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Implications of the Υ(9.5) for Gauge Theories of Weak Interactions†

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ABSTRACT

We analyse the implications of the discovery of the Υ(9.5), assumed to be a heavy quark-antiquark bound state, for gauge theories of weak interactions and especially for neutrino reactions. Taking into account all present experimental constraints, we study the consequences of particular multiplet assignments of the new heavy quark in models based on the group $SU(2) \otimes U(1)$ and higher gauge groups, in particular $SU(3) \otimes U(1)$. In neutrino and antineutrino reactions in which this new quark is produced, its semileptonic decays will provide a key signal for its discovery. We specialize to the $SU(3) \otimes U(1)$ model and give a careful analysis of multimuon events arising from the production of this heavy quark and the concomitant heavy lepton.

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INTRODUCTION

Recently a resonance in the reaction $p + (Be \text{ or } Cu) \rightarrow \mu^+\mu^- + \text{anything}$ has been observed in the Columbia-Fermilab-Stony Brook experiment [1] at Fermilab with a mass $M_{\mu^+\mu^-} \cong 9.5\ GeV/c^2$ and width(FWHM) $\Gamma \simeq 1.2$ GeV. In analogy with the corresponding narrow resonance[2] at $M_{\mu^+\mu^-} = 3.1\ GeV/c^2$ we shall assume that this new phenomenon is due to the production and decay into dimuons of a $J^P=1^-$ bound state Υ of heavy quarks Q and $\bar{Q}$ together with its radial excitations Υ', Υ'' etc.[3]

Clearly, one of the most important properties of the Υ is the charge of its constituent quarks Q and $\bar{Q}$. In order to determine this charge from the proton experiment one must have available accurate calculations of the hadronic production cross section σ and the leptonic and hadronic decay widths Γ_e and Γ_h. Although σ and Γ_h have been intensively analysed in the case of the J/ψ and its excitations[4], there is no very reliable model which can be used in the case of the Υ. Hence it is difficult to use the present data to extract Γ_e or the heavy quark charge in a decisive way. We shall therefore consider various charge assignments in this paper.

Turning now to other ways of studying the $\Upsilon(9.5)$, we note that the mass of this state is too large to be observed by the presently available e^+e^- colliding ring machines, and it will be some time before the higher energy machines are ready. However, hadrons composed of the new heavy quark Q can possibly be produced in neutrino and antineutrino beams at Fermilab and CERN. Thus it is imperative to analyse the implications of the heavy quarks Q and $\bar{Q}$ for gauge models of weak interactions.

Since most recent gauge models contain new heavy quarks there are abundantly many ways to identify them with the constituents of the $\Upsilon(9.5)$. However such an investigation must be carried out within the comprehensive framework of the latest available weak interaction data[5]. Hence we are obliged to consider three recent developments: (1) the new measurements of the y distributions in antineutrino scattering, (2) the observation of trimuon events and (3) the absence of sizeable parity violations in atomic physics experiments.

Regarding the first point, the recent experiments by the Caltech-Fermilab[6] (CF) and CERN-Dortmund-Heidelberg-Saclay[7] (CDHS) groups indicate the absence of the high-y anomaly and associated rise in the ratio $\sigma^{\bar{\nu}N}/\sigma^{\nu N}$ reported by the Harvard-Pennsylvania-Wisconsin-Fermilab[8] (HPWF) group. This rules out models with full-strength right-handed currents coupling $u \to b$ or $d \to x$ (where u and d are the usual up and down quarks, and b and x are new-flavored quarks with charges - 1/3 and -4/3 respectively). We stress however, that this data certainly does <u>not</u> rule out right-handed charged currents connecting the new heavy quark with another heavy quark or with a light sea quark.

As far as the second point is concerned, there have now been three observations of trimuon events: by the CF group,[9] by the Fermilab-Harvard-Pennsylvania-Rutgers-Wisconsin (FHPRW) group,[10] and by the CDHS group[11]. We are fully aware that the characteristics of these events vary from one experiment to the next leading the authors to propose different explanations. We shall tentatively accept here the interpretation chosen by the FHPRW group for their trimuon data, namely the production and cascade decay of a heavy lepton[12]. Detailed numerical studies of this production mechanism have shown that it is successful in accounting for both the magnitude and the kinematic characteristics of the events.[13,14] As far as models are concerned, the large rate for trimuon production implies that either additional multiplets with new quarks and leptons have to be added to a $SU(2) \otimes U(1)$ gauge theory[14,15], or that the gauge group must be enlarged to allow a full strength coupling between ν_μ and a new heavy lepton M^-.[13]

Although certain aspects of the simultaneous production of new heavy quarks and heavy leptons were discussed in Ref. 13, the effects of the semi-leptonic decays of the new quark were not considered. Now that a new quark has presumably been discovered and its mass determined, it is possible to make our previous analysis more precise. Furthermore, there is now strong reason to expect tetramuon events in neutrino and antineutrino reactions. We stress that a serious effort should therefore be made to increase the world sample of multimuon events since the analysis of these events could yield the correct assignment of the heavy quark within gauge theory models.

The third point concerns the recent results of the University of Washington and Oxford University experiments[16] which seem to indicate that parity violation in the atom Bi^{209}, if it is present at all, is smaller by roughly an order of magnitude than the value predicted by the best available calculations[17] for the Weinberg-Salam (WS) model.[18] It is true that there is some uncertainity inherent in these calculations of the atomic physics aspects of the prediction; however it is hard to see how this can be so large as to make up for the significant disagreement noted above. These results of atomic physics experiments therefore rule out models which predict parity violation in heavy atoms to be as large as that in the WS model, including, in particular, $SU(2) \otimes U(1)$ models with purely left-handed doublets.

The type of questions which can hopefully be answered by existing and forthcoming neutrino data concern:

(1) the mass of the new heavy quark Q;

(2) the charge of the Q,

(3) the nature of the weak charged and neutral currents which involve Q Included within question (3) are:

(a) the chirality of the weak currents involving Q;

(b) the possible existence of nondiagonal neutral currents coupling to Q;

(c) the issue of whether Q couples via charged or neutral currents to sea quarks, or alternatively to valence quarks;

(d) the mass, gauge transformation properties, and leptonic couplings of the gauge bosons which enter into the production and the partial and total decay rates of hadrons containing Q; in particular the question of whether the production of Q is necessarily accompanied by the production of a heavy lepton;

(e) related to point (d), the role of possible mixing angles in moderating heavy quark production.

In addressing these questions, we make crucial use of the important connections between lepton and quark multiplet assignments in gauge models. These connections arise from such constraints as (1) cancellation of Adler-Bell-Jackiw anomalies[19] and (2) quark-lepton universality. Hence an analysis of properties for the heavy quark Q necessarily requires simultaneous consideration of the lepton sector. This will prove to be of significance for our study of multimuon events.

Concerning point (1), it is well known but perhaps merits emphasis that the mass of the quark cannot be defined in the usual way as the pole of the full on-shell renormalized propagator if, as is presumably the case in the standard model of quantum chromodynamics, there do not exist any free quarks. Hence an off-shell definition must be used. For phenomenological applications, the mass of the new quark is approximately known from the measured dimuon invariant mass. We shall assume, henceforth, that the effective mass is 4.75 GeV/c^2.

Regarding point (2), it is necessary to make an assumption. Within the context of the quark-parton model with regular up and down valence quarks, it is very difficult to excite new quarks unless their charges are 5/3, 2/3, - 1/3 or -4/3. Of course single or pair production of arbitrarily charged quarks can occur from the sea, but all analyses of experimental data find that sea contributions are small. Most gauge models do not incorporate exotic quarks (with charges other than 2/3 or - 1/3), so we will specialize to new quarks of charge 2/3 or -1/3. For economy of notation we shall label a new heavy quark as Q(2/3) or Q(-1/3) if its charge is 2/3 or -1/3, respectively. Of course there may be several such quarks; in such a case we shall identify each quark individually. Hence the question becomes: is the new heavy quark Q(2/3) or Q(-1/3)?

With the above assumptions, the only valence strength reactions which are allowed are

$$\nu_\mu + d \to \mu^- + Q(2/3) \tag{1.1a}$$

$$\nu_\mu + u \to \nu_\mu + Q(2/3) \tag{1.1b}$$

$$\nu_\mu + d \to M^- + Q(2/3) \tag{1.1c}$$

$$\nu_\mu + u \to M^o + Q(2/3) \tag{1.1d}$$

$$\bar{\nu}_\mu + u \to \bar{\nu}_\mu + Q(2/3) \tag{1.1e}$$

$$\bar{\nu}_\mu + u \to \bar{M}^o + Q(2/3) \tag{1.1f}$$

and

$$\bar{\nu}_\mu + u \to \mu^+ + Q(-1/3) \tag{1.2a}$$

$$\bar{\nu}_\mu + d \to \bar{\nu}_\mu + Q(-1/3) \tag{1.2b}$$

$$\bar{\nu}_\mu + u \to M^+ + Q(-1/3) \tag{1.2c}$$

$$\bar{\nu}_\mu + d \to \bar{M}^o + Q(-1/3) \tag{1.2d}$$

$$\nu_\mu + d \to \nu_\mu + Q(-1/3) \tag{1.2e}$$

$$\nu_\mu + d \to M^o + Q(-1/3) \tag{1.2f}$$

where M^- and M^o denote new heavy leptons. We consider these to be the dominant reactions for the production of the new quark Q. Reactions involving sea quarks or antiquarks of the form $\nu(\bar{\nu})+s \rightarrow \ell(\bar{\ell}) + Q$ or $\nu(\bar{\nu}) + \bar{q} \rightarrow \ell(\bar{\ell}) + \bar{Q}$, where ℓ represents some lepton and $\bar{q}$ denotes any flavor of antiquark, will make a small contribution and are therefore not considered. Furthermore, other mechanisms such as associated production, whether diffractive[20] or not (which of course necessarily occur from the sea) are excluded from our analysis. Reactions (1.1a), (1.1c), (1.2a) and (1.2c) are charged-current reactions which possibly involve the exchange of a new gauge boson. Reactions (1.1b) and (1.2b) involve flavor-changing hadronic neutral currents, while reactions (1.1d) and (1.2d) involve both flavor-changing leptonic, and flavor-changing hadronic, neutral currents. We shall discuss all these reactions in greater detail in the next Section. However it is convenient to introduce a classification of reactions (1.1) - (1.2) based on mass. We call (ℓ,q) reactions those involving only light quarks and light leptons. It is convenient for our purposes to classify c as a light quark since $m_c^2 << m_Q^2$. Class (ℓ,Q) denotes reactions involving light leptons and heavy quarks. Class (L,q) consists of reactions involving heavy leptons and light quarks, and finally class (L,Q) denotes reactions which involve heavy leptons and heavy quarks.

Regarding question (3a) which involves the chirality of the current, we note that all reactions (1.1) - (1.2) could, in principle, allow both right-handed and left-handed couplings to the new quarks. This question will be taken up more fully in Sec. II. where we discuss the implications of the absence of the high-y anomaly. However, if the new quark is Q(2/3) or Q(-1/3) and if it is coupled (1) with a full strength weak coupling constant $g_{\nu e}$, (2) via a gauge boson of reasonable mass and (3) to a valence quark, we expect to see some signal of its presence in neutrino, or in antineutrino reactions.

The question of the couplings of new bosons enters into both the production and the decay of the new quarks. Because reactions (1.1) - (1.2) cannot be seen directly but rather must be inferred from a study of their decay products the branching ratios into different decay modes are of prime importance. In order to compute these branching ratios one must specialize to a given gauge model; unfortunately even when one does this there are generally still several parameters such as mixing angles and fermion masses which enter into the calculation.

Reactions of the type (L,Q) need to be considered because it is still not known whether new heavy quarks must be produced with new heavy leptons. Roughly speaking, the kind of $SU(2) \otimes U(1)$ models which have been proposed recently[14] allow the possibility of coupling the ν_μ to M^- with strength comparable to that of the coupling of ν_μ to μ^-. Heavy quarks can then be produced without the necessity of heavy leptons. In most of the higher gauge groups[21-26] if a heavy quark is produced, the gauge boson which mediates this transition necessarily couples a light lepton, e.g., a neutrino, to a heavy lepton. Thus reactions like (1.1a), (1.2a) fit naturally into extended $SU(2) \otimes U(1)$ models while (1.1c) and (1.2c) fit more naturally into gauge models such as $SU(3) \otimes U(1)$[21,22] but can also be incorporated into the more conventional models.

Previous discussions of heavy lepton production have been made by several authors,[13,14,26] in order to explain the FHPRW trimuon events. The first explanation involves a cascade decay process such as $M^- \rightarrow \mu^- \nu_\mu M^o$ followed by $M^o \rightarrow \mu^- \mu^+ \bar{\nu}_\mu$ (note that in one model[21] of the $SU(3) \otimes U(1)$ type the neutrinos may be replaced by massive neutral stable leptons). The second explanation[22,26] involves the decay of a neutral heavy lepton $M^o \rightarrow \nu_\mu \mu^- \mu^+$ together with the decay of a heavy quark of the type $Q(-1/3) \rightarrow u + \mu^- + \bar{\nu}_\mu$. This explanation involves both flavor-changing leptonic and hadronic neutral currents. A discussion is given in Ref. 22 of the multimuon branching ratios which enter crucially into the calculation of the trimuon rate. We should also mention that an examination of a cascade decay of a heavy quark through a lighter mass quark was also considered in Ref. 26 (and independently by Soni[27]). However this model does not seem to fit the characteristics of the trimuon events.

Clearly the answers to all the questions raised in point (3) require a very detailed study of neutrino physics data and a careful analysis of available models. In the next Section we analyse some of the restrictions imposed upon models and discuss possible assignments for the new quark. To conclude our preliminary discussion we stress that one way to find the correct assignment of the new heavy quark Q is through a detailed study of neutrino-and anti-neutrino-induced multilepton events. Since the neutrino experiments with the highest statistics are counter experiments which cannot distinguish electrons from hadronic debris, we shall concentrate in this paper on multimuon modes. When sufficient data becomes available from bubble chamber experiments it will also be of interest to compare the predictions of various models for events containing muons and/or electrons and positrons. Two of us (in collaboration with J. Vermaseren) have already carried out such a study in a general, largely model-independent, manner.[28]

Confining our attention to the muon modes, we expect to see up to four or more muons from reactions (1.1c) and (1.2c). Although it may seem premature to discuss such possibilities at a time when the rate for trimuon events is uncertain, this seems to be one of the few reliable ways of gaining useful information. Models of the class (L,Q), lead to well defined ratios of dimuon, trimuon and tetramuon events, which can be checked experimentally.

In Sec. II. we discuss constraints on model building implied by the new experimental data. An analysis is given of possible heavy quark and heavy lepton assignments in gauge models based on the groups $SU(2) \otimes U(1)$ and $SU(3) \otimes U(1)$. This work includes only brief discussions of the various models; the reader should consult the papers listed in the References 21-26 for a more complete treatment. The next section contains a discussion of the general features of reactions (1.1) through (1.2). We concentrate on a careful analyses of the kinematics and dynamics of (L,Q) reactions. The discussion is not necessarily tied to a specific gauge model. Then in Sec. IV we turn to a particular SU(3) ⊗ U(1) model, in which all non-singlet fermions are assigned to $\underline{3}$ representations of SU(3)[21] and compute the multimuon event rates for neutrino and antineutrino beams. We concentrate mainly on rates and on the question of how cuts and misidentification problems change the raw theoretical results. Some distributions relevant to experimental searches are also included. Finally in Sec. V we give a summary of our results and present our conclusions.

II. GAUGE MODELS

In this section we shall briefly describe the gauge models which will be used as a theoretical framework in our analysis of the implications of the heavy quark Q for neutrino reactions. The existing weak interaction data places a number of constraints upon such models. A list of these follows: (1) quark-lepton universality, i.e., the fact that $G_\beta \sec\theta_c = G_\mu$; (2) $\mu - e$ universality, in particular the equality of the $\nu_\mu - \mu$ and $\nu_e - e$ couplings; (3) the absence of right-handed currents which involve only light fermions and could appear in neutron and hyperon (weak semileptonic) decay or μ decay; (4) retention of the successful Cabibbo theory of neutron and hyperon decays and the Glashow-Iliopoulos-Maiani (GIM) extension[29] which is necessary for the elimination of the neutral strangeness-changing current in the original WS model. It is useful to note that the GIM prediction that the ratio of charmed quark decays to d, versus s, quarks should be in the ratio $\tan^2\theta_c$ has received tentative verification from the SPEAR data on D meson decays[30]; (5) strong suppression of μ- and e- number violating processes[31] such as the decays $\mu \to e\gamma$ and $\mu \to ee\bar{e}$ or the reactions $\mu + N \to e + N$ and $\nu_\mu + N \to e + X$ (where the electron originates at the leptonic vertex) to the respective levels presently established by experiment; (6) the absence to order $G_F\alpha$, and the suppression to order G_F^2, of neutral strangeness-changing currents which could contribute to the transitions $K^o \leftrightarrow \overline{K^o}$, $K_L \to \mu\bar{\mu}$, and $K^\pm \to \pi^\pm e\bar{e}$ [32]; (7) the requirement that the weak contributions to the anomalous magnetic moments of the electron and muon be sufficiently small that they do not upset the very precise agreement between the QED predictions and the experimental measurements[33]; (8) in models which incorporate CP violation, the necessity that CP-violating quantities such as the neutron electric dipole moment be predicted to be in agreement with the present experimental bounds[34]; and (9) retention of successful current algebra results such as the relation between the $K \to 2\pi$ and $K \to 3\pi$ decay amplitudes[35]. A further general requirement is that these constraints be met naturally[36], i.e., in a way which depends only on the group structure and representation content of the theory, and not upon the values taken by the parameters of the theory. In addition to these constraints, there are well known theoretical requirements on the Lagrangian of an admissible theory upon which we shall not dwell here, except to mention that a theory must not contain any Adler-Bell-Jackiw triangle anomalies[19] since these would spoil its renormalizability.

These standard experimental constraints (1) - (9) have been supplemented in recent years by a growing body of data from neutrino reactions, including quasi-elastic charged current processes, and elastic leptonic and semileptonic, as well as deep inelastic, neutral current reactions. In addition, there is pion production in the resonance region by charged and neutral weak currents. Among the experimental results on these processes three important recent developments were singled out in the previous section, viz. the FHPRW trimuons, the apparent absence of a high-y anomaly, and the smallness or absence of parity violation in heavy atoms. This data serves to determine an admissable gauge group, the choices of fermion and Higgs multiplet assignments, the allowed types of fermion (and gauge boson) mixing, and various adjustable parameters within a particular model, such as ratios of vacuum expectation values of different Higgs fields and ratios of the gauge coupling constants for the factor groups in the case where the gauge group is not semi-simple.

The discovery of trimuon events by the FHPRW experiment[10] has important implications for the gauge group to be considered for the placement of the new heavy quark Q. We shall provisionally accept the conclusion reached by this collaboration that the various hadronic backgrounds are not sufficient to account for the events.[12] Detailed quantitative studies have demonstrated that heavy lepton production and sequential decay is in fact a viable explanation for both the rate and the spectral characteristics of the FHPRW trimuon events.[13,14] As has already been discussed in the literature,[13] in order to account for the observed (uncorrected) trimuon production rate

$$\left.\frac{R(\nu_\mu + N \to \mu^-\mu^-\mu^+ + X)}{R(\nu_\mu + N \to \mu^- + X)}\right|_{E_\nu > 100\text{ GeV}} \simeq 5 \times 10^{-4}$$

it is necessary to have the ν_μ couple to a heavy lepton M^- via a weak vertex of essentially full strength, and furthermore, to have this coupling mediated by a vector boson which does not possess too large a mass. Taken together, these two properties then imply that the amplitude for the reaction $\nu_\mu + N \to M^- + X$ is of order G_F. This is not possible in the original $SU(2) \otimes U(1)$ WS model which of course contains no heavy leptons.

There are essentially two ways to render such a full strength coupling possible. The first is to retain the minimal group, $SU(2) \otimes U(1)$, but enlarge its particle content by adding new doublets of fermions. There will in general then be mixing among the various (nondegenerate) physical fermion fields to form the states with definite transformation properties under the action of the weak gauge group. The crucial point, as discussed in Ref. 15, is that the experimental constraints listed above do allow large mixing, as long as this mixing is sufficiently symmetric with respect to e and μ and quarks versus leptons.

Thus in Ref. 15 large lepton mixing was suggested as a mechanism contributing to trimuon production; a particular model due to Kobayashi and Maskawa[37] (KM) having three left-handed quark and three left-handed lepton doublets was used as an illustrative example. This model naturally and automatically satisfies constraints

(3) and (5) - (9); in general, however, it allows small violations of quark-lepton and e - μ universality, and of the Cabibbo structure for charged light-quark currents. These violations can be made as small as desired by appropriate choices of mixing angles because nothing within the theory fixes them. This model does not predict a high-y anomaly in charged current antineutrino reactions, in agreement with the CF and CDHS experiments. However, in view of the recent impressive increase in the accuracy of the atomic parity violation experiments, it is necessary to modify the model by the addition of right-handed doublets in order to suppress the amount of parity violation expected in heavy atoms. As will be seen, when such right-handed leptonic (and consequently, also quark) currents are added, it becomes very difficult to satisfy, in a natural way, all of the experimental constraints. In order to show how these problems arise, we shall sketch the framework of an SU(2) $\otimes$ U(1) model with substantial mixing and minimal parity violation in heavy atoms. We do not consider this particular model very appealing, but do consider the general class of which it is an example to have distinctive experimental predictions which justify its inclusion here.

A. SU(2) ⊗ U(1) MODEL.

We proceed, then, to give a general classification of the possible multiplet assignments of Q within an SU(2) ⊗ U(1) model. We shall first specify the lepton multiplet assignments. Since our focus is on neutrino-induced reactions, it is convenient to use a convention whereby the $T_3 = +\frac{1}{2}$ weak eigenstates are simultaneously mass eigenstates (this is of course automatic for the massless neutrinos) while the $T_3 = -\frac{1}{2}$ eigenstates are linear combinations of mass eigenstates. (The opposite convention was used in Ref. 15 since it was more suitable for the processes $\mu \to e\gamma$, $\mu \to ee\bar{e}$, etc. studied there.) The left-handed sector of the model consists of three neutral singlets N_ℓ and six doublets, comprised of pairs of e, μ, and τ-type leptons, where τ denotes the heavy lepton discovered at SPEAR[38]. These may be written as

$$(N_\ell)_L \; ; \quad \begin{pmatrix} \nu_\ell \\ \ell^- \cos\chi_L + L^- \sin\chi_L \end{pmatrix}_L \; ; \quad \begin{pmatrix} L^o \\ -\ell^- \sin\chi_L + L^- \cos\chi_L \end{pmatrix}_L \qquad (2.1)$$

where

$$\begin{aligned} \ell &= e,\ \mu,\ \tau \\ L^- &= E^-,\ M^-,\ T^- \\ L^o &= E^o,\ M^o,\ T^o \end{aligned} \qquad (2.2)$$

and χ_L is a mixing angle, assumed to be the same for e, μ and τ doublets. This is, of course, quite a special assumption, but is required, at least for the e and μ doublets by e-μ universality. We recognize that the necessity of this assumption means that this model, like the KM model, fails to satisfy e-μ universality naturally. We denote, in accordance with the notation of Ref. 15, the transformations which map the chiral weak fermion eigenstates ξ_j onto the chiral mass eigenstates ψ_i as $U^{(\ell,h)}_{L,R}$ where ℓ,h label the leptonic and hadronic reactions;

$$\left(\psi_{L,R}^{(\ell,h)}\right)_i = \sum_j \left(U_{L,R}^{(\ell,h)}\right)_{ij} \left(\xi_{L,R}^{(\ell,h)}\right)_j \tag{2.3}$$

A necessary condition in order for the theory to have diagonal neutral currents to order $G_F\alpha$ is that $U^{\dagger}_{L,R} = U^{-1}_{L,R}$ (where the ℓ, h label is understood). Clearly the mixing scheme indicated in Eqs. (2-1) - (2.2), which is a slight generalization of one used recently by Barger, et al.,[14] for a study of trimuon production, represents a very special case of the most general such unitary transformation $U_L^{(\ell)}$. To guarantee this form one can use appropriate discrete or global symmetries to prevent mixing of e-type leptons with μ- or τ-type leptons, and to prevent mixing of fermions of equal charge and chirality between the singlets and doublets. Since these are of peripheral interest here, we shall not discuss them in detail.

In order to eliminate enhanced parity violation in heavy atoms, we shall render the electron neutral current purely vector by adding an appropriate right-handed doublet. A particularly symmetric choice is indicated in an obvious notation by

$$\begin{pmatrix} N_\ell \\ \ell\cos\chi_R + L^-\sin\chi_R \end{pmatrix}_R \; ; \quad \begin{pmatrix} L^o \\ -\ell\sin\chi_R + L^-\cos\chi_R \end{pmatrix}_R \tag{2.4}$$

where ℓ = e, μ, τ etc. as in Eq. (2.2) and χ_R is another mixing angle. Note that because there are nine left-handed, but only six right-handed, neutral chiral components of leptons, together with the discrete symmetries which are assumed to prevent mixing of neutral leptons between the (left-handed) singlet and doublet, the neutrinos ν_e, ν_μ, and ν_τ are naturally massless.

Unfortunately, unless χ_L and/or χ_R is very small, this multiplet assignment leads to an intolerably large weak contribution to the anomalous magnetic moment of the muon. The reason for this is the presence of "LR" and "RL" graphs in which a μ_L, say, makes a transition into a virtual M_L^o with emission of a virtual W^-, which couples to the photon. The M_L^o changes to an M_R^o through a mass insertion and absorbs the virtual W^- to become a μ_R. This, along with the analogous graph involving an initial μ_R and final μ_L (which gives the same contribution) yields a weak contribution to the anomalous magnetic moment

$$a_\mu^{wk} = -\frac{G_F m_\mu m_{M^o}}{2\sqrt{2}\pi^2} \sin\chi_L \sin\chi_R \quad . \tag{2.5}$$

In order for a_μ^{wk} to satisfy the bound resulting from the very precise agreement between the QED prediction and the present experimental measurement, it is necessary that $\sin\chi_L \sin\chi_R$ be quite small. Specifically, the allowed range for the weak contribution to the anomalous magnetic moment of the muon is given by[39]

$$-9.5 \times 10^{-9} < a_\mu^{wk} < 17 \times 10^{-9} \tag{2.6}$$

Together with Eq. (2.5) this required that

$$-0.39 < \left(\frac{m_{M^o}}{1\text{GeV}}\right) \sin\chi_L \sin\chi_R < 0.22 \tag{2.7}$$

Using the value $m_{M^o} \simeq 4$ GeV which is inferred from an analysis of the FHPRW trimuon data, we then find the stringent bound

$$-9.8 \times 10^{-2} < \sin\chi_L \sin\chi_R < 5.5 \times 10^{-2} \tag{2.8}$$

(The bound arising from a_e^{wk} is not so restrictive.) Since it is necessary that $\sin^2\chi_L \gtrsim 0.1$[14], in order for the model to predict a large enough trimuon rate, we shall take Eq. (2.8) to imply that $\sin^2\chi_R$ must be quite small.

The various experimental constraints, in particular, those of quark-lepton universality, retention of Cabibbo structure for the charged weak currents involving light quarks, and natural diagonality to order $G_F\alpha$ of the weak neutral hadronic current, determine the left-handed quark sector essentially completely. In particular, the last property requires that all quarks with the same charge and chirality have the same values of weak T and T_3 (where the T_i, i = 1, 2, 3 generate the weak SU(2) group). The resulting left-handed quark multiplets must all be doublets; they are:

$$\begin{pmatrix} u \\ d_\theta \cos\chi_L + Q_d \sin\chi_L \end{pmatrix}_L \qquad \begin{pmatrix} c \\ s_\theta \cos\chi_L + Q_s \sin\chi_L \end{pmatrix}_L$$

$$\begin{pmatrix} Q_u \\ -d_\theta \sin\chi_L + Q_d \cos\chi_L \end{pmatrix}_L \qquad \begin{pmatrix} Q_c \\ -s_c \sin\chi_L + Q_s \cos\chi_L \end{pmatrix}_L \tag{2.9}$$

where

$$\begin{pmatrix} d_\theta \\ s_\theta \end{pmatrix}_L = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}_L \tag{2.10}$$

We shall take the right-handed quark sector to be

$$\begin{pmatrix} u \\ Q_d \cos\chi_R + Q_s \sin\chi_R \end{pmatrix}_R \qquad \begin{pmatrix} c \\ -Q_d \sin\chi_R + Q_s \cos\chi_R \end{pmatrix}_R$$

$$\begin{pmatrix} Q_u \\ d\cos\chi_R + s\sin\chi_R \end{pmatrix}_R \qquad \begin{pmatrix} Q_c \\ -d\sin\chi_R + s\cos\chi_R \end{pmatrix}_R \tag{2.11}$$

This then renders the model quasi-vectorlike, i.e., fermions are distributed symmetrically into left- and right-handed chiral multiplets, except for possible asymmetrically arranged fermions which are singlets under the gauge group. Typically, as in the present model, the quark sector is purely vector like while the lepton sector includes extra left-handed singlets to ensure the masslessness of the neutrinos. As this model is arranged, all right-handed charged current transitions connect light quarks with heavy quarks. Furthermore for the quark as well as lepton sectors the neutral current is naturally diagonal to order $G_F\alpha$.

In passing, we observe that if one allows CP violation in this model, then the magnitude predicted for the electric dipole moment of the neutron is very large. The cause is the same as in the case of the muon anomalous magnetic moment, viz. the occurence of LR and RL graphs. These give an electric dipole moment in the free quark approximation, which is of order

$$D_n \sim \varepsilon\left(\frac{eG_F m_Q}{\sqrt{2}\,\pi^2}\right)\cdot \sin\chi_L \cos\chi_R \qquad (2.12)$$

where ε represents a generic CP–violating phase of order 10^{-3}. Numerically, without the mixing factors, if $m_Q = 4.75$ GeV/c^2, $D_n \sim 10^{-22}$e–cm; larger by about two orders of magnitude than the experimental result $D_n^{exp} = (0.4\pm1.1) \times 10^{-24}$e-cm.[34] Hence, roughly, it is necessary to assume that ε is smaller than 10^{-3} and/or

$\sin\chi_L \cos\chi_R \lesssim 10^{-2}$.

Concerning the multiplet assignment of the heavy quark Q, we consider the four possibilities and their consequences for neutrino reactions. In order to show the effects of Q production we give in Table 1 the resulting expressions for σ, $d\sigma/dy$, $\langle y\rangle$ and $R \equiv \sigma^{\bar{\nu}N}/\sigma^{\nu N}$. In the reactions listed in this table the chiralities of the quarks are indicated in the usual way by subscripts L or R; the chiralities of the leptons are always left-handed and hence are not explicitly indicated. Since we are concerned with gross effects here, we use the valence quark model with θ_c=0, which implies charge symmetry for light quark transitions, and neglect asymptotic freedom scaling deviations.[40] It is particularly difficult to calculate such deviations in the presence of heavy quark production. The units used for $\sigma^{(\nu,\bar{\nu})N}$ and $d\sigma^{(\nu,\bar{\nu})N}/dy$ are

$\sigma_0 = \left(\frac{G_F^2 m_N E}{\pi}\right) \int_0^1 F_2^{\nu N}(x)\,dx$ where $F_2^{\nu N} = F_2^{\bar{\nu} N}$ is the structure function for the light-quark transition .

The results given in Table 1 for an isoscalar target entail the assumption of the Callan Gross relation $F_2(x) = 2xF_1(x)$ and the relation $F_2(x) = x|F_3(x)|$ for each quark transition. Also they assume that $E >> E_{th,Q}$ where

$$E_{th,Q} = (W^2_{th,Q} - m^2_N)/2m_N \tag{2.13}$$

and $W_{th,Q} \simeq m_Q + m_N$ represents an approxmiate mass for the lightest Q-flavored final hadronic state. At lower energies, where quark masses are non-negligible and the structure functions do not scale, phase space factors and effective scaling variables must be included. These modifications express the rescaling behavior of the cross section and other kinematic quantities such as $\langle y \rangle$ above the threshold for heavy quark production. Unfortunately, the present neutrino energies are not really asymptotic so we have added a factor ρ to the terms involving light-to-heavy quark transitions. Numerical results are used to estimate a value for ρ in the reactions considered. Previous studies[41] have shown that for $m_Q \simeq 4 - 5$ GeV/c^2 the heavy quark contributions reach nearly their asymptotic magnitudes by $E \simeq 150$ GeV. From the CF data we observe that $(\sigma^{(\nu,\bar{\nu})N}/E)$ are both roughly constant from $E \simeq 30$ GeV to $E \simeq 190$ GeV; indeed $\sigma^{\nu N}/E$ exhibits an apparent slight decrease. The CDHS data indicates that $\sigma^{\bar{\nu}N}/\sigma^{\nu N}$ is flat over the same energy range, in agreement with the CF results.

Let us consider then the different charge options for Q, which will affect ν and $\bar{\nu}$ induced reactions. We assume that $\sin^2\chi_L \gtrsim 0.1$ to give reasonable agreement with the FHPRW trimuon rate, and $\sin^2\chi_R \lesssim 0.03$ to fit the bound (2.8), which implies that the CP violating parameter ε must be much smaller than 10^{-3}. Because the experimental data on $\sigma^{\nu N}$ and $\sigma^{\bar{\nu}N}$ increases almost linearly with energy, we tentatively assume that the slopes of these cross sections are constant to within 10 percent over the energy range $30\ \text{GeV} \leq E_{\nu,\bar{\nu}} \leq 200\ \text{GeV}$.

(1) $Q = Q_u$.

From table 1 we see that the total cross section for the regular neutrino interaction $\nu_\mu + d_L \to \mu^- + u_L$ now receives additional contributions from the channels $\nu_\mu + d_{L,R} \to \mu^- + Q_{u(L,R)}$. The asymtotic form of the cross section is not really applicable at present accelerator energies due to the heavy mass of the quark. As a rough approximation, which is reasonable for our purposes, we multiply the additional contribution to the cross section by a phase space suppression factor of $\rho = \frac{1}{2}$ when we compare it with the corresponding light quark transition. Using the above values of $\sin^2\chi_L$ and $\cos^2\chi_R$ we therefore expect a 20% increase in $\sigma^{\nu N}$ which is not seen in the CF and CDHS data. It is true that asymptotic freedom scaling deviations reduce $\sigma^{\nu N}/E$ slightly; however, a detailed calculation shows that this reduction is less than $\sim 10\%$ at $E \simeq 190$ GeV (and commensurately smaller at lower energies). Therefore the choice $Q=Q_u$ is strongly disfavored.

(2) $Q = Q_c$

From table 1, we see that the total cross section for the regular neutrino reaction $\nu_\mu + d_L \to \mu^- + u_L$ now receives an additional contribution from the channel $\nu_\mu + d_R \to \mu^- + Q_{cR}$. However, the magnitude of the contribution is only 5×10^{-3} so the choice $Q = Q_c$ is allowed.

(3) $Q = Q_d$

This choice for the new quark only affects the antineutrino reaction because the usual reaction $\bar{\nu}_\mu + u_L \to \mu^+ + d_L$ is supplemented by a contribution $\bar{\nu}_\mu + u_{L,R} \to \mu^+ + Q_{d(L,R)}$. The additional contribution now amounts to 60% increase, in clear contradiction with experiment. Thus we must exclude the choice $Q = Q_d$.

(4) $Q = Q_s$

As in the previous case, this choice only affects antineutrino interactions. The additional contribution is only 1.5 percent which is within the experimental bounds. Hence this choice is allowed.

Therefore we see that the neutrino and antineutrino data still allow a charge 2/3 quark with right-handed coupling to the d quark and a charge - 1/3 quark with right-handed coupling to the u quark. However both choices imply that $\cos^2\chi_R = 0.97$, which predicts too large a magnitude for D_n, the electric dipole moment of the neutron. Thus the introduction of right-handed currents in this model, which are necessary to fit the results of the atomic physics experiments, is not a viable approach to fit all the known weak interaction results.

Our analysis of the SU(2) ⊗ U(1) model was presented to illustrate the difficulties in pursuing this approach. We do not seriously advocate this as a realistic model. Note that, in addition to these (ℓ,Q) reactions, the new heavy quark can also be produced via (L,Q) reactions but with a much larger threshold. Hence, within the framework of *this* model, the latter type of reactions make a contribution which is negligible compared with that of the (ℓ,Q) reactions and therefore are not considered further.

Thus in summary, it is interesting that the standard experimental constraints do allow large fermion mixing which, in the leptonic sector, is sufficient to explain the FHPRW trimuon events via heavy lepton production and cascade decay. However if a purely left-handed model such as that used for illustration in Ref. 15 is modified to include right- handed currents, as is necessary in order to account for the more accurate results of recent atomic parity violation experiments,[16] then one encounters a number of serious problems such as described above. For this reason, in view of the recent and crucial null result of the search for enhanced atomic parity violation in heavy atoms, this enlarged model does not seem too likely a candidate for the correct weak gauge group. Since the high-y anomaly reported by the FHPRW group has not been confirmed by the CF or CDHS experiments the sole remaining reason for right-handed weak currents in $SU(2) \otimes U(1)$ gauge models is the apparent absence of enhanced parity violation in heavy atoms. However, this latter result seems to us to be decisive and consequantly at present we would reject any model with only left-handed weak currents.[42]

B. $SU(3) \otimes U(1)$ MODEL.

We turn next to theories based on enlarged gauge groups, which constitute the alternate, and perhaps more natural, way of accounting for FHPRW trimuons. We shall concentrate on $SU(3) \otimes U(1)$ gauge groups, in particular a recently developed model,[21] in which all nonsinglet fermions are assigned to $\underline{3}$ representations of SU(3). This model will serve as the main theoretical framework for our discussion of (L,Q) reactions. However, since it has been analysed in detail in Ref. 21 our treatment here will be quite brief. The quark content of the theory consists of the light quarks u,d, and s, the charmed quark c, (considered here as a light quark), and four still heavier quarks: t and g with charge 2/3, and b and h, with charge - 1/3. These are arranged in two families, as shown in Fig. 1; each quark family is comprised of two singlets and two triplets, one of each chirality.

The various discrete symmetries of the Lagrangian and/or vacuum allow mixing in the quark sector to occur only between u_L and c_L and separately, t_R and g_R. The expressions for the resulting weak eigenstates, which are primed, in terms of the mass eigenstates are:

$$\begin{pmatrix} u' \\ c' \end{pmatrix}_L = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} u \\ c \end{pmatrix}_L \qquad (2.14)$$

$$\begin{pmatrix} t' \\ g' \end{pmatrix}_R = \begin{pmatrix} \cos\theta' & \sin\theta' \\ -\sin\theta' & \cos\theta' \end{pmatrix} \begin{pmatrix} t \\ g \end{pmatrix}_R \qquad (2.15)$$

where θ is the Cabibbo angle and θ' is an analogous mixing angle.

The leptons are similarly arranged in three families, as shown in Fig. 2, each lepton family consisting of a left-handed singlet and triplets of both chiralities. As in the quark sector, only the first members of the triplets are allowed to mix. For simplicity the discrete symmetry denoted by S in Ref. 21 allows E^o and M^o to mix with each other but not with T^o; thus we have

$$\begin{pmatrix} E^{o'} \\ M^{o'} \end{pmatrix}_R = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} E^o \\ M^o \end{pmatrix} \tag{2.16}$$

In Eqs. (2.14)-(2.16) we have suppressed possible CP-violating phases since these will play no role in the neutrino production of heavy quarks and multi-muon final states which are of interest here. As will be seen, the mixing of E^o and M^o is crucial for trimuon production; without it the rate for this process would vanish to leading order.

The absence of a right-handed singlet, together with a certain discrete symmetry which prevents undesirable fermion mixing, guarantees that the neutrino in each family is naturally massless. Thus there are the three leptons e,μ,τ, their associated neutrinos ν_e, ν_μ, and ν_τ, and six charged and neutral heavy leptons L^- and L^o, L = E,M,T. The mass of τ is measured by SPEAR to be $m_\tau = 1.9$ GeV/c^2; the masses of M^- and M^o are inferred from the trimuon data[12] to be $m_{M^-} \simeq 8$ GeV/c^2 and $m_{M^o} \simeq 4$ GeV/c^2. These are the values which will be used here. Because of an exact discrete symmetry, the E^o is absolutely stable; the bound on the size of its contribution to the mean mass density of the universe then limits its mass to be either less than about 40 eV or greater than 1-4 GeV.[43] In order for the $SU(3) \otimes U(1)$ model to yield a large enough trimuon production rate to agree with the FHPRW data one must choose the heavy mass option. As in Ref. 21, we shall take $m_{E^o} = 1$ GeV/c^2 for our analysis here.

The original nine massless gauge bosons of the unbroken SU(3) ⊗ U(1) group gain masses in the usual spontaneous symmetry breakdown via their couplings to Higgs fields consisting of two triplets with U(1) hypercharge y = - 2/3 and a complex Higgs octet with y = 0. In a generalized version of the model there is also a y = 1/3 Higgs triplet which couples to gauge bosons but not fermions; however, we shall restrict our consideration here to the minimal SU(3) ⊗ U(1) model which does not contain this last triplet. The resulting physical vector boson sector of the model consists of the photon, and eight massive vector bosons, $W^{\pm}$, $U^{\pm}$, $X_{1,2}$, Y, and Z. The $W^{\pm}$ and $U^{\pm}$ effect transitions in the 1±i2 and 4±i5 directions (the $T_{\pm}$ and $V_{\pm}$ directions in conventional notation) in SU(3) space respectively. The X_1 and X_2 bosons are certain linear combinations, corresponding to definite mass, of the X^o and $\bar{X}^o$, which effect transitions in the 6±i7 SU(3) directions. Finally there are the two vector bosons Y and Z which are neutral both with respect to charge and SU(3). Of the neutral currents J^{μ}_{Y} and J^{μ}_{Z}, the former (1) is purely axial-vector and (2) contains no neutrino term. Furthermore, the $X_{1,2}$ vector bosons do not couple to neutrinos, and hence, since both J^{μ}_{Y} and J^{μ}_{Z} are diagonal (and remain so to order $G_F\alpha$), there are no non-diagonal neutrino neutral currents.

This model is quite appealing for a number of reasons. It naturally accounts for both the magnitude and kinematic characteristics of the FHPRW trimuon events. Furthermore, it naturally satisfies all of the experimental constraints listed at the beginning of this section, including quark-lepton and e-μ universality, absence of right-handed currents in β and μ decay, proper suppression of neutral strangeness-changing currents to order G_F^2 in $K^o \leftrightarrow \bar{K}^o$ and $K_L \rightarrow \mu\bar{\mu}$, proper suppression of μ- and e-number nonconservation, and acceptably small values for $a^{wk}_{\mu,e}$ and the CP violating neutron electric dipole moment. The model predicts no sizeable high-y anomaly in deep inelastic $\bar{\nu}N$ reactions, in agreement with the preponderance of the data on this issue. Its neutral current predictions depend on two parameters, ℓ, which measures the relative size of the vacuum expectation values of the Higgs octet(s) and Higgs triplet(s), and w, which is a certain function of $(g'/g)^2$, where g' and g are the gauge coupling constants for the U(1) and SU(3) factor groups, respectively. For $\ell \approx 0.18$ and $w \approx 0.25$, the SU(3) $\otimes$ U(1) model gives a satisfactory simultaneous fit to all available neutral current data, including inclusive and elastic semileptonic reactions, elastic leptonic scattering, and parity-violation by neutral currents in (heavy) atoms. The reader is referred to Ref. 21 for further details.

It is an important feature of this SU(3) $\otimes$ U(1) model that in neutrino reactions heavy quark and heavy lepton production necessarily occur together. Thus there are only (ℓ,q) and (L,Q) neutrino reactions, in contrast to the case with SU(2) $\otimes$ U(1) gauge models, which, in general allow all four types (ℓ or L, q or Q). In the SU(3) $\otimes$ U(1) model it is not the W boson but rather the U boson which effects the transition of the ν_μ to M^-. In the minimal version of the SU(3) $\otimes$ U(1) model which we are considering here $m_U = m_W$; furthermore the fermion coupling constant is the same for the W and U bosons. Hence, other things being equal, such as fermion masses, initial quark parton distributions, and chiralities of the relevant currents, reactions which proceed via W-exchange and U-exchange will have the same cross section. In fact , in this model W bosons link light (left-handed) fermions with other light (left-handed) fermions or heavy (right-handed) ones with other heavy (right-handed) ones. In contrast the U bosons necessarily link light fermions with heavy ones.

We next analyze which reactions will show manifestations of the production of the heavy quark Q, depending on its charge assignment, 2/3 or - 1/3. If the charge of Q is 2/3 it could be identified, in the framework of this model, with either of the t or g quark. It could be produced, at sufficiently high energies, through the reactions

$$\nu_\mu + d_R \to M^- + t_R \tag{2.17}$$

or

$$\nu_\mu + d_R \to M^- + g_R \; . \tag{2.18}$$

The effects of these new transitions on σ, $d\sigma/dy$, $\langle y \rangle$ and R are shown in Table 2. At asymptotically large energy, $E/E_{th,LQ} >> 1$, where

$$E_{th,LQ} = [(m_{M^-} + m_{t,g} + m_N)^2 - m_N^2]/(2m_N), \tag{2.19}$$

the cross sections for these reactions, in the units used in Table 1, are

$$\sigma(\nu_\mu + d_R \to M_L^- + t_R) = \frac{1}{3}\sigma_0 \cos^2\theta' \tag{2.20}$$

and

$$\sigma(\nu_\mu + d_R \to M_L^- + g_R) = \frac{1}{3}\sigma_0 \sin^2\theta' \tag{2.21}$$

respectively. The factors of $\frac{1}{3}$ in these asymptotic formulas result from the helicity suppression, and the factors of $\cos^2\theta'$ and $\sin^2\theta'$ reflect the mixing of t_R and g_R to form t_R'. Unfortunately, the mixing angle θ' is arbitrary, so that we cannot give more definite predictions for the magnitudes of the asymptotic cross sections in (2.20) or (2.21). Note that formulae (2.20) and (2.21) cannot be compared with present data because the neutrino and antineutrino energies are too small. Indeed a phase space suppression factor as small as 1% must be included when comparing these (L,Q) reactions with regular (ℓ,q) reactions.

In Ref. 21 estimates of trimuon production were made with the assumptions that θ' was small and $m_t \simeq 4$ GeV/c^2. Of these assumptions, the one pertaining to θ' was included in order to simplify the problem by making $\sigma(\nu_\mu + N \rightarrow M^- + X)$ depend on only m_t rather than m_t, m_g, and θ'. However the assumption regarding m_t was important since a significantly larger t quark mass, unless compensated for by a smaller M^- mass, would reduce the cross section for M^-, and hence trimuon, production. It is thus very tempting to presume that Q might be identified with t or g.

If, however, the charge of Q is $-1/3$ then, again in the context of the $SU(3) \otimes U(1)$ model, one may identify it with either the b or h quarks. If Q = b, it can be produced via the process

$$\bar{\nu}_\mu + u_L \rightarrow M^+ + b_L \tag{2.22}$$

In contrast to the case with t and g quarks there is, by necessity, no mixing of b and h quarks. Thus, if Q = h, the only reaction that will produce it is

$$\bar{\nu}_\mu + c_L \rightarrow M^+ + h_L \tag{2.23}$$

which gives a very small contribution. As in the ν_μ reactions, the transitions (2.22) and (2.23) are helicity-suppressed so that asymptotically the reaction (2.22) will yield a cross section

$$\sigma(\bar{\nu}_\mu + N \rightarrow M^+ + X) \simeq \frac{1}{3}\sigma_0 \tag{2.24}$$

for $m_b = m_t = m_g$ on an isoscalar target (denoted as N). On such a target the $\bar{\nu}_\mu$-induced heavy quark production reaction (2.22) gives three times as large a cross section, relative to the corresponding regular charged current process, than do the ν_μ-induced reactions (2.17) and (2.18). Thus, indicating the quark transition by subscripts,

$$\frac{\sigma_{u\to b}(\bar{\nu}_\mu+N\to M^++X)}{\sigma(\bar{\nu}_\mu+N\to\mu^++X)} = 3\sec^2\theta', \quad \frac{\sigma_{d\to t}(\nu_\mu+N\to M^-+X)}{\sigma(\nu_\mu+N\to\mu^-+X)} = 3\csc^2\theta', \quad \frac{\sigma_{d\to g}(\nu_\mu+N\to M^-+X)}{\sigma(\nu_\mu+N\to\mu^-+X)} \tag{2.25}$$

As will be seen, however, this advantage by a factor of 3 is more than outweighed by the fact that the $\bar{\nu}_\mu$ flux at high energies is considerably smaller than the ν_μ flux.

If the charge of Q is-1/3 then one can derive a rough lower bound on the mass of the Q(2/3) quark from the Columbia-Fermilab-Stony Brook data. The reason for this is that the value of $\sigma(p+p\to\Upsilon + X) \times BR(\Upsilon \to \mu^+\mu^-)$ depends on $(q_Q)^n$ where $n>2$ (the value of n being determined by the relative importance of various hadronic production mechanisms). Hence, given the observed magnitude of the $\Upsilon(9.5)$resonance, another $Q'\bar{Q}'$ state with $q_{Q'} = 2/3$ would, at the same mass, produce more than 4 times as large a signal. The apparent absence of any such hugh dimuon resonances in the Columbia-Fermilab-Stony Brook data up to $M_{\mu^+\mu^-} \simeq 11$ GeV/c^2, could then be taken to imply that if $q_Q = -1/3$ then for $q_{Q'} = 2/3$, $m_{Q'} \gtrsim 5.5$ GeV/c^2. In the SU(3)$\otimes$U(1) model, if we retained the original value of m_{M^-}, this would imply a significant decrease in the trimuon production rate. However, one could, of course, reduce m_{M^-} from the value of 8 GeV/c^2 used in Refs. 13 and 21 to 7 GeV/c^2 (the experimentally inferred value being $7^{+3.0}_{-1.5}$ GeV/c^2) and obtain essentially the same trimuon production rate as before.

To summarize, the LW SU(3) $\otimes$U(1) model can incorporate a new quark Q(2/3) as either the t or g quark, or a new quark Q(-1/3) as either the b or h quark. In all cases there is a small contribution to the inclusive total ν or $\bar{\nu}$ cross section which cannot be larger than one percent. The only decisive way to decide between these options is therefore to examine the multimuon rates and compare them with experiment, Theoretical predictions for this model will be given in Sect. IV, where we discuss both the possibility that the quark has charge 2/3 and that it has charge -1/3.

Although the SU(3) ⊗ U(1) model of Ref. 21 will serve as the main framework for our numerical work, it is of interest to mention another type of SU(3) ⊗ U(1) model because it gives distinctively different predictions. Recall that the flavor-changing neutral currents $J^{\mu}_{X_1}$ and $J^{\mu}_{X_2}$ in the SU(3) ⊗ U(1) model discussed above do not play any role in neutrino reactions. However, if one assigns the leptons to $\underline{3}^*$ representations of SU(3) rather than $\underline{3}$ representations, then the analogous neutral currents which effect transitions in the 6±i7 direction in SU(3) space do play an important role in neutrino reactions and in particular, those leading to heavy quark production. In one model of Langacker and Segrè[22] (LS) for example, the lepton multiplets are as shown in Fig. 3. In this model the quarks are assigned to $\underline{3}$ representations of SU(3) in a manner similar to that of Fig. 1 except for the very important difference that among the right-handed chiral quarks, u_R and c_R are assigned to the $\underline{3}$ representations, rather than to singlets. There are thus no t and g quarks in this model. This choice of quark sector leads to a full strength high-y anomaly, a prediction which at present is in conflict with the CF and CDHS data (which data had not been reported when the model was proposed). There are also problems with giving the u and c quarks masses while keeping the neutrinos massless in this model.

Because this LS SU(3) ⊗ U(1) model has no t or g quarks there is no place for the new heavy quark to be assigned if its charge turns out to be 2/3. One could, of course, devise a new model with additional triplets;

we shall not dwell upon the details here. The neutrino reactions which lead to the production of Q in the case where it has charge - 1/3 and is accordingly identified with b or h are the (L,Q)-type transitions involving both flavor changing leptonic and hadronic neutral currents

$$\nu_\mu + d_L \rightarrow N_{1\mu} + b_L \tag{2.26}$$

and

$$\nu_\mu + s_L \rightarrow N_{1\mu} + h_L \quad . \tag{2.27}$$

Reaction (2.26) is an explicit example of the general class (1.2f). The antineutrino reactions which can produce b or h (of the type (1.2d)) are similar, i.e.,

$$\bar{\nu}_\mu + d_R \rightarrow \bar{N}_{1\mu} + b_R \tag{2.28}$$

and

$$\bar{\nu}_\mu + s_R \rightarrow \bar{N}_{1\mu} + h_R \tag{2.29}$$

In both the ν_μ- and $\bar{\nu}_\mu$-induced processes the helicities of the leptons and quarks match, and hence there is no factor of 1/3 suppression as in the SU(3) ⊗U(1) model discussed in Ref. 21. In both SU(3)⊗U(1) models the heavy quark is necessarily produced together with a heavy lepton, so that the threshold is commensurately higher than in the SU(2)⊗U(1) class of models. Thus the kinematic suppression factor is the same for both models and there is little chance one can distinguish between them on the basis of total cross section measurements. We therefore turn to the multimuon decay modes where there are distinct differences between the models.

Reactions (2.26) and (2.28) which involve valence quarks, produce trimuon events when the neutral heavy lepton N_1 decays via the mode

$$N_{1\mu} \to \mu^- + \mu^+ + \nu_\mu \tag{2.30}$$

and the heavy quark b simultaneously decays via the mode

$$b \to u + \mu^- + \bar{\nu}_\mu \tag{2.31}$$

Thus in both neutrino and antineutrino reactions the decays of the N_1 and b lead to $\mu^-\mu^-\mu^+$ events. The LW SU(3) ⊗U(1) model on the other hand yields $\mu^-\mu^-\mu^+$ events in neutrino reactions and $\mu^-\mu^+\mu^+$ events in antineutrino reactions. A study of the trimuon distributions from the decays (2.30) and (2.31) has been made by Barnett and Chang[26] assuming a b quark mass of 5-6 GeV/c^2 and a M^o mass of 3 GeV/c^2. It is tempting to identify this b quark with the constituent of the $\Upsilon(9.5)$. If one does so then reaction (2.26) must account for both the rate, and the spectral characteristics of the FHPRW events.

In contrast to the model in Ref. 21, however, the LS SU(3) ⊗U(1) model predicts semileptonic decay branching rates for b and h quarks which may comprise a small fraction, perhaps 15%-20%, of the total decay rate. The heavy lepton has a similar leptonic branching ratio. Hence, taking into account that there is a large phase space suppression for the simultaneous production of the N_1 and b, and that the muon from the b quark is relatively soft so that the minimum energy cut of 4 GeV will seriously reduce the rate for trimuon production, it is not obvious that the model can give a large enough $\mu^-\mu^-\mu^+$ event rate to fit the FHPRW result. More theoretical work on multimuon decay modes will help to pinpoint the differences between these two SU(3) ⊗U(1) models.

III. GENERAL FEATURES OF (L,Q) REACTIONS

We would like to consider here a number of distinctive features of reactions (1.1) – (1.2) which serve as diagnostic evidence for demonstrating that heavy quarks Q(2/3) and/or Q(−1/3) are produced. Our aim is to differentiate between reactions of the type (ℓ,Q), (L,q) and (L,Q). For completeness some reasonably well known results are included. The consideration of reaction (L,Q) is new and goes beyond previous discussions of heavy lepton, heavy-quark production.

In general our comments fall into two distinct categories. First of all, there is the rather obvious question of how the kinematics of production and decay are influenced by the presence of heavy quark and heavy lepton masses. The second question is how the dynamics influences cross sections and decay rates. Although some general features can be discussed, this question can only be answered by taking a specific gauge model and performing the relevant calculations. Accordingly, in the latter part of this work, we shall use the SU(3) $\otimes$ U(1) model of Ref. 21, since this model has been analyzed in detail and found to agree with all presently available experimental data. Therefore we concentrate on general points in this section and leave model dependent features to Section IV.

We begin our discussion of the heavy lepton and heavy quark production processes by writing the differential cross section

$$\frac{d^2\sigma^{\nu,\bar\nu}}{dq^2 d\nu} = \frac{G^2}{8\pi E^2}\left\{2(q^2+m^2)W_1^{\nu,\bar\nu} + [4E(E-\nu)-(q^2+m^2)]W_2^{\nu,\bar\nu} \mp \frac{1}{M}[2Eq^2-\nu(q^2+m^2)]W_3^{\nu,\bar\nu} + \frac{m^2}{M^2}[(q^2+m^2)W_4^{\nu,\bar\nu}-2MEW_5^{\nu,\bar\nu}]\right\} \quad (3.1)$$

in terms of the time-reversal-invariant structure functions $W_i = W_i(q^2,\nu)$. M is the nucleon mass, m is the heavy lepton mass, $q^2 > 0$ is the modulus of the four momentum transfer squared and ν is the energy transfer to the hadrons. The W_i are assumed to scale for (ℓ,q) reactions and become functions of $x = q^2/2M\nu$ ie, in the limit that $E\to\infty$, $q^2\to\infty$, $\nu\to\infty$ where x is fixed

$$\lim MW_1 = F_1(x)$$

$$\lim \nu W_k = F_k(x) \qquad k = 2,3,4,5 \tag{3.2}$$

The positivity condition on the W's can be translated into certain restrictions on the F's in the scaling region and we can show that the Callan-Gross relation together with the Gross-Llewellyn Smith relation imply that

$$xF_1(x) = F_2(x)$$

$$xF_3(x) = \mp F_2(x)$$

$$F_4(x) = 0$$

$$x\,F_5(x) = F_2(x) \tag{3.3}$$

where the $-$ sign holds for left-handed quarks and the $+$ sign for right-handed quarks. Finally the structure function $F_2(x)$ can be approximately expressed in terms of valence and sea parton distributions. Eq. (3.1) holds true for (L,q) reactions because we have retained the mass of the heavy lepton, and by changing the scaling variable also describes light-to-heavy quark transitions fo the type (ℓ,Q) and (L,Q).

Obviously the existence and location of energy thresholds are crucial in testing whether Q(2/3)/Q(-1/3) production in $\nu_\mu/\bar{\nu}_\mu$ reactions is necessarily accompanied by M^-/M^+ production, respectively. We take the mass of Q(=t or b, generically) to have been determined as $\sim$4.75 GeV/c^2; however, the actual threshold for the reaction which produces Q is set by $W_{th,Q}$ the mass of the lightest physical hadronic final state containing this heavy quark. We approximate this as

$$W_{th,Q} \simeq m_Q + m_N.$$

Taking $W_{th,Q} = 5.5$ GeV and $m_{M^-} = 8$ GeV/c^2, $E_{th}(\ell = \mu, Q) \simeq 16$ GeV whereas $E_{th}(L=M^-, q) \simeq 50$GeV and $E_{th}(L=M^-, Q) \simeq 96$ GeV. The (ℓ, Q) reaction therefore has a substantially lower threshold than the (L,Q) reaction.

In the energy region slightly beyond threshold, the cross section obviously does not scale exactly. However, from a study of quark mass effects in the free field limit of the light cone operator production expansion[40] it has been shown that these effects can be taken into account in the form of an effective scaling variable ξ. For the case of interest here

$$\xi \simeq x + m_Q^2/(2m_N Ey). \tag{3.4}$$

It is ξ, and not x, which represents the fraction of the total nucleon momentum carried by the struck quark. The deep inelastic $(\nu,\bar{\nu})N$ structure functions $F_i(x)$ are expected to be approximately functions of this variable for values of ξ which are sufficiently large so as not to be in the Regge region and sufficiently small so as not to be in the resonance region. (In the latter two regions, conventional nonscaling and nonperturbative effects pertaining to Regge exchange or multiple gluon exchange and hadron binding, respectively, will play a dominant role; these are not included within the above analysis).

As was discussed previously, although the actual threshold in incident ν_μ or $\bar{\nu}_\mu$ energy is set by $W_{th,Q}$ and m_{M^-}, the rapidity of rescaling for energies beyond this threshold is largely determined by the size of the quark mass. It has been shown that for $m_Q \cong 4$–5 GeV/c^2, depending on the chirality of the currents involved, the onset of new heavy quark production can have quite noticeable effects on σ, $d\sigma/dy$, $\langle y \rangle$ and $R = \sigma^{\bar{\nu}N}/\sigma^{\nu N}$. As the analysis of Ref. 40 has demonstrated, in the (ℓ,Q) case even though $E_{th} \sim 16$ GeV, the effect of the new quark production on the inclusive cross section sets in rather slowly and is only sizeable for $E_\nu \gtrsim 30$ GeV. Similarly, as the study of Ref. 13 showed the (L,Q) reaction has a cross section which approaches its asymptotic slope (σ/E) only for $E_\nu \gtrsim 150$ GeV. For the other reaction, namely (L,q) (σ/E) reaches its asymptotic value near $E_\nu \sim 100$ GeV.

The threshold question can be studied by exploiting the differences between broad-band and dichomatic neutrino beams. For (L,Q) reactions the threshold is so large that neutrinos from charged pion decays are completely ineffective and only neutrinos from kaon decays can initiate the reaction. Broad band beams essentially have more high energy neutrinos (with $E_\nu > 100$ GeV) which are necessary to study the (L,Q) reactions. In Figure 4 we compare the event rate curves for the neutrino production of muons in (ℓ,q) reactions and heavy leptons in (L,Q) reactions calculated using Eqs. (3.1)-(3.4) with the masses given above. We have folded the cross sections with the best available estimates for the quadrupole triplet spectrum of the FHPRW group and the dichromatic spectrum of the CDHS group. The absolute normalization is not well known and the relative positions of the curves could change by 10%. Even though the actual threshold is at $E_\nu \gtrsim 100$ GeV, there are very few events until $E_\nu \gtrsim 150$ GeV. The advantages in having a long high energy tail in the neutrino spectrum are clearly demonstrated, remembering that all the cross sections are rising essentially linearly in E. As an example we find for the quadrupole triplet spectrum

$$\left.\frac{\sigma(\nu_\mu + N \to M^- + X)}{\sigma(\nu_\mu + N \to \mu^- + X)}\right|_{E_\nu > 100 \text{ GeV}} = 1.7 \times 10^{-2}$$

in the LW SU(3) $\otimes$ U(1) model.

The analogous results for the antineutrino case are shown in Fig. 5. Taking the FHPRW quadrupole-triplet spectrum we compute that

$$\left.\frac{\sigma(\bar{\nu}_\mu + N \to M^+ + X)}{\sigma(\bar{\nu}_\mu + N \to \mu^+ + X)}\right|_{E_{\bar{\nu}} > 100 \text{ GeV}} = 3.6 \times 10^{-2}$$

The branching ratio calculations to be discussed in Sec. IV give $BR(M^+ \to \mu^+ + \ldots) \lesssim 30\%$. Hence, at the most, M^+ production will contribute a 1% increment to the cross section for the reaction $\bar{\nu}_\mu + N \to \mu^+ + X$. This has a negligible effect on $d\sigma^{\bar{\nu}N}/dy$ and $\sigma^{\bar{\nu}N}$. Hence only a small fraction of the decays will simulate regular charged current reactions through the chain $\bar{\nu}_\mu + N \to M^+ + X$, $M^+ \to \mu^+ + \ldots$, and the model therefore predicts no sizable high-y anomaly, in agreement with the present data from the CF and CDHS experiments.

The higher thresholds for (L,q) and (L,Q) reactions are also reflected in the hadronic energy threshold. In the light-to-heavy-quark transition, the actual threshold is determined by

$$E_{th,had} = (W^2_{th,Q} + q^2_{min} - M^2)/2M \tag{3.5}$$

Hence, if a light quark converts into a heavy quark with a mass 4.75 GeV/c^2 and $W_{th,Q}$ = 5.5 GeV, Eqn. (3.5) implies that $E_{th,had} \cong 15$ GeV when we flux average. Of course this value for the hadronic energy threshold assumes that no neutral stable massive leptons are emitted, since they can carry away some of the "hadronic" energy and make $E_{th,had}$ smaller. In the LW $SU(3) \otimes U(1)$ model such leptons do exist and this phenomenon occurs.

The (L,q) reaction involving heavy leptons and light quarks has already been examined in Refs. 13 and 14 so there is no need to discuss it extensively here. However, we would like to point out one interesting fact. Even if the charged current is unable to excite the heavy quark directly so that (ℓ,Q) reactions are forbidden, the heavy quark could be produced via the decay of the heavy lepton in an (L,q) reaction. Depending on the masses involved, part of the heavy lepton cascade would therefore involve the heavy quark. This situation can be distinguished from regular quark decay by a careful study of the decay distributions. For instance, the hadronic spray coming from the heavy lepton decay will have different characteristics from the regular hadronic spray; in particular it will be more peaked along the beam direction and involve higher energy secondaries.

Continuing our discussion of effects in the total cross section, we recall the key results from the previous studies of (ℓ,Q) reactions including charm temporarily in this category. As is well known, charmed particle production in neutrino reactions consists of two components, one of which occurs off valence quarks but is Cabibbo suppressed, viz. $\nu_\mu + d_L \to \mu^- + c_L$ and the other occurs off sea quarks but is Cabibbo favored, viz. $\nu_\mu + s_L \to \mu^- + c_L$. When an even heavier quark is produced, the chirality of the weak current manifests itself in the differential distributions, in perticular, the y distribution. Assuming that the neutrino energy is so large that threshold effects are no longer important, and lepton masses are negligible, we can use the results from Tables 1 and 2 for the asymptotic increments in σ, $d\sigma/dy$, $\langle y\rangle$ and R for typical (ℓ,Q) and (L,Q) reactions, and for reactions

$$\nu_\mu + d_R \to \mu^- + (t,g)_R \tag{3.6}$$

and

$$\bar{\nu}_\mu + u_L \to \mu^+ + b_L \tag{3.7}$$

in particular. If the helicities of the quarks and leptons are equal (opposite) then the y distributions are 1, $((1-y)^2)$ respectively. Since the leptons are always left-handed in the reactions, there is no need to add a helicity subscript. As remarked previously, the increments in the total inclusive cross sections (and in the distributions) are already constrained by the new data from the CF and CDHS experiments. Unfortunately the increments are generally so small for (L,Q) reactions that this is not a good way to differentiate between models.

We now turn to a discussion of the kinematic characteristics of the (L,Q) reactions in the (q^2,ν) and (x,y) planes, and compare them with the (ℓ,Q) and (L,q) reactions. We begin by giving the event rate distributions in the (q^2,ν) plane, where the boundary is determined by the equations

$$2M\nu - q^2 + M^2 = W^2_{th,Q} \tag{3.8}$$

and

$$\nu = E - \frac{q^2 + m^2}{4E} - \frac{m^2 E}{q^2 + m^2} \tag{3.9}$$

In Figs. 6, 7, and 8 we show the results of a Monte Carlo calculation for (ℓ,Q), (L,q) and (L,Q) reactions respectively. The masses were given previously and all the double differential cross sections are folded with the FHPRW neutrino spectrum. The actual boundary determined from Eq. (3.9) is not included.

If we translate the (q^2,ν) plots into the more usual (x,y) plots then the following features are apparent. The (ℓ,Q) reaction, for fixed and low beam energy, starts at small x and large y, whereas the (L,q) reaction starts at large x and small y. The (L,Q) reaction therefore starts at roughly equal values of x and y, depending on the relative sizes of the heavy lepton and heavy quark masses. When we flux average then the distributions of events in the (ℓ,Q) and (L,q) cases move towards the center of the (x,y) plane. Figs. 9, 10, and 11 show the resulting double differential distributions. Clearly the $\langle x \rangle$ tends to be rather large and the corresponding $\langle \xi \rangle$ is similar in magnitude. Unfortunately, it is difficult to exploit these distributions because only the decay leptons from the heavy quark and/or heavy lepton are visible. Hence $\mu^{\pm}$ arising from the single or cascade decay of the initially produced heavy lepton will carry only a small fraction (very roughly, on the average, $1/n$, where n is the number of final stable leptons produced) of the energy of the intial heavy lepton. Accordingly, visible y, y_{vis} is greater than the true y. Note that there is a countervailing tendency which arises from the fact that several of the leptons from the decay of the initial heavy lepton (and the neutrino or neutral stable heavy lepton from the decay of the heavy quark, if there is one produced) carry away a considerable amount of undetected energy. Hence E_{vis}, defined experimentally as the sum of $E_{had} \equiv \nu$, the energy deposited by the hadronic spray in the calorimeter, plus $\sum_i E_{\mu i}$, the sum of the muon energies, is significantly smaller than the true incident ν or $\bar{\nu}$ energy. If we define the variable $y_{vis} = 1 - E_\mu/E_{vis}$ for a specific muon, the fact that $E_{vis} < E$ means that $y_{vis} < y$. However this effect is generally a small perturbation. Basically the muon energies are so small that the resulting y_{vis} variable tends to be sharply peaked near unity for reactions in which heavy leptons are produced.

A similar feature holds for muons produced when the heavy quark decays. During the production process the heavy quark variables are well defined. However, the decay involves a physical hadron state rather than a quark and there is some uncertainity as to the energy carried by the hadron at the instant of decay. To account for this, one introduces a quark fragmentation function D(z) depending upon z defined to be the ratio of the energy carried by the physical hadron to the energy of the quark. The functional form of D(z) is not known for heavy quarks. Studies of D(z) for production of pions (i.e. light mass quarks) and for production of charmed hadrons indicate that D(z) is peaked near $z \sim 0$ and falls rather sharply as z increases. The form $D(z) = e^{-3z}$ has been advocated to fit the data in the production of charmed particles by neutrinos i.e., the opposite sign dimuon events[44]. If such a form is used for a heavier mass quark then the resulting decay muon will be rather slow and, to a large degree will not escape the cut of E = 4 GeV imposed by the counter experiments. This means that the resulting value of y_{vis} will again be peaked near unity. One way to increase the decay muon energy is to increase the initial energy of Q so that the D(z) distribution is not so important. Other authors[26] who have studied this effect claim that there is little dependence on D(z) in heavy quark production, primarily because they choose such a large mass that there is little phase space inihibition for the decay muons and, on the average, they can be reasonably fast.

Similar features occur with the variable x_{vis}. One can define a quantity $q_i^2 = (k-p_i)^2$ where k is the neutrino four momentum and p_i is the four momentum of say, the fast μ^- arising from a heavy lepton decay. The q^2_{vis} tends to be smaller than q^2_i because $q^2_{vis} \equiv 4E_{vis}E_\mu \sin^2\theta/2$, where E_μ is the energy of the muon and θ is its angle with respect to the beam direction. Thus $q^2_{vis} < q^2_i$ and $\nu_{vis} > \nu$ means that $x_{vis} < x$.

To illustrate these remarks the reader should compare the results for the (x,y) distributions in Figs. 9, 10, and 11 with the (x_{vis}, y_{vis}) distributions in Fig. 12 for a typical (L,Q) reaction, involving the leptonic cascade decay of the M^- and the semileptonic decay of a heavy quark with left handed coupling. We defined x_{vis} and y_{vis} with respect to the fast μ^- which derives from the cascade decay of the M^-, and avoids the necessity of choosing a functional form for D (z).

The peaking in the x_{vis} and y_{vis} distributions changes as we switch the definitions to refer to the slower muons. For instance if we define analogous variables for the slow μ^+ coming from the decay of the heavy quark then $y_{vis} = 1 - \frac{E_\mu}{E_{vis}} \simeq 1$ and $q^2_{vis} = 4E_{vis}E_\mu \sin^2\theta/2 \simeq 0$ while $\nu = E_{had}$ is still quite large. Thus the peaks in the distributions move rather dramatically to very small x and very large y. In fact the resulting y_{vis} distribution is only sizable in a region where the average counter experiment has very poor acceptance.

In general, the distributions in the decay muon variables for (ℓ,Q) reactions are similar to those expected from charmed particle decays. Thus for neutrino induced reactions $\langle E_{\mu-}\rangle/\langle E_{\mu+}\rangle$ is large, $M_{\mu-\mu+}$ is energy dependent, the muons tend to be emitted back-to-back in the plane perpendicular to the neutrino beam, the $\langle p_\perp \rangle$ distribution for the μ^+ is large reflecting the mass of the charmed hadron, and both polar angles for the μ^- and the μ^+ are large. Similar features hold for antineutrino interactions where the decay muon is now the μ^-

so $\langle E_{\mu+}\rangle/\langle E_{\mu-}\rangle$ is large, etc. The transition from charmed particle production to the production of even heavier mass objects means that the limits of distributions are now controlled by m_Q and so the averages can be much larger. We refer the reader to References 41 and 44 for further details.

In general, all (ℓ,Q), (L,q) and (L,Q) reactions will lead to opposite sign dimuon pairs. The largest potential signal is clearly from charm production so it may be difficult to extract dimuons from (L,q) and (L,Q) reactions in the opposite sign dimuon channel. A complete study of the expected signals for $\mu^-\mu^+$ and $\mu^-\mu^-$ events in a heavy lepton cascade reaction was made in Ref. 13, where we pointed out that the $\mu^-\mu^+$ events would be difficult to see above the charm background. In the $\mu^-\mu^-$ chain, the muons were binned into a fast μ^- and a slow μ^- and we showed that there was very little probability of finding two fast μ^- particles. Interestingly, the $\mu^-\mu^-$ energy distribution reported from the CDHS experiment does not show the presence of two fast μ^- particles, so this data cannot be used to either confirm or reject the heavy lepton cascade hypothesis. Regarding dimuon production in the LW SU(3) $\otimes$ U(1) model, we would like to point out that a complete classification of all decays leading to dimuon events of the type $\mu^-\mu^+$, $\mu^-\mu^-$ and $\mu^+\mu^+$ has already been made in Ref. 21.

We have concentrated up to now on the kinematics of the heavy lepton and heavy quark decays. The rates for the decays bring in another problem. Fortunately, the leptonic decays for heavy leptons can be calculated with reasonably accuracy. However, the semileptonic decay branching ratio for $Q \to q + \ell^+ + \nu_e$, with q a lighter mass quark, is more difficult to estimate. The ratio of leptonic to nonleptonic decay rates for heavy quarks depends on the question of the short distance enhancement of the nonleptonic channels and the chirality of the coupling. For rough estimates one can ignore such effects and use the free quark model counting rules.

To illustrate our general comments we now turn to one specific model, namely the LW $SU(3) \otimes U(1)$ model and compute the neutrino and antineutrino induced multimuon event rates, assuming that the new heavy quark is either the t quark or the b quark. With more data from the present counter experiments, it will be possible to check these particular assignments for Q.

IV. MULTIMUON EVENTS

We have emphasized in the previous Sections that a study of neutrino-and antineutrino-induced multimuon events will help to elucidate the assignment of the heavy quark Q in gauge theory models. Now we turn to specific details in the SU(3) $\otimes$U(1) gauge model of Lee and Weinberg. The simplest reaction to consider from the theoretical point of view is the production of four muons because this mode arises essentially from the leptonic decay of the heavy lepton together with the semileptonic decay of the heavy quark. Thus we first study the features of tetramuon events and then make comments on trimuon and dimuon events.

We concentrate on neutrino production of multimuon events because antineutrino beams have less flux at high energies. Also results are only given for counter experiments. The rates are so small that it seems premature to discuss the modes leading to electrons and positrons which can only be detected in a bubble chamber. However, we stress that bubble chamber experiments are uniquely well designed to be sensitive to multilepton final states in (L,Q) reactions, since they have much better detection efficiency for slow leptons. Electron and/or positron events will occur in the counter experiments but these particular events will be included as part of the hadronic shower energy. They will then be misclassified into another type of multimuon decay mode. Thus we identify the charge 2/3 t quark as the constituent of the $\Upsilon(9.5)$ and look for its presence in neutrino induced multimuon events.

In the SU(3) $\otimes$ U(1) model there are many diagrams leading to multimuon decay modes. Fortunately the decay rates for the heavy leptons have been calculated in Ref. 21 as functions of one parameter, conveniently called $\sin^2\beta$, and quark and lepton masses. The dominant trimuon decay modes are

$$M^- \rightarrow \mu^-\mu^-\mu^+E^o\bar{\nu}_\mu \qquad (4.1)$$

and

$$M^- \rightarrow \mu^-\mu^-\mu^+\bar{E}^o\nu_\mu \qquad (4.2)$$

Remember that E^o, M^o mixing is allowed and E^o is the lightest stable member of the pair. There are other trimuon decay modes such as

$$M^- \rightarrow \mu^-\mu^-\mu^+E^o\bar{\nu}_\mu\nu_\tau\bar{\nu}_\tau \qquad (4.3)$$

and

$$M^- \rightarrow \mu^-\mu^-\mu^+E^o\bar{\nu}_\mu s_\theta\bar{s}_\theta \qquad (4.4)$$

which involve the production of τ-leptons and strange quarks respectively. However, these constitute a very small fraction of the decay rate and can therefore be safely ignored. The heavy t quark always decays semileptonically, via the modes

$$t_R \rightarrow d_R + E^o + \mu^+ \qquad (4.5)$$

and

$$t_R \rightarrow d_R + E^o + e^+ \qquad (4.6)$$

with equal probability, so that reactions (4.1) and (4.2), in conjunction with (4.5) are responsible for the neutrino induced tetramuon events in the model.

If some of the muons in the heavy lepton decay are replaced by electrons and/or positrons the experimental signature changes from the identification of tetramuon events, to trimuon, dimuon or single muon events. For instance if the M^- particle decays via the mode

$$M^- \rightarrow \mu^-\mu^-e^+E^o\bar{\nu}_\mu \tag{4.7}$$

and the t quark decays via (4.5), there will be a trimuon $\mu^-\mu^-\mu^+$ signal. Other decay modes such as

$$M^- \rightarrow \mu^-\mu^+e^-E^o\bar{\nu}_e \tag{4.8}$$

yield a trimuon $\mu^-\mu^+\mu^+$ signal. Continuing in the same way, the decays

$$M^- \rightarrow \mu^-E^o\bar{\nu}_\mu$$

and

$$M^- \rightarrow \mu^+e^-e^-E^o\bar{\nu}_e \tag{4.9}$$

give dimuon $\mu^-\mu^+$ and $\mu^+\mu^+$ events. There are many such decay modes and we refer the reader to Ref. 21 for a complete classification. The dependence of all the heavy lepton decay branching ratios on $\sin^2\beta$ is shown in Fig. 13. From the figure one sees that the purely hadronic decays amount to approximately 60% of the total branching ratio. Next in magnitude is the single μ^- decay amounting to 30% - 35% depending on the value of $\sin^2\beta$. Indeed, there is the possibility that the $\mu^-\mu^-$, $\mu^-\mu^+$, $\mu^-\mu^-\mu^+$ and μ^+ modes all have zero branching ratios. From our previous calculations of the trimuon event rate for the FHPRW experiment, we have estimated that $\sin^2\beta \doteq 0.2$. We shall adopt this value for the remainder of the analysis.

We should note here that the use of trimuon and dimuon event rates to establish the existence of the t quark suffers from the following difficulty. As we will show, the muon from the t quark is always rather slow while the muons from the heavy lepton decays are generally much faster. Trimuon events can arise from either the trimuon decay of the heavy lepton together with the t quark decay mode (4.6), or from a dimuon decay of the heavy lepton together with the t quark decay mode (4.5). Unless one event rate is much larger than the other it will prove difficult to disentangle the one trimuon signal from the other.

The actual leading diagrams which yield the four muon decay modes are given in Fig. 14. These diagrams also lead to trimuon and dimuon events when one μ^+ or μ^- is replaced by an undetected e^+, or e^- and one or more of the other muons is so low in energy that it cannot pass our minimum energy cut of $E_\mu > 4$GeV. We proceed to first discuss the case when all four muons are detected.

The calculation of the tetra muon decay mode follows rather straight forwardly from our previous work. We have incorporated the spin of the M^- into the production exactly and then taken the M^o to decay with no preferred spin orientation. An examination of the latter effect reveals that it is small enough to be safely neglected. The masses of the M^-, M^o, t and hadronic energy threshold $W_{th,Q}$ are selected to be 8 GeV/c^2, 4 GeV/c^2, 4.75 GeV/c^2 and 5.5 GeV respectively. The fragmentation function for the heavy quark is chosen to be $(1-z)/z$ where $z = p_h/p_Q$ is the fraction of the quark momentum carried by the physical hadron. The heavy quark is assumed to become a spin one-half particle which subsequently decays into three fermions. All event rates are given for the quadrupole triplet neutrino spectrum used by the FHPRW group, and their cuts are imposed.

A Mônte Carlo calculation of the diagram in Fig. 14.(a) has been carried out with minimum muon energies of 4 GeV and angular cuts of 400 milliradians. The energy cut is rather severe, especially in the case of the μ^+ coming from the heavy quark decay. In our previous trimuon calculations approximately 33% of the cross section was lost due to similar cuts on the muon momenta and angles. Now we lose approximately 90% of the events. Without cuts the average energy of the slow μ^+ is less than 4 GeV so we only pick up the tail of the energy distribution when we insist on $E_\mu > 4$ GeV.

To check whether our results depend significantly on our choice for D(z) we have rerun our program with D(z) = constant and with D(z) = z(1+z). The latter distributions yield harder momentum spectra for the μ^+ emitted during the heavy quark decay so there is a corresponding increase in the fraction of tetramuon events which survive the cut. However, the actual D(z) distribution is unknown and further theoretical work needs to be done on this subject so we only present results for the choice given above.

An analogous calculation has been made for the diagram shown in Fig. 14(b) to see if there are any significant changes in the distributions. We found them to be so small that they are not worth commenting upon. The reason is that the topology with these masses is essentially the same and kinematics rather than dynamics are the most important aspects of the reaction. Hence we will only give distributions for the diagram in Fig. 14(a).

Fig. 15 gives the energy spectra for the negatively charged muons as they are identified in the theoretical calculation and Fig. 16 shows the same distributions for the positively charged muons. For this situation we know which muon is which so we can plot their individual spectra. Experimentally, the two negatively charged muons and the two positively charged muons will be ordered according to their energies. When we simulate this effect in the Monte Carlo calculation for the two positive muons there is only a small change because the muon from the heavy lepton is nearly always so much faster than the muon from the heavy quark. On the other hand the negative muons have similar theoretical distributions so when we order them into fast and slow on an event by basis, then all the fast muons are pushed into the higher energy bins and all the slow muons into the lower energy bins. This causes a signficant change in these distributions. Figs. 15 and 16 also show the muon spectra ordered according to their energies.

We now turn to the other energy spectra, namely E_{had} , E_{vis}, E_{miss} and E_{tot}, which we give in Fig. 17. These spectra stretch out to much higher energies than the muon spectra. We note in particular the position of the threshold in the E_{had} distribution. The fact that there are missing neutral stable leptons with mass m_{E^o}=1 GeV/c^2 in the final state causes this threshold to move to a lower value. When neutrinos are the only missing particles this threshold position is higher, as was shown in Ref. 13. Hence a careful study of the threshold behaviour may help to distinguish between competing theoretical models. The visible energy, E_{vis}, is defined to include the hadronic energy and the energy of the four identified muons. The energy carried away by the neutrino and E^o leptons is summed in E_{miss}. E_{tot} is given out of theoretical interest to check that the distribution resembles the event rate curve shown previously in Fig. 14. It is interesting to note that the E_{tot} distribution is only sizable for reasonably large values of the energy above approximately 140 GeV. Even though the physical threshold for the reaction occurs at $E_{th} \approx$ 100 GeV, the cross section is extremely low at that point. We conclude that neutrino energies larger than 140 GeV are required before the event rate becomes measureable. This again emphasizes the need to use the highest possible neutrino energies to study these phenomena.

If four muons are identified then there are a considerable number of distributions which can be examined. Rather than give these plots we prefer to limit ourselves to some observations. Clearly these events should have typically two relatively fast muons of opposite charge and two slower muons. The energies are expected to be ordered in the sequence $E^-_{fast} > E^+_{fast} > E^-_{slow} > E^+_{slow}$. We expect the distributions for the negative muons and the faster μ^+ to show typical heavy lepton features, i.e., to be uncorrelated in the (x,y)-plane perpendicular to the neutrino beam and have larger transverse momenta than, say, the muons in the charm generated dimuon events. The slow μ^+ should follow the hadron direction and be oppositely correlated in the (x,y)-plane to the other muons. Double differential distributions in the energy, or in the energy versus $\Delta\phi$ angle, should also have heavy lepton like features. Remember $\Delta\phi$ is the angle between the muon transverse momentum vectors projected on the plane perpendicular to the neutrino axis. We also expect the transverse momentum distributions to reflect the fact that the slow μ^+ comes from the hadron shower. If we define an apparent W boson direction, defined with respect to the three fast muons, as discussed in Ref. 13, then the transverse momentum distribution of the slow μ^+ along the axis is controlled by the mass of the t quark. We therefore anticipate that the $\langle p_\perp \rangle$ will be larger than the corresponding $\langle p_\perp \rangle$ in the case of charm generated μ^+ events.

The influence of the various cuts has serious consequences for the so-called misidentification problem. In Table 3 we show the percentage of the $\mu^-\mu^-\mu^+\mu^+_Q$ events which survive the energy cuts. The subscript Q denotes that the muon comes from heavy quark decay. We also show the percentage of events when the slow μ^+ is below the energy cut but the other three muons are identified as a $\mu^-\mu^-\mu^+$ event. Similarly in the next column we give the relative probability that the slow μ^- is not detected so that the four muon event looks like a $\mu^-\mu^+\mu^+$ event. Then there are finally the percentages that some permutation of two muons are not detected but the other two are identified. If we summarize the first row of Table 3, then it is evident that 69% of the four muon events are actually misidentified as trimuon $\mu^-\mu^-\mu^+$ events and only 8% are actually measured as genuine tetramuon events. These numbers depend sensitively on the choice of distribution function D(z). If D(z) peaks at large z then the slow μ^+ receives more energy and the fraction of tetramuon events which survive the cut is correspondingly larger.

The actual branching ratio for the $\mu^-\mu^-\mu^+\mu^+$ decay mode is the product of the heavy lepton $\mu^-\mu^-\mu^+$ branching ratio taken from Fig. 13, with $\sin^2\beta = 0.2$, i.e. 1.3×10^{-2}, multiplied by the branching ratio of 50% for the (4.5) decay mode of the t quark. For convenience we given in Table 3 the product of these two numbers, i.e., 6.5×10^{-3} for the theoretical branching ratio, and the corresponding fractions for the other channels. Note that we have not given the fraction for the $\mu^+\mu^-$ channel because this is where the regular neutrino production of charmed particles gives a large contribution so it will be difficult to see any signal from the (L,Q) reaction.

Further complicating the analysis, there is also the question of electron and/or positron production. If the heavy quark decay involves a positron then the counter experiment classifies that event as a $\mu^-\mu^-\mu^+$ event when all the muons survive the energy cut which they do 76% of the time. The theoretical branching ratio for the $\mu^-\mu^-\mu^+e^+$ channel is also 6.5×10^{-3}. The other events look like either $\mu^-\mu^-$ or $\mu^-\mu^+$ events because one muon does not survive the cut and it is interesting that 11% of these events fall into the $\mu^-\mu^-$ class. Continuing in the same fashion, there are the other decay modes of the heavy leptons which involve electrons or positrons. The third row gives these results showing that $\sim$ 9% of these events are classified as $\mu^-\mu^-\mu^+$ and 78% as $\mu^-\mu^-$. The other two rows show how frequently one misidentifies other $\mu^-\mu^+\mu^+$, $\mu^-\mu^-$ and $\mu^+\mu^+$ events. Note that the percentages in the rows do not add up to exactly 100% because there is a small probability that only one single muon will escape the cuts. Finally in the last row we add up the partial rates multiplied by the cut survival fraction. Hence from a four muon decay chain we see that the largest fraction of the events actually get detected as $\mu^-\mu^-$ events followed closely by $\mu^-\mu^-\mu^+$, then $\mu^-\mu^+\mu^+$, $\mu^+\mu^+$ and finally $\mu^-\mu^-\mu^+\mu^+$.

Switching to antineutrino induced (L,Q) reactions we have also studied the corresponding decay rates for M^+ decay in association with a heavy quark Q(-1/3). The production and decay chain can be read off from Fig. 13 by switching particles into antiparticles. Our Monte Carlo calculations of the $\mu^+\mu^+\mu^-\mu_Q^-$ chain show that most of these events are misidentified as $\mu^+\mu^+\mu^-$ events. In Table 4 we give the percentage fraction of misidentified events and their branching ratios. We have refrained from giving the corresponding numbers for events with electrons and/or positrons because the rate for the antineutrino initiated (L,Q) reaction is so small.

The experimental signal for the multimuon decay modes can be found by folding together our production cross section value with the branching ratios. From our calculations we find the result that

$$\sigma(\nu_\mu + N \to M^- + t)/\sigma(\nu_\mu + N \to \mu^- + X)\Big|_{E_\nu > 100 \text{ GeV}} = 0.0167 \qquad (4.10)$$

for the model under consideration. The event rates for the largest multimuon modes are then summarized in Table 5. We remark that the $\mu^-\mu^-\mu^+$ event rate is only 2×10^{-4} which is lower than the number 5×10^{-4} reported by the FHPRW group. The latter number has not been corrected in any way. Hence the theoretical prediction and the experimental result should be considered in reasonable agreement with each other. Our prediction for the $\mu^-\mu^+\mu^+$ event rate is 5×10^{-5}. The same sign dimuon event rates are also interesting from an experimental point of view. In particular the value 5×10^{-4} for the $\mu^-\mu^-$ event rate gives a relative ratio $\sigma(\mu^-\mu^-)/\sigma(\mu^-\mu^-\mu^+) = 2.5$. Previous estimates for this ratio were given in Ref. 13, and were slightly higher ranging from 4 - 8. We have also carried out the calculation of the production cross section using the flux spectrum of the CDHS experiment and find that the ratio (4.10) is equal to ~ 0.009, roughly half of the FHPRW value. Therefore, the event rates given above should be scaled by roughly a factor of 1/2 for application to the CDHS experiment.

There are other contributions to trimuon and dimuon production involving semileptonic decays of the M^- lepton which have not been included in the above analysis. Estimates for these event rates can be found from Fig. 13, Eqn. (4.8) and the fact that the t quark decays 50% of the time into muons and 50% of the time into electrons. The numbers must then be corrected for the fraction of events which survive the cuts. As a rough estimate, each individual muon from a heavy lepton decay has an 85% probability to survive the cut, whereas the slow muon from the heavy quark decay only has a 40% probability to do so.

We close this section by giving an estimate for the like sign dimuon rate in this model, $\sigma(\mu^-\mu^-)/\sigma(\mu^-)$. From Fig. 13, we see that the relative rate for $M^- \to \mu^-$ is 30%. Hence

$$\left.\frac{\sigma(\mu^-\mu^-)}{\sigma(\mu^-)}\right|_{E>100\text{ GeV}} = \left.\frac{\sigma(M^-)}{\sigma(\mu^-)}\right|_{E>100\text{ GeV}} \times \left[BR(M^-\to\mu^-) \times BR(t\to\mu^-) \times \rho + a_1 + a_2 + a_3 \right] \tag{4.11}$$

where ρ is the acceptance factor representing the effects of the cuts and a_1, a_2 and a_3 are the like sign dimuon rates coming from misidentified dimuon trimuon , and tetramuon events. The first term in the brackets gives 0.5×10^{-2}. The other factors are summed in Table 3 (excluding small contributions from other semileptonic channels) and yield 2.7×10^{-2}. Hence $\sigma(\mu^-\mu^-)/\sigma(\mu^-) = 3.2 \times 10^{-2} \times 1.7 \times 10^{-2} \approx 5 \times 10^{-4}$, for a neutrino energy above 100 GeV. This number is close to the limit set by the recent CDHS experiment.

V. CONCLUSIONS

Assuming that the discovery of the $\Upsilon(9{\cdot}5)$ implies the existence of a new heavy quark Q, we have discussed the consequences for weak interactions, in particular neutrino physics. We have considered various multiplet assignments for the new quark according to its charge and chirality in $SU(2)\otimes U(1)$ and $SU(3) \otimes U(1)$ models. The principle emphasis behind the paper is to stress the importance of neutrino data, to determine the type of weak coupling for the new quark, together with its charge. Not only can the present data be used to severely restrict models containing the new quark, we also expect that the future data on neutrino produced multimuons will guide the way to the correct model. In particular, it is imperative to determine whether the new quark can be produced with regular muons or is necessarily produced in association with new heavy leptons.

In Sec. II we have given a rather exhaustive discussion of the constraints on gauge models imposed by recent experimental data. The discovery of trimuon events, together with the lack of parity violating effects in atomic physics, and the disappearance of the high-y anomaly, place restrictions on presently available models. We considered the consequences of this new experimental information together with all the additional constraints imposed by precision measurements of low energy weak interactions, as they relate to the construction of gauge theory models. The assignment of leptons and hadrons in an $SU(2)\otimes U(1)$ model was found to be severly constrained. In particular, the new leptons, which are presumably responsible for the neutrino induced trimuon events, and the new heavy quark must be assigned to representations in a manner which obeys lepton-hadron symmetry. The allowed ranges for mixing angles were examined and we commented on the consequences of identifying the heavy quark as Q(2/3) or as Q(- 1/3). Models with only left-handed quarks and leptons can explain the large rate for trimuon production by allowing a large fermion mixing angle. However, such models cannot explain the results of the recent atomic physics experiments,which show a lack of sizable parity violation.

Turning to the higher dimensional groupswhich have recently been discussed in the literature, we chose the $SU(3) \otimes U(1)$ model of Lee and Weinberg and assigned the new quark to a triplet representation of that group. The model has already been investigated from a phenomenological point of view and is known to fit all the presently available data. Once the quark structure is fixed the consequences for neutrino interactions can be worked out. In particular we propose the study of neutrino induced multimuon events to test the model. The $SU(3)\otimes U(1)$ model has the interesting feature that the new quark can only be excited concomitantly with the charged heavy lepton. Also the heavy t quark may have no non-leptonic decay modes. When both particles decay we therefore expect a rich spectrum of multimuon decay channels.

In view of the fact that the kinematics of simultaneous heavy lepton, heavy quark reactions have not been considered before in any detail, we examined in Sec. III particular features of these so called (L,Q) reactions. The choice of effective scaling variable ξ determines the shape of the total cross section near threshold. Obviously the threshold energy for (L,Q) reactions is large and should be studied by neutrinos with the highest available energy, ie.,using broad band beams rather than dichromatic beams. The discussion of threshold effects was followed by a consideration of the x,y distributions for production and x_{vis}, y_{vis} distributions for one of the final detected muons. We also mentioned tests to distinguish between the (ℓ,Q) and (L,Q) reactions. The discussion was kept as model independent as possible.

We then turned to one specific model to illustrate the consequences of picking the assignment of the heavy quark. If its charge is 2/3 then we can conveniently choose it to be the t quark in the LW SU(3) $\otimes$ U(1) model. The consequences for neutrino interactions will be a rich spectrum of multimuon decay modes, arising from the heavy quark decay with the concomitant heavy lepton decay. If the charge of the Q is $-1/3$ then it can be identified with the b quark of the WS SU(3) $\otimes$ U(1) model. Then, however, it will be much harder to see its presence, because the large threshold is difficult to attain with available antineutrino beams so the event rates will be very low.

In Sec. IV we presented the results of an investigation of multimuon decay modes in the Lee-Weinberg SU(3) $\otimes$ U(1) model. Not only have we given the raw theoretical rates, we also calculated distributions and investigated the effects of imposing cuts on the muon energies and angles. Such cuts complicate the analysis because they allow tetramuon events to be misclassified as trimuon events or as dimuon events. Also events with electrons and/or positrons cannot be identified in counter experiments so these events are automatically classified as multimuon modes. We have given results in Table 3 to show how the true branching fraction gets modified due to these effects. In Table 4 we showed some of the corresponding results for antineutrinos. Combining the results for the production cross section and the decay branching ratios we then gave results for multimuon event rates in Table 5. The event rates predicted by the model are within easy reach of experimental groups at CERN and Fermilab, so we should soon find out whether the model correctly describes the multimuon events.

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TABLE CAPTIONS

Table 1. The asymptotic values for σ, $d\sigma/dy$, $\langle y \rangle$ and R for the reactions listed. The four blocks reflect the choice $Q = Q_u$, Q_c, Q_d, and Q_s, respectively in the $SU(2) \otimes U(1)$ model. Normalization and notation are explained in the text.

Table 2. The asymptotic values for σ, $d\sigma/dy$, $\langle y \rangle$ and R for the reactions listed. The three blocks reflect the choice Q = t, g and b, respectively, in the $SU(3) \otimes U(1)$ model. Normalization and notation are explained in the text.

Table 3. Branching ratios for specific decay modes of a neutrino produced heavy lepton and heavy quark after the energy and angle cuts given in the text have beem imposed. In the first row we give the percentage of the time each four lepton channel is identified or misidentified as a trimuon or dimuon mode due to one of the leptons failing to satisfy the cuts. The second row gives the branching ratio, as determined from Fig. 14 with $\sin^2\beta = 0.2$, together with the partial branching ratios into the channels indicated. These numbers are summed in the last row to give the total branching ratios.

Table 4. Same as Table 3 for an antineutrino produced heavy lepton and heavy quark. We only give the first row of numbers to show that the effects of the cuts are similar in the ν and $\bar{\nu}$ induced reactions.

Table 5. Event rates for multimuons in the FHPRW quadrupole-triplet neutrino beam relative to the regular μ^- production cross section with $E_\nu > 100$ GeV. We use the masses of M and t as in the text and the calculated cross section $\sigma_{(d\to t)}(\nu+N\to M^-+X)/\sigma_{(d\to u)}(\nu+N\to\mu^-+X)\Big|_{E_\nu>100\text{GeV}} = 0.0167$.

TABLE 1

(1) $Q = Q_u$

reactions: $\nu_\mu + d_L \to \mu^- + u_L$

$\nu_\mu + d_{L,R} \to \mu^- + Q_{u(L,R)}$

$$\sigma^{\nu N}/\sigma_0 = 1 + \rho\left[\sin^2\chi_L + \frac{1}{3}\cos^2\chi_R\right] ,$$

$$\frac{1}{\sigma_0}\frac{d\sigma^{\nu N}}{dy} = 1 + \rho\left[\sin^2\chi_L + (1-y)^2\cos^2\chi_R\right] ,$$

$$\langle y\rangle^{\nu N} = \frac{\left(\frac{1}{2} + \rho\left[\frac{1}{2}\sin^2\chi_L + \frac{1}{12}\cos^2\chi_R\right]\right)}{\sigma^{\nu N}/\sigma_0} , \qquad R = \frac{\frac{1}{3}\sigma_0}{\sigma^{\nu N}} .$$

(2) $Q = Q_c$

reactions: $\nu_\mu + d_L \to \mu^- + u_L$

$\nu_\mu + d_R \to \mu^- + Q_{cR}$

$$\sigma^{\nu N}/\sigma_0 = 1 + \rho\left[\frac{1}{3}\sin^2\chi_R\right] ,$$

$$\frac{1}{\sigma_0}\frac{d\sigma^{\nu N}}{dy} = 1 + \rho(1-y)^2\sin^2\chi_R ,$$

$$\langle y\rangle^{\nu N} = \frac{\left(\frac{1}{2} + \rho\frac{1}{12}\sin^2\chi_R\right)}{\sigma^{\nu N}/\sigma_0} , \qquad R = \frac{\frac{1}{3}\sigma_0}{\sigma^{\nu N}} .$$

TABLE 1 (CONTINUED)

(3) $Q = Q_d$

reactions: $\bar{\nu}_\mu + u_L \to \mu^+ + d_L$

$\bar{\nu}_\mu + u_{L,R} \to \mu^+ + Q_{d(L,R)}$

$$\sigma^{\bar{\nu}N}/\sigma_0 = \frac{1}{3} + \rho\left[\frac{1}{3}\sin^2\chi_L + \cos^2\chi_R\right] ,$$

$$\frac{1}{\sigma_0}\frac{d\sigma^{\bar{\nu}N}}{dy} = (1-y)^2 + \rho\left[(1-y)^2 \sin^2\chi_L + \cos^2\chi_R\right] ,$$

$$\langle y\rangle^{\bar{\nu}N} = \frac{\left(\frac{1}{12} + \rho\left[\frac{1}{12}\sin^2\chi_L + \frac{1}{2}\cos^2\chi_R\right]\right)}{\sigma^{\bar{\nu}N}/\sigma_0} , \qquad R = \frac{\sigma^{\bar{\nu}N}}{\sigma_0} .$$

(4) $Q = Q_s$

reactions: $\bar{\nu}_\mu + u_L \to \mu^+ + d_L$

$\bar{\nu}_\mu + u_R \to \mu^+ + Q_{sR}$

$$\sigma^{\bar{\nu}N}/\sigma_0 = \frac{1}{3} + \rho\left[\sin^2\chi_R\right] ,$$

$$\frac{1}{\sigma_0}\frac{d\sigma^{\bar{\nu}N}}{dy} = (1-y)^2 + \rho\left[\sin^2\chi_R\right] ,$$

$$\langle y\rangle^{\bar{\nu}N} = \frac{\left(\frac{1}{12} + \rho\left[\frac{1}{2}\sin^2\chi_R\right]\right)}{\sigma^{\bar{\nu}N}/\sigma_0} , \qquad R = \frac{\sigma^{\bar{\nu}N}}{\sigma_0} .$$

TABLE 2

(1) $Q = t$

reactions: $\nu_\mu + d_L \rightarrow \mu^- + u_L$

$\nu_\mu + d_R \rightarrow M^- + t_R$

$$\sigma^{\nu N}/\sigma_0 = 1 + \frac{1}{3}\rho\cos^2\theta'$$

$$\frac{1}{\sigma_0}\frac{d\sigma^{\nu N}}{dy} = 1 + \rho(1-y)^2\cos^2\theta'$$

$$\langle y\rangle^{\nu N} = \frac{\left[\frac{1}{2} + \frac{1}{12}\rho\cos^2\theta'\right]}{\sigma^{\nu N}/\sigma_0} \quad , \quad R = \frac{\frac{1}{3}\sigma_0}{\sigma^{\nu N}} \quad .$$

(2) $Q = g$

reactions: $\nu_\mu + d_L \rightarrow \mu^- + u_L$

$\nu_\mu + d_R \rightarrow M^- + g_R$

$$\sigma^{\nu N}/\sigma_0 = 1 + \frac{1}{3}\rho\sin^2\theta' \quad ,$$

$$\frac{1}{\sigma_0}\frac{d\sigma^{\nu N}}{dy} = 1 + \rho(1-y)^2\sin^2\theta' \quad ,$$

$$\langle y\rangle^{\nu N} = \frac{\left[\frac{1}{2} + \frac{1}{12}\rho\sin^2\theta'\right]}{\sigma^{\nu N}/\sigma_0} \quad , \quad R = \frac{\frac{1}{3}\sigma^0}{\sigma^{\nu N}} \quad .$$

(3) $Q = b$

reactions: $\bar{\nu}_\mu + u_L \to \mu^+ + d_L$

$\bar{\nu}_\mu + u_L \to M^+ + b_L$

$$\sigma^{\bar{\nu}N}/\sigma_0 = \frac{1}{3} + \frac{1}{3}\rho \quad ,$$

$$\frac{1}{\sigma_0}\frac{d\sigma^{\bar{\nu}N}}{dy} = (1-y)^2 + \rho(1-y)^2 \quad ,$$

$$\langle y\rangle^{\bar{\nu}N} = \frac{\left[\frac{1}{12} + \frac{\rho}{12}\right]}{\sigma^{\bar{\nu}N}/\sigma_0} \quad , \qquad R = \frac{\sigma_0}{\sigma_{\bar{\nu}N}} \quad .$$

TABLE 3

Mode	B.R.	$\mu^-\mu^-\mu^+\mu^+$	$\mu^-\mu^-\mu^+$	$\mu^-\mu^+\mu^+$	$\mu^-\mu^-$	$\mu^+\mu^+$
$\mu^-\mu^-\mu^+\mu_Q^+$		0.078	0.69	0.021	0.10	0.0001
	6.5×10^{-3}	5.1×10^{-4}	4.5×10^{-3}	1.4×10^{-4}	6.5×10^{-4}	6.5×10^{-7}
$\mu^-\mu^-\mu^+e_Q^+$			0.76		0.11	
	6.5×10^{-3}		4.9×10^{-3}		7.2×10^{-4}	
$\mu^-\mu^-e^+\mu_Q^+$			0.086		0.78	
	1.6×10^{-2}		1.4×10^{-3}		1.2×10^{-2}	
$\mu^-\mu^-e^+e_Q^+$					0.87	
	1.6×10^{-2}				1.4×10^{-2}	
$e^-\mu^-\mu^+\mu_Q^+$				0.10		0.020
	3×10^{-2}			3×10^{-3}		6×10^{-4}
$e^-e^-\mu^+\mu_Q^+$						0.090
	6.5×10^{-3}					5.9×10^{-4}
Total B.R.		5.1×10^{-4}	1.1×10^{-2}	3.1×10^{-3}	2.7×10^{-2}	1.2×10^{-3}

TABLE 4

Mode	BR	$\mu^+\mu^+\mu^-\mu^-$	$\mu^+\mu^+\mu^-$	$\mu^+\mu^-\mu^-$	$\mu^+\mu^+$	$\mu^-\mu^-$
$\mu^+\mu^+\mu^-\mu^-_Q$		0.068	0.71	0.027	0.09	∿0
	6.5×10^{-3}	4.4×10^{-4}	4.6×10^{-3}	1.8×10^{-4}	5.8×10^{-4}	∿0

TABLE 5

Multimuon Mode	$\mu^-\mu^-\mu^+\mu^+$	$\mu^-\mu^-\mu^+$	$\mu^-\mu^+\mu^+$	$\mu^-\mu^-$	$\mu^+\mu^+$
Relative Event Rate	8.5×10^{-6}	1.8×10^{-4}	5.2×10^{-5}	4.5×10^{-4}	2.0×10^{-5}

FIGURE CAPTIONS

Fig. 1. Quark multiplet structure in the LW SU(3) $\otimes$ U(1) model. The mixing of the Q = 2/3 members of the triplet is specified in the text.

Fiq. 2. Lepton multiplet structure in the LW SU(3) $\otimes$ U(1) model. The mixing of the Q = 0 and Q = 1 members of the triplet is specified in the text.

Fig. 3. Lepton multiplet structure in one of the LS SU(3) $\otimes$ U(1) models. The notation is explained in the text.

Fig. 4. Event rate curves for the neutrino production of muons in the (ℓ,q) reaction, and concomitant heavy lepton, heavy quark production in the LW model. The total cross sections for $\nu_\mu+N\rightarrow\mu^-+X$ and $\nu_\mu+N\rightarrow M^-+X$ have been folded with the FHPRW quadrupole triplet neutrino spectrum and the CDHS dichromatic neutrino spectrum.

Fig. 5. The same as Fig. 4 for antineutrino production of muons with light quarks and heavy leptons with heavy quarks.

Fig. 6. The distribution of events in the (q^2,ν) plane for an (ℓ,Q) reaction involving a heavy quark with mass 4.75 GeV/c^2.

Fig. 7. The distribution of events in the (q^2,ν) plane for an (L,q) reaction with an 8 GeV/c^2 heavy lepton mass.

Fig. 8. The distribution of events in the (q^2,ν) plane for an (L,Q) reaction with m_L = 8 GeV/c^2 and m_Q = 4.75 GeV/c^2.

Fig. 9. The distribution of events in the (x,y) plane for an (ℓ,Q) reaction involving a heavy quark with mass 4.75 GeV/c^2.

Fig. 10. The distribution of events in the (x,y) plane for an (L,q) reaction with an 8 GeV/c^2 heavy lepton mass.

Fig. 11. The distribution of events in the (x,y) plane for an (L,Q) reaction with m_L = 8 GeV/c^2 and m_Q = 4.75 GeV/c^2.

Fig. 12. The distribution of events in the variable x_{vis} and y_{vis} for the fast μ^- produced in the (L,Q) reaction.

Fig. 13. Branching ratios for muonic final states in M^- decay, as a function of $\sin^2\beta$. The mass of the E^o is taken to be 1 GeV/c^2. For clarity the BR($\mu^-\mu^-$) decay is dashed.

Fig. 14. Feynman diagrams for the two multimuon decay modes discussed in the text. The chirality at each vertex is given as well as the type of boson which is exchanged.

Fig. 15. The theoretical and experimental spectra of the negative muons produced via the (L,Q) reaction.

Fig. 16. The theoretical and experimental spectra of the positive muons produced via the (L,Q) reaction.

Fig. 17. The distributions in E_{had}, E_{vis}, E_{miss} and E_{tot} in the (L,Q) reaction.

Fig 1

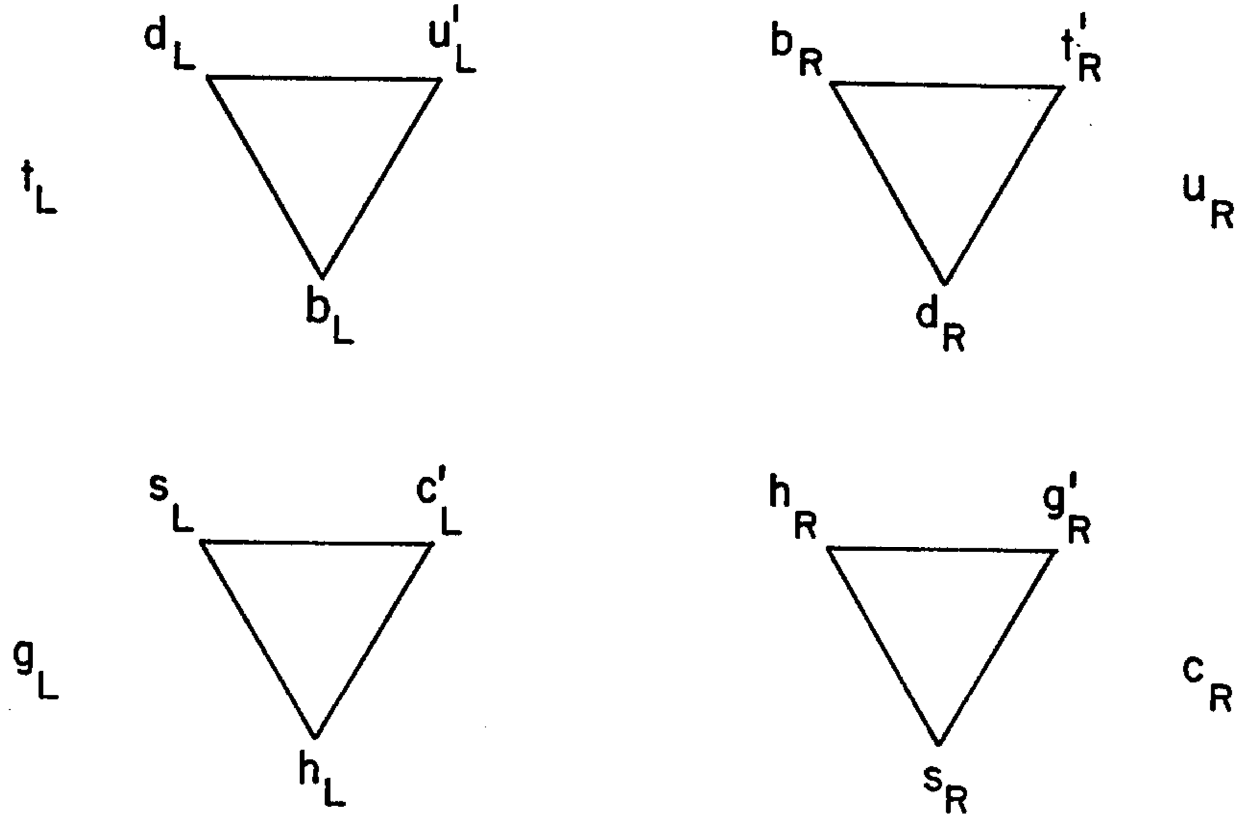
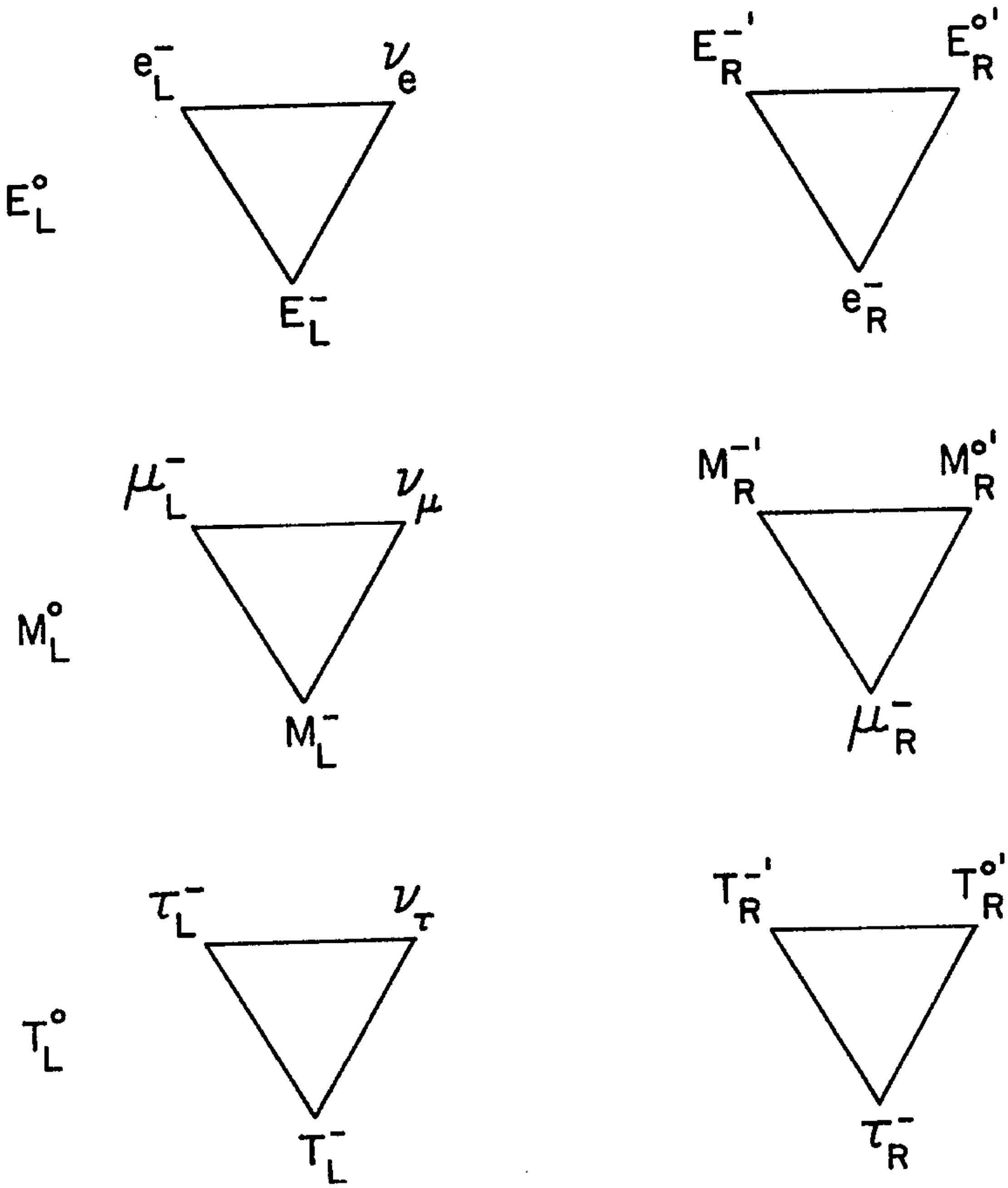


Fig. 2



e_L^-
ν_e
E_L^0
E_L^-
$E_R^{-\prime}$
$E_R^{0\prime}$
e_R^-
μ_L^-
ν_μ
M_L^0
M_L^-
$M_R^{-\prime}$
$M_R^{0\prime}$
μ_R^-
τ_L^-
ν_τ
T_L^0
T_L^-
$T_R^{-\prime}$
$T_R^{0\prime}$
τ_R^-

Fig. 3

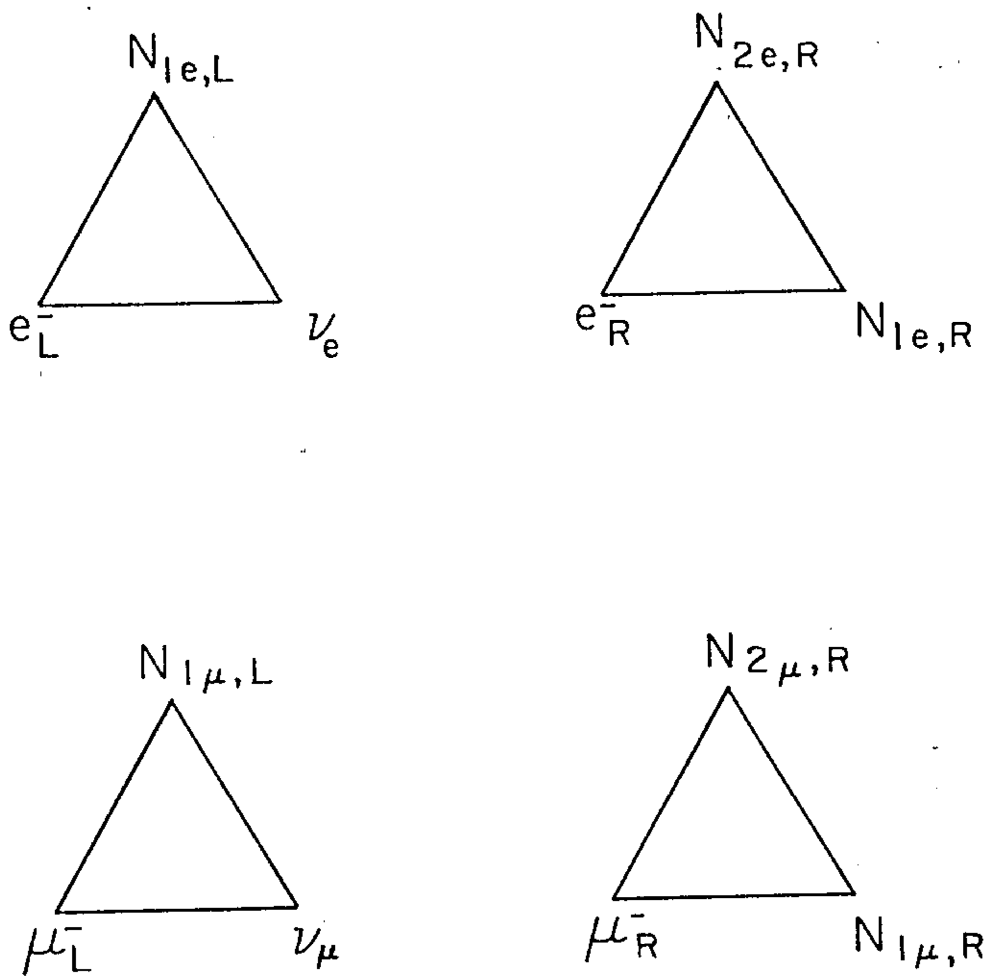


Fig 4

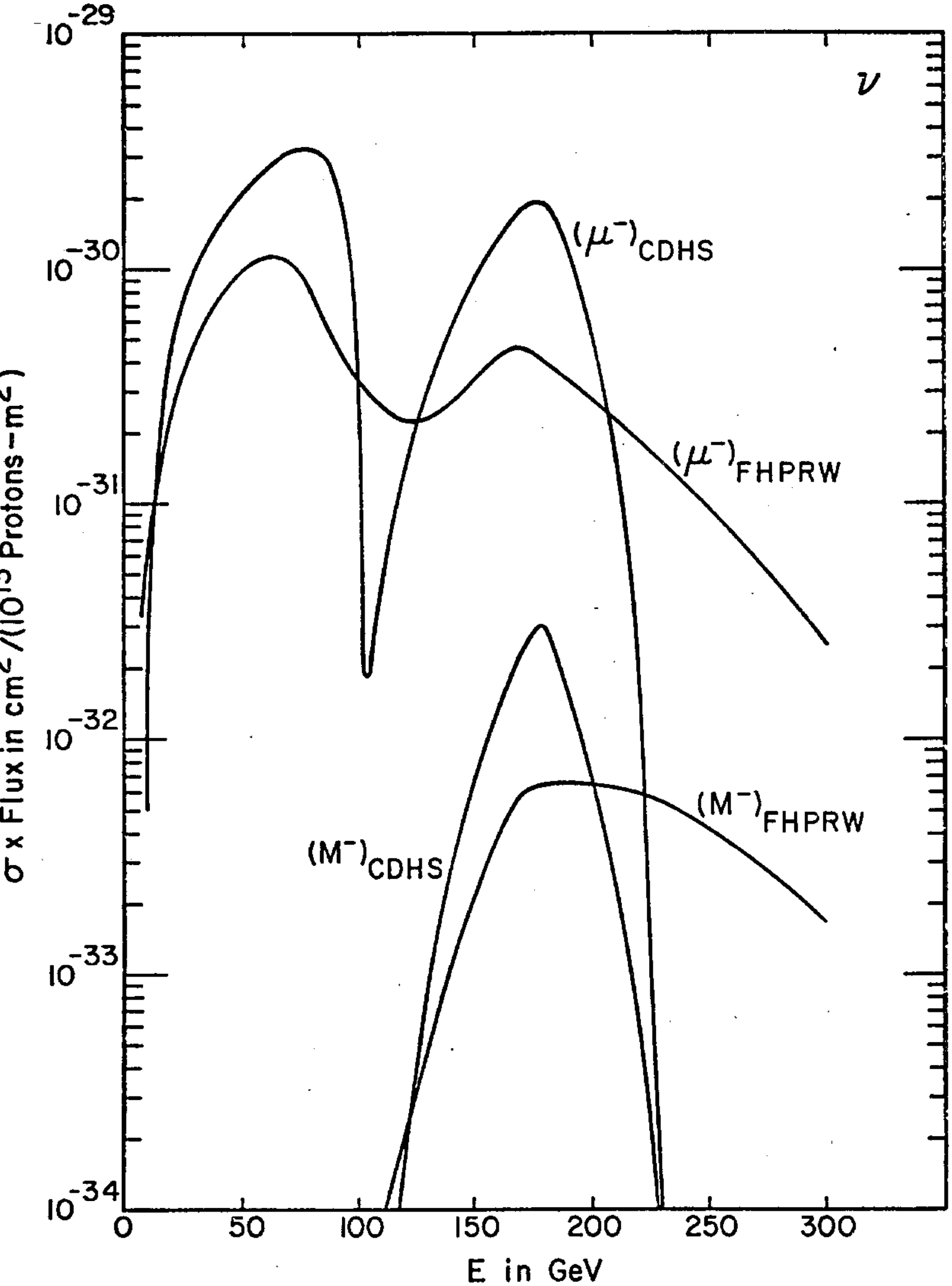

ν
$(\mu^-)_{CDHS}$
$(\mu^-)_{FHPRW}$
$(M^-)_{CDHS}$
$(M^-)_{FHPRW}$
10^{-29}
10^{-30}
10^{-31}
10^{-32}
10^{-33}
10^{-34}
0
50
100
150
200
250
300
E in GeV
σ x Flux in $cm^2/(10^{13}$ Protons$-m^2)$

Fig 5

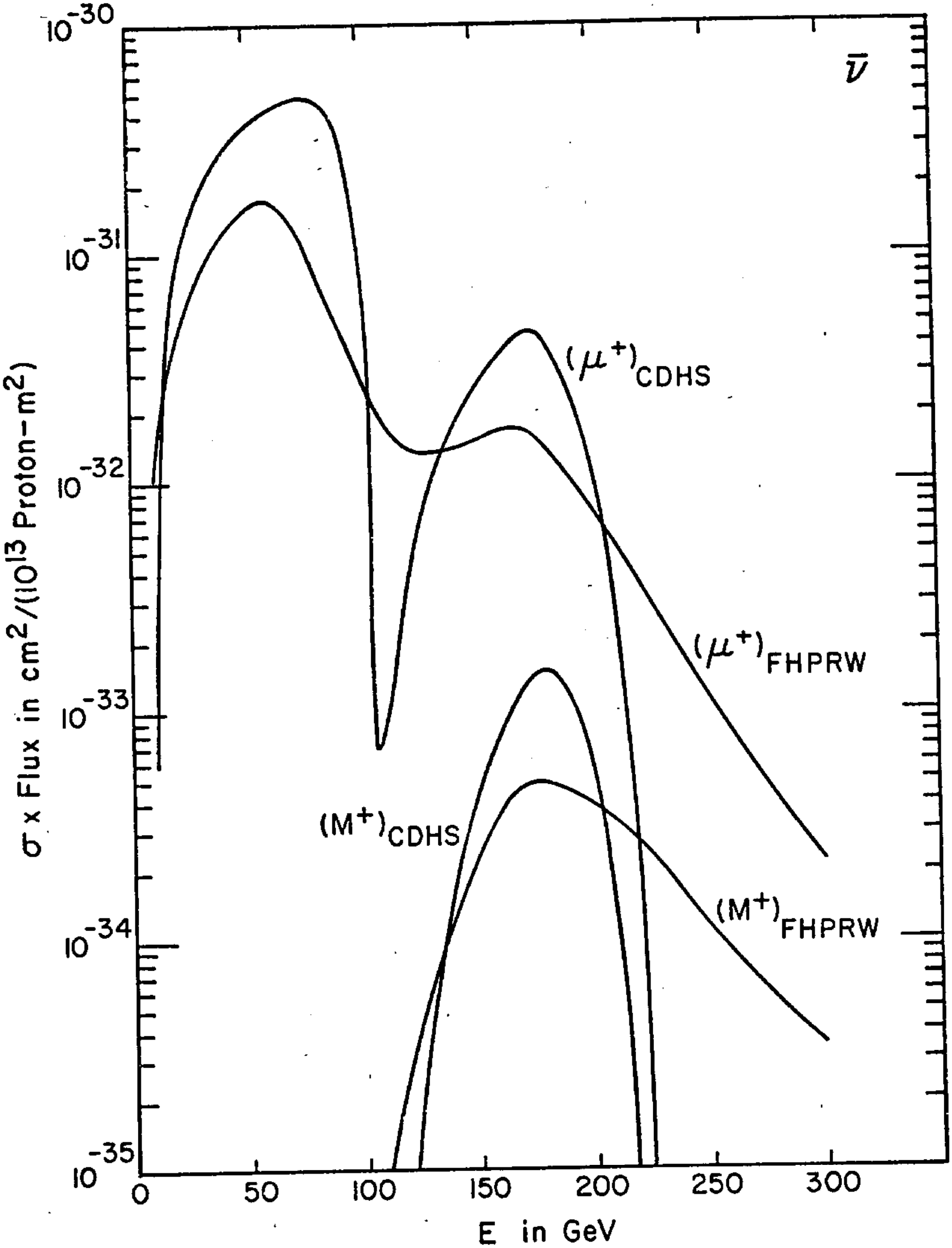
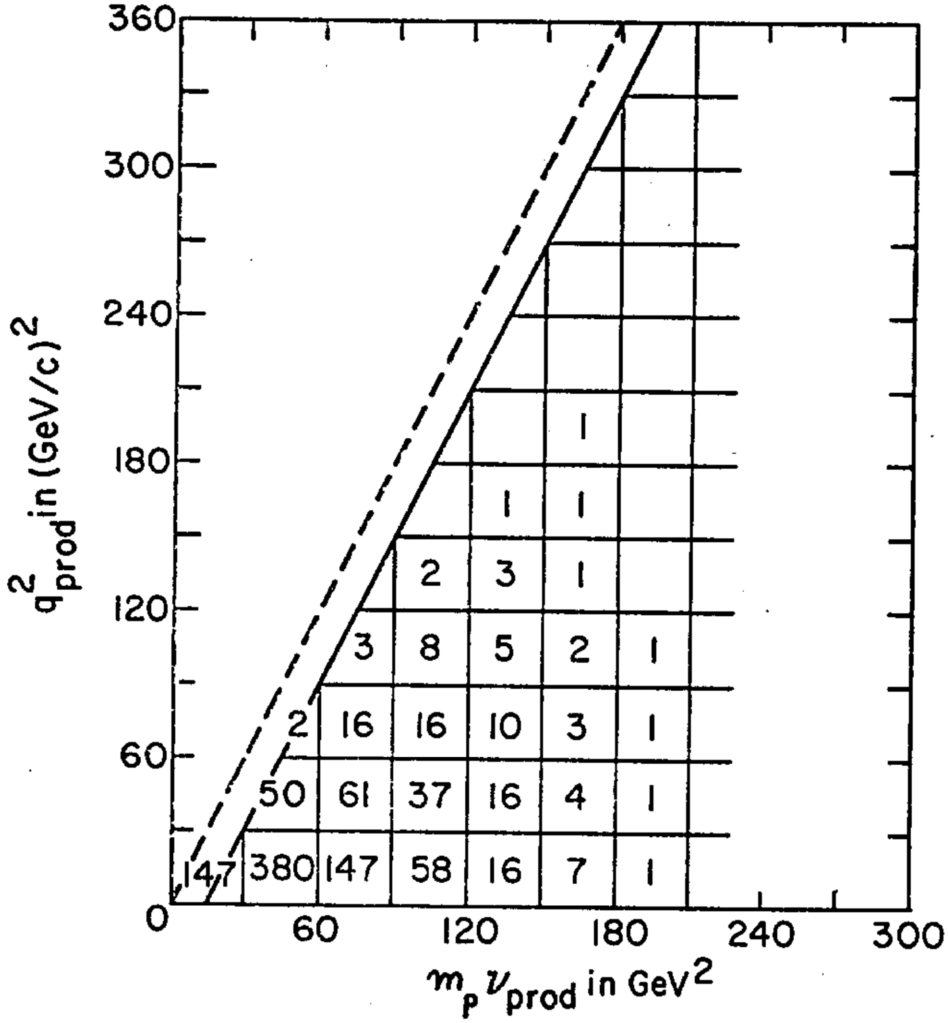
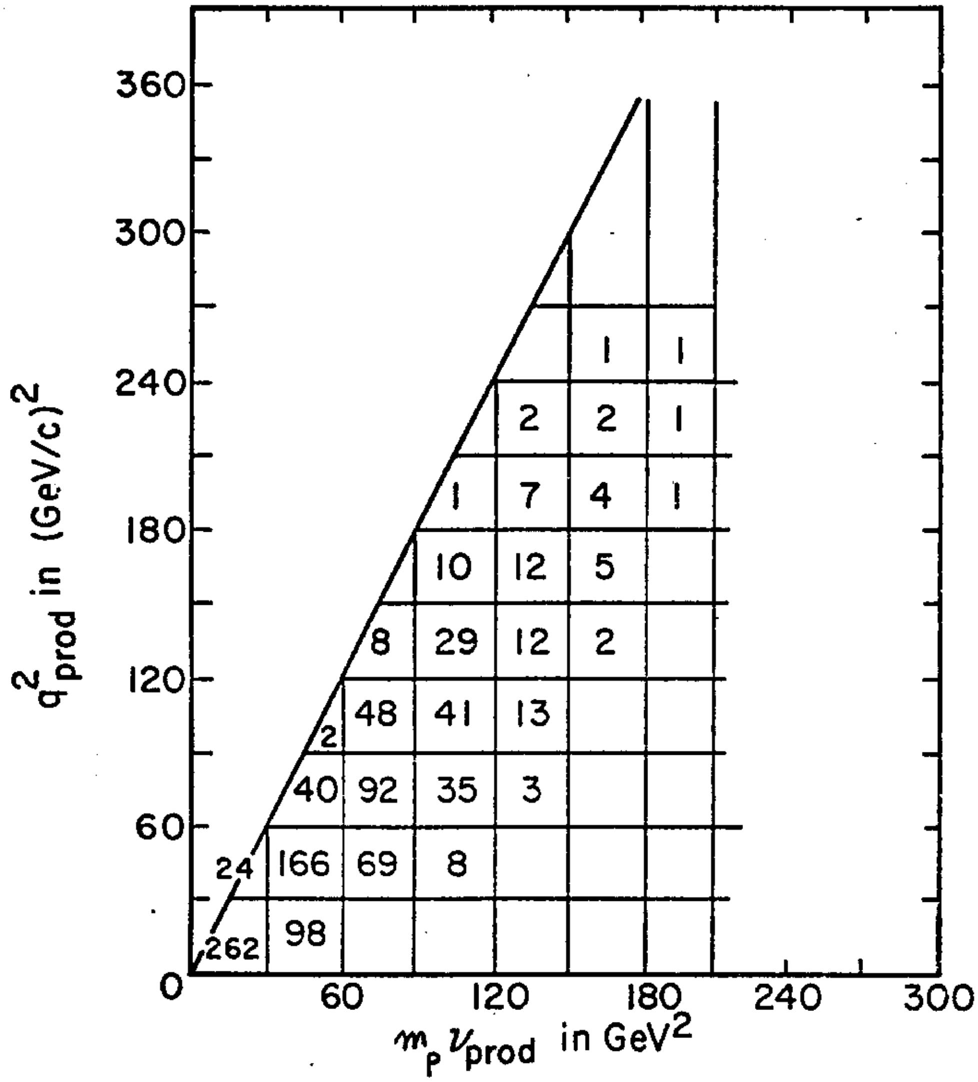

10^{-30}
10^{-31}
10^{-32}
10^{-33}
10^{-34}
10^{-35}
$\bar{\nu}$
$(\mu^+)_{CDHS}$
$(\mu^+)_{FHPRW}$
$(M^+)_{CDHS}$
$(M^+)_{FHPRW}$
σ x Flux in $cm^2/(10^{13}$ Proton$-m^2)$
0
50
100
150
200
250
300
E in GeV

Fig. 6



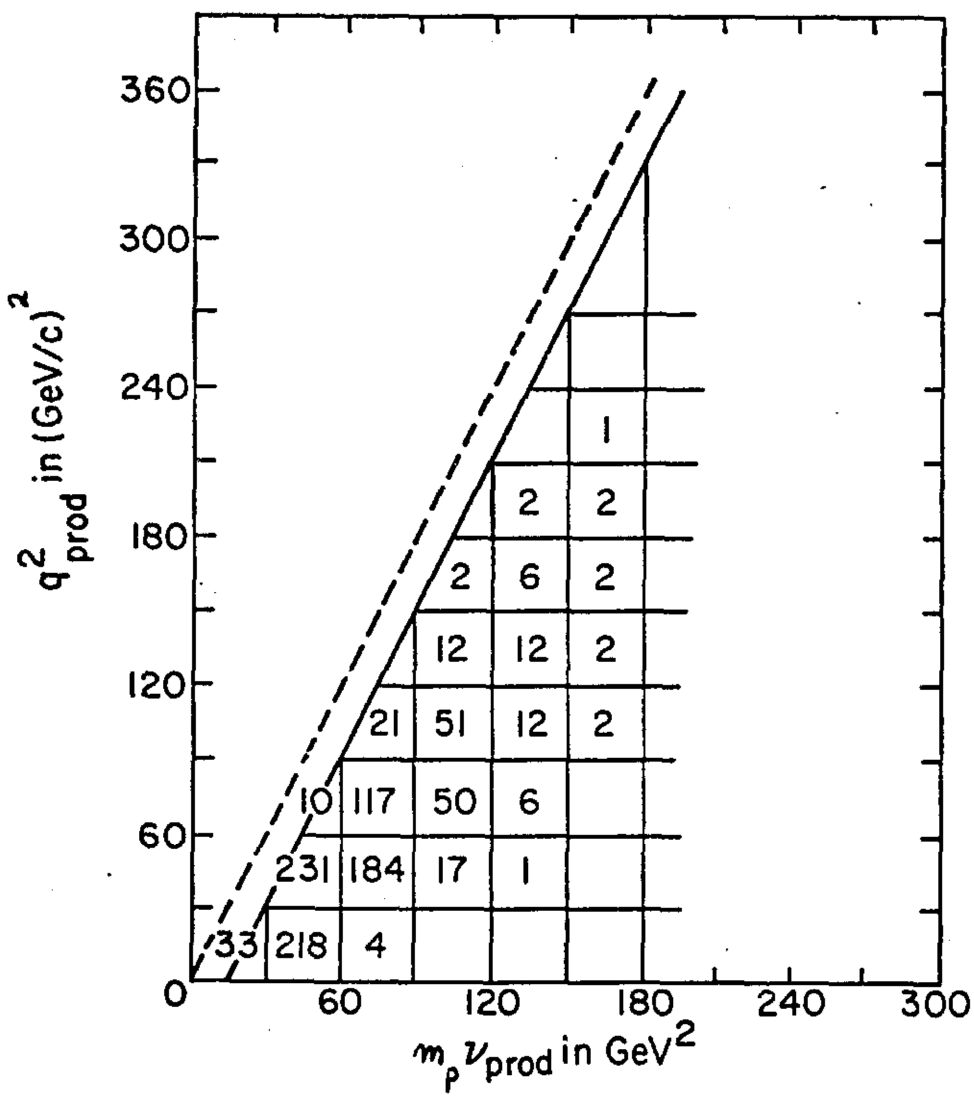
360
300
240
180
120
60
0
60
120
180
240
300
q^2_{prod} in $(GeV/c)^2$
$m_p \nu_{prod}$ in GeV^2
147
380
147
58
16
7
1
50
61
37
16
4
1
2
16
16
10
3
1
3
8
5
2
1
2
3
1
1
1
1

Fig 7



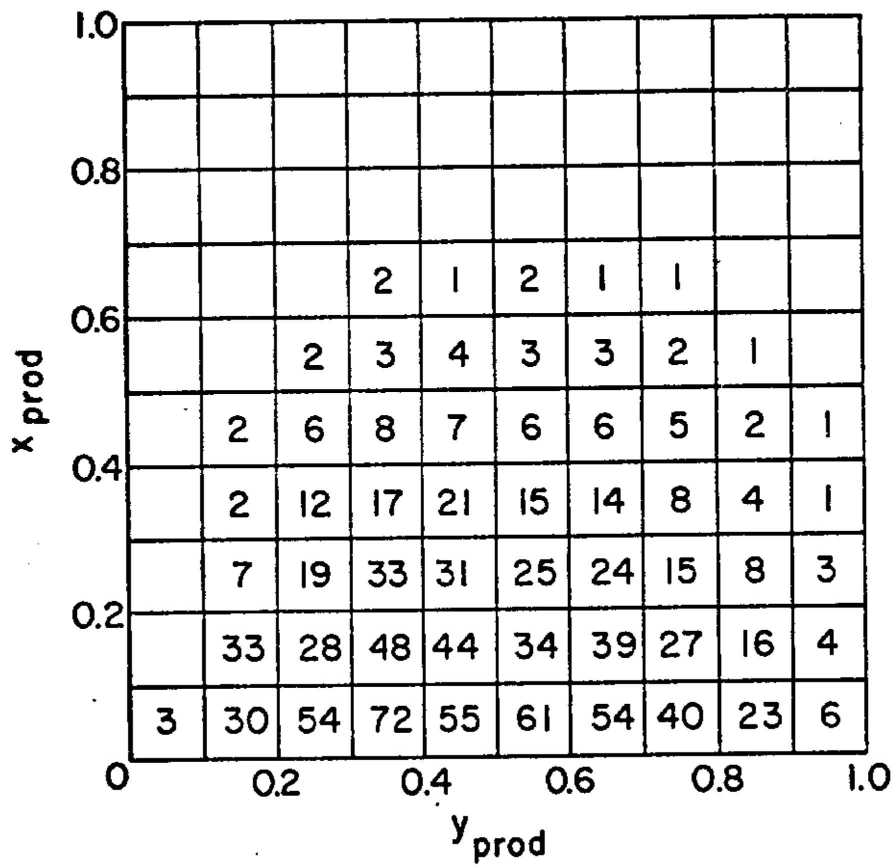
360
300
240
180
120
60
0
60
120
180
240
300
q^2_{prod} in $(GeV/c)^2$
$m_p \nu_{prod}$ in GeV^2
262
98
24
166
69
8
40
92
35
3
2
48
41
13
8
29
12
2
10
12
5
1
7
4
1
2
2
1
1
1

Fig. 8



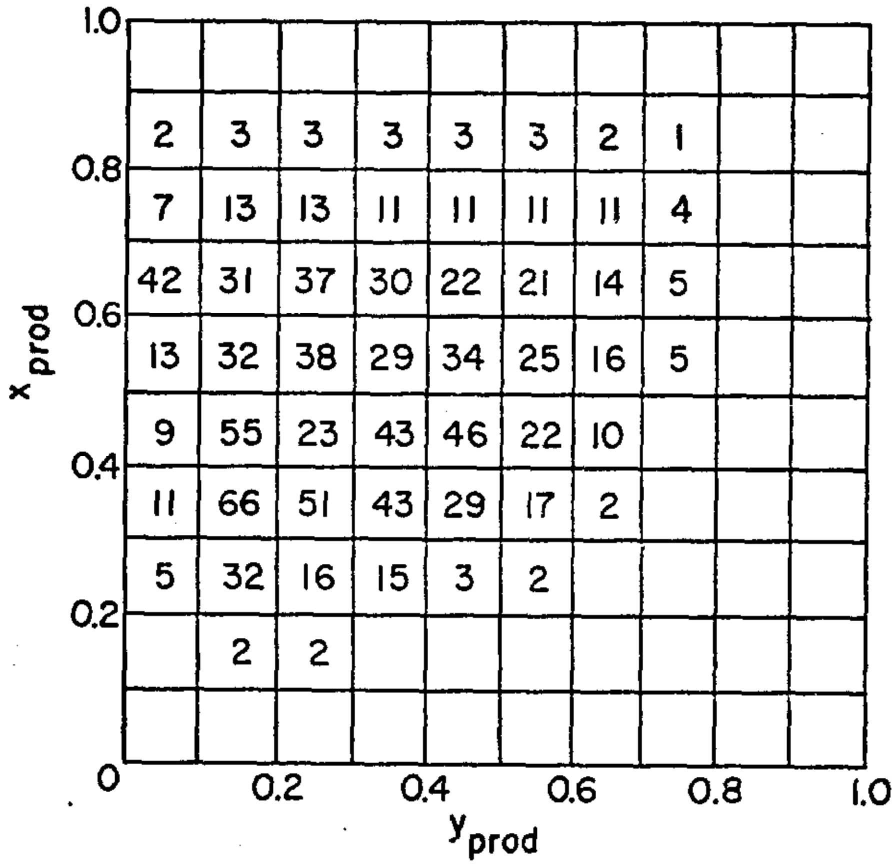
q^2_{prod} in $(GeV/c)^2$
360
300
240
180
120
60
0
60
120
180
240
300
$m_p \nu_{prod}$ in GeV^2
1
2
2
2
6
2
12
12
2
21
51
12
2
10
117
50
6
231
184
17
1
33
218
4

Fig. 9



1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
x_{prod}
y_{prod}
2 1 2 1 1
2 3 4 3 3 2 1
2 6 8 7 6 6 5 2 1
2 12 17 21 15 14 8 4 1
7 19 33 31 25 24 15 8 3
33 28 48 44 34 39 27 16 4
3 30 54 72 55 61 54 40 23 6

Fig 10



1.0
0.8
0.6
0.4
0.2
0
x prod
2 3 3 3 3 3 2 1
7 13 13 11 11 11 11 4
42 31 37 30 22 21 14 5
13 32 38 29 34 25 16 5
9 55 23 43 46 22 10
11 66 51 43 29 17 2
5 32 16 15 3 2
2 2
0.2 0.4 0.6 0.8 1.0
y prod

Fig 11

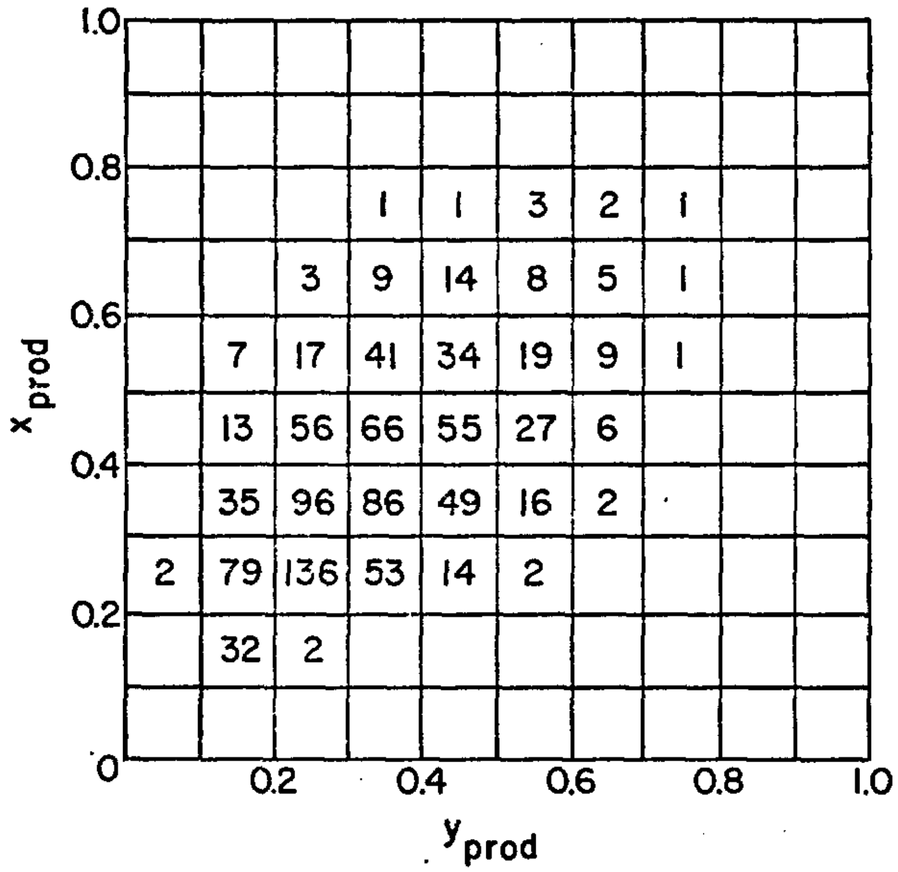

1.0
0.8
0.6
0.4
0.2
0
x_{prod}
1 1 3 2 1
3 9 14 8 5 1
7 17 41 34 19 9 1
13 56 66 55 27 6
35 96 86 49 16 2
2 79 136 53 14 2
32 2
0.2 0.4 0.6 0.8 1.0
y_{prod}

Fig 12

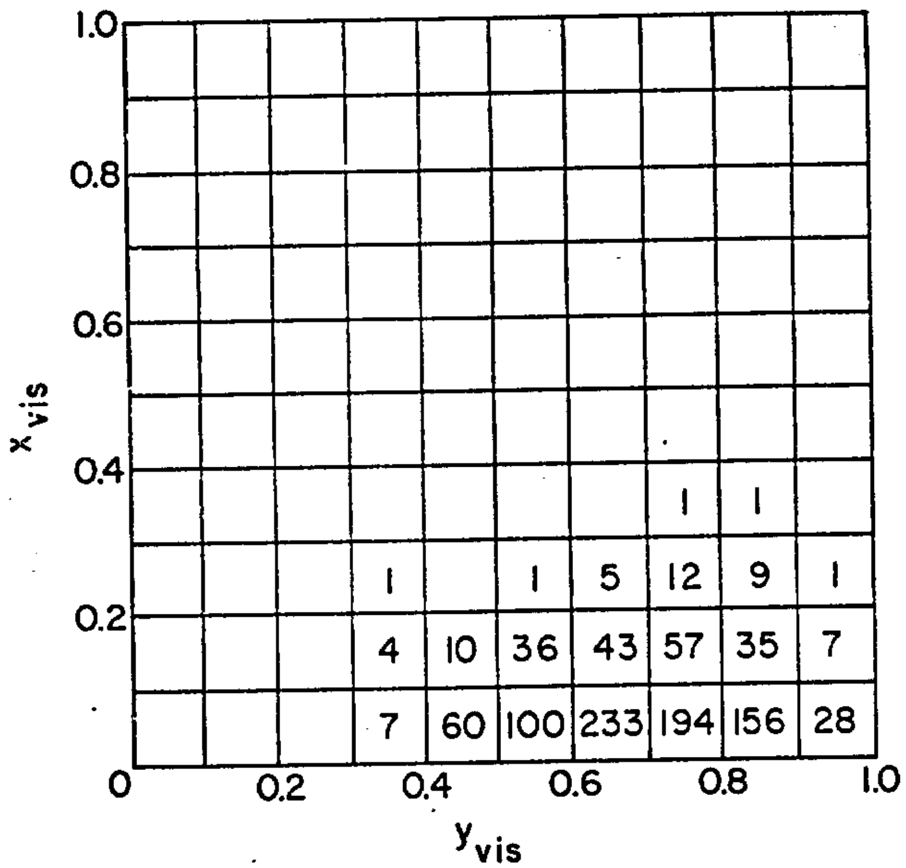
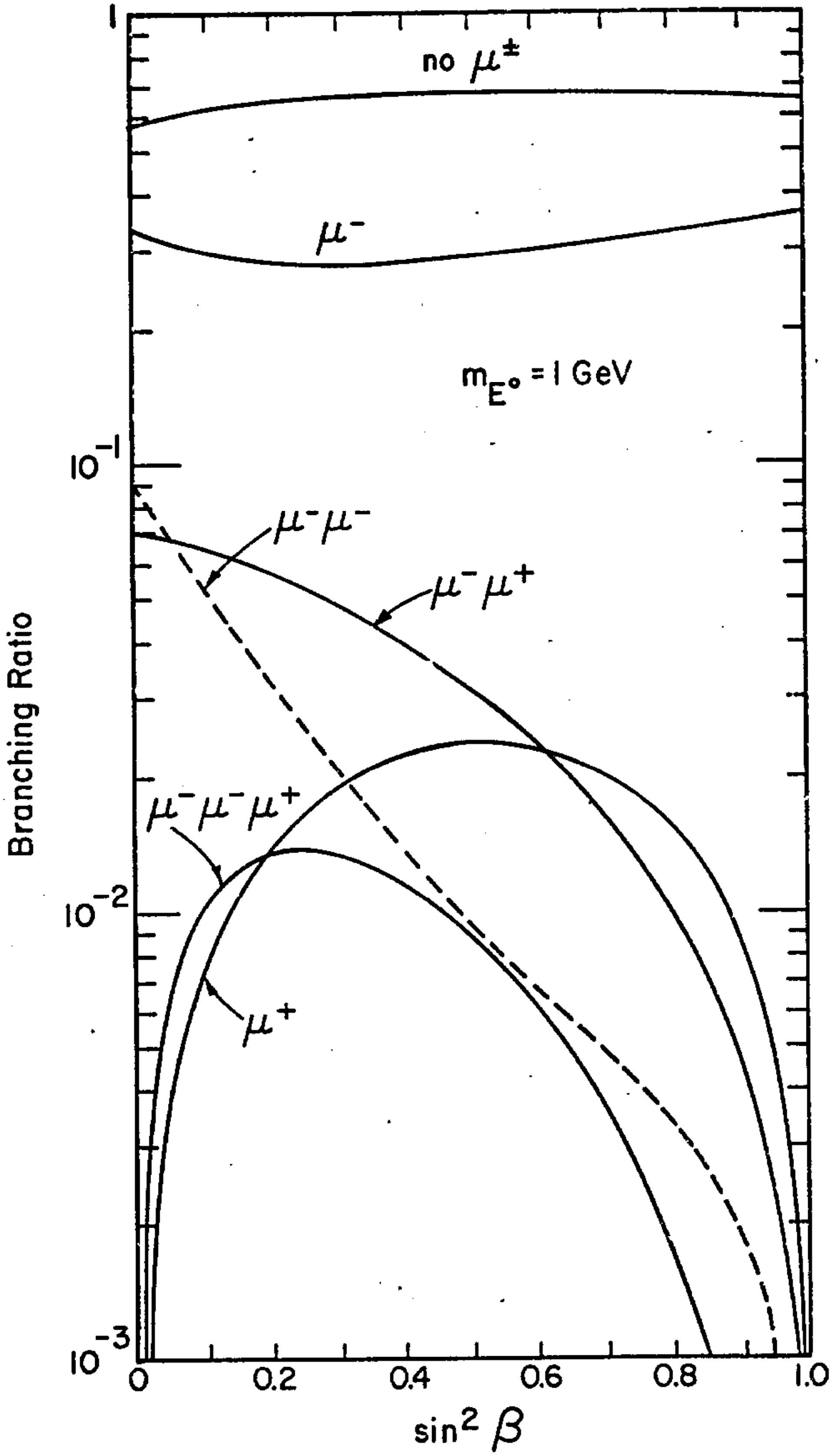
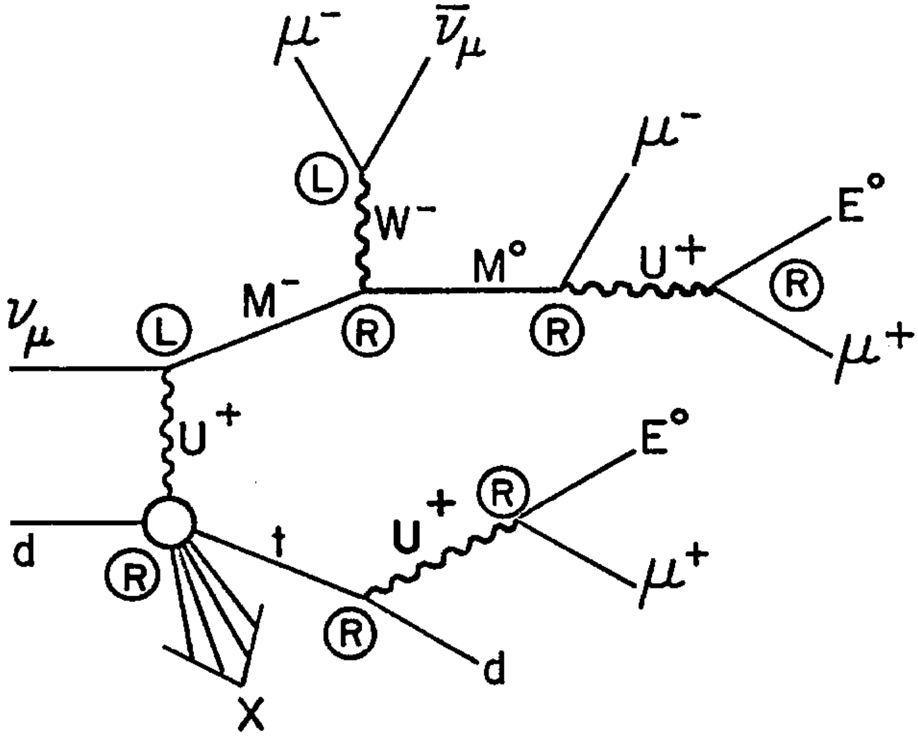

1.0
0.8
0.6
0.4
0.2
0
x_{vis}
1 1
1 1 5 12 9 1
4 10 36 43 57 35 7
7 60 100 233 194 156 28
0.2
0.4
0.6
0.8
1.0
y_{vis}

Fig 13



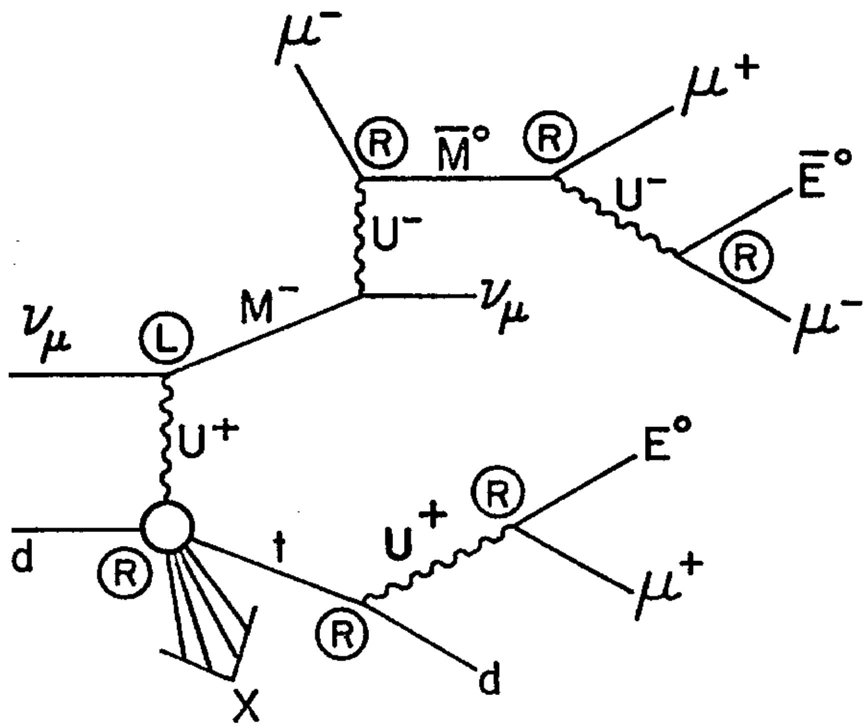
no $\mu^{\pm}$
μ^-
m_{E° = 1 GeV
$\mu^-\mu^-$
$\mu^-\mu^+$
$\mu^-\mu^-\mu^+$
μ^+
Branching Ratio
1
10^{-1}
10^{-2}
10^{-3}
0
0.2
0.4
0.6
0.8
1.0
$\sin^2\beta$

Fig 14



μ^-
$\bar{\nu}_\mu$
L
W^-
μ^-
E°
M°
U^+
R
M^-
R
R
ν_μ
L
μ^+
U^+
E°
R
U^+
d
t
R
μ^+
R
d
X

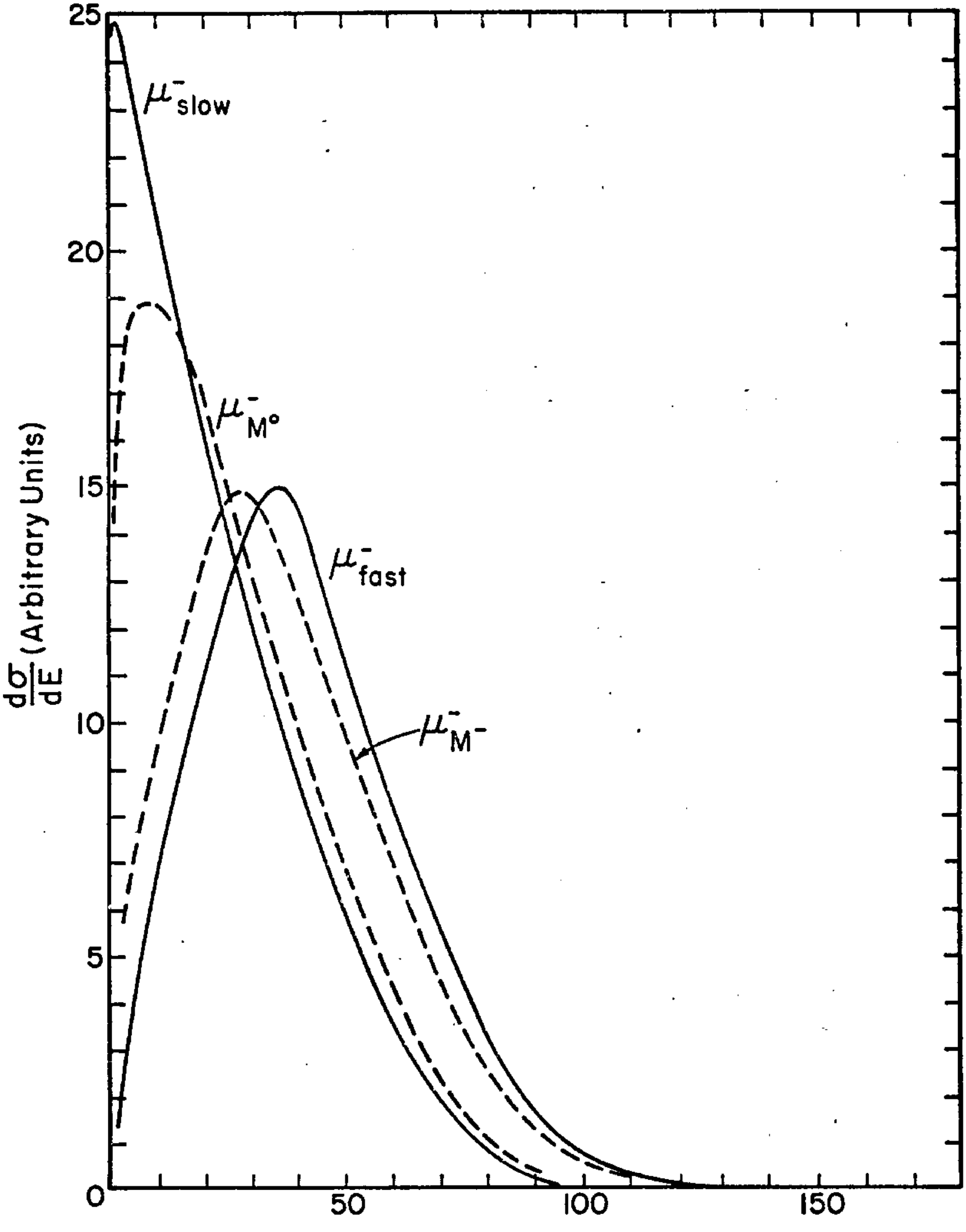
(a)



μ^-
μ^+
R
$\bar{M}^\circ$
R
$\bar{E}^\circ$
U^-
U^-
R
M^-
ν_μ
μ^-
ν_μ
L
U^+
E°
R
U^+
d
t
R
μ^+
R
d
X

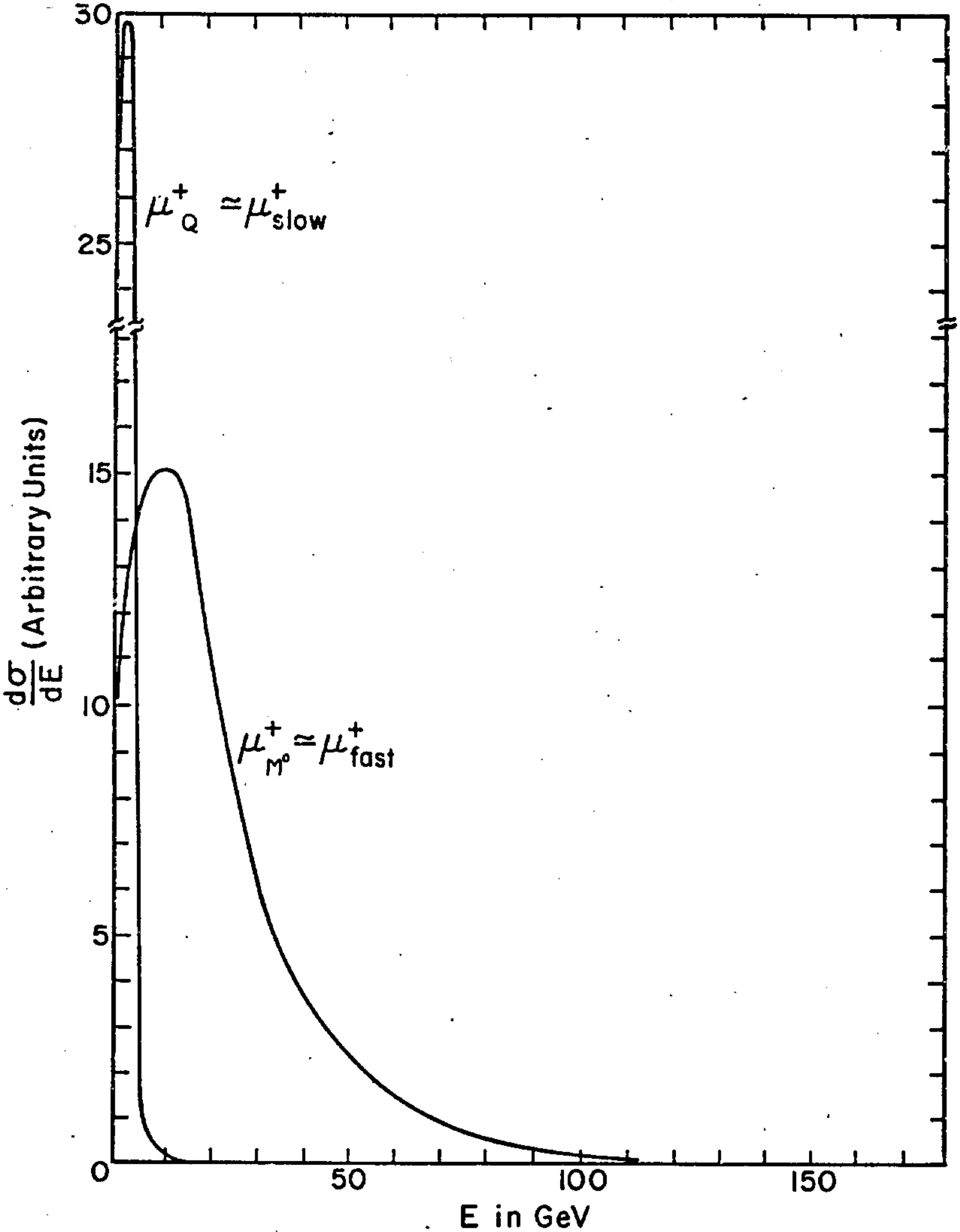
(b)

Fig. 15



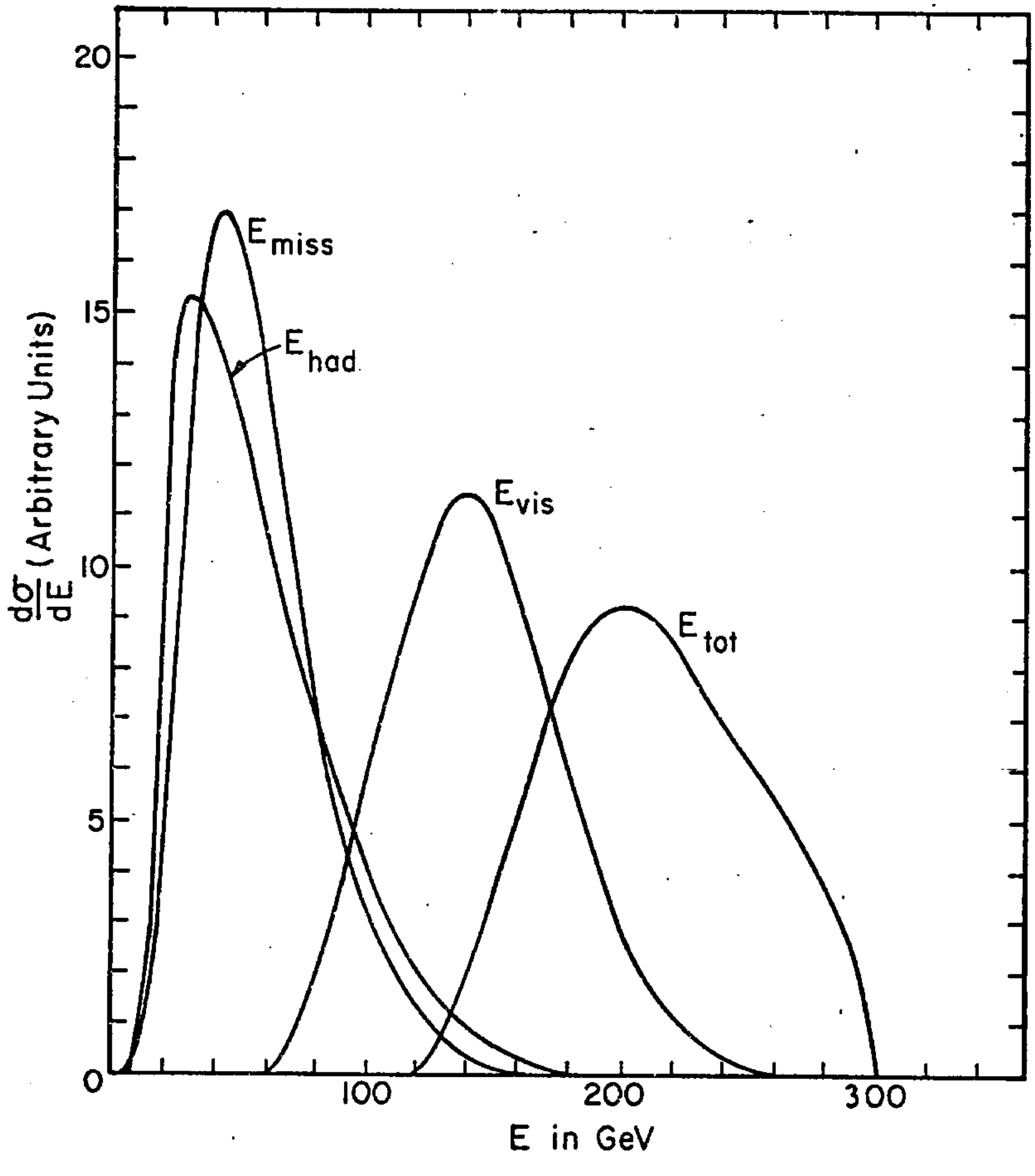
25
20
15
10
5
0
50
100
150
μ^-_{slow}
$\mu^-_{M^\circ}$
μ^-_{fast}
$\mu^-_{M^-}$
$\frac{d\sigma}{dE}$ (Arbitrary Units)
E in GeV

Fig 16



30
25
15
10
5
0
50
100
150
E in GeV
$\frac{d\sigma}{dE}$ (Arbitrary Units)
$\mu^+_Q \simeq \mu^+_{slow}$
$\mu^+_{M^\circ} \simeq \mu^+_{fast}$

Fig 17



20
15
10
5
0
100
200
300
E in GeV
dσ/dE (Arbitrary Units)
E miss
E had.
E vis
E tot