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Violation of the Leggett-Garg Inequality in neutrino oscillations

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Abstract. The Leggett-Garg inequality, an analogue of Bell's inequality involving correlations of measurements on a system at different times, stands as one of the hallmark tests of quantum mechanics against classical predictions. The phenomenon of neutrino oscillations should adhere to quantum-mechanical predictions and provide an observable violation of the Leggett-Garg inequality. We demonstrate how oscillation phenomena can be used to test for violations of the classical bound by performing measurements on an ensemble of neutrinos at distinct energies, as opposed to a single neutrino at distinct times. A study of the MINOS experiment's data shows a greater than 6σ violation over a distance of 735 km, representing the longest distance over which either the Leggett-Garg inequality or Bell's inequality has been tested.

1. Introduction

Perhaps one of the most counterintuitive aspects of quantum mechanics is the principle of superposition, which stipulates that an entity can exist simultaneously in multiple different states. Bell indicated how experiments could distinguish between classical systems and those that demonstrate quantum superposition [1]. The Leggett-Garg inequality (LGI), sometimes referred to as the "time-analogue" of Bell's inequality, concerns correlations among measurements performed at different times and allows for a complementary test of quantum mechanics [2].

The original goal of LGI tests was to demonstrate macroscopic coherence—that is, that quantum mechanics applies on macroscopic scales up to the level at which many-particle systems exhibit decoherence [2, 3]. However, LGI tests have another purpose: to test "realism," the notion that physical systems possess complete sets of definite values for various parameters prior to measurement. LGI violations imply that hidden-variable (or "realistic") alternatives to quantum mechanics cannot adequately describe a system's time evolution [3].

Neutrino flavor oscillations provide an interesting system with which to test the LGI. Neutrino oscillations may be treated with the same formalism that is typically used to describe other systems displaying quantum coherence, such as squeezed atomic states. However, because the coherence length of neutrino oscillations extends over vast distances, an LGI experiment using neutrinos presents a stark contrast to other types of LGI tests, which typically use photons, electrons, or nuclear spins [3]. Although the idea of testing the LGI with neutrinos has been proposed, we believe this is the first such empirical test to be performed [4].

2. Formalism and Assumptions

We consider a dichotomic observable $\hat{Q}$ (with realizations ± 1) that may be measured at various times t_i. The correlation between measurements at times t_i and t_j can be written as $\mathcal{C}_{ij} \equiv \langle \hat{Q}(t_i)\hat{Q}(t_j) \rangle$, where $\langle ... \rangle$ indicates averaging over many trials. For measurements at n distinct times, we may define the Leggett-Garg parameter K_n for $n \geq 3$ as

$$K_n \equiv \sum_{i=1}^{n-1} \mathcal{C}_{i,i+1} - \mathcal{C}_{n,1}. \tag{1}$$

"Realistic" systems obey the Leggett-Garg bound [2], which is given by $K_n \leq n-2$.

We may calculate an expected value for K_n according to quantum mechanics, K_n^Q, by time-evolving $\hat{Q}$ under the unitary operator $\mathcal{U}(t)$: $\hat{Q}(t_i) = \mathcal{U}^\dagger(t_i)\,\hat{Q}\,\mathcal{U}(t_i)$. If one artificially imposes that $\hat{Q}(t_i)$ and $\hat{Q}(t_j)$ must commute, then one recovers the classical prediction for K_n, which we denote K_n^C. We see that K_n^C satisfies the LGI, whereas $K_n^Q \leq n\cos(\pi/n)$ may violate the LGI. The discrepancy between these predictions provides an opening for experimental testing [3].

The original derivation of the LGI assumed that measurements of $\hat{Q}$ are made in a non-invasive manner [2]. The LGI may be derived instead under the assumption of "stationarity," such that the correlations $\mathcal{C}_{ij}$ depend only on the time difference $\tau \equiv t_j - t_i$ between measurements. In this case, the LGI applies to the class of "realistic" models that are Markovian, for which the evolution of the system after some time t is independent of the means by which the system arrived in its state at t [3]. We may then consider measurements performed on distinct members of an identically prepared ensemble, each of which begins in some known initial state. When paired with the "prepared ensemble" condition, stationarity acts as a substitute for measurement schemes intended to be noninvasive, because wave function collapse and classical disturbance in a given system do not influence measurements performed on distinct members of the ensemble. Unlike the assumption of noninvasive measurability, moreover, stationarity may be subjected to independent testing [3]. As we will see, this condition may be fulfilled for neutrino oscillations.

3. LGI Violation Using Neutrinos

The observable $\hat{Q}$ measures neutrino flavor as projected along a particular axis: $\hat{Q} \equiv \sigma_z$, with eigenvalues $\hat{Q}|\nu_\mu\rangle = |\nu_\mu\rangle$ and $\hat{Q}|\nu_e\rangle = -|\nu_e\rangle$. The time evolution of a neutrino with energy E_a and oscillation frequency ω_a is governed by the unitary operator $\mathcal{U}$, which is related to the oscillation term of the two-flavor neutrino Hamiltonian in vaccuum $\mathcal{H}_{\rm osc} \equiv \vec{r}\cdot\vec{\sigma}/2$ via

$$\mathcal{U}(\omega_a; t_i, t_j) \equiv \mathcal{U}(\psi_{a;ij}) = \exp\left[-i\int_{t_i}^{t_j} \mathcal{H}_{\rm osc}(\omega_a)dt\right] \simeq \cos(\psi_{a;ij})\mathbb{1} - i\sin(\psi_{a;ij})(\hat{r}(\omega_a)\cdot\vec{\sigma}), \quad (2)$$

where $\psi_{a;ij} \simeq \omega_a(t_j - t_i)/2 = \frac{\Delta m_{21}^2}{4E_a}(t_j - t_i)$ is the phase accumulated while propagating from t_i to t_j. A neutrino's evolution depends only on $\psi_{a;ij}$, rather than on individual times. Moreover, the phases obey a sum rule: $\sum_{i=1}^{n-1}\psi_{a;i,i+1} = \psi_{a;1n}$. Given the unitary operator in Eq. (2), the correlation $\mathcal{C}_{ij}$ simplifies to $\mathcal{C}_{ij}(\omega_a) = 1 - 2\sin^2 2\theta\sin^2\psi_{a;ij}$, where θ is the vacuum mixing angle.

For a pair of measurements that depend on ω_a and a time interval $\tau = t_j - t_i$, $\mathcal{C}_{ij}$ depends only on $\psi_{a;ij} = \omega_a\tau/2$, consistent with the stationarity condition. Furthermore, when measurements occur at a fixed distance δL from the source, $\psi_{a;ij} \to \psi_a = \omega_a\delta L/2$ varies only with energy E_a. We use measurements at different *energies* E_a, as opposed to different *times*, to probe the LGI. Assuming a beam that begins in the pure $|\nu_\mu\rangle$ state and is measured at two locations separated by δL, each correlation simplifies to $\mathcal{C}(\omega_a) = 2\mathcal{P}_{\mu\mu}(\psi_a) - 1$, where $P_{\mu\mu}$ is the neutrino survival probability. We may therefore construct the Leggett-Garg parameter as a sum of measured values $\mathcal{P}_{\mu\mu}(\psi_a)$:

$$K_n^Q = (2-n) + 2\sum_{a=1}^{n-1} P_{\mu\mu}(\psi_a) - 2P_{\mu\mu}\left(\sum_{a=1}^{n-1}\psi_a\right). \quad (3)$$

For non-zero mixing angles θ, violations of $K_n \leq n-2$ are expected in neutrino oscillations.

4. Results and Discussion

We test for LGI violations using $P_{\mu\mu}$ data from MINOS, which extends across a 735 km fixed baseline from the NuMI complex at Fermilab to Soudan, MN. The MINOS collaboration recently released preliminary results for energies of 0.5-50 GeV [6]; in this interval, LGI violations for a quantum system are expected to be near maximal. More than 98% of neutrinos measured at the Near Detector are found to be in the $|\nu_\mu\rangle$ state, consistent with the identically prepared ensemble condition. Moreover, the data exhibit stationarity, as verified by tests of Lorentz invariance [5].

To construct K_3 (K_4), we select 82 (715) $P_{\mu\mu}$ groups that satisfy the appropriate sum rule, e.g. $\psi_a + \psi_b = \psi_c$, to within 0.5%. Of these groups, 64 (577) explicitly violate the LGI bound. We use

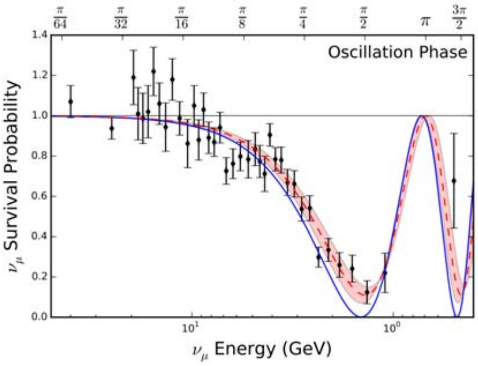

Figure 1. $P_{\mu\mu}$ as measured by the MINOS experiment [6]. The solid (blue) curve indicates the prediction for oscillations assuming global values of $\Delta m^2_{\rm atm}$ and $\sin^2 2\theta_{\rm atm}$, while the dashed (red) curve indicates the prediction fitting directly to measured MINOS values of $P_{\mu\mu}$. The red band indicates a 1σ confidence interval around the fitted prediction. The data are readily consistent with the existing quantum mechanical oscillation model, but we use them to test and constrain alternative explanations [7].

the STAN simulation package to generate pseudodata (accounting for statistical correlations) and construct distributions of the expected number of LGI violations (n_{obs}) based on K_n^C and K_n^Q. To determine the prevalence of false positive violations due to statistical fluctuations, we fit the classical distribution to a beta-binomial function. We find that n_{obs} represents a 6.2σ (7σ) deviation from the number expected to arise from a "realistic" model. In addition, observed K_3 values generally agree with the quantum model of Eq. (3) (χ^2_Q=104.8 for 81 degrees of freedom).

These results strongly constrain alternatives to quantum mechanics, such as classical Markovian models. Our method employs projective measurements on individual neutrinos from an ensemble, minimizing the opportunity for one measurement to affect the evolution of other neutrinos in a quantum or classical manner. We demonstrate an LGI violation in neutrino oscillations across 735 km: the longest range over which a Bell-like test of quantum mechanics has been carried out to date. The observation stands as further affirmation that quantum coherence applies broadly to microscopic systems, including neutrinos, across macroscopic distances.

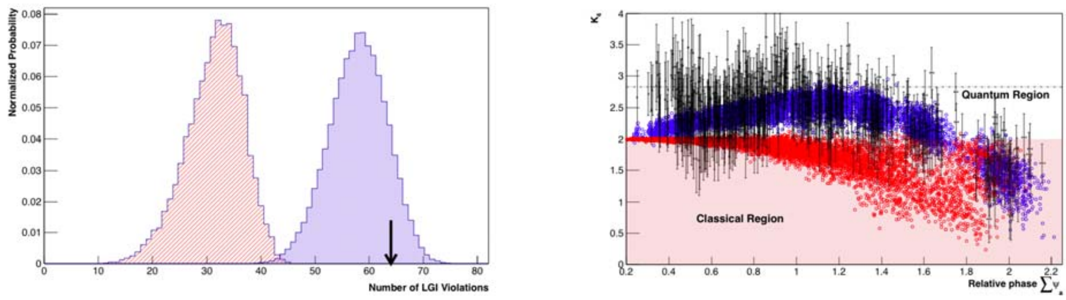

Figure 2. *Left* – Classical (red) and quantum (blue) expected distributions for the number of K_3 values which violate the LGI. The arrow indicates the observed number of violations. *Right* – K_4 versus the sum of the phases as reconstructed from $P_{\mu\mu}$ at various energies. The data (black) cluster above the LGI bound. Also shown: expected distributions for classical (red) and quantum (blue) theoretical predictions.

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[7] In order for a hidden-variable theory to replicate the $P_{\mu\mu}$ curve in Figure 1, each neutrino measured at the Far Detector would need to have access to most of the measurement outcomes on an ensemble of neutrinos at the Near Detector—including Near Detector measurements that had not yet been performed.