2	1	1	1
	$L \oplus N \oplus V \oplus E$	3	1	1	3	1	2	2	1	1

Table 4. Techni-pion content of color neutral golden-class composite DM models.

5.3 Higgs-mediated direct detection of Dark Matter

In both cases (real and complex DM) many golden-class composite DM models contain Yukawa couplings to the Higgs in order to break species number symmetries that would lead to unwanted stable particles. Such Yukawa couplings give rise to an extra Higgs-mediated contribution to the spin-independent cross section for direct DM searches, given by

$$\sigma_{\rm SI} = \frac{g_{\rm DM}^2 m_N^4 f_N^2}{2\pi v^2 M_h^4} \tag{5.11}$$

for DM with any spin. Here $f_N \approx 0.3$ is a nuclear form factor, $v \approx 174\,{\rm GeV}$ is the Higgs vev, and $g_{\rm DM}$ is the dimension-less coupling of the TCb DM candidate with mass $M_{\rm DM}(h)$ to the higgs, defined as

$$g_{\rm DM} = \frac{\partial M_{\rm DM}}{\partial h} \tag{5.12}$$

and roughly given by the Yukawa couplings of the Higgs to techni-quarks. The size of these Yukawa couplings is unknown. The LUX bound on $\sigma_{\rm SI}$ implies $g_{\rm DM} < \sqrt{M_{\rm DM}/75\,{\rm TeV}}$.

5.4 Techni-pions at colliders

As explained in eq. (5.1), cosmology suggests two possible values for $M_{\rm DM}$: 100 TeV or 3 TeV depending on whether DM has a TCb asymmetry. In both cases TCb DM is out

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of reach from LHC. Furthermore, DM production at colliders gives missing energy signals which, especially at hadron colliders, can be undetectably below the neutrino background.

Composite DM models predict a richer collider phenomenology: a general prediction is the existence of many resonances of various spin charged under the SM and with quantum numbers that can be determined from the ones of the constituents. Techni-pions are the lightest states in the theory so they are the most promising particles to be produced at colliders.[8] The anomalous coupling of some TCπ with SM vectors V gives rise to single production of TCπ, $V^* \to \pi V$ and $V^* V^* \to \pi$. Techni-pions can also be produced in pairs via their SM gauge interactions, with cross sections determined by their gauge quantum numbers and summarised e.g. in [43]. SM gauge interactions and techni-quark masses determine TCπ masses as in eq. (2.8) and (2.9). For an electro-weak triplet 3_0 the gauge contribution alone is $M_{3_0} \approx 0.1 M_{\rm DM}$ in a QCD-like $\mathrm{SU}(3)_{\rm TC}$.

The two values for $M_{\rm DM}$, 100 TeV or 3 TeV, correspond to $M_{3_0} \approx 10\,\mathrm{TeV}$ (significantly above LHC capabilities) or $M_{3_0} \approx 300\,\mathrm{GeV}$ (observable at LHC).

The only exception to the rule above is TCπ SM singlets η that do not receive mass from SM gauge interactions. Their mass is entirely determined by the constituent techni-quark masses, such that these TCπ could be very light. Usually such singlets undergo decays into pairs of SM gauge bosons through chiral anomalies [4]; when present their axion-like couplings to photons provides a mild constraint on their mass (that need to be larger than a keV) and a production mechanism at colliders.

Each composite DM model predicts a distinctive set of TCπ, as summarised in table 4.

The collider TCπ phenomenology can in principle discriminate golden-class from silver-class models [44]. In both cases TCπ without species number undergo anomalous decays into pairs of SM weak vectors,

$$\pi_{1_0}, \pi_{3_0}, \pi_{5_0} \to WW, ZZ, \gamma\gamma \tag{5.13}$$

(models with coloured TCπ, omitted from table 4, also predict anomalous decays into gluon pairs). In models with G-parity ($\Psi = V$) the π_{3_0} is stable. Techni-pions made of different species decay via couplings that violate species number.

In silver-class models such couplings are provided by higher dimension operators involving SM particles (for example 4-fermion operators), giving decays into such SM particles. If these operators are suppressed by a large scale, the decay is slow leading to displaced vertices or apparently stable particles on collider length scales, see [4] for a detailed discussion.

In golden-class models, species number and G-parity can be broken by Yukawa couplings with the SM Higgs boson. As a consequence, TCπ made of different species undergo decays into same specie TCπ (possibly off-shell) emitting one or more Higgs doublets H. For example a doublet with $Y = 1/2$ and a singlet with $Y = 1$ can decay as

$$\pi_{2_{1/2}} \to H \pi_{1_0}, \qquad \pi_{1_1} \to HH\pi_{1_0} \tag{5.14}$$

and π_{1_0} in turn decays into SM bosons through anomalies. Thereby, unlike in silver-class models, the SM fermions exhibit peaks in their invariant-mass distributions at the

[8] Heavier spin-1 resonances can be singly produced through the mixing with SM gauge bosons. They will then mostly decay in pairs of TCπ.

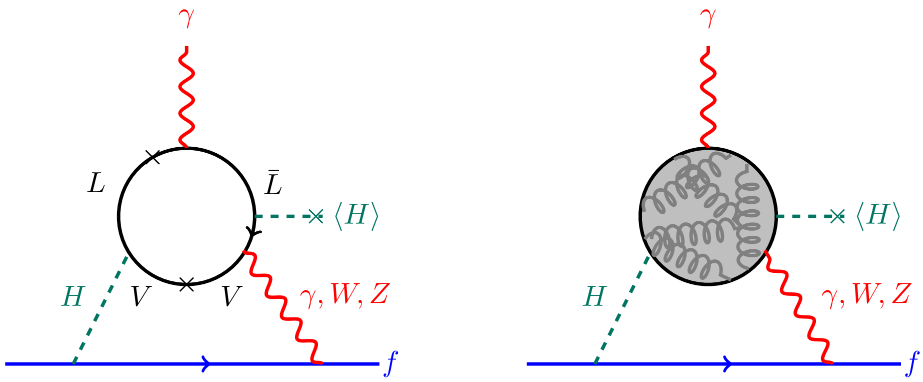

Figure 4. Electron dipole moment generated by complex Yukawa couplings of techni-quarks, neglecting techni-color interactions (left) and including techni-color interactions (right).

h, W, Z masses (the Goldstone components of the Higgs doublet become the longitudinal components of the W, Z vectors) [44].

In models with Yukawa couplings the lighter technipions could also give interesting corrections to precision observables. The loop corrections to electro-weak precision tests are universal and can be encoded in the $\hat{S}, \hat{T}, W, Y$ parameters [48], that can be computed generalising section 2.1 of [49], finding corrections of order $\alpha m_W^2/4\pi m_{TC\pi}^2$. Concerning precision Higgs physics, $h \to \gamma\gamma$ gets corrected as [50]

$$\frac{\Gamma(h \to \gamma\gamma)}{\Gamma(h \to \gamma\gamma)_{\rm SM}} \simeq 1 - 0.072 \sum_i \frac{v q_i^2 g_{hii}}{m_{TC\pi_i}^2} \tag{5.15}$$

where $v \approx 174\,\mathrm{GeV}$ is the Higgs vev and g_{hii} is the trilinear coupling between the Higgs boson and the techni-pion mass eigenstate i with electric charge q_i and mass $m_{TC\pi_i}$ (here assumed to be much heavier than m_h, for simplicity). The elemenatary Yukawa couplings $y\,H\Psi\Psi$ give rise to cubic interactions such us $y\Lambda_{\rm TC}\,H\pi_2\pi_3$ which generates $g_{hii} \sim y^2\Lambda_{\rm TC}$ in the formula above. This effect might only be relevant in the asymmetric scenario where TCπ can be as light as few hundred GeV. A comprehensive study of precision observables will appear in [44].

5.5 Electron electric dipole

Many models contain Yukawa couplings of techni-quarks with un-eliminable complex phases, generating electric dipole moments for light SM fermions. Let us consider for example the model $\Psi = L + V$ with $\mathrm{SO}(N)_{\rm TC}$. The techni-quark Lagrangian contains schematically,[9]

$$m_L \bar{L}L + \frac{m_V}{2} VV + y_L H^\dagger V L + y_R^* H V \bar{L} + \text{h.c.} \tag{5.16}$$

It contains one physical CP-violating phase corresponding to $\arg\,[m_L m_V y_L^* y_R^*]$. Ignoring technicolor interactions, an EDM is generated through the diagrams in the left panel of

[9]The structure is analogous to the Higgsino/wino system in split-supersymmetry [51]. The same would work for $\mathrm{SU}(N)$ models with the difference that the triplet would be a Dirac fermion.

figure 4, giving, in leading log approximation [51],

$$d_f \sim N e\, Q_f \frac{\alpha\, \mathrm{Im}[y_L^* y_R^*]}{16\pi^3} \frac{m_f}{m_L\, m_V} \ln \frac{m_L m_V}{m_H^2}\,. \tag{5.17}$$

For the electron one finds

$$d_e \sim 10^{-27}\, e\, \mathrm{cm} \times \mathrm{Im}[y_L y_R] \times \frac{N}{3} \times \frac{\mathrm{TeV}^2}{m_L m_V} \tag{5.18}$$

to be compared with the experimental bound $d_e < 8.7 \times 10^{-29}$ e cm at 90% C.L [52].

However, the approximation of neglecting technicolor interactions is only reliable for $m_{L,V} > \Lambda_{\mathrm{TC}}$. In the more interesting regime $m_{L,V} < \Lambda_{\mathrm{TC}}$ techni-color effects cannot be neglected and the loops will be dominated by the hadrons of the theory, as depicted in the right-handed panel of figure 4. A detailed study will appear in [44].

5.6 Gravitational waves

Confining gauge theories can give rise to first order phase transitions. For SU(N) with N_{TF} massless flavours this is believed to happen in the window $3 \leq N_{\mathrm{TF}} \leq 4N$ and $N > 3$ [45]. The phase transition occurs, within our framework, at a temperature $T \sim \Lambda_{\mathrm{TC}}$ (in the thermal dark matter scenario $\Lambda_{\mathrm{TC}} \sim \mathcal{O}(10\ \mathrm{TeV})$) and can lead to large anisotropic fluctuations in the energy momentum tensor sourcing the gravitational waves (GW) in the early universe. Following [46], we estimate the frequency of the peak in the GW signal as a function of the phase transition temperature T as:

$$f_{\mathrm{peak}} = 3.3 \times 10^{-3}\ \mathrm{Hz} \times \left(\frac{T}{10\,\mathrm{TeV}}\right) \times \left(\frac{\beta}{10H}\right) \tag{5.19}$$

where β is the duration of the phase transition which is usually taken in the range 1-100 of a Hubble time H. For the reference values of the parameters, the amplitude of the expected GW signal is $h^2\Omega_{\mathrm{GW}} \sim 10^{-9}$ [46] which is in the range that can be probed by future satellite experiments such as (E)LISA [47].

5.7 Unification of SM gauge couplings

Throughout the paper we assumed that techni-quarks belong to fragments of unified SU(5) representations. We here study if they can improve unification of SM gauge couplings. The large number of independent masses allows for considerable freedom; we make the extra assumption that the missing members of the unified SU(5) multiplets have a common mass M_X, below the GUT scale and above the TC scale Λ_{TC}. Furthermore we make the rough assumption that the strong dynamics does not contribute to the running of the SM gauge couplings below the $\Lambda_{\mathrm{TC}} \sim 100\,\mathrm{TeV}$, ignoring threshold effects including those of TCπ. With this mass ordering, in 1-loop approximation the running of gauge couplings is given by

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_{\mathrm{GUT}}} + \frac{b_i^{\mathrm{SM}}}{2\pi} \log \frac{M_{\mathrm{GUT}}}{M_Z} + \frac{\Delta b_i}{2\pi} \log \frac{M_X}{\Lambda_{\mathrm{TC}}} + \frac{\Delta b}{2\pi} \log \frac{M_{\mathrm{GUT}}}{M_X} \tag{5.20}$$

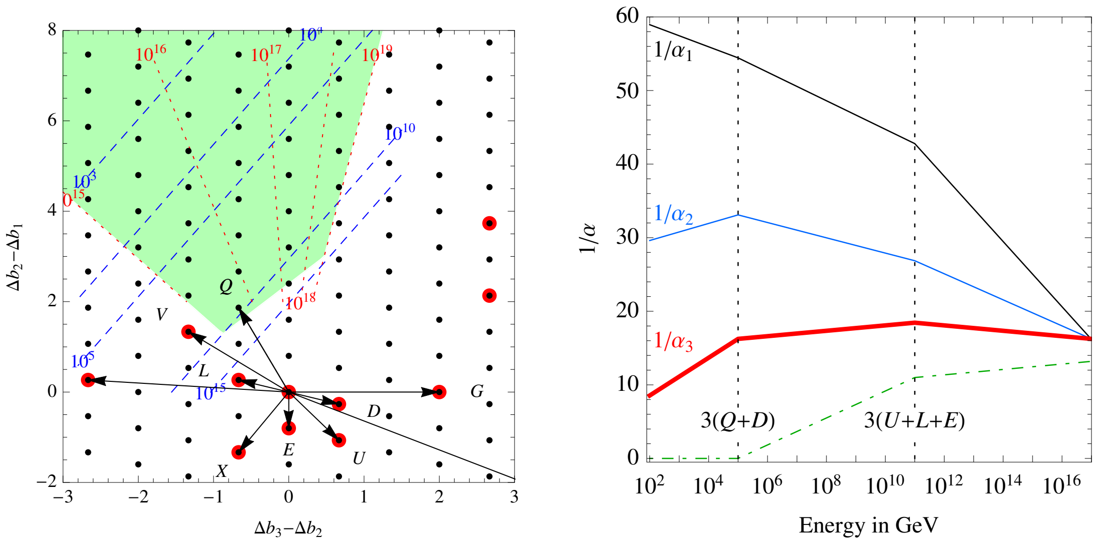

Figure 5. *Left*: General analysis of unification, as described in the main text. We show contour-lines of $M_{\rm GUT}/\,{\rm GeV}$ ($M_X/\Lambda_{\rm TC}$) in red (blue). *Right*: Running of the gauge couplings in the golden-class $\mathrm{SU}(3)_{\rm TC}$ model $\Psi = Q \oplus \tilde{D}$.

where Δb_i is the contribution from techni-quarks (listed in table 1), and Δb is the contribution from the full SU(5) multiplets above M_X. The three unification conditions determine the values of the three high-scale parameters $\alpha_{\rm GUT}$, $M_{\rm GUT}$ and M_X. Inserting the SM values we find

$$\ln \frac{M_X}{\Lambda_{\rm TC}} = \frac{68}{\Delta b_{21} - 1.9\Delta b_{32}}, \qquad \ln \frac{M_{\rm GUT}}{M_X} = \frac{35.3\Delta b_{21} - 49.2\Delta b_{32}}{\Delta b_{21} - 1.9\Delta b_{32}}. \tag{5.21}$$

figure 5a shows contour-values of $M_{\rm GUT}/\,{\rm GeV}$ (dotted red lines) and of $M_X/\Lambda_{\rm TC}$ (blue dashed lines) as function of $\Delta b_{21} = \Delta b_2 - \Delta b_1$ and of $\Delta b_{32} = \Delta b_3 - \Delta b_2$. The dots in the figure are the grid of β-function coefficients allowed by SU(5) group theory [54], and the arrows are the contributions to $(\Delta b_{32}, \Delta b_{21})$ from the fragments of SU(5) representations listed in table 1. The total β-function coefficient in any given model is obtained summing the contributions of each techni-quark taking into account their techni-color multiplicity N.

We see that models that can provide successful unification must contain a V or a Q in order to obtain the desired sign of Δb_{21}. For example:

- The golden-class $\mathrm{SU}(3)_{\rm TC}$ model $\Psi = Q \oplus \tilde{D}$, with techni-quarks coming from unified $5 \oplus 10 + \text{h.c.}$ multiplets of SU(5), provides successful unification

$$\alpha_{\rm GUT} \approx 0.06, \qquad M_{\rm GUT} \approx 2 \times 10^{17}\ \text{GeV}, \qquad M_X \approx 2 \times 10^{11}\ \text{GeV} \times \frac{\Lambda_{\rm TC}}{100\ \text{TeV}} \tag{5.22}$$

  having assumed $\Lambda_{\rm TC} \approx 100\,\text{TeV}$. The running of the couplings is shown in figure 5b.

- The golden-class $\mathrm{SO}(3)_{\rm TC}$ model $\Psi = V$, with V coming from an adjoint of SU(5), provides

$$\alpha_{\rm GUT} \approx 0.065, \qquad M_{\rm GUT} \approx 3 \times 10^{14}\ \text{GeV}, \qquad M_X \approx 4 \times 10^{7}\ \text{GeV} \times \frac{\Lambda_{\rm TC}}{100\ \text{TeV}}. \tag{5.23}$$

Such a low unification scale would be excluded by proton decay. However, given the large uncertainties (we performed a one-loop analysis, ignoring threshold effects that could be sizeable at the technicolor scale, in view of the light TCπ) such a model could still be viable.

- The silver-class $\mathrm{SU}(3)_{\mathrm{TC}}$ model with $\Psi = Q \oplus D \oplus U \oplus L$ coming from $\bar{5} \oplus 10 + \mathrm{h.c.}$ multiplets of SU(5). We have,

$$\alpha_{\mathrm{GUT}} \approx 0.085, \qquad M_{\mathrm{GUT}} \approx M_X \approx 4 \times 10^{17}\ \mathrm{GeV} \tag{5.24}$$

having assumed $\Lambda_{\mathrm{TC}} \approx 200\,\mathrm{TeV}$. The DDU TCb can provide the observed Dark Matter, as discussed in appendix B, model $\Psi = D \oplus U$.

6 Conclusions

Extensions of the SM with new strong interactions are interesting from the point of view of Dark Matter. First, they naturally provide new stable particles, thanks to accidental symmetries analogous to baryon number that guarantees the stability of the proton within the SM: DM could be the lightest techni-baryon (TCb) or techni-pion (TCπ). Second, the lightest among the many TCb tends to be the one with least SM gauge interactions, thereby explaining why DM has no color, no electric charge, and at most a small hypercharge.

The models that we propose are compatible with all present bound from collider and precision experiments because, with techni-quarks in a real representation of the SM gauge group, the new strong interactions do not break the electroweak symmetry. The Higgs doublet is elementary and we do not address the hierarchy problem here. We use the old name 'techni-color' in order to emphasize that we do not postulate desired good properties of effective Lagrangians. On the contrary, we propose fundamental theories where all the good properties follow from an appropriate choice of the quantum numbers: a concrete 'techni-color' gauge group and a concrete set of techni-quarks.

In the simplest 'golden-class' of models, everything follows from a renormalizable Lagrangian. In 'silver-class' models, mild assumptions on non-renormalizable interactions are needed in order to break accidental symmetries and get rid of unwanted stable particles. The list of 'golden-class' models is meant to be exhaustive, within some assumptions: no techni-scalars, only techni-fermions that transform in the fundamental representations of the technicolor gauge group, and in representations of the SM gauge group which are compatible with SU(5) unification. We found successful models with both $\mathrm{SU}(N)_{\mathrm{TC}}$ and $\mathrm{SO}(N)_{\mathrm{TC}}$ techni-color groups. We did not explore exceptional groups.

In $\mathrm{SO}(N)_{\mathrm{TC}}$ theories DM is a TCb, stable thanks to a $\mathbb{Z}_2 = \mathrm{O}(N)/\,\mathrm{SO}(N)$ symmetry: there is no conserved techni-baryon number, such that DM is a real particle (a Majorana fermion for odd N, a real scalar for even N) with no TCb asymmetry, no magnetic nor electric dipole. Assuming that its cosmological abundance comes from thermal freeze-out of techni-strong annihilations into TCπ, the DM mass is expected to be around 100 TeV. TCb mix once the Higgs boson acquires its vacuum expectation value (somehow analogously to the Wino/Bino/Higgsino system in supersymmetry), giving the following

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phenomenology: in some regions of the parameter space DM can have an axial coupling to the Z, detectable in direct-detection signals; in other regions of the parameter space it behaves as inelastic DM.

In $\mathrm{SU}(N)_{\mathrm{TC}}$ theories, the lightest TCb is a complex particle, stable thanks to conservation of an accidental $\mathrm{U}(1)_{\mathrm{TC}}$ techni-baryon number. The DM mass could again be around 100 TeV: a Dirac fermion however can give sizable magnetic and electric dipole moments, giving direct-detection cross-sections enhanced in a characteristic way at low recoil energy with respect to the case of a standard spin-independent cross section. A large θ_{TC}-angle of the new strong sector can give an electric dipole such that direct detection is just below present bounds; while a magnetic dipole cross section (suppressed at low DM velocities) is within the capabilities of future direct detection experiments. Alternatively, the cosmological DM abundance could be due to a TCb asymmetry, with a DM mass around 3 TeV.

In both cases, successful DM models often need Yukawa couplings with the Higgs boson in order to break unwanted techni-flavor symmetries, leading to extra spin-independent direct detection signals. CP-violating phases also lead to a possibly detectable electric dipole moment for the SM particles, such as the electron.

In some models composite DM has spin 1 or higher.

Concerning collider experiments, each model predicts a distinctive set of techni-pions, summarised in table 4, which are at most a factor 10 lighter than DM itself, than techni-baryons and than other vector composite resonances. Some techni-pions undergo anomalous decays into SM vectors (and can be singly produced via the inverse process), others decay into lighter techni-pions (and can be doubly produced via their SM gauge interactions) emitting one or more Higgs doublets (i.e. h, W, Z), or, in silver-class models, emitting other SM particles.

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A Computing techni-baryons

In section 2.1 we computed the multiplets of lighter TCb in $\mathrm{SU}(N)_{\mathrm{TC}}$ models. The SM gauge interactions break explicitly the techni-flavor symmetry: here we outline how we compute the decomposition of the lightest TCb multiplet under the SM gauge group. We label the SM quantum numbers of each state under $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ as $(n_c, n_L)_Y$.

Let us first consider $\mathrm{SU}(3)_{\mathrm{TC}}$ theories with two species: $\Psi = \Psi_1 + \Psi_2$. Models with more species can be solved by iteration. The lightest TCb fill a $\yng(2,1)$ representation of the techni-flavor group $\mathrm{SU}(d_1 + d_2)_{\mathrm{TF}}$, where $d_{1,2}$ are the dimensions of the $\Psi_{1,2}$ SM representations. We proceed in steps: first decompose the TCb multiplet under $\mathrm{SU}(d_1) \times$

SU(d_2), with the embedding $(d_1,1)\oplus(1,d_2)$, then decompose each component under the SM group and finally identify the SU$(3)_c$ and SU$(2)_L$ factors.

From the first step we get:

$$\begin{aligned}\ydiagram{2,1} = {}&\left(\ydiagram{1,1}\,,\ydiagram{1}\right)_{2Y_1+Y_2}\oplus\left(\ydiagram{1}\,,\ydiagram{1,1}\right)_{2Y_2+Y_1}\oplus\left(\ydiagram{2}\,,\ydiagram{1}\right)_{2Y_1+Y_2}\oplus\left(\ydiagram{1}\,,\ydiagram{2}\right)_{2Y_2+Y_1}\\ &\oplus\left(\ydiagram{2,1}\,,1\right)_{3Y_1}\oplus\left(1\,,\ydiagram{2,1}\right)_{3Y_2}.\end{aligned}\tag{A.1}$$

The last two terms $(\ydiagram{2,1},1)$, $(1,\ydiagram{2,1})$ correspond to TCb made only by Ψ_1 or Ψ_2 respectively and they reduce to one specie problems. The first four terms describe TCb composed of both species. For example, $(\ydiagram{1,1},\ydiagram{1})_{2Y_1+Y_2}$ describes TCb made of $\Psi_1\Psi_1\Psi_2$.

We next decompose each component on the right hand side of eq. (A.1) under the SM gauge group. This can require non-trivial group theory computations: for example a techni-quark V (triplet under SU$(2)_L$) lies in the fundamental representation of techni-flavor SU$(3)_{\rm TF}$: TCb lie in higher representations of SU$(3)_{\rm TF}$ that need to be decomposed under SU$(2)_L$. In general, we need to decompose a given representation with K boxes of SU$(n_c n_L)$ under SU$(n_c)\times$ SU(n_L), where the fundamental of SU$(n_c n_L)$ is now embedded as (n_c, n_L). This can be done writing all the representations of SU(n_c) and SU(n_L) with K boxes. From group theory we know that each tableau is associated with a representation of the permutation group S_K with a given symmetry. Then (D_1, D_2) appears in the decomposition if the product of D_1 and D_2 representations contains a component with the S_K symmetry of the initial representation. Here is the decomposition of the two-index symmetric and antisymmetric tensors under SU(n_c) and SU(n_L):

$$\ydiagram{1,1}=\left(\ydiagram{1,1}\,,\ydiagram{2}\right)\oplus\left(\ydiagram{2}\,,\ydiagram{1,1}\right)\qquad\qquad \ydiagram{2}=\left(\ydiagram{1,1}\,,\ydiagram{1,1}\right)\oplus\left(\ydiagram{2}\,,\ydiagram{2}\right).\tag{A.2}$$

To be concrete, consider the techni-quark $\Psi=(3,2)$. The decomposition of the two index tensors above under SU$(3)_c\times$ SU$(2)_L$ become:

$$\ydiagram{1,1}=(6,1)\oplus(\bar{3},3),\qquad\qquad \ydiagram{2}=(6,3)\oplus(\bar{3},1).\tag{A.3}$$

If with respect to any SM group factor the techni-quarks transform in a representation n_i higher than the fundamental, we can embed it into the fundamental of SU(n_i) and decompose representations of this larger group under the SM group. For example in the $\Psi = V$ model, the techni-quark is a vector of SU$(2)_L$: we can think of the two-index symmetric 3 of SU$(2)_L$ as the fundamental of SU(3) into which SU(2) is embedded symmetrically. With simple group algebra we find:

$$\begin{aligned}&\mathrm{SU}(3): 3\times 3 = 6\oplus\overline{3}\,,\quad 3\times\overline{3}=1\oplus 8\\ &\mathrm{SU}(2): 3\times 3 = 1\oplus 3\oplus 5\,,\end{aligned}\tag{A.4}$$

from which we get the decomposition rules $6 = 5\oplus 1$ and $8 = 5\oplus 3$ for the SU(3) representations under the SU(2) group.

After this step, each state in eq. (A.1) is labeled by the quantum numbers $(n_{c1}, n_{L1}, n_{c2}, n_{L2})_Y$. To obtain the final representation under the SM group we have to identify SU$(3)_c$ and SU$(2)_L$ factors, taking the tensor product $n_{c1}\otimes n_{c2}$ and $n_{L1}\otimes n_{L2}$.

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For $\mathrm{SU}(4)_{\mathrm{TC}}$, we can proceed analogously. First, we decompose the lightest TCb multiplet of $\mathrm{SU}(d_1+d_2)$ under $\mathrm{SU}(d_1)\times\mathrm{SU}(d_2)$:

$$\begin{aligned}\yng(2,2) = &\left(\yng(2,1)\,,\,\yng(1)\right)_{3Y_1+Y_2} \oplus \left(\yng(1,1)\,,\,\yng(1,1)\right)_{2Y_2+2Y_1} \oplus \left(\yng(2)\,,\,\yng(2)\right)_{2Y_1+2Y_2} \\ &\oplus \left(\yng(1)\,,\,\yng(2,1)\right)_{Y_1+3Y_2} \oplus \left(\yng(2,2)\,,\,1\right)_{4Y_1} \oplus \left(1\,,\,\yng(2,2)\right)_{4Y_2},\end{aligned} \tag{A.5}$$

then we decompose each representation under the SM group and identify the $\mathrm{SU}(3)_c$ and $\mathrm{SU}(2)_L$ factors respectively.

As discussed in section 3.1, the $\mathrm{SO}(N)_{\mathrm{TC}}$ theories can be analyzed starting from the results of the $\mathrm{SU}(N)_{\mathrm{TC}}$ models.

B Silver-class composite DM models

We here list silver-class $\mathrm{SU}(N)_{\mathrm{TC}}$ and $\mathrm{SO}(N)_{\mathrm{TC}}$ models restricted for simplicity to $N=3,4$ techni-colors and $N_s\le 2$ species of techni-quarks. These models satisfy TC asymptotic freedom and do not give rise to sub-Planckian Landau poles. But, besides to acceptable DM candidates, they give rise to unwanted stable states, that are TCπ with hypercharge or color, stable because of accidental symmetries such as species number or G-parity. They can be made unstable with extra model building, for example adding higher dimension operators that break the accidental symmetries, as explained in section 1.

B.1 $\mathrm{SU}(N)_{\mathrm{TC}}$ silver-class models

$\mathrm{SU}(N)_{\mathrm{TC}}$ model $\Psi = N\oplus E$. This model has $N_S = N_{\mathrm{TF}} = 2$. TC$\pi$ fill the adjoint of $\mathrm{SU}(2)_{\mathrm{TF}}$:

$$\mathrm{TC}\pi:\ 3 = 1_{0,\pm1} \qquad \text{under}\ \ \mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y. \tag{B.1}$$

TCπ made by both techni-quarks have non zero hypercharge and are stable because of species number. If we want to make the model phenomenologically viable, we need to break species number by ad hoc assumptions. For $N=3$, the lightest TCb live in the fundamental of $\mathrm{SU}(2)_{\mathrm{TF}}$, that is

$$\mathrm{TCb}:\ 2 = 1_{\pm1} \qquad \text{under}\ \ \mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\,. \tag{B.2}$$

The DM candidate is the spin 3/2 singlet NNN^* that belongs to the symmetric representation $\yng(3)$ of $\mathrm{SU}(2)_{\mathrm{TF}}$. It can be the lightest TCb if $m_N \ll m_E$. The same conclusion is valid for $N=4$, where the DM candidate is the spin 2 singlet $NNNN^*$ that lives in the symmetric representation $\yng(4)$ of $\mathrm{SU}(2)_{\mathrm{TF}}$.

$\mathrm{SU}(N)_{\mathrm{TC}}$ model $\Psi = E\oplus\tilde{E}$. This model with $N_{\mathrm{TF}}=2$ can give rise to a neutral TCb for $N=4$. It presents a Landau pole for g_Y slightly above the Planck scale and gives rise to the following TCπ

$$\mathrm{TC}\pi:\ 3 = 1_{0,\pm2} \quad \text{under}\ \ \mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y\,. \tag{B.3}$$

The $1_{\pm2}$ TCπ made by both species are stable, so that we need to break species number. The model gives only one lighter TCb, that is a SM singlet made by $EE\tilde{E}\tilde{E}$ and is a good DM candidate.

$\mathrm{SU}(N)_{\mathrm{TC}}$ model $\Psi = L \oplus \tilde{L}$. The TC$\pi$ of this model with $N_{\mathrm{TF}} = 4$ are:

$$\mathrm{TC}\pi\,:\; 15 = 1_{0,\pm 1} \oplus 3_{2\times 0,\pm 1} \quad \text{under } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \tag{B.4}$$

where states with hypercharge are stable, unless the species number symmetry is broken. Analogously to the previous model, it can provide a DM candidate for $N = 4$, where the lighter TCb fill a $20'$ of $\mathrm{SU}(4)_F$, that decomposes as

$$\mathrm{TCb}\,:\; 20' = 1_{2\times 0,\pm 1,\pm 2} \oplus 3_{0,\pm 1} \oplus 5_0 \quad \text{under } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y\,. \tag{B.5}$$

The list contains two singlets $LL\tilde{L}\tilde{L}$ that are good DM candidates.

$\mathrm{SU}(N)_{\mathrm{TC}}$ model $\Psi = L \oplus E$. This model, studied in detail in section 4.1, has $N_{\mathrm{TF}} = 3$ and for $N = 3$ gives rise to the successful DM candidate LLE. In this case, both TCπ and TCb live in the adjoint of $\mathrm{SU}(3)_{\mathrm{TF}}$, that decomposes as

$$8 = 1_0 \oplus 2_{\pm 3/2} \oplus 3_0 \quad \text{under } \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y\,. \tag{B.6}$$

TCπ made of $L\bar{E}$ are $2_{\pm 3/2}$ states, stable because of the unbroken species number symmetry. One can get rid of the unwanted stable particles by ad-hoc model building, breaking accidental symmetries with higher dimensional operators or adding new particles. For example one can add a scalar doublet H' with $|Y| = 3/2$ such that the Yukawa coupling $H'LE$ is allowed.

The TCb DM candidate is the singlet LLE. As explained in section 4.1, techni-quark masses favor LLL or LEE as the lightest state, so that the LLE singlet can be the stable DM candidate if gauge interactions contribute to mass splitting more than techni-quark masses.

$\mathrm{SU}(N)_{\mathrm{TC}}$ model $\Psi = V \oplus E$. This model with $N_{\mathrm{TF}} = 4$ is allowed only for $N = 3$ and gives rise to the following TCπ:

$$\mathrm{TC}\pi\,:\; 15 = 1_0 \oplus 3_{0,\pm 1} \oplus 5_0 \quad \text{of } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \tag{B.7}$$

with the $3_{\pm 1}$ states stable because of species number. The lightest multiplet of TCb decomposes as

$$\mathrm{TCb}\,:\; \overline{20} = 1_1 \oplus 3_{0,1,2} \oplus 5_{0,1} \quad \text{under } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \tag{B.8}$$

and the DM candidate is the triplet made by VVV.

$\mathrm{SU}(N)_{\mathrm{TC}}$ model $\Psi = D \oplus N$, $\Psi = U \oplus N$ and $\Psi = Q \oplus N$. We can study together the first two models ($N_{\mathrm{TF}} = 4$) defining $Y = 1/3, -2/3$ for D and U respectively. For the $\Psi = D(U) \oplus N$ models we get the following TCπ:

$$\mathrm{TC}\pi\,:\; 15 = 1_0 \oplus \bar{3}_Y \oplus 3_{-Y} \oplus 8_0 \quad \text{of } \mathrm{SU}(3)_c \otimes \mathrm{U}(1)_Y\,. \tag{B.9}$$

Analogously, for the $\Psi = Q \oplus N$ ($N_{\mathrm{TF}} = 7$) model we get:

$$\mathrm{TC}\pi\,:\; 48 = (1,1)_0 \oplus (1,3)_0 \oplus (3,2)_{1/6} \oplus (\bar{3},2)_{-1/6} \oplus (8,1)_0 \oplus (8,3)_0 \tag{B.10}$$

of $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. Because of species number symmetry, we have stable TCπ with color and/or hypercharge, so that we need to break this accidental symmetry to avoid the strong experimental bounds. As in the other models containing the singlet N, the TCb DM candidate is an higher spin state made only by N techni-quarks. For $N = 3$ and $N = 4$ it has spin 3/2 and 2 respectively and it can be the lightest if the other techni-quark is sufficiently heavier.

SU(N)$_{\mathrm{TC}}$ model $\Psi = D \oplus V$, $\Psi = U \oplus V$. As before, we study together the two models defining $Y = 1/3, -2/3$ for D and U respectively. Because of the presence of the techni-quark V, these models are allowed only for $N = 3$. They have $N_{\mathrm{TF}} = 6$ and give rise to the following states:

$$\begin{aligned} \mathrm{TC}\pi &: 35 = (1,1)_0 \oplus (1,3)_0 \oplus (\bar{3},3)_Y \oplus (3,3)_{-Y} \oplus (1,5)_0 \oplus (8,1)_0 \\ \mathrm{TCb} &: 70 = (\bar{3},1)_Y \oplus (1,3)_0 \oplus (\bar{3},3)_Y \oplus (3,3)_{2Y} \oplus (1,5)_0 \oplus (\bar{3},5)_Y \oplus (8,1)_{3Y} \oplus (\bar{6},3)_{2Y} \end{aligned} \tag{B.11}$$

under $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. We need to break species number in order to avoid stable TCπ made by both species that are colored and have hypercharge. The DM candidate is the triplet VVV: from the DM point of view these models are a trivial extension of the $\Psi = V$ model described in section 2.2.

SU(N)$_{\mathrm{TC}}$ model $\Psi = D \oplus U$. This model has $N_{\mathrm{TF}} = 6$, so the TCπ live in the adjoint of $\mathrm{SU}(6)_{\mathrm{TF}}$:

$$\mathrm{TC}\pi : 35 = 1_{0,\pm1} \oplus 8_{2\times0,\pm1} \quad \text{under } \mathrm{SU}(3)_c \otimes \mathrm{U}(1)_Y . \tag{B.12}$$

TCπ with non zero species number are colored and have hypercharge, so we need to break the species number symmetry. The model gives a good TCb DM candidate for $N = 3$, where the lightest TCb are

$$\mathrm{TCb} : 70 = 1_{0,-1} \oplus 8_{2\times0,1,2\times(-1),-2} \oplus \overline{10}_{0,-1} \quad \text{under } \mathrm{SU}(3)_c \otimes \mathrm{U}(1)_Y , \tag{B.13}$$

and the DM candidate is the singlet made by DDU.

B.2 SO(N)$_{\mathrm{TC}}$ silver-class models

SO(N)$_{\mathrm{TC}}$ model $\Psi = E$. This model has $N_{\mathrm{TF}} = 2$ and it is free from Landau poles up to $N = 8$. The unwanted stable TCπ are singlets with $Y = \pm 2$. The model can provide a TCb DM candidate for even N. For $N = 4$, there is only one TCb with spin 0 that is a singlet and thus a good DM candidate.

SO(N)$_{\mathrm{TC}}$ model $\Psi = E \oplus N$. This $N_{\mathrm{TF}} = 3$ model does not allow for Yukawa couplings and contains unwanted stable TCπ with hypercharge, that belongs to the 5 of $\mathrm{SO}(3)_{\mathrm{TF}}$:

$$\mathrm{TC}\pi : 5 = 1_{0,\pm1,\pm2} \quad \text{of } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y . \tag{B.14}$$

The model can be extended up to $N = 8$. For $N = 3$ the lightest TCb is a singlet $NE\bar{E}$ that lives in the $\square$ representation of $\mathrm{SO}(3)_{\mathrm{TF}}$, while the heavier TCb are

$$\mathrm{TCb} : \begin{array}{|c|c|}\hline & \\ \hline \end{array}\!\!\begin{array}{l}\\ \square\end{array} = 5 = 1_{0,\pm1,\pm2} \quad \text{of } \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y . \tag{B.15}$$

For $N = 4$, the DM candidate is a singlet linear combination of $E\bar{E}NN$ and $E\bar{E}E\bar{E}$. In this case the $\ydiagram{2,2}$ representation is absent, so the full set of TCb is already specified.

SO(N)$_{\rm TC}$ model $\Psi = E \oplus V$. In this model $N_{\rm TF} = 5$ so that TCπ compose a 14 of SO(5)$_{\rm TF}$ that is

$$\text{TC}\pi:\ 14 = 1_{0,\pm 2} \oplus 3_{\pm 1} \oplus 5_0 \quad \text{of } \text{SU}(2)_L \otimes \text{U}(1)_Y\,, \tag{B.16}$$

where the states with hypercharge, made by EE, $\bar{E}\bar{E}$ or EV, $\bar{E}V$ are stable. Extra assumptions are needed to break the accidental symmetries and remove unwanted stable states. The model is valid up to $N = 7$. For $N = 3$ the TCb DM candidate is the 3_0 state $V(E\bar{E} + VV)$ belonging to the representation $\square$ of the unbroken flavor group. The other TCb are:

$$\text{TCb}:\ \ydiagram{2,1} = 35 = 1_{\pm 1} \oplus 3_{2\times 0,\pm 1,\pm 2} \oplus 5_{0,\pm 1} \quad \text{of } \text{SU}(2)_L \otimes \text{U}(1)_Y\,. \tag{B.17}$$

For $N = 4$ the DM candidate is a singlet $(E\bar{E} + VV)^2$, while the remaining TCb are given by

$$\text{TCb}:\ \ydiagram{2,2} = 35 = 1_0 \oplus 3_{0,\pm 1} \oplus 5_{0,\pm 1,\pm 2} \quad \text{of } \text{SU}(2)_L \otimes \text{U}(1)_Y\,, \tag{B.18}$$

plus a set of states living in the same representations as the TCπ above.

SO(N)$_{\rm TC}$ model $\Psi = L$. This model has $N_{\rm TF} = 4$, the TCπ lie in a 9 of SO(4)$_{\rm TF}$ that decomposes as

$$\text{TC}\pi:\ 9 = 3_{\pm 1} \oplus 3_0 \qquad \text{under } \text{SU}(2)_L \otimes \text{U}(1)_Y. \tag{B.19}$$

TCπ made by LL and $\bar{L}\bar{L}$ have non zero hypercharge and are stable because of accidental U(1) species symmetry. The extra physics needed to avoid unwanted stable TCπ can be nicely realised considering the golden-class model $\Psi = L \oplus N$ in the limit where $m_N \gg \Lambda_{\rm TC}$, such that the $(LH)^2$ effective operator is generated at low energy.

TCb can contain a DM candidate for N even. For $N = 4$, this is the singlet $(L\bar{L})^2$. The other TCb that need to be specified are

$$\text{TCb}:\ \ydiagram{2,2} = 10 = 5_0 \oplus 1_{\pm 2,\pm 1,0} \qquad \text{under } \text{SU}(2)_L \otimes \text{U}(1)_Y\,. \tag{B.20}$$

Landau poles are avoided for $N \leq 14$.

SO(N)$_{\rm TC}$ model $\Psi = L \oplus E$. This model with $N_{\rm TF} = 6$ allows for two Yukawa couplings, leaving an unbroken U(1) species number, rotating $L, \bar{E}$ with a common phases, and $\bar{L}, E$ with the opposite phase. Thereby TCπ made by $L\bar{E}$, $\bar{L}E$ are stable and have hypercharge $\pm 3/2$. The full list of TCπ is:

$$\text{TC}\pi:\ 20 = 1_{\pm 2,0} \oplus 2_{\pm 3/2,\pm 1/2} \oplus 3_{\pm 1,0} \qquad \text{under } \text{SU}(2)_L \otimes \text{U}(1)_Y\,. \tag{B.21}$$

Again, we need to break the unwanted accidental symmetry in some way. The techni-color theory is asymptotically free only for $N \geq 4$ and sub-Planckian Landau poles are avoided for $N \leq 5$. The model gives a singlet TCb DM candidate for $N = 4$, that is $(L\bar{L} + E\bar{E})^2$.

To obtain the full list of TCb we need to decompose the multiplet of heavier TCb under $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$:

$$\begin{aligned}\mathrm{TCb}: \begin{array}{|c|c|}\hline & \\ \hline & \\ \hline\end{array} = 84 = 5_0 \oplus 4_{\pm 3/2, \pm 1/2} \oplus 3_{\pm 3, \pm 2, 2\times(\pm 1), 2\times 0} \\ \oplus 2_{\pm 5/2, 2\times(\pm 3/2), 3\times(\pm 1/2)} \oplus 1_{\pm 2, 2\times(\pm 1), 3\times 0}.\end{aligned} \tag{B.22}$$

$\mathrm{SO}(N)_{\mathrm{TC}}$ models $\Psi = U$, $\Psi = D$. We study the two models jointly, defining $Y = -2/3$ for $\Psi = U$ and $Y = 1/3$ for $\Psi = D$. The model is asymptotically free for $N \geq 4$ and Landau poles are avoided up to $N = 6$ for U and to $N = 14$ for D. In both cases $N_{\mathrm{TF}} = 6$ and TCπ lie in the 20 of $\mathrm{SO}(6)_{\mathrm{TF}}$ that decomposes as

$$\mathrm{TC}\pi: \ 20 = 8_0 \oplus \left(\bar{6}_{2Y} \oplus \mathrm{h.c.}\right) \qquad \text{under } \mathrm{SU}(3)_c \otimes \mathrm{U}(1)_Y. \tag{B.23}$$

Because of the U(1) accidental symmetry there are unwanted stable colored TCπ made by $D\bar{D}$ or $U\bar{U}$. The models provide singlets TCb DM candidates only for even N. For $N = 4$ the full TCb list contains the multiplet

$$\mathrm{TCb}: \begin{array}{|c|c|}\hline & \\ \hline & \\ \hline\end{array} = 84 = 1_0 \oplus 8_0 \oplus 27_0 \oplus \left(3_{2Y} \oplus 6_{4Y} \oplus 15_{2Y}\right) \quad \text{under } \mathrm{SU}(3)_c \otimes \mathrm{U}(1)_Y. \tag{B.24}$$

$\mathrm{SO}(N)_{\mathrm{TC}}$ models $\Psi = D \oplus N$, $\Psi = U \oplus N$. A trivial extension of the previous models is given by the $N \oplus D$ and $N \oplus U$ models, with $N_{\mathrm{TF}} = 7$. They give rise to an extended list of TCπ:

$$\mathrm{TC}\pi: \ 27 = 1_0 \oplus 8_0 \oplus \left(3_{-Y} \oplus 6_{-2Y} \oplus \mathrm{h.c}\right) \qquad \text{under } \mathrm{SU}(3)_c \otimes \mathrm{U}(1)_Y\,, \tag{B.25}$$

where the extra state made by N and D or U are unwanted stable particles. For $N = 4$ the lightest TCb DM candidate is again a singlet, made by $D\bar{D}(D\bar{D}+NN)$ or $U\bar{U}(U\bar{U}+NN)$.

$\mathrm{SO}(N)_{\mathrm{TC}}$ models $\Psi = D \oplus V$, $\Psi = U \oplus V$. These are less trivial extensions of the D and U models, with $N_{\mathrm{TF}} = 9$. The list of TCπ is

$$\mathrm{TC}\pi: \ 44 = (1,1)_0 \oplus (1,5)_0 \oplus (8,1)_0 \oplus \left((3,3)_{-Y} \oplus (6,1)_{-2Y} \oplus \mathrm{h.c.}\right) \tag{B.26}$$

under $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$, including stable unwanted states with color and hypercharge. Asymptotic freedom requires $N \geq 4$, the model can be extended up to $N = 7$ for $D \oplus V$ and up to $N = 6$ for $U \oplus V$. For $N = 4$, the lightest TCb DM candidate is a singlet $(D\bar{D} + VV)^2$ or $(U\bar{U} + VV)^2$, while the heavier TCb contain the multiplet

$$\begin{aligned}\mathrm{TCb}: \begin{array}{|c|c|}\hline & \\ \hline & \\ \hline\end{array} = 495 = 2 \times (1,1)_0 \oplus (1,3)_0 \oplus (1,5)_0 \oplus 2 \times (8,1)_0 \oplus (8,3)_{0,3Y} \oplus (8,5)_0 \\ \oplus (27,1)_0 \oplus \Big((3,1)_{2Y} \oplus (3,3)_{2Y, 2\times -Y} \oplus (\bar{6},1)_{2Y,-4Y} \oplus (3,5)_{-Y} \\ \oplus (\bar{6},3)_{-Y} \oplus (\bar{6},5)_{2Y} \oplus (15,1)_{2Y} \oplus (15,3)_{-Y} \oplus \mathrm{h.c.}\Big) \\ \text{under } \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y\,.\end{aligned} \tag{B.27}$$

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SO(N)$_{\rm TC}$ models $\Psi = D \oplus E$. This model with $N_{\rm TF} = 8$ is valid from $N = 4$ up to $N = 6$, while the analogous model $U \oplus E$ suffers by a sub-Planckian Landau pole for g_Y. The decomposition of the TCπ multiplet under $\mathrm{SU}(3)_c \otimes \mathrm{U}(1)_Y$ is

$$\mathrm{TC}\pi\,:\,35 = 1_{2\times 0,\pm 2} \oplus 8_0 \oplus \left(3_{2/3,-4/3} \oplus 6_{-2/3} \oplus \text{h.c.}\right). \tag{B.28}$$

The list includes stable TCπ with color and hypercharge, so that we need to break the accidental symmetries to remove unwanted stable particles. For $N = 4$ the DM candidate is a TCb singlet $(D\bar{D} + E\bar{E})^2$. To complete the list of TCb we need the decomposition of the multiplet

$$\begin{aligned}\mathrm{TCb}\,:\,\yng(2,2) = 300 = 3\times 1_0 \oplus 8_{5\times 0,2\times(\pm 2)} \oplus 27_0 \oplus \Big(&3_{4\times 2/3,2\times(-4/3)}\\ &\oplus\bar{6}_{8/3,2\times 2/3,3\times(-4/3)} \oplus 15_{2\times 2/3,-4/3} \oplus \text{h.c.}\Big)\end{aligned} \tag{B.29}$$

under $\mathrm{SU}(3)_c \otimes \mathrm{U}(1)_Y$.

SO(N)$_{\rm TC}$ models $\Psi = D \oplus L$ and $\Psi = U \oplus L$. These models can be analysed jointly defining $Y = 1/3$ and $Y = -2/3$ for D and U respectively. They are characterized by $N_{\rm TF} = 10$, the model with D is allowed for $4 \leq N \leq 9$, while the model with U is allowed only for $N = 4$. TCπ fill a 54 dimensional representation of $\mathrm{SO}(10)_{\rm TF}$:

$$\mathrm{TC}\pi\,:\,54 = (1,1)_0 \oplus (1,3)_{0,\pm 1} \oplus (8,1)_0 \oplus \left((3,2)_{1/2-Y,-1/2-Y} \oplus (\bar{6},1)_{2Y} \oplus \text{h.c.}\right) \tag{B.30}$$

under $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. Accidentally stable TC$\pi$ have color and/or hypercharge, so that we need to break these accidental symmetries to make the model phenomenologically viable. The lightest TCb for $N = 4$ is a singlet $(D\bar{D} + L\bar{L})^2$ or $(U\bar{U} + L\bar{L})^2$ and it is a good DM candidate. The set of heavier TCb contains the following states

$$\begin{aligned}\mathrm{TCb}\,:\,770 \;&= (1,1)_{4\times 0,2\times(\pm 1),\pm 2} \oplus (1,3)_{2\times 0,\pm 1} \oplus (1,5)_0 \oplus (8,1)_{3\times 0,\pm 1}\\ &\oplus (8,2)_{\pm(1/2+3Y),\pm(1/2-3Y)} \oplus (8,3)_{2\times 0,\pm 1} \oplus (27,1)_0 \oplus \Big((3,1)_{1+2Y,2\times 2Y,-1+2Y}\\ &\oplus (3,2)_{3/2-Y,3\times(1/2-Y),3\times(-1/2-Y),-3/2-Y} \oplus (3,3)_{2Y} \oplus (\bar{6},1)_{2Y,-4Y}\\ &\oplus (3,4)_{1/2-Y,-1/2-Y} \oplus (\bar{6},2)_{1/2-Y,-1/2-Y} \oplus (\bar{6},3)_{1+2Y,2Y,-1+2Y}\\ &\oplus (15,1)_{2Y} \oplus (15,2)_{1/2-Y,-1/2-Y} \oplus \text{h.c.}\Big),\end{aligned} \tag{B.31}$$

under $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$.

SO(N)$_{\rm TC}$ models $\Psi = G \oplus N$ and $\Psi = G \oplus E$. These are simple extensions of the $\Psi = G$ model described in section 3.2, with $N_{\rm TF} = 9$ and $N_{\rm TF} = 10$ respectively, allowed only for $N = 4$. Because of species number TCπ made by different species are stable and since they have hypercharge and/or color, they are excluded by DM direct search bounds. We need ad hoc assumptions to break the accidental symmetry and make them unstable. The lists of TCπ for the $G \oplus N$ and $G \oplus E$ models respectively are:

$$\mathrm{TC}\pi\,:\,44 \;= 1_0 \oplus 2\times 8_0 \oplus 27_0 \tag{B.32}$$

$$\mathrm{TC}\pi\,:\,54 \;= 1_{0,\pm 2} \oplus 8_{0,\pm 1} \oplus 27_0 \quad \text{under } \mathrm{SU}(3)_c \otimes \mathrm{U}(1)_Y\,. \tag{B.33}$$

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In both cases, the DM candidate is a singlet, in the first model it is made by $GG(GG{+}NN)$, in the second by $(GG+E\bar{E})^2$. Here we present the decomposition under $\mathrm{SU}(3)_c \otimes \mathrm{U}(1)_Y$ of the heavier TCb multiplet for the $G \oplus N$ and $G \oplus E$ models respectively:

$$\mathrm{TCb}: \begin{array}{|c|c|}\hline & \\ \hline & \\ \hline\end{array} = 495 = 1_0 \oplus 4\times 8_0 \oplus 6\times 27_0 \oplus 64_0 \oplus \Big(2\times 10_0 \oplus 28_0 \oplus 2\times 35_0 \oplus \mathrm{h.c}\Big) \tag{B.34}$$

$$\mathrm{TCb}: \begin{array}{|c|c|}\hline & \\ \hline & \\ \hline\end{array} = 770 = 2\times 1_0 \oplus 8_{\pm 2, 3\times(\pm 1), 0} \oplus 27_{\pm 2, 2\times(\pm 1), 4\times 0} \oplus 64_0 \oplus \Big(10_{\pm 1, 2\times 0} \oplus 28_0 \oplus 35_{\pm 1, 0} \oplus \mathrm{h.c.}\Big)\,. \tag{B.35}$$

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