Ernst Potential of Near-Horizon Extremal Kerr Black Holes

M F A R Sakti¹, A Irawan¹, A Suroso^{1,2}, and F P Zen^{1,2}

 1 Theoretical Physics Division, Institut Teknologi Bandung, Jl. Ganesha 10 Bandung 40132, Indonesia

 2 Indonesia Center for Theoretical and Mathematical Physics (ICTMP), Jl. Ganesha 10 Bandung 40132, Indonesia

E-mail: m.fitrah@students.itb.ac.id

Abstract. One way to find the solution of black holes is through the Ernst equations that is quite simple instead of solving the Einstein equation. Solution of Ernst equations for Kerr and Kerr-Newman black holes have been achieved in the last century. The magnetized case for those black holes and their Ernst potentials can be found using Harrison transformation. Herein the Ernst potential for extremal rotating Kerr and its magnetized solution is shown. In the end, we also extend this fashion for extremal Kerr-Newman black hole.

1. Introduction

The Einstein field equation has been applied to many physical phenomena in our universe. The space-time metric of our accelerated expansion universe itself is the solution of this equation called by Friedmann-Lemaître-Robertson-Walker metric.¹ It is believed that this expansion is accelerated due to the presence of dark energy that might be modeled by the scalar field² and other fields (see [3–5] for some examples). We could also study about the smaller scale of the gravitational field using Einstein equation for instance are Neutron stars⁶ and Boson stars which contain scalar field within the theory (see [7–10] for some examples). For the absence of a gravitational field, we could find the investigation of the so-called Q-balls.^{7,11,12} The other thing which is quite important is the black hole especially for rotating black hole solution because it is believed that this type is the most physical black hole in our universe.¹³

Black hole solutions can be obtained from Ernst formulation.^{14,15} In this formulation, it is introduced the Ernst potentials which are the solutions to the Ernst equations which are equivalent to the Einstein field equation in the stationary case. In Ernst formalism, we work on general rotating black hole metric¹⁶ called by Papapetrou metric which corresponds to the existence of Killing vector fields $\hat{\xi}_t$ and $\hat{\xi}_{\phi}$. This formalism has been applied to the more general black holes, Kerr-Newman black holes.¹⁵ For an uncharged solution, the intrinsic charge of the Kerr-Newman space-time is taken to be zero and will reduce to the Kerr space-time.

Tools of Ernst formalism are also applied to the Kerr black hole which is immersed by the external magnetic field. The magnetized Kerr metric firstly is proposed by Melvin that solves the Maxwell-Einstein system of equations. In addition, such physical magnetic fields are believed to exist in the active galactic nuclei that have an important role in explaining the physical aspects of the supermassive black holes in the center of the galaxies.²¹ The magnetized metric also

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can be obtained from the method proposed by Harrison²² called by Harrison transformation. In [23], the Harrison transformation has been applied to the Kerr-Newman space-time metric. Therein it investigates the correspondence between the gravity and the CFTs. In addition, Near-Horizon Extremal Magnetized Kerr (NHEMK) metric of the magnetized Kerr black holes has been obtained in [23, 26, 31] by applying Harrison transformation.

In this paper, we compute the Ernst potentials for the near-horizon and extremal case of Kerr black hole. We can see that the Ernst potential for the extremal case depends on the angular coordinate only. We also calculate the Ernst potential of the magnetized solution. For more general case, the Ernst potentials for Near-Horizon Extremal Kerr-Newman black hole and its magnetized case are calculated. The aim of computing all of this Ernst potentials is to provide another way of deriving charged rotating black hole solution without employing Einstein-Maxwell system of equations, yet through the Ernst equations which are simpler.

We organize the remaining parts of the paper as follows. In section 2, we review the part of the derivation of Kerr metric from the Ernst equations. In section 3, we show the nearhorizon and extremal Kerr metric and its new Ernst potential. In the next section, we find the Ernst potential related to the NHEMK metric. The general Ernst potentials for Near-Horizon Extremal Kerr-Newman black hole are then shown in section 5. Finally, we summarize the entire paper in the last section.

2. Review on Ernst solution for Kerr black holes

We wish to briefly review the derivation of the rotating black hole solutions from the Ernst equations.¹⁶ Ernst equations are equivalent to the Einstein field equation in the stationary case. The original papers^{14,15} use this Ernst formalism to provide a simpler way of deriving black hole solutions of stationary space-time. To obtain stationary black hole solutions, we may apply Ernst formalism that has Ernst potentials as the solutions. Solving Ernst equations is simpler than solving the Einstein field equation because one only needs to solve a second differential order equation. In addition, when employing Einstein equation, we need to solve more than one second differential order equation.

The classical stationary axially symmetric metric is described by the Lagrangian density that is independent of time \hat{t} and azimuth coordinate $\hat{\phi}$ so it has Killing vectors $\hat{\xi}_t$ and $\hat{\xi}_{\phi}$. So, it will be invariant under transformations $\hat{t} \to -\hat{t}$ and $\hat{\phi} \to -\hat{\phi}$. This can be expressed in Papapetrou form of metric defined by

$$ds^{2} = f^{-1} \left[e^{2\gamma} (d\rho^{2} + dz^{2}) + \rho^{2} d\hat{\phi}^{2} \right] - f (d\hat{t} - \omega d\hat{\phi})^{2}.$$
(1)

Then the well-known Ernst equation for axially symmetric metric without intrinsic charge is given by

$$(\operatorname{Re} \varepsilon)\nabla^2 \varepsilon = \nabla \varepsilon. \nabla \varepsilon, \qquad (2)$$

for which ε is the Ernst potential where $\varepsilon = f + i\tilde{\varphi}$ with twist potential $\tilde{\varphi}$ (see 16 for detail derivation). We apply the following operator

$$\nabla^2 = \frac{\partial^2}{\partial \rho^2} + \frac{\partial^2}{\partial z^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho}, \quad \nabla = \frac{\partial}{\partial \rho} \hat{\rho} + \frac{\partial}{\partial z} \hat{z}, \tag{3}$$

which indicates cylinder coordinate. To produce Kerr space-time metric, we start with the Papapetrou metric (1) and define $e^{2\gamma} = -P^{-2}$, $\zeta = (z + i\rho)/\sqrt{2}$. Hence Eq. (1) will become

$$ds^{2} = f^{-1} \left(-2P^{-2}d\zeta . d\zeta^{*} + \rho^{2}d\hat{t}^{2} \right) - f \left(d\hat{\phi} - \omega d\hat{t} \right)^{2}.$$
 (4)

Note that we have changed $\hat{t} \leftrightarrow \hat{\phi}$ just for the convention. For the case of the metric consists of the radial and angular coordinates, we can choose

$$d\zeta = \frac{1}{\sqrt{2}} \left(\frac{d\hat{r}}{\sqrt{\Delta}} + id\theta \right),\tag{5}$$

where $\Delta = \hat{r}^2 - 2M\hat{r} + a^2$. Hence, Eq. (4) becomes

$$ds^{2} = f^{-1} \left[-P^{-2} \left(\frac{d\hat{r}^{2}}{\Delta} + d\theta^{2} \right) + \rho^{2} d\hat{t}^{2} \right] - f \left(d\hat{\phi} - \omega d\hat{t} \right)^{2}.$$
 (6)

By choosing

$$\rho = \sqrt{\Delta} \sin\theta, \ P = (\sqrt{A} \sin\theta)^{-1}, \ f = -\frac{A \sin^2\theta}{\Sigma}, \ \omega = \frac{2Ma\hat{r}}{A},$$
(7)

we straightforwardly obtain Kerr metric in Boyer-Lindquist coordinates that is given by

$$ds^{2} = \Sigma \left(-\frac{\Delta}{A} d\hat{t}^{2} + \frac{d\hat{r}^{2}}{\Delta} + d\theta^{2} \right) + \frac{A \sin^{2} \theta}{\Sigma} \left(d\hat{\phi} - \frac{2Ma\hat{r}}{A} d\hat{t}^{2} \right)^{2}, \tag{8}$$

where

$$\Sigma = \hat{r}^2 + a^2 \cos^2\theta, \ A = (\hat{r}^2 + a^2)^2 - \Delta a^2 \sin^2\theta$$

where ω is the angular velocity, M is black hole mass, J is angular momentum, and a = J/M. Then the Ernst potential for Kerr space-time metric as the solution of Eq. (2) is

$$\varepsilon = -(\hat{r}^2 + a^2)\sin^2\theta + 2iMa\cos\theta(3 - \cos^2\theta) - \frac{2Ma^2\sin^4\theta}{\hat{r} + ia\cos\theta}.$$
(9)

For more general black hole, Kerr-Newman black hole, Ernst equation becomes more complex because of the presence of the intrinsic charge q. The Ernst equations are now

$$(\operatorname{Re} \varepsilon + |\Phi|^2) \nabla^2 \varepsilon = (\nabla \varepsilon + 2\Phi^* \nabla \Phi) \cdot \nabla \varepsilon, \qquad (10)$$

$$(\operatorname{Re} \varepsilon + |\Phi|^2)\nabla^2 \Phi = (\nabla \varepsilon + 2\Phi^* \nabla \Phi) \cdot \nabla \Phi, \qquad (11)$$

where $\varepsilon = f - |\Phi|^2 + i\tilde{\varphi}$. For Kerr-Newman, the Ernst potentials are

$$\varepsilon = -[(\hat{r}^2 + a^2)\sin^2\theta + q^2\cos^2\theta] + 2iMa\cos\theta(3 - \cos^2\theta) - 2a\frac{Ma\sin^2\theta + iq^2\cos\theta}{\hat{r} + ia\cos\theta}\sin^2\theta,$$

$$\Phi = -iq\cos\theta + \frac{qa}{\hat{r} + ia\cos\theta}\sin^2\theta.$$
(12)

It is obviously seen that for $\Phi = 0$ ($\hat{q} = 0$), it will reduce to Kerr black hole.

2.1. Magnetized Kerr black holes

In this subsection, we briefly review the way to find magnetized black holes solution by employing Harrison transformation²² on the Ernst potentials. Harrison transformation satisfies

$$\varepsilon' = \frac{\varepsilon}{\Lambda}, \ \Phi' = \Lambda^{-1} \left(\Phi - \frac{1}{2} B \varepsilon \right), \ \Lambda = 1 + B \Phi - \frac{1}{4} B^2 \varepsilon,$$
 (13)

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where B is the axial magnetic field parameter. Several Papapetrou potentials in Eq. (1) also change simultaneously

$$f' = |\Lambda|^{-2} f, \quad \nabla \omega' = |\Lambda|^2 \nabla \omega - \rho f^{-1} \left(\Lambda^* \nabla \Lambda - \Lambda \nabla \Lambda^* \right).$$
(14)

If we perform the Harrison transformation to the Kerr metric (8), we will produce magnetized Kerr metric that is given as follows

$$ds^{2} = \Sigma |\Lambda|^{2} \left(-\frac{\Delta}{A} d\hat{t}^{2} + \frac{d\hat{r}^{2}}{\Delta} + d\theta^{2} \right) + \frac{A \sin^{2} \theta}{\Sigma |\Lambda|^{2}} \left(d\hat{\phi} - \omega' d\hat{t} \right)^{2}.$$
 (15)

This is the electrovacuum solution of the Einstein field equation. The potential ω' is obtained from (14) and can be separated to this relations³⁸

$$\frac{\partial \omega'}{\partial r} = |\Lambda|^2 \frac{\partial \omega}{\partial r} - \frac{2\Sigma}{A \sin \theta} \left(\operatorname{Im} \Lambda \frac{\partial \operatorname{Re} \Lambda}{\partial \theta} - \operatorname{Re} \Lambda \frac{\partial \operatorname{Im} \Lambda}{\partial \theta} \right), \tag{16}$$

$$\frac{\partial \omega'}{\partial \theta} = |\Lambda|^2 \frac{\partial \omega}{\partial \theta} + \frac{2\Sigma \Delta}{A \sin \theta} \left(\operatorname{Im} \Lambda \frac{\partial \operatorname{Re} \Lambda}{\partial r} - \operatorname{Re} \Lambda \frac{\partial \operatorname{Im} \Lambda}{\partial r} \right).$$
(17)

By solving Eqs. (16) and (17), we will find

$$\omega' = \frac{16M\hat{r}a + \omega_b B^4}{8A}, \text{ where}$$
(18)

$$\omega_b = 4a^3 M^3 \hat{r}(3 + \cos^4\theta) + 2aM^2 [\hat{r}^4 \{ (\cos^2\theta - 3)^2 - 6 \} + 2a^2 \hat{r}^2 (3 - 3\cos^2\theta - 2\cos^4\theta) - a^4 (1 + \cos^4\theta)] + aM\hat{r}(\hat{r}^2 + a^2) \{ \hat{r}^2 (3 + 6\cos^2\theta - \cos^4\theta) - a^2 (1 - 6\cos^2\theta - 3\cos^4\theta) \}.$$

This is the Kerr black hole solution immersed by the axially external magnetic field. This transformation actually generates non-asymptotically flat black holes solution but may be a good theoretical model for the strong magnetic field around the black holes. However, it is found in 39 that magnetized Kerr metric (15) suffers conical singularity in the axial coordinate $\hat{\phi}$ hence one rotation of $\hat{\phi}$ is not 2π anymore but $2\pi |\Lambda_0|^2$ where Λ_0 is equal to $\Lambda|_{\theta=0}$. In order to solve this problem, we need to rescale the axial coordinate, i.e. $\hat{\phi}' \to |\Lambda_0|^2 \hat{\phi}'$. After rescaling, the magnetized Kerr metric (15) becomes

$$ds^{2} = \Sigma |\Lambda|^{2} \left(-\frac{\Delta}{A} d\hat{t}^{2} + \frac{d\hat{r}^{2}}{\Delta} + d\theta^{2} \right) + \frac{A \sin^{2} \theta}{\Sigma |\Lambda|^{2}} \left(|\Lambda_{0}|^{2} d\hat{\phi} - \omega' d\hat{t} \right)^{2},$$
(19)

for which the event horizon is similar with the Kerr black hole solution. If we take B = 0, it will reduce to Kerr metric. The more general magnetized Kerr solution, i.e. magnetized Kerr-Newman metric also has been obtained in [40].

3. Ernst Potentials for Near-Horizon Extremal Kerr Black Holes

We will see the corresponding Ernst potential for the Near-Horizon Kerr solution. We will also extend to the extremal condition. The event horizon of Kerr black hole is located on

$$r_{+} = M + \sqrt{M^2 - a^2}.$$
 (20)

To produce the Near-Horizon extremal Kerr (NHEK) metric, we work with new coordinates defined by 18

$$\hat{t} = \frac{r_0}{\lambda}t, \ \hat{\phi} = \phi + \omega_H \frac{r_0}{\lambda}t, \ \hat{r} = \lambda r_0 r + r_+,$$
(21)

where $r_0^2 = r_+^2 + a^2$. By adjusting that appropriate coordinates transformation, the Ernst potential (9) changes to be

$$\varepsilon = -\left[\left(\lambda r_0 r + r_+\right)^2 + a^2\right]\sin^2\theta + 2iMa\cos\theta(3 - \cos^2\theta) - \frac{2Ma^2\sin^4\theta}{(\lambda r_0 r + r_+) + ia\cos\theta}.$$
 (22)

In this circumstance, we can call this Ernst potential as the potential for the Near-Horizon Kerr black holes. However, it doesn't prevail for the extremal case. For Kerr black holes, extremality occurs when $M^2 = a^2$ and in coordinates transformation (21) we have to take $\lambda \to 0$. In fact, this constant is a perturbation constant of the near-horizon and we have neglected the higher terms. Potential (22) is a solution of the Ernst equation (2) where Eq. (3) has to transform using Eq. (21). So we find this Ernst potential for Near-Horizon Kerr which is by finding this, we could derive near-horizon Kerr black hole solution without performing the tensorial calculation and using near-horizon coordinates transformation. We have to mention also that the near-horizon form of the Ernst potential is never shown before because the near-horizon metric is just recently discussed.³²

After the near-horizon case, we will add another feature which is the extremal condition. To gain the extremal and Near-Horizon Kerr black hole, we may apply (21) and put $\lambda \to 0$ to the Kerr metric. Then the NHEK black holes metric is finally given by

$$ds^{2} = 2J\Omega^{2} \left[-r^{2}dt^{2} + \frac{dr^{2}}{r^{2}} + d\theta^{2} + \Lambda^{2} \left(d\phi + rdt \right)^{2} \right],$$
(23)

where

$$J = M^2, \Omega^2 = \frac{1 + \cos^2\theta}{2}, \Lambda = \frac{2\sin\theta}{1 + \cos^2\theta},$$

The NHEK metric is asymptotically AdS and possesses $SL(2, R) \times U(1)$ isometry. This property has brought the opportunity to study the correspondence between AdS space-time and the CFTs on the boundary of the black holes.^{17, 23–29} After taking $\lambda \to 0$, the corresponding Ernst potential is then³³

$$\varepsilon = -2M^2 \sin^2\theta - \frac{2M^2 \sin^4\theta}{1 + \cos^2\theta} + 2iM^2 \cos\theta \left((3 - \cos^2\theta) + \frac{\sin^4\theta}{1 + \cos^2\theta} \right).$$
(24)

Interestingly, it is not the solution of Eq. (2) anymore. One could check by inserting (24) to Eq. (2). The AdS structure on the metric (23) may be the reason of that. Because in [41], it is found that for black hole solution in AdS background, we apply the other different Ernst equations that come from the reality of the non-zero value of the cosmological constant. This constant appears in the Lagrangian besides the scalar Ricci as the gravitational part. The main point is to check whether this extremal potential matches the Ernst equation for the case of non-vanishing cosmological constant or not.

4. Near-Horizon Extremal Magnetized Kerr Black Holes

The Harrison transformation does not replace the location of the event horizon of the Kerr black hole, so the extremal condition occurs also when $M^2 = a^2$. So, to gain the NHEK for magnetized metric, we work with these new coordinates transformations^{26,42}

$$\hat{t} = \frac{2M^2 t}{\lambda}, \ \hat{\phi} = \phi + \frac{(1+2B^4M^4)Mt}{(1+B^4M^4)\lambda}, \ \hat{r} = \lambda r + r_+.$$
(25)

We are not going to show the form of near-horizon Ernst potential of the magnetized solution because the mathematical formula is quite long. So we just show the potential and the spacetime metric when the extremal limit is also taken. By taking the limit $\lambda \to 0$, we obtain Near-Horizon Extremal Magnetized Kerr (NHEMK) metric from Eq. (19) that is defined by

$$ds^{2} = \Sigma |\Lambda|^{2} \left(-r^{2} dt^{2} + \frac{dr^{2}}{r^{2}} + d\theta^{2} \right) + \frac{A \sin^{2} \theta |\Lambda_{0}|^{4}}{\Sigma |\Lambda|^{2}} \left(d\phi + \frac{(1 - B^{4} M^{4})}{|\Lambda_{0}|^{2}} r dt \right)^{2},$$
(26)

It is similar with NHEK metric, the Ernst potentials can be obtained from the corresponding coordinate transformations (25). The new Ernst potentials for NHEMK are

$$\varepsilon' = \frac{-4M^2[1+3M^2B^2+(B^2M^2-1)^2(2\cos^2\theta-1)]+16iM^2\cos\theta}{3+2B^2M^2+3B^4M^4+(B^2M^2-1)^2(2\cos^2\theta-1)},$$
(27)

$$\Phi' = \frac{2M^2B[1+3M^2B^2+(B^2M^2-1)^2(2\cos^2\theta-1)]-8iM^2B\cos\theta}{3+2B^2M^2+3B^4M^4+(B^2M^2-1)^2(2\cos^2\theta-1)},$$
(28)

which reduce to the Ernst potentials of the NHEK metric by taking the magnetic field parameter B to 0. The remaining orthonormal components of the electromagnetic field for the locally non rotating observer, are only

$$H_r = \frac{4B[1 + 6B^2M^2 + B^4M^4 - (B^2M^2 - 1)^2(2\cos^2\theta - 1)]}{[3 + 2B^2M^2 + 3B^4M^4 + (B^2M^2 - 1)^2(2\cos^2\theta - 1)]^2},$$
(29)

$$E_r = \frac{16B(1 - B^4 M^4)\cos\theta}{[3 + 2B^2 M^2 + 3B^4 M^4 + (B^2 M^2 - 1)^2 (2\cos^2\theta - 1)]^2},$$
(30)

and the angular components vanish. In [44], they have shown the orthonormal components for extremal magnetized Kerr-Newman. However we will also show the similar Ernst potentials for the Kerr-Newman solution in the next section.

5. Near-Horizon Extremal Kerr-Newman Black Holes

Before we summarize our result, we will show the Ernst potentials for the more general extremal black hole, i.e. extremal Kerr-Newman black holes. The Kerr-Newman black holes has the event horizon located on

$$r_{+} = M + \sqrt{M^2 - a^2 - q^2},\tag{31}$$

which comes from the root of $\Delta = \hat{r}^2 - 2M\hat{r} + a^2 + q^2 = 0$. The Ernst potential for the near-horizon Kerr-Newman black hole is given by

$$\varepsilon = -q^2 \cos^2 \theta - \left[(\lambda r_0 r + r_+)^2 + a^2 \right] \sin^2 \theta + 2i M a \cos \theta (3 - \cos^2 \theta) - \frac{2M a^2 \sin^4 \theta + 2i a q^2 \cos \theta \sin^2 \theta}{(\lambda r_0 r + r_+) + i a \cos \theta},$$
(32)

$$\Phi = \frac{aq\sin^2\theta - iq\cos\theta(r_+ + \lambda r_0 r + ia\cos\theta)}{r_+ + \lambda r_0 r + ia\cos\theta}.$$
(33)

In the extremal limit, we have to take $M^2 = a^2 + q^2$. Near-Horizon Extremal Kerr-Newman black hole has the form

$$ds^{2} = \sigma \left(-r^{2}dt^{2} + \frac{dy^{2}}{r^{2}} + d\theta^{2} \right) + \frac{(2a^{2} + q^{2})^{2} \sin^{2}\theta}{\sigma} \left(d\phi + \frac{2a\sqrt{q^{2} + a^{2}}}{q^{2} + 2a^{2}} r dt \right)^{2},$$
(34)

where $\sigma = q^2 + a^2(1 + \cos^2\theta)$. The solution of Eq. (34) is related to these Ernst potentials

$$\varepsilon = \frac{-\sqrt{q^2 + a^2}(4a^2 + q^2) + ia(4a^2 + 3q^2)\cos\theta}{\sqrt{q^2 + a^2} + ia\cos\theta},\tag{35}$$

$$\Phi = \frac{q\left(a - i\sqrt{q^2 + a^2}\cos\theta\right)}{\sqrt{q^2 + a^2} + ia\cos\theta}.$$
(36)

The solution of Near-Horizon extremal magnetized Kerr-Newman has also been derived.²⁴ Here we show the related Ernst potentials which are given by

$$\varepsilon' = \frac{-4\sqrt{q^2 + a^2}(4a^2 + q^2) + 4ia(4a^2 + 3q^2)\cos\theta}{\chi_1 - i\chi_2},\tag{37}$$

$$\Phi' = \frac{4qa + (8a^2 + 2q^2)B\sqrt{q^2 + a^2} - 2i(4a^3B + 3aq^2B + 2q\sqrt{q^2 + a^2})\cos\theta}{\chi_1 - i\chi_2}, \quad (38)$$

where

$$\chi_1 = 4aqB + 4a^2B^2\sqrt{q^2 + a^2} + \sqrt{q^2 + a^2}(4 + q^2B^2),$$

$$\chi_2 = \left[4a^3B^2 + 4qB\sqrt{q^2 + a^2} + a(3q^2B^2 - 4)\right]\cos\theta.$$

It is worth noting that it is needed to check the Ernst potentials of the near-horizon form of extremal Kerr-Newman black hole. As the extremal Kerr black hole and its magnetized solution, we need to check whether it satisfies the Ernst equations such in [41] or not. So we will address these extremal results to be checked in the next paper.

6. Summary

Black hole solutions are related to the arbitrary Ernst potentials which is solved from the Ernst equation. Ernst equation is a simpler way to find a new black hole solution because we only need to solve two complex differential equations because we know that Einstein equation is tensorial equation with second order differential equation. Herein we have studied the Kerr black hole and its Ernst potentials. We have shown the Ernst potential for the near-horizon and extremal Kerr black holes metric. The aim of this calculation is to provide the simpler calculation to find near-horizon and extremal Kerr black hole solution without solving the Einstein equation and using the coordinates transformation because it has been shown within this paper.

Tools of Ernst formalism are also applied to the magnetized Kerr black holes. Mathematically, this magnetized metric is produced from applying Harrison transformation. By applying it, Ernst potentials for this magnetized case are found. The non-zero Cartan components of the NHEMK metric are also shown. Note that by taking B to be zero will reduce to the case of NHEK solution. Later we finish our result by showing the Ernst potentials for the extremal Kerr-Newman metric and its magnetized case. As the extremal Kerr solution, we calculate the Ernst potential to provide the simpler way to find the near-horizon and extremal Kerr-Newman black hole solution and its magnetized case.

For further studies, it is interesting to verify whether the Ernst potential we find here are related or matching with the solutions of the Ernst equation when the cosmological constant is not vanishing.

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