Cosmological Evolution in the Background of Non-Minimally Coupled Gravity

By

Muhammad Zeeshan

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Name	Registration Number
Muhammad Zeeshan	CIIT/FA16-RMT-010/LHR

Supervisor

DR. MUHAMMAD ZUBAIR
Assistant Professor
Department of Mathematics,
COMSATS University Islamabad (CUI)
Lahore Campus
June, 2018

Final Approval

This thesis titled

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By

Muhammad Zeeshan

CIIT/FA16-RMT-010/LHR

has been approved

for the MS Mathematics in the

COMSATS University Islamabad, Lahore Campus

External Examiner : ________________________

Supervisor : ________________________

(Dr. Muhammad Zubair)

HoD/Incharge : ________________________

(Dr. Sarfraz Ahmad)

(Mathematics Department, CUI, Lahore Campus)

<u>Declaration</u>

I, Muhammad Zeeshan with registration number **CIIT/FA16-RMT-010/LHR** hereby declare that I have produced the work presented in this thesis, during the scheduled period of study. I also declare that I have not taken any material from any source except referred to wherever due, that amount of plagiarism is within acceptable range. If a violation of HEC rules on research has occurred in this thesis, I shall be liable to punishable action under the plagiarism rules of the HEC.

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(Muhammad Zeeshan)

(CIIT/FA16-RMT-010/LHR)

Certificate

It is certified that Muhammad Zeeshan with CIIT/FA16-RMT-010/LHR has carried out all the work related to this thesis under my supervision at the Department of Mathematics, COMSATS University Islamabad, Lahore campus and the work fulfills the requirement for award of MS degree.

Dated :____________________

Supervisor:____________________

(Dr. Muhammad Zubair)

Assistant Professor

Head/Incharge Of Department

____

(Dr. Sarfraz Ahmad)

Associate Professor

Department of Mathematics

DEDICATED To

My Family

And

Khayam Javed
(My Brother in Law)

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Abstract

In current cosmic scenario, an accelerated expansion era is being reported by various observations. The reasoning of such scenario is still unknown, the presence of an unknown energy component is seen which is named as DE. There are various approaches to discuss the existence of DE and present acceleration of universe. One of such attempts is the modification of Einstein's gravity, here we attempt to explore this problem within the modified gravity based on non-minimal matter-geometry coupling. We examine $f(R, T, Q)$ theory (where R is the Ricci Scalar, T is the trace of EMT T^{uv} and $Q = R_{uv}T^{uv}$ is interaction of EMT $T^{\mu\nu}$ and Ricci Tensor R_{uv}). We formulate the dynamical equations in the background of FLRW model and find the result of non-conserved EMT using the divergence of the field equations. In this scenario test particles deviates from geodesic motion and an extra force is there due to non-minimal coupling. We applied this result to find an expression for energy density ρ for particular choice of Lagrangian. Furthermore, we discuss the energy bound on the model parameters and discuss the late time cosmic acceleration for best suitable parameters in accordance with recent observations.

We also study the cosmic evolution of non-minimally coupled $f(R, T)$ gravity (where R stands for Ricci scalar and T for trace of EMT) with matter formed of CM and radiations. We find the cosmic evolution in the background of CM and compare the results with NCM and ΛCDM model. In this study, we consider the flat FLRW metric and formulate the dynamical equations. Here, we choose

two models of non-minimal coupled $f(R,T)$ gravity (already reconstructed in [1]), and discuss the evolution of cosmological parameters, the effective EoS ω_{eff} and the deceleration parameter $q(z)$ in the universe containing self-interacting CM and radiations. In graphical description of these parameters we establish the comparison of results for self-interacting CM, NCM and ΛCDM model. Our results are consistent with the observational data.

Abbreviations

In this thesis, the convention to be used for the metric signatures will be $(+, -, -, -)$ and Greek indices will vary from 0 to 3, if different it will be mentioned. Also, we shall use the following list of abbreviations.

GR:	General Relativity
DE:	Dark Energy
DM:	Dark Matter
EMT:	Energy Momentum Tensor
EoS:	Equation of State
CMBR:	Cosmic Microwave Background Radiations
BAO:	Baryon Acoustic Oscillations
LSS:	Large Scale Structure
FLRW:	Friedmann-Lema$\hat{i}$tre-Robertson-Walker
Λ:	Cosmological Constant
ΛCDM:	Λ-Cold Dark Matter
NEC:	Null Energy Condition
WEC:	Weak Energy Condition
SEC:	Strong Energy Condition
DEC:	Dominant Energy Condition
SNeIa:	Supernovae Type Ia

BD: Brans Dicke

EFE: Einstein Field Equation

GSLT: Generalized Second Law of Thermodynamics

CM: Collisional Matter

NCM: Non-Collisional Matter

MTG: Modified Theories of Gravity

WMAP: Wilkinson Microwave Anisotropy Probe

LHS: Left Hand Side

RHS: Right Hand Side

WIMP: Weakly Interacting Massive Particles

MACHO: Massive Compact Halo Objects

Contents

Chapter 1

Introduction

Currently our universe is experiencing an accelerated expansion phase and multiple astrophysical researches have been conducted to observe this cosmic scenario. It is highly assumed and considered that this cosmic acceleration is the consequence of an anonymous energy named as DE [2]. Antagonistic to the gravitational pull, the DE is expanding the universe by having a negative pressure which is completely opposite to the ordinary matter. Many attempts have been made to unveil the reason for accelerated cosmic expansion. The major findings [3] enlists DE as the major candidate with overall contribution of 68.3%, the other significant 26.8% contribution is from DM despite its elusive and un-explored nature. Baryon, is the major part of visible cosmos which accounts for 4.9% among cosmic ingredients. Despite tremendous researches and observations, late time cosmic acceleration is still a significant as well as challenging area for cosmologists. However, attention is attached to the confirmation through measurements from temperature anisotropies of the existence of DE as puzzling cosmic ingredient with reference to cosmic acceleration by CMBR [3], BAO [4], LSS [5], weak lensing [6] and most recent Planck's data [7]. Furthermore, to describe the nature of DE several theoretical models are proposed like phantom [8], quintessence [9] and fluids with anisotropic EoS [10]. In ΛCDM model, the role of DE in GR is played by Λ. Yet the origin of cosmological constant Λ is still under question and Λ has two well-known problems known as coincidence and fine-tuning. The EoS is proposed to describe the properties of DE which is stated as $\omega_{DE} = \frac{p_{DE}}{\rho_{DE}}$ where p stands for cosmic pressure and ρ for energy density. The EoS is evaluated by considering that universe is isotropic and homogeneous, and taking the FLRW space-time at the background. In ΛCDM model ω_{DE} is exactly equal to -1 whereas in quintessence model ω_{DE} is dynamical quantity and $-1 < \omega_{DE} < \frac{-1}{3}$. Moreover ω_{DE} varies with time and $\omega_{DE} < -1$ in phantom model. It concludes that different model descriptions such as fluid description and the description of a scalar field theory might describe cosmic picture. DE models can also be establish by reconstruction of Einstein Hilbert action that further guide to the modified gravity models. The preliminary step is to substitute Einstein Hilbert term by scalar curvature and it results in the formation of $f(R)$ gravity [11]. In this theory, the general non-linear function f depends on the Ricci scalar R and if we replace this generic function f by $f \equiv R - 2\Lambda$ then we will get the classic ΛCDM model. This theory is also interesting due to the fact that for a specific BD parameter [12] it develops correspondence with the BD theory that include non-minimal coupling of scalar field and geometry. This coupling is also further constructed in $f(R)$ gravity [13, 14]. Bertolami et al. [14] gave another direction to $f(R)$ gravity where they coupled matter Lagrangian density $\mathcal{L}_m$ with the Lagrangian as a function of scalar curvature. In [15], authors developed the equivalence of scalar tensor theories with this theory which involves non-minimal scalar curvature term and a non-minimal coupling of the matter and scalar curvature. This non-minimal coupling further lead to non-conserved matter EMT which shows that test particles deviates from geodesic motion [16]. In [17], Harko generalized this non-minimal coupling by introducing a function of matter Lagrangian. Later Wu [18] further extended this work by studying few forms of curvature components and forming the thermodynamic laws. Harko along with the contributions of Lobo [19] proposed another induced form of $f(R)$ by involving curvature matter coupling incorporating matter langrangian $\mathcal{L}_m$ and defined generic function $f(R, \mathcal{L}_m)$. In [20], Sharif and Zubair discussed the non-equilibrium thermodynamics in $f(R, \mathcal{L}_m)$ gravity, and develop constraints on two specific gravitational models $f(R, \mathcal{L}_m) = \lambda \exp\left(\frac{1}{2\lambda}R + \frac{1}{\lambda}\mathcal{L}_m\right)$ and $f(R, \mathcal{L}_m) = \alpha R + \beta R^2 + \gamma \mathcal{L}_m$ to secure the validity of GSLT in this theory. In [21], authors presented the torsion-matter coupling and inclusion of boundary term to discuss different cosmic issues.

The selection of matter Lagrangian density has an issue in modified theories, specifically for those which involve non-minimal coupling with matter Lagrangian. For the natural conservation of matter we are restricted to take matter Lagrangian as $\mathcal{L}_m = p$ then extra force will be vanished [22] or for the sake of effective non-minimal coupling we can also take $\mathcal{L}_m = -\rho$ [15]. Alternative method to modify the Lagrangian of Einstein's equations is to take the function which depends on trace T of the EMT [23], so ΛCDM model can be taken of the form $R + 2\Lambda(T)$. Finally, by using this idea Harko et al. [24] proposed the extension of $f(R)$ gravity by replacing the function f with the new dependent parameters R and T and non-minimal coupling of geometry and matter allowed in astonishing theory known as $f(R,T)$ gravity. The coinciding with constructive geometry matter coupling shows the deviation of test particles from geodesic motion which ruled to additional force as proposed in different theories [14, 17, 22, 24].

Due to remarkable growing interest in this theory the efficacy of thermodynamics laws in $f(R,T)$ theory have been discussed by Sharif and Zubair [25] and it is concluded that the equilibrium picture of thermodynamics cannot be achieved due to matter geometry interaction. Attempts to reconstruct $f(R,T)$ Lagrangian has also been made under various considerations like the family of holographic DE models by supposing the FLRW universe [26], considering an auxiliary scalar field [27] and anisotropic solutions [28]. Jamil et al. [29] worked on the reconstruction of cosmological models and they showed that the dust fluid reproduce Einstein static universe, ΛCDM and de sitter Universe. Alvarenga et al. [30] discussed the development of matter density perturbations in this theory and they presented the required constraints to get the standard continuity equation in $f(R,T)$ theory. Other way, Sharif and Zubair [1] reconstructed cosmological models by applying additional constraints for the conserved EMT and studied the stability of the constructed models. Furthermore, the dynamical systems in $f(R,T)$ gravity were explored by Shabani and Farhoudi [31] that resulted in the development of a vast scale of considerable cosmological solutions. Other cosmic issues including compact stars, wormholes and gravitational instability of collapsing stars have been discussed in literature [32].

Lately, the non-minimal coupling of the EMT and Ricci tensor is introduced, resulting in the modified yet more complicated theory known as $f(R,T,Q)$ gravity [33, 34]. Due to complicated non-minimal matter-geometry coupling EMT is generally non-conserved and additional force is there. Therefore, it proposes a vast range to explore different cosmic features as thermodynamics properties have already been studied by Sharif and Zubair [35] and they [36] also discussed the energy conditions for particular models of $f(R,T,Q)$ gravity. They found that the non-minimal coupling becomes the reason of the deviation of test particles from geodesic motion and that gives strength to the non-equilibrium representation of thermodynamics. This induced the idea that the efficacy of GSLT in an expanding universe might lead to the thermal equilibrium in future. E. H. Baffou et al. [37] studied the stability of de-sitter and power law solution by using perturbation scheme for particular models.

In this thesis, we are motivated to discus the cosmological evolution in $f(R,T,Q)$ theory, which is based on more general matter-geometry coupling. We pick two particular model of the form $f(R,T,Q) = R + \alpha Q + \beta T$ and $f(R,T,Q) = R(1+\alpha Q)$. We solve the matter conservation equation to find the explicit expression of energy density. Evolution of EoS parameter ω_{eff} and deceleration parameter is discussed employing the power law cosmology. We also discuss CM within $f(R,T)$ theory and discuss the deceleration parameter by considering CM plus radiations. This thesis is organized as follows: In chapter **II**, we defined the all important definitions related to this work. In chapter **III**, we discuss the evolution of two significant models in $f(R,T,Q)$ gravity and present the expressions for ρ_{eff}, p_{eff} and ω_{eff}. The model parameters are constrained using the energy bounds. Chapter **IV** is devoted to CM in $f(R,T)$ theory, we discuss the significant models of $f(R,T)$ theory by considering CM+radiations and compare their results with NCM and ΛCDM model. Chapter **V**, summarizes the important results and comparison with observational data.

Chapter 2

Preliminaries

In this chapter, we will explain the basic definitions which are necessary to understand this thesis.

2.1 The Big Bang Theory

In physical cosmology, most of the cosmologists believe that the universe has a beginning and the universe was infinitely dense when time was zero. Although, many people believe that universe has no beginning and no end but in 1929, Hubble [38] observed that cosmos is expanding, and galaxies are moving away from each other. He also suggested that if we go in past almost 10 to 20 million years ago then the cosmos has infinite density which means that there was a singularity. Hawking and Penrose [39] were also believer of the big bang cosmology. They proved in their theorem that the time and universe had a beginning in terms of big bang explosion. The big bang theory predicts that at the beginning, the universe would have been infinitely hot and dense. At the big bang singularity, GR and all other physical laws would have been broken. There was no one to predict what would come out of the singularity. We cannot predict the events before the big bang since we have no observational consequences. As universe expands, temperature of universe decreases and this was started just after the big bang.

2.1.1 Inflation

The inflationary era was started just after 10^{-36} seconds of the big bang singularity and some believed that it was about 10^{-33} and 10^{-32} seconds after the big bang. After the big bang universe continues to expand but at less acceleration rate as compare to current acceleration rate. The basically inflationary theory is about exponential expansion of cosmos at the very beginning time of the universe. In other words, we can say that the era of repulsive gravity which is responsible for early expansion is known as inflation. Just after the repulsive gravity era, the era of attractive gravity began with the creation of those particles which can maintain there state of equilibrium at the highest temperature. As time passes, temperature of the cosmos decreases and eventually it reaches at the stage which is enough to create first nuclei of the cosmos. If we accept this theory then it gives strength to the hot big bang.

2.2 The Cosmological principle

In the cosmological principle, we assumes that universe is isotropic and homogenous. The key idea of this principle is that universe represents the same picture at any particular epoch in whichever direction we may look from whatever position. Copernicus observed that the Earth occupies a relatively unimportant position rather at the center of the universe. The same is true for the Sun which looks similar to all other stars. The cosmological principle is the modified form of the Copernican principle. This principle suggests that all the positions in the universe are equivalent, also the physical properties are position independent. This assumption implies that our universe is homogeneous. The second assumption is concerned with the equivalence of all spatial directions. The space around us looks to be isotropic. It is to be noted that the cosmological principle is not based on observational consequences but it was an assumption to apply the theory of GR on the structure of the universe. This assumption helped scientists (Friedmann, 1920; Lemaître, 1927) to construct the universe models since at that time there was no observational data to contradict it. Cosmological models are constructed following the idea that universe is filled with matter of the same average density and same average pressure, both of these quantities vary with time uniformly throughout the universe.

2.3 Cosmic Observations

The following are the different cosmic observations.

2.3.1 Type Ia Supernovae Observation

Perlmutter et al. [Supernova Cosmology Project (SCP)] [3] and Riess et al. [High-redshift Supernova Search Team (HSST)] [40] separately announced the late-time cosmic acceleration by observing distant supernovae of Type Ia (SNeIa). Till 1998, in the redshift range $(z = 0.18 - 0.83)$ Perlmutter et al. had discovered the 42 supernovae and Riess et al. had also found 16 high-redshift SNeIa together with 34 nearby supernovae. The history of supernovae is extremely bright and cause a blast of radiations. The supernovae can be restricted by the absorption lines of chemical elements. The spectral line of hydrogen element is restricted by Type II. Other than spectral line of hydrogen it is restricted by Type I. If a supernovae have a absorption line of only ionized silicon then it is restricted by Type Ia. If a supernovae have a absorption line of helium then it is restricted by Type Ib. Type Ic supernovae has a absorbtion line of both silicon and helium. When the mass of the white dwarf in a binary system overtake the Chandrasekhar limit [41] by absorbing gas from other star then explosion of Type Ia developed. That's why, the peak of brightness occur when the absolute luminosity of Type Ia is almost constant, the distance to a SNeIa can be calculated by its apparent luminosity. In this manner, the SNeIa plays the role of standard candle and we can measure the luminosity distances practically.

2.3.2 Cosmic Microwave Background Radiation

The temperature of the universe would have fallen about ten thousand million degrees just one second after the big bang. At that time, universe would have contained particles like photons, electrons and neutrinos (the particles which are affected by the weak force and gravity) and their antiparticles, also protons and neutrons. As the universe cooled down to about one billion degrees, protons and neutrons would have begun combining to form the nuclei of helium, hydrogen and other light particles. After that atoms would have formed due to the combination of electrons and nuclei [39]. Gamow investigated the nuclear fusion process, i.e., formation of nuclei together with neutrons and protons. In 1948, Gamow and his student Alpher presented the picture of dense and hot earlier universe. Alpher and Hamer made prediction that radiation of the early era still exist at present. In 1965, Penzias and Wilson confirmed their prediction, as they observed cosmic microwave background radiation. In 1992, a satellite named Cosmic Background Explorer (COBE) measured the spectrum of the CMB radiation and detected slight fluctuations of the temperature of CMB [42]. The results from WMAP [43] reveals that temperature variations in CMB follow a distinctive pattern predicted by cosmological theory. The anisotropies in the CMB are confirmed from the recent observations and this behavior is helpful to tell us about the present and past history of the universe. The discovery of CMB was the strongest evidence in the favor of hot big bang.

2.3.3 Planck's Observation

Planck's high-precision cosmic microwave background map [7] has offers scientists to select the most clear value of the cosmos ingredients. The simple matter forms galaxies and stars is just contribute 4.9% of the cosmos. DM which is marked indirectly by its gravitational impact on closely mater contributes 26.8% of the cosmos and the remaining part of the cosmos is 68.3% which is responsible of accelerated expansion of cosmos known as DE.

2.4 Dark Matter

DM is a matter in clusters, galaxies and possibly between clusters, that cannot be observed directly but that can be detected by its gravitational effects [39]. In 1933, Zwicky was interested to measure the mass of coma cluster. He showed that velocity dispersion of galaxies in cluster exceeds the expected for a gravitationally bound system. Zwicky postulated the presence of missing mass. WMAP data reveals that total density of our universe contains 23% of DM. Cosmologists postulate that DM is present in the outer regions of galaxies but its nature is still unknown. Usually DM was considered as ordinary matter in some undetectable form like gas clouds or MACHOs like neutron stars, black holes and white dwarfs. Recent observations show that presence of DM may be due to masses of light elementary particles such as neutrinos or axions. It may consist of exotic particles such as WIMPs and yet such particles are to be detected experimentally [39].

2.5 Dark Energy

In cosmology, DE is mysterious type of energy which is supposed to fill all of the space, reasoning to accelerate the cosmic expansion. DE is a mysterious force which is responsible for driving the galaxies away from each other against the force of gravity. The evidence of DE has not been detected directly. It appeared as the anti-gravity force whose properties are still unknown. The recognition that DE appears to exist has completely altered the landscape of theoretical physics and driving a host of astrophysicists to launch new cosmic probes to detect its nature. Significant number of papers have been written on this subject, while it is still a debate to understand much about the nature of DE.

2.5.1 Quintessence

Wetterich [44], Caldwell [45], and Ratra and Peebles [46], introduced the quintessence to solve the fine tuning problem of cosmological constant Λ at present era. Evolution of quintessence model is depends on its potential which is described by scalar field. This is a dynamical scalar field which can explain the role of DE in an accelerating universe. The cosmological constant is attributed to the vacuum energy with constant energy density ρ, pressure p and an EoS $\omega = -1$ whereas quintessence is a time varying inhomogeneous field with an EoS $\omega > -1$ [44]-[48]. In quintessence, DE dominates the cosmic acceleration of the universe for future evolution. The dominance of quintessence field increases with the increase in ω.

2.5.2 Phantom

Phantom energy is a hypothetical form of DE with EoS $\omega < -1$. The difference between the DE with $\omega > -1$ and $\omega < -1$ becomes apparent if we consider the expansion of the universe. Phantom energy [8] violates the dominant energy condition [49] that might result in the existence of wormholes. For phantom energy, the energy density emerge and becomes unbounded in a bounded time. Phantom energy increases the gravitational repulsion that will destroy the galaxies and then any bound system including elementary particles [8, 50]. Expansion factor of the universe dominated by the phantom energy diverges in a finite time to approach the future singularity [50, 51]. This situation is also termed as cosmic doomsday when all the objects, from galaxies to nucleons will be ripped apart. According to Baushev [52], phantom energy is not enough to produce big rip because ω does not seem to be constant throughout the evolution of the universe.

2.5.3 Quintom

Quintom is a unified DE model with EoS parameter getting across the cosmological constant boundary $\omega = -1$ from either side. Feng et al. [53] considered the effects of cosmic age and Supernovae Ia limits on the variation of the EoS parameter ω. They found that age limits can lower the variation of amplitude on the EoS parameter. Current Supernovae Ia data favors the transition of ω from quintessence to phantom. The quintom model predicts some interesting features related to the evolution and fate of the universe. In quintom scenario, the universe would avoid the singularities such as big bang, big rip [54, 55].

2.6 The Cosmological Constant

Einstein involved the Λ in the field equations to obtain a static and finite cosmological solution [56]. He thought that universe is positively curved and attractive gravity is balanced by repulsive gravity of Λ. However, the cosmological constant was neglected with the discovery of expansion of the cosmos. The simplest form of DE is the energy associated with the vacuum, i.e., the cosmological constant which has EoS $\omega = -1$. The matter is responsible to slow down the expansion but DE is playing the role of opponent to the matter and accelerate the cosmic expansion. In 1983, Linde presented a model of the expanding universe known as chaotic inflationary model. He suggested that there is no phase transition and cooling. Quantum theory implies that spacetime is filled with quantum fluctuations. According to spectrometric theory, the infinite positive and negative energies of the ground states would cancel out between particles of different spin. But all these energies would not cancel out because quantum fluctuations would have large values in some regions of the early universe. The vacuum energy of those regions would behave like cosmological constant and hence would expand in an inflationary manner due to the repulsive gravitational effect of vacuum energy [39].

2.7 ΛCDM Model

The first type of explanation accommodates this acceleration with GR by invoking DE as a strange cosmic fluid having large negative pressure [57]. The EoS relating pressure and energy density of this fluid is defined as $p_{DE} = \omega_{DE}\rho_{DE}$ where $\omega_{DE} < \frac{-1}{3}$ is necessitated in the EFE to provide comic acceleration. For the particular case $\omega_{DE} = -1$, this fluid behaves precisely as a cosmological constant Λ. Within this approach, the so-called concordance ΛCDM model is best fitted to the recent observational data by WMAP [43]. In the scenario of ΛCDM, the universe is spatially flat, dominated by DE (cosmological constant or vacuum energy) and cold dark matter which is made up of weakly interacting particles yet to be discovered. Except this, the curious ΛCDM consists of CMB particles (photons and neutrino) and baryonic matter (protons and nuclei plus electrons) making up only 4% of the total energy. It is based on the following two assumptions:

- GR is a correct theory of gravity at all scales,
- Universe is homogeneous and isotropic.

Although ΛCDM is in good agreement with current observations, however, problems related to a pure cosmological constant make it an unappealing solution from theoretical viewpoint.

2.8 Cosmological parameters

The following are the significant cosmological parameters.

2.8.1 Hubble parameter

The Hubble parameter H the most important quantity in cosmology as it is used to guess the age and the size of the cosmos. It represents the rate at which the cosmos is expanding. Although the Hubble parameter changes with time and given as

$$H = \frac{\dot{a(t)}}{a(t)}, \tag{2.1}$$

where dot represents the derivative with respect to time. $a(t)$ would be zero at the time of big bang and it would be increasing when we consider expanding universe.

2.8.2 Deceleration parameter

Deceleration parameter q is one of the classical parameters in cosmology. The point is that if we can measure the change in the Hubble parameter then we have important information to explain the nature and fate of the cosmos. The deceleration parameter "q" describes the change of rate at which the cosmic expansion is going to slow down because of self-gravitation. It is stated as

$$q = -\frac{\ddot{a}a}{\dot{a}^2}, \tag{2.2}$$

herein, above equation $a(t)$ indicates scale factor. In this context, $\frac{\dot{a}}{a}$ is the Hubble parameter denoted by H, and its present value is H_0, the Hubble constant. In accordance to the latest examinations, the expansion rate of cosmos is currently accelerating, it is because of the effects of DE. This yields negative values for the deceleration parameter. Deceleration parameter is also described in terms of Hubble parameter as

$$q = \frac{d}{dt}(\frac{1}{H}) - 1. \tag{2.3}$$

2.8.3 Equation of State

The EoS in cosmology described by dimensionless number ω as, $\omega = \frac{p}{\rho}$ where p stands for pressure and ρ for energy density. If we evaluate the EoS by considering that universe is isotropic, homogeneous, and taking the FLRW space-time at the background then EoS for ω_{DE} is exactly equal to -1 in ΛCDM model whereas in quintessence model ω_{DE} is dynamical quantity and $-1 < \omega_{DE} < \frac{-1}{3}$. Moreover ω_{DE} varies with time and $\omega_{DE} < -1$ in phantom model.

2.9 The Energy Momentum Tensor

In GR, the EMT $T^{\mu\nu}$ represents the source of the gravitational field. The relation of Einstein tensor and EMT is given as $G^{\mu\nu} = 8\pi G T^{\mu\nu}$ The components of the EMT can be written into a matrix with the property that $T^{\mu\nu} = T^{\nu\mu}$. For arbitrary manifold, the EMT is defined as

$$T^{\mu\nu} = \rho u^{\mu} u^{\nu} + \sigma^{lm} \delta^{\mu}_{l} \delta^{\nu}_{m}, \tag{2.4}$$

where u^{μ} is for four velocity, ρ is for matter density and σ^{lm} is the stress density given as

$$\sigma^{lm} = \frac{dF^{l}}{dS_{m}}, \qquad (l, m = 1, 2, 3) \tag{2.5}$$

Here dF^{l} is representing the force acting on the area element dS_{m}.

The different components of EMT describe the following meanings

- Energy density ρ is represented by T_{00} component of EMT.
- Energy flow across the surface x^{i} is represented by T_{0i} component of EMT.
- Flow of momentum across the surface is represented by T_{i0} component of EMT.
- Stress is represented by spatial components T_{ij} of EMT.

2.9.1 Perfect fluid

A fluid which shows no heat conduction and viscosity is known as perfect fluid. These type of fluid are only described by their pressure p and mass density ρ. The EMT that describes a perfect fluid in the local frame is for signature $(-,+,+,+)$

$$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} + pg^{\mu\nu}, \tag{2.6}$$

if we use this signature $(+,-,-,-)$ then EMT for perfect fluid takes the following form

$$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} - pg^{\mu\nu}, \tag{2.7}$$

if we will take $p = 0$, then EMT represents the only dust case of the universe and EMT takes the following form

$$T^{\mu\nu} = \rho u^{\mu}u^{\nu}. \tag{2.8}$$

The components of EMT for perfect fluid in the local frame can be written as

$$T^{\mu\nu} = diag[\rho, p, p, p]. \tag{2.9}$$

2.10 Non-Minimal coupling

In a simple wording, we can differentiate minimal and non-minimal coupling by saying that minimal coupling is a weak coupling and non-minimal coupling is a strong coupling. If we discuss mathematically then, we know that coupling is based on different mathematical models, if we choose the model in terms of addition then it is known as minimal coupling like, following are the examples of minimal coupling,

- $f(R,T) = R + \alpha T$
- $f(R,T) = f(R,T) = \beta_1 R^{\mu} + \beta_2 R^{\nu} + \frac{2k^2}{-1+3\omega}T + \beta_3 T^{\frac{1}{2} - \frac{\sqrt{-1+\omega(3+\omega-3\omega^2)}}{(1+\omega)\sqrt{-2+6\omega}}}$

If we choose the models in term of product then coupling is known as non-minimal coupling like, following are the examples of non-minimal coupling

- $f(R,T) = R(1 + \alpha T^n)$
- $f(R,T,R_{\mu\nu}T^{\mu\nu}) = R + \alpha R_{\mu\nu}T^{\mu\nu} + \beta T.$

2.11 FLRW Metric

To arrive at the form of the FLRW metric, we first choose the time coordinate so that space-time slices of fixed t are homogeneous and isotropic. In other words, physical conditions on each slice are the same at every position, and in every direction. We choose the threading to be orthogonal to the slicing, corresponding to $g_{0i} = 0$. Isotropy requires that an observer moving with the threading measures zero velocity for the cosmic fluid, or in other words zero momentum density. The threading is therefore comoving (moving with the fluid flow). Homogeneity demands that the proper time interval between slices is independent of position, which means that we can choose t as proper time corresponding to $g_{00} = -1$. Homogeneity and isotropy require that the distance between nearby threads is proportional to a universal scale factor $a(t)$. Putting all this together, the FLRW line element takes the form

$$ds^2 = -dt^2 + a^2(t)g_{ij}(x_1, x_2, x_3)dx_i dx_j \tag{2.10}$$

The Universe is observed to be expanding, corresponding to $a(t)$ increasing with time.

2.12 Conservation Equation

The conservation of energy and momentum is the important limitations on the RHS of the EFE in the form of EMT, which can be expressed by the relation

$$\nabla_\mu T^{\nu\mu} = 0 \tag{2.11}$$

This constraint means that $R_{uv} \propto T_{\mu\nu}$ is not true because $\nabla_\mu R^{\nu\mu} \neq 0$. By using contracted Bianchi identities $\nabla_\mu T^{\nu\mu} = \frac{1}{2} g^{\mu\nu} \nabla_\mu R$, we can use the Einstein tensor on the LHS, this will satisfy the conservation of energy and momentum. That is, we set

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \tag{2.12}$$

and then finally arrive at the following EFE

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \tag{2.13}$$

where $\kappa = 8\pi G$.

2.13 Einstein Field Equations

The EFE $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ describe the relation between space-time geometry and matter content. We can also write EFE as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{2.14}$$

The LHS of this equation represent geometry and RHS corresponds the matter distribution. The suppressed idea is that the energy matter distribution tells us that how space time is curved and how gravity plays its role. Therefore, if we apply any condition on $T_{\mu\nu}$ then it will be immediately referred to the conditions on Einstein Tensor $G_{\mu\nu}$ [49]. Matter energy distribution is responsible for casual and geodesic structure of space-time. For this purpose energy conditions ensure that the casuality principle is appreciated and acceptable physical sources have to be studied [49, 58]. In GR, the metric tensor $g_{\mu\nu}$ plays the same role as does the scalar potential ϕ in Newtonian theory of gravitation. The EFEs reduce to Poisson's equations in the weak field approximation (Newtonian limit). These are the second order partial as well as non-linear differential equations for the metric components [58].

In GR, we solve the field equations simultaneously for the space-time metric and matter distribution. In particular, these equations for the vacuum solutions are obtained by choosing $T_{\mu\nu} = 0$. The matter distribution in these equations satisfies the principle of local conservation of energy and momentum, i.e. $T_{\mu\nu;\mu} = 0$. This equation gives information about the behavior of matter [58].

2.14 Modified Theories of gravity

After Edwin Hubble's theory of expanding cosmos, latest examinations from Supernovae Type Ia and CMBR [3], have confirmed the phenomena of accelerated expanding cosmos. Although, GR is the most elementary theory that is compatible with experimental data of gravity but still it leaves various eras like DE and DM to be explained, which led a way to the alternative theories of gravity. The modified theories are really helpful to explain the reasonable cosmic expansion history. MTG being a large distance modifications of GR are reconstructed by adding an extra degree of freedom that may be scalar, vector or tensor field. There are several approaches to extend Einstein formulation of gravity, e.g., by the inclusion of spin-0 particles, i.e., scalar field, higher order terms of curvature and non-Christoffel connections in its gravitational sector. A successful modification of GR is the one for which the predicted measurements of solar system tests do not deviate much from the corresponding estimations of GR. In 1980, $f(R)$ gravity was introduced by Starobinsky [59]. The theory of

$f(R)$ gravity is famous because of cosmological importance of its models. $f(R)$ gravity can be directly found by varying the Ricci scalar R by the function $f(R)$ in the Einstein-Hilbert action and its defined as

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}f(R) + \int d^4x \pounds_M(g_{\phi\varphi}, \Psi_M), \tag{2.15}$$

where $f(R)$ is the function of Ricci scalar R, $\pounds_M$ denotes matter Lagrangian, $g_{\phi\varphi}$ is the metric and Ψ_M represents the matter field.

2.15 $f(R,T)$ Gravity

In the presence of ordinary matter, we will discuss the formulation of $f(R,T)$ gravity for FLRW space-time as well as filed equation. We have the following action of this gravity [24]

$$S = \frac{1}{\kappa^2}\int f(R,T)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x, \tag{2.16}$$

where R is Ricci Scalar and T is the trace of EMT $T = g^{\mu\nu}T_{\mu\nu}$. Here matter lagrangian represented by $\mathcal{L}_m$. we have the following set of equations by varying the action (2.16)

$$8\pi T_{\mu\nu} - f_T T_{\mu\nu} - f_T\Theta_{\mu\nu} = f_R R_{\mu\nu} - \frac{1}{2}fg_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\nu\nabla_\mu)f_R. \tag{2.17}$$

We have a following relation between Ricci scalar R and the trace T of the EMT by contraction of above equation

$$8\pi T - f_T T - f_T\Theta = f_R R + 3\Box f_R - 2f. \tag{2.18}$$

The ∇ and $\Box$ stands for the covariant derivative and d'Alembert operator respectively in the above equation. Furthermore, f_R and f_T represents the function derivatives with respect to R and T, respectively. The term $\Theta_{\mu\nu}$ is defined by

$$\Theta_{\mu\nu} = \frac{g^{\alpha\beta}\delta T_{\mu\nu}}{\delta g^{\mu\nu}} = -2T_{\mu\nu} + g_{\mu\nu}L_m - 2g^{\alpha\beta}\frac{\partial^2 L_m}{\partial g^{\mu\nu}\partial g^{\alpha\beta}}.$$

Here, we choose $\mathcal{L}_m = -P_m$, which gives us the following expression for $\Theta_{\mu\nu}$

$$\Theta_{\mu\nu} = -2T_{\mu\nu} - P_m g_{\mu\nu}. \tag{2.19}$$

With the help of equation (2.19), the field equation (2.17) can be converted in the following form

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = k^2_{eff}T^{eff}_{\mu\nu}, \tag{2.20}$$

with $k^2_{eff} = \frac{k^2+f_T}{f_R}$ is the effective gravitational constant and

$$T^{eff}_{\mu\nu} = T_{\mu\nu} + \frac{1}{k^2+f_T}\left[\frac{1}{2}g_{\mu\nu}(f - Rf_R) + f_T P_m g_{\mu\nu} - (g_{\mu\nu} - \nabla_\mu\nabla_\nu)f_R\right] \tag{2.21}$$

is the effective EMT. In this study, we consider the flat FLRW geometry described by the metric

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2,$$

where $a(t)$ is the scale factor and $d\mathbf{x}^2$ comprises the spatial part of the metric.

$$R = -6(2H^2 + \dot{H}),$$

where H stands for Hubble parameter and dot for the differentiation with respect to time t.

2.16 $f(R, T, R_{\mu\nu}T^{\mu\nu})$ Gravity

The $f(R, T, Q)$ theory is the most generalized theory among other modified theories like $f(R)$ and $f(R, T)$ and this theory is very effective for non-minimal coupling of geometry and matter. The action of this complicated theory takes the following form [33, 34]

$$\mathcal{A} = \frac{1}{2\kappa^2}\int dx^4\sqrt{-g}\,[f(R, T, R_{\mu\nu}T^{\mu\nu}) + \mathcal{L}_m], \tag{2.22}$$

where $\kappa^2 = 8\pi G$, $f(R, T, Q)$ is a general function which depends on three components, the Ricci scalar R, trace of the EMT T, product of the EMT $T^{\mu\nu}$ to Ricci tensor $R_{\mu\nu}$, and $\mathcal{L}_m$ shows the matter Lagrangian. The EMT for matter is defined as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}. \tag{2.23}$$

If the dependence of the matter action only on metric tensor then the EMT yields

$$T_{\mu\nu} = g_{\mu\nu}\mathcal{L}_m - \frac{2\partial\mathcal{L}_m}{\partial g^{\mu\nu}}. \tag{2.24}$$

The field equations in $f(R,T,Q)$ theory can be found by varying the action (2.22) with respect to $g_{\mu\nu}$ as

$$R_{\mu\nu}f_R - \{\frac{1}{2}f - \mathcal{L}_m f_T - \frac{1}{2}\nabla_\alpha\nabla_\beta(f_Q T^{\alpha\beta})\}g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_R + \frac{1}{2}$$
$$\Box(f_Q T_{\mu\nu}) + 2f_Q R_{\alpha(\mu}T^\alpha_{\nu)} - \nabla_\alpha\nabla_{(\mu}[T^\alpha_{\nu)}f_Q] - G_{\mu\nu}\mathcal{L}_m f_Q - 2(f_T g^{\alpha\beta} + f_Q$$
$$R^{\alpha\beta})\frac{\partial^2\mathcal{L}_m}{\partial g^{\mu\nu}\partial g^{\alpha\beta}} = (1 + f_T + \frac{1}{2}Rf_Q)T_{\mu\nu}. \tag{2.25}$$

The subscripts shows the derivatives with respect to R, T, Q, and box function defined as $\Box = \nabla^\beta\nabla_\beta$, ∇_μ represent covariant derivative. If we will choose the particular form of Lagrangian then Equation (2.25) can be shifted towards the well known field equations in $f(R)$ and $f(R,T)$ theories. The field equation (2.25) can be rewritten into the form of effective EFE as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T^{eff}_{\mu\nu}. \tag{2.26}$$

This effective form of EFE is similar to GR's standard field equations. Here $T^{eff}_{\mu\nu}$, the effective EMT in $f(R,T,Q)$ theory is found to be as

$$T^{eff}_{\mu\nu} = \frac{1}{f_R - \mathcal{L}_m f_Q}[T_{\mu\nu}(\frac{1}{2}f_Q R + 1 + f_T) + g_{\mu\nu}\{\frac{1}{2}(f - f_R R) - f_T\mathcal{L}_m\frac{1}{2}\nabla_\alpha$$
$$\nabla_\beta(f_Q T^{\alpha\beta})\} - (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_R - \frac{1}{2}\Box(T_{\mu\nu}f_Q)2f_Q R_{\alpha(\mu}T^\alpha_{\nu)} + \nabla_\alpha$$
$$\nabla_{(\mu}[T^\alpha_{\nu)}f_Q] + 2(g^{\alpha\beta}f_T + R^{\alpha\beta}f_Q)\frac{\partial^2\mathcal{L}_m}{\partial g^{\mu\nu}\partial g^{\alpha\beta}}]. \tag{2.27}$$

Applying the covariant divergence to the field equation (2.25), we get

$$\nabla^\mu T_{\mu\nu} = \frac{2}{2(1 + f_T) + Rf_Q}[\nabla_\mu(f_Q R^{\alpha\mu}T_{\alpha\nu}) + \nabla_\nu(\mathcal{L}_m f_T) - \frac{1}{2}(f_Q R_{\sigma\zeta}f_T$$
$$g_{\sigma\zeta})\nabla_\nu T^{\sigma\zeta} - G_{\mu\nu}\nabla^\mu(f_Q\mathcal{L}_m) - \frac{1}{2}[\nabla^\mu(Rf_Q) + 2\nabla^\mu f_T]T_{\mu\nu}]. \tag{2.28}$$

It is important to see that any modified theory which involve non-minimal coupling between geometry and matter does not obey the ideal continuity equation. This complicated theory $f(R,T,Q)$ also involves this type of non-minimal coupling so it also deviate from standard behavior of continuity equation. Here, non-minimal coupling between geometry and matter produce extra force acting on massive particles, whose equation of motion is given by [34]

$$\frac{d^2x^\lambda}{ds^2}+\Gamma^\lambda_{\mu\nu}u^\mu u^\nu=f^\lambda,$$

where

$$f^\lambda=\frac{h^{\lambda\nu}}{(\rho+p)(1+2f_T+Rf_{RT})}\Big[(f_T+Rf_{RT})\nabla_\nu\rho-(1+3f_T)\nabla_\nu p-\\(\rho+p)f_{RT}R^{\sigma\rho}(\nabla_\nu h_{\sigma\rho}-2\nabla_\rho h_{\sigma\nu})-f_{RT}R_{\sigma\rho}h^{\sigma\rho}\nabla_\nu(\rho+p)\Big]. \tag{2.29}$$

It has been found that the impact of non-minimal coupling is always present independent of the choice matter Lagrangian, the extra force does not vanish even with the Lagrangian $\mathcal{L}_m=p$ as compared to the results presented in [14, 60]. In [34], authors also presented the Lagrange multiplier approach and found the conservation of matter EMT. Moreover, if one eliminates the dependence of Q, it results in divergence equation of $f(R,T)$ theory as given below

$$\nabla^\alpha T_{\alpha\beta}=\frac{f_T}{1-f_T}\left[(\Theta_{\alpha\beta}+T_{\alpha\beta})\nabla^\alpha lnf_T-\frac{1}{2}g_{\alpha\beta}\nabla^\alpha T+\nabla^\alpha\Theta_{\alpha\beta}\right].$$

In [30], Alvarenga et al. shown that choice of a specific model within these theories can guarantee the conservation of EMT and continuity equation is valid for the model $f(R,T)=f_2(T)+f_1(R)$, where $f_2(T)=T^{\frac{1+3\omega}{2(1+\omega)}}\alpha+\beta$.

In flat FLRW background, and considering the perfect fluid ρ_{eff} and p_{eff} can be found as

$$\rho_{eff} = \frac{1}{f_R - \mathcal{L}_m f_Q}[\rho + f_T(\rho - \mathcal{L}_m) + \frac{1}{2}(f - f_R R) + \frac{3}{2}(\dot{H} - 3H^2) f_Q \rho$$
$$-3H\partial_t f_R - \frac{3}{2} p f_Q(3H^2 + \dot{H}) + \frac{3}{2} H \partial_t [f_Q(p - \rho)]],$$
$$p_{eff} = \frac{1}{f_R - \mathcal{L}_m f_Q}[p + f_T(p + \mathcal{L}_m) + \frac{1}{2}(f_R R - f) + \frac{1}{2}(3H^2 + \dot{H}) f_Q(-p$$
$$+\rho) + 2H\partial_t f_R + \partial_{tt} f_R + \frac{1}{2}\partial_{tt}[f_Q(-p + \rho)] + 2H\partial_t[f_Q(p + \rho)]], \tag{2.30}$$

where $R = -6(2H^2 + \dot{H})$ and upper dot for the time derivative. Here, we ignored those terms which involved the second derivative of matter Lagrangian with respect to $g_{\mu\nu}$. In the case of perfect fluid the matter Lagrangian can either be $\mathcal{L}_m = \rho$ or $\mathcal{L}_m = -p$.

Chapter 3

Cosmic evolution in the background of non-minimal coupling in $f(R, T, R_{\mu\nu}T^{\mu\nu})$ Gravity

We are interested to explore the cosmic evolution using matter conservation equation of more generic modified theory. We choose two significant models, first is of the form $f(R,T,Q) = R + \alpha Q + \beta T$ and second is $f(R,T,Q) = R(1+\alpha Q)$. We formulate the dynamical equations for both two models in the background of FLRW space-time and find the result of non-conserved EMT using the divergence of the field equations. Furthermore, we discuss the energy bound on the model parameters and discuss the late time cosmic acceleration for best suitable parameters in accordance with recent observations.

3.1 $f(R,T,Q) = R + \alpha Q + \beta T$

Here, we will set $\mathcal{L}_m = \rho$ and we will take the simplest model $f(R,T,Q) = R + \alpha Q + \beta T$ where α, β are coupling parameters. In this model, choice of $\alpha = 0$ results in minimal coupling of the form $f(R,T) = R + \beta T$ [24], such model has been widely studied in the formalism of $f(R,T)$ gravity (for review see [32]). Moreover, the choice of $\alpha = \beta = 0$, results in Eisntein's formalism of GR.

For a flat FLRW universe, the non-zero components of FLRW equation for $p_{eff} = p + p_{DE}$ and $\rho_{eff} = \rho + \rho_{DE}$ are

$$3H^2 = \rho_{eff},$$
$$-2\dot{H} - 3H^2 = P_{eff}, \tag{3.1}$$

where dots being time derivative and components of ρ_{DE} and p_{DE} are given as follows

$$\rho_{DE} = \frac{1}{2\alpha\rho - 2}[3\beta p - (\beta - 12\alpha H^2)\rho - 2\alpha\rho^2 + 3\alpha H(\dot{\rho} - \dot{p})],$$
$$p_{DE} = \frac{1}{2\alpha\rho - 2}[-\rho(\beta + 6\alpha H^2 + 4\alpha\dot{H}) + p(-5\beta + 12\alpha H^2 - 2\alpha\rho + 4\alpha$$
$$\dot{H}) - \alpha(4H(\dot{p} + \dot{\rho}) - \ddot{p} + \ddot{\rho})], \tag{3.2}$$

and effective EoS ω_{eff} is

$$\omega_{eff} = [-\rho(\beta + 6\alpha H^2 + 4\alpha\dot{H}) + p(-2 - 5\beta + 4\alpha(3H^2 + \dot{H})) - \alpha(4H \\ (\dot{\rho} + \dot{p}) + \ddot{\rho} - \ddot{p})][3\beta p - \rho(2 + \beta - 12\alpha H^2) + 3\alpha H(-\dot{p} + \dot{\rho})]^{-1}. \tag{3.3}$$

The EoS of DE is, $\omega_{DE} = \frac{p_{DE}}{\rho_{DE}}$

$$\omega_{DE} = [-\rho(\beta + 6\alpha H^2 + 4\alpha\dot{H}) + p(-5\beta + 12\alpha H^2 - 2\alpha\rho + 4\alpha\dot{H}) - \alpha(4 \\ H(\dot{\rho} + \dot{p}) + \ddot{\rho} - \ddot{p})][3\beta p - \rho(\beta - 12\alpha H^2) - 2\alpha\rho^2 + 3\alpha H(-\dot{p} + \dot{\rho})]^{-1} \tag{3.4}$$

and conservation equation (2.28) takes the form

$$\dot{\rho} + 3H(\rho + p) = [-18\alpha H^3(\rho + p) - 6\alpha H(p + \rho)\dot{H} + 3\dot{p}(\beta - \alpha\dot{H}) + \dot{\rho}(\beta \\ -3\alpha\dot{H}) - 9\alpha H^2(\dot{\rho} + \dot{p})][2(1 + \beta - 6\alpha H^2 - 3\alpha\dot{H})]^{-1}. \tag{3.5}$$

Now the above equations are expressed in terms of redshift by using relation $a(t) = \frac{1}{1+z}$, where $\frac{d}{dt} = -(1+z)H\frac{d}{dz}$ whereas $p = p(z)$ and $\rho = \rho(z)$. Where prime is for derivative with respect to redshift parameter z.

$$3H(p + \rho) - (1+z)H\rho' = \frac{1}{2(1 + \beta + 3\alpha H(-2H + (1+z)H'))}[-(H \\ (1+z)\beta(3p' + \rho') + 9\alpha H^2(2p + 2\rho - (1+z)(p' + \rho')) + 3(1+z)\alpha H H' \\ (-2p - 2\rho + (1+z)(p' + \rho')))]. \tag{3.6}$$

The revolutionary field equation $G^{\nu}_{\mu} = 8\pi G T^{\nu}_{\mu}$ shows the connectedness of geometry of the fabric of space-time with matter content of cosmos, represented in GR. The LHS of the previously stated field equation show the Einstein tensor, which satisfy the Bianchi identities $\nabla_{\nu} G^{\nu}_{\mu} \equiv 0$ and RHS shows the EMT. If the covariance derivative of EMT is zero ($\nabla_{\mu} T^{\nu}_{\mu} = 0$) then it shows the conservation of matter in every part of the universe. EFE can be explored on different choices of metric g^{μ}_{ν} and EMT T^{μ}_{ν}. Although matter and geometry are on same footing but GR does not allow us to check the possible effects of non-minimal coupling between them. These limitations of GR vanished in recently developed theories like $f(R,T)$ and $f(R,T,Q)$ theories. In these theories EMT is not conserved ($\nabla_\mu T^\nu_\mu \neq 0$), we use this result to find the value of energy density. Such formation of energy density from the nonconserved EMT helps to study the role of non-minimal coupling in cosmic expansion. Before finding the value of $\rho(z)$ we should know the relation of $H(z)$. Many theoretical relations exists in literature which are observationally consistence. But here we will take the power law expansion in terms of red shift defined as $H(z) = H_0(1+z)^{\frac{1}{m}}$, where m is the power law exponent.

Power law cosmology appears as a good phenomenological explanation of the evolution of universe, it can explain the cosmic history including radiation era, the DM era and the accelerating DE dominated era. The evolution of the scale factor for the standard fluids provided by theses solutions such as dust matter case ($m = 2/3$) or radiation dominated eras ($m = 1/2$). Also, $m \gtrsim 1$ shows a late-time accelerating cosmos. It provides an interesting alternative to deal with the problems like (horizon, flatness and age problems) associated with the standard model. Evolution of power law model has been discussed in various articles [61], for instance it talks about the the flatness, age and horizon problems for the parametric value $m \geq 1$ [62]. These type of solutions are found to be consistent with various data sets including nucleosynthesis [63, 64], with the age of high-redshift objects such as globular clusters [63, 64], with the SNeIa data [62, 65], and with X-ray gas mass fraction measurements of galaxy clusters [66]. In the framework of power law cosmology, authors have discussed the angular size-redshift data of compact radio sources [67], the gravitational lensing statistics and SNeIa magnitude-redshift relation [64, 68].

In this scenario, energy density $\rho(z)$ is found by solving the Eq. (3.6) as

$$\rho(z) = e^{-\frac{(1+\omega)(6(1+\beta)\log(1+z)+\frac{m(-1+3m+\beta)(1+3\omega)\log[m(2+\beta-3\beta\omega)+3H_0^2(1+z)^{\frac{2}{m}}\alpha(1-\omega+m(-1+3\omega))]}{-1+m+\omega-3m\omega})}{-2+\beta(-1+3\omega)}} c \quad (3.7)$$

where c stands for constant of integration. As energy density is found to be in an exponential form so it will remain positive for all values of unknowns parameters like α, β, ω, m. It will only depend on constant of integration c when we take negative value of c then energy density will be negative or less than zero otherwise for all positive values of c energy density will remain positive. One can also get the relation between time and redshift as

$$t = \left(\frac{1}{1+z}\right)^{\frac{1}{m}}. \tag{3.8}$$

Using the value of $\rho(z)$, one can get ρ_{eff} and p_{eff} in terms of redshift as and we take $c = 10$ and $\omega = 1$

$$\rho_{eff} = \frac{10(6H_0^2 m(1+z)^{\frac{2}{m}}\alpha + m(2-2\beta))^{\frac{4(-1+3m+\beta)}{-2+2\beta}}(-1+6H_0^2(1+z)^{\frac{2}{m}}\alpha+\beta)}{-(1+z)^{\frac{12(1+\beta)}{-2+2\beta}} + 10\alpha(6H_0^2 m(1+z)^{\frac{2}{m}}\alpha + m(2-2\beta))^{\frac{4(-1+3m+\beta)}{-2+2\beta}}}, \tag{3.9}$$

$$p_{eff} = 2^{3+\frac{6m}{-1+\beta}} 5(m(1+3H_0^2(1+z)^{\frac{2}{m}}\alpha-\beta))^{1+\frac{6m}{-1+\beta}}(3H_0^4(-16+21m)$$
$$(1+z)^{\frac{4}{m}}\alpha^2 - 12H_0^2 m(1+z)^{\frac{2}{m}}\alpha(2+\beta) - m(-1+\beta)(1+3\beta))[(1+$$
$$z)^{\frac{6(1+\beta)}{-1+\beta}} - 10\alpha(6H_0^2 m(1+z)^{\frac{2}{m}}\alpha + m(2-2\beta))^{\frac{4(-1+3m+\beta)}{-2+2\beta}}]^{-1}, \tag{3.10}$$

and effective EoS in term of redshift can be written as

$$\omega_{eff} = [48H_0^4(1+z)^{\frac{4}{m}}\alpha^2 + m(-1-63H_0^4(1+z)^{\frac{4}{m}}\alpha^2 - 2\beta + 3\beta^2 + 12$$
$$H_0^2(1+z)^{\frac{2}{m}}\alpha(2+\beta))][m(1+3H_0^2(1+z)^{\frac{2}{m}}\alpha-\beta)(-1+6H_0^2(1+z)^{\frac{2}{m}}\alpha$$
$$+\beta)]^{-1}. \tag{3.11}$$

Cosmic acceleration can be defined through a dimensionless cosmological function known as the deceleration parameter q. Here, q is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{1}{m} - 1, \tag{3.12}$$

q characterizes the accelerating or decelerating behavior of cosmos, here, $q < 0$ describe an accelerating epoch, whereas $q > 0$ shows decelerating epoch. In

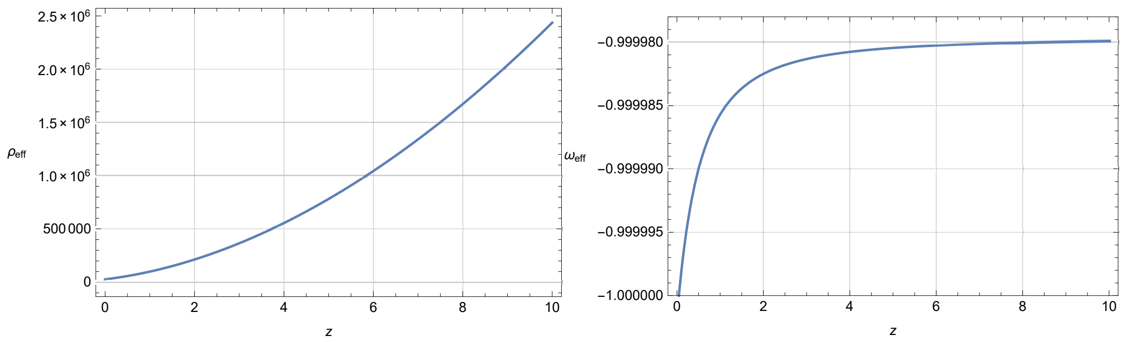

Figure 3.1: The LHS figure shows the evolution of ρ_{eff} whereas the RHS figure shows the behavior ω_{eff}. Herein, we set $\alpha = 10$, $\beta = -5$, $m = 1.066658$, and $H_0 = 67.3$ [7].

power law cosmology, we require $m > 1$ to restrict q as $q > -1$. Graphical representation of effective components ρ_{eff}, EoS ω_{eff} are shown in Fig. **3.1**. In this discussion, we choose the following values of unknown parameters $\alpha = 10$, $\beta = -5$, and $m = 1.066658$. For this value of m, deceleration parameter is -0.0624924 which favors the expanding behavior of cosmos. It can be seen that ρ_{eff} is positive and increasing function as shown on right plot and ω_{eff} approaches to -1 at $z = 0$ representing the ΛCDM epoch in accordance with recent observations from Planck's data [7].

3.2 Energy conditions for $f(R,T,Q) = R + \alpha Q + \beta T$ Gravity

The EFE $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ describe the relation between space-time geometry and matter content. The LHS of this equation represent geometry and RHS corresponds the matter distribution. The suppressed idea is that the energy matter distribution tells us that how space time is curved and how gravity plays his role. Therefore, if we apply any condition on $T_{\mu\nu}$ then it will be immediately referred to the conditions on Einstein Tensor $G_{\mu\nu}$ [49]. Matter energy distribution is responsible for casual and geodesic structure of space-time. For this purpose energy conditions ensure that the casuality principle is appreciated and acceptable physical sources have to be studied [49, 58]. The energy conditions are based on Raychaudhuri equations and can be taken from the expansion given by

$$\frac{d\theta}{d\tau} = -\frac{\theta^2}{2} - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}k^{\mu}k^{\nu} \tag{3.13}$$

where θ, $\sigma_{\mu\nu}$ and $\omega_{\mu\nu}$ shows expansion, shear and rotation respectively. These parameters are related to the congruence explained by the null vector field k^{μ}. The shear is a spatial tensor with $\sigma^2 \equiv \sigma_{\mu\nu}\sigma^{\mu\nu} \geq 0$, thus it is obvious from Raychaudhuri equations that for any hypersurface orthogonal congruences, which forces $\omega \equiv 0$, the condition for attractive gravity reduce to $R_{\mu\nu}k^{\mu}k^{\nu} \geq 0$. However, in GR, through the EFE we can write $T_{\mu\nu}k^{\mu}k^{\nu} \geq 0$. In the context of modified theory we used the effective EMT which is shown in equation (2.27) and positivity condition, $R_{\mu\nu}k^{\mu}k^{\nu} \geq 0$ in the Raychaudhuri equations gives the following form of NEC $T^{eff}_{\mu\nu}k^{\mu}k^{\nu} \geq 0$ and for ordinary matter we can also write $T^{mat}_{\mu\nu}k^{\mu}k^{\nu} \geq 0$. It is simple to prove that the previous conditions impose energy density positive in all local frame of references by using local lorentz transformation. Energy conditions describes the behavior of the similarity of lightlike, timelike and spacelike curves. It is generally used in GR to find and study the

$\rho_{eff} \geq 0$			$\rho_{eff} + p_{eff} \geq 0$		$\rho_{eff} + p_{eff} \geq 0$, $\rho_{eff} \geq 0$	
m	α	β	α	β	α	β
1.1		$1.1 \leq \beta \leq 270$ $-550 \leq \beta \leq -2.8$		$-1.3 \leq \beta \leq 0.9$	$0 \leq \alpha \leq 0.00000016$	$-0.64 \leq \beta \leq 0.11$
2	$\alpha \geq 1.01$	$1.1 \leq \beta \leq 1500$ $-3000 \leq \beta \leq -8$	$\alpha \geq 1.01$	$-2.9 \leq \beta \leq 0.9$	$0 \leq \alpha \leq 0.0000013$	$-0.58 \leq \beta \leq 0.29$
10		$1.1 \leq \beta \leq 9000$ $-17900 \leq \beta \leq -50$		$-17.8 \leq \beta \leq 0.9$	$0 \leq \alpha \leq 0.00001$	$-0.55 \leq \beta \leq 0.22$

Table 3.1: Validity ranges of parameters α and β for the model $R + \alpha Q + \beta T$.

singularities of space time. The energy conditions, NEC, WEC, SEC and DEC in terms of EMT are given by

$$\begin{aligned}
NEC\textbf{:} &\qquad \rho_{eff} + p_{eff} \geq 0, \\
WEC\textbf{:} &\qquad \rho_{eff} \geq 0, \;\; \rho_{eff} + p_{eff} \geq 0, \\
SEC\textbf{:} &\qquad \rho_{eff} + 3p_{eff} \geq 0, \;\; \rho_{eff} + p_{eff} \geq 0, \\
DEC\textbf{:} &\qquad \rho_{eff} \geq 0, \;\; \rho_{eff} \pm p_{eff} \geq 0.
\end{aligned} \tag{3.14}$$

Now, we will discus the energy conditions for our first model of $f(R, T, Q)$ gravity which is $R + \alpha Q + \beta T$ by considering FLRW metric.
WEC is found to be of the following form

$$\rho_{eff} = \rho + \frac{1}{2\alpha\rho - 2}[3\beta p - (\beta - 12\alpha H^2)\rho - 2\alpha\rho^2 + 3\alpha H(\rho' - p')], \tag{3.15}$$

NEC yields as

$$\begin{aligned}
\rho_{eff} + p_{eff} = &\frac{1}{2\alpha\rho - 2}[-2p(1 + \beta - 6\alpha H^2 + 2(1+z)\alpha H H') + \rho(-2 \\
&(1+\beta) + 6\alpha H^2 + 4(1+z)\alpha H H') + (1+z)\alpha H((1+z)H'(-\rho' + p') \\
&+(8p' + (p'' - \rho'')(1+z))H)] \geq 0.
\end{aligned} \tag{3.16}$$

SEC yields as

$$\rho_{eff}+3p_{eff}=\frac{1}{2\alpha\rho-2}[-2\rho(1+2\beta+3\alpha H^2-6(1+z)\alpha HH')-6p$$
$$(1+2\beta-6\alpha H^2+2(1+z)\alpha HH')+3(1+z)\alpha H((1+z)H'(p'-\rho')+$$
$$H(6p'+2\rho'+(1+z)(p''-\rho'')))]\geq 0. \tag{3.17}$$

DEC yields as

$$\rho_{eff}-p_{eff}=\frac{1}{2\alpha\rho-2}[2\rho(-1+9\alpha H^2-2(1+z)\alpha HH')+2p(1+4\beta$$
$$-6\alpha H^2+2(1+z)\alpha HH')-(1+z)\alpha H((1+z)H'(p'-\rho')+H(2p'+$$
$$6\rho'+(1+z)(p''-\rho'')))]\geq 0. \tag{3.18}$$

The inequalities (3.15-3.18) have dependence on these parameters m, α and β. We fixed two parameters z, c and find the valid regions by varying the ranges of remaining parameters. We fix the constant of integration as $c=10$ and range of z will be from -0.9 to 10 and show the results for WEC and NEC. The validity region for different cases are shown in Table. **3.1** in which we took the different values of m to show the relation between α, β and m. Initially, we fix the value of $m=1.1$ for WEC then range for alpha is $\alpha\geq 1.01$ and for beta is $(1.1\leq\beta\leq 270)$ and $(-550\leq\beta\leq -2.8)$. If we take value of $m=10$ then the ranges of β will also increase like for $\alpha\geq 1.01$, it requires $(1.1\leq\beta\leq 9000)$ and $(-17900\leq\beta\leq -50)$. We can see that WEC is valid only for positive values of α whereas β needs some particular range for different values of α and m. If we take small value of m then validity range is also small for β, likewise if we increase the starting value of α then range of β also increases. Choice of m and particular range of β are directly proportional to each other while $\alpha\geq 1.01$ and α has also direct relation with β. If we will take larger value of α then we have to choose the larger value for β and vice versa, like if we choose $m=2$ then $\alpha=10$ and $\beta=-5$ but if decrease the value of alpha as $\alpha=1.001$ then β will be -8. $\rho_{eff}+p_{eff}\geq 0$ is also valid for positive values of α. In this setup, we show different ranges of β depending

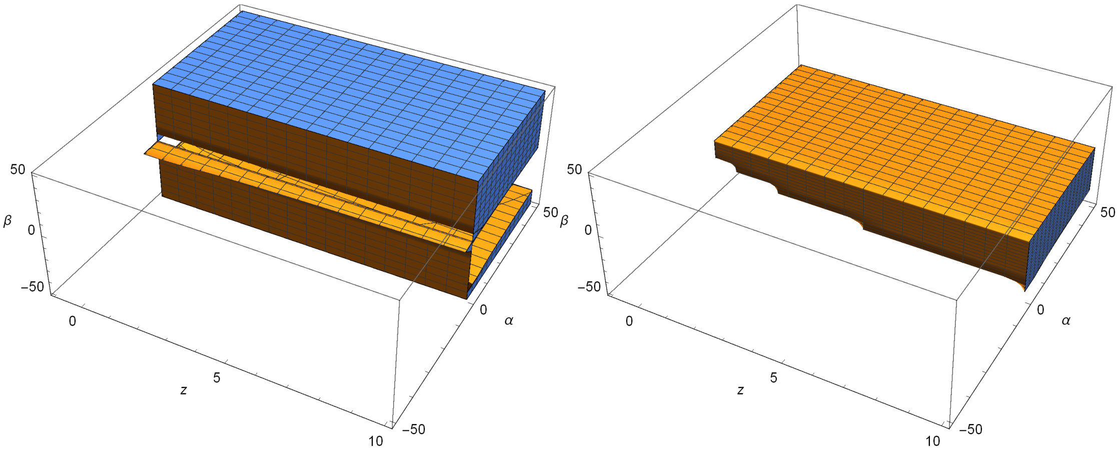

Figure 3.2: The LHS figure shows the validity region for $\rho_{eff} \geq 0$ whereas the RHS figure shows the validity region for $\rho_{eff} + p_{eff} \geq 0$. Herein, we set $H_0 = 67.3$.

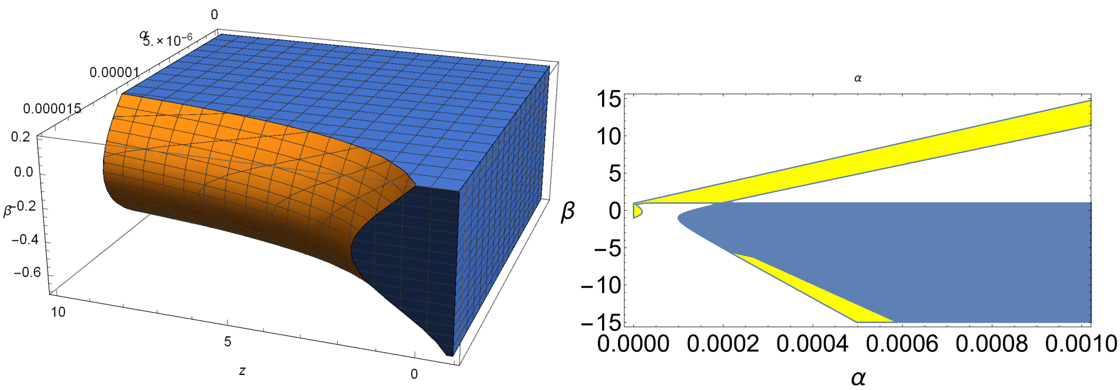

Figure 3.3: The LHS figure shows the validity region for WEC ($\rho_{eff} \geq 0$, $\rho_{eff} + p_{eff} \geq 0$) in $3D$ whereas the RHS figure shows the validity region for WEC in $2D$. Herein, we set $H_0 = 67.3$, and for $2D$ plot we set $z = 0$, $m = 10$.

on the particular ranges of α and results are shown in Table. **3.1**. If we choose $m = 1.1$ with $\alpha \geq 1.01$ then range of β is ($-1.3 \leq \beta \leq 0.9$). If we fix $m = 2$ and $\alpha \geq 1.01$ then range of β is ($-2.9 \leq \beta \leq 0.9$). From this discussion we can conclude that range of β for $\rho_{eff} + p_{eff} \geq 0$ lies between -2.9 and 0.9 for any value of $\alpha \geq 1.01$ and $m > 1$. Finally, in the last two columns of Table. **3.1** we show the combine validity region for WEC and NEC. Same in this case if we increase the value of m then the ranges of α also increases. Keep in notice that in common region, range of α is also restricted and very short. For $m = 1.1$ range of α is ($0 \leq \alpha \leq 0.00000016$) and range of β is ($-0.64 \leq \beta \leq 0.11$). If we fix $m = 10$ then range of α is ($0 \leq \alpha \leq 0.00001$) and range of β is ($-0.55 \leq \beta \leq 0.22$) for ρ_{eff}.

We show the graphical description for validity regions of $\rho_{eff} \geq 0$ and $\rho_{eff} + p_{eff} \geq 0$ in Figs. **3.2-3.3**. In Fig. **3.2**, we show the validity region for $\rho_{eff} \geq 0$ $\rho_{eff} + p_{eff} \geq 0$ for the particular choice $m = 10$. Fig. **3.3**, shows the region which validate the NEC for $m = 10$. Right side of Fig. **3.3** presents the common region for both $\rho_{eff} \geq 0$ and $\rho_{eff} + p_{eff} \geq 0$ at $z = 0$ and $m = 10$. In $2D$ regional plot yellow color shows the region of $\rho_{eff} \geq 0$ and blue color shows the region for $\rho_{eff} + p_{eff} \geq 0$. The validity regions of energy conditions are shown in Table. **3.1**.

We can write the energy conditions in the combined form as

$$\beta A_1 + \alpha H A_2 \geq A_3 \tag{3.19}$$

where $A_{i,s}$ purely depend on energy conditions which are under discussion for WEC, we found the values of $A_{i,s}$

$$\begin{aligned} A_1^{WEC} &= \frac{3p - \rho}{2}, \\ A_2^{WEC} &= 6\rho H + \frac{3}{2}(1+z)H(p' - \rho'), \\ A_3^{WEC} &= \rho, \end{aligned} \tag{3.20}$$

for NEC, we can found as

$$
\begin{aligned}
&A_1^{NEC} = -\rho - p, \\
&A_2^{NEC} = p(6H - 2(1+z)H') + \rho(3H + 2H'(z+1)) + \frac{z+1}{2}[H'(z+1) \\
&(-\rho' + p') + H(8p' + (z+1)(-\rho'' + p''))], \\
&A_3^{NEC} = p + \rho, \qquad (3.21)
\end{aligned}
$$

for SEC, we can found as

$$
\begin{aligned}
&A_1^{SEC} = -2\rho - 6p, \\
&A_2^{SEC} = 3p(6H - 2(1+z)H') + \rho(-3H + 6(z+1)H') + \frac{3(z+1)}{2} \\
&[(z+1)H'(p' - \rho') + H(6p' + 2\rho' + (1+z)(p'' - \rho''))], \\
&A_3^{SEC} = \rho + 3p, \qquad (3.22)
\end{aligned}
$$

for DEC, we can found as

$$
\begin{aligned}
&A_1^{DEC} = 4p, \\
&A_2^{DEC} = p(-6H + 2H'(z+1)) + \rho(9H - 2H'(z+1)) - \frac{z+1}{2}[(z+1) \\
&(p' - \rho')H' + H(2p' + 6\rho' + (1+z)(p'' - \rho''))], \\
&A_3^{DEC} = \rho - p. \qquad (3.23)
\end{aligned}
$$

3.3 $f(R,T,Q) = R(1+\alpha Q)$

We intend to dicuss the cosmic evolution using matter conservation equation of more generic modified theory. Here, we will set $\mathcal{L}_m = \rho$ and we will take the second model $f(R,T,Q) = R(1+\alpha Q)$ where α is coupling parameter. In this model, the choice of $\alpha = 0$, results in Eisntein's formalism of GR.

For a flat FLRW universe, the non-zero components of FLRW equation for $p_{eff} = p + p_{DE}$ and $\rho_{eff} = \rho + \rho_{DE}$ are

$$3H^2 = \rho_{eff},$$
$$-2\dot{H} - 3H^2 = P_{eff}, \tag{3.24}$$

where dots being time derivative and components of ρ_{DE} and p_{DE} are given as follows

$$\rho_{DE} = \frac{-1}{1+3\alpha(p+\rho)(3H^2+\dot{H})}[3\alpha(-18H^4(\rho+p)+3H^2(\rho(\rho+p)+ \dot{H}(5p-7\rho)) + \dot{H}(\rho(\rho+p)+3(\rho-p)\dot{H}) + 3H^3(5\dot{p}-3\dot{\rho}) + 6H\dot{H} (\dot{p}-\dot{\rho}) + (p-\rho)\ddot{H})], \tag{3.25}$$

$$p_{DE} = \frac{-1}{1+3\alpha(p+\rho)(3H^2+\dot{H})}[3\alpha(6H^4(-p+\rho)+2H^3(\dot{p}+5\dot{\rho})+4 (-\dot{p}+\dot{\rho})\ddot{H}) + 2H(9\dot{H}(\dot{\rho}-\dot{p}) + 2\ddot{H}(-2p+3\rho)) + \dot{H}(p(\rho+p)+(-11 p+7\rho)\dot{H} - 2\ddot{p} + 2\ddot{\rho}) + H^2(3p(p+\rho) - (p-25\rho)\dot{H} - 5\ddot{p} + 3\ddot{\rho}) + 2(-p+\rho)H^3], \tag{3.26}$$

and effective EoS ω_{eff} is

$$\omega_{eff} = [-3\alpha(6H^4\rho + 2H^3(\dot{p}+5\dot{\rho}) + 4(-\dot{p}+\dot{\rho})\ddot{H} + 6H(3\dot{H}(-\dot{p}+\dot{\rho}) + 2\rho\ddot{H}) + \dot{H}(7\rho\dot{H} - 2\ddot{p} + 2\ddot{\rho}) + H^2(25\rho\dot{H} - 5\ddot{p} + 3\ddot{\rho}) + 2\rho H^3) + p(1+3\alpha (6H^4 + H^2\dot{H} + 11\dot{H}^2 + 8H\ddot{H} + 2H^3))][\rho + 9\alpha(6H^4(p+\rho) + H^2 (-5p+7\rho)\dot{H} + (p-\rho)\dot{H}^2) + H^3(-5\dot{p}+3\dot{\rho}) + 2H(\dot{H}(-\dot{p}+\dot{\rho}) + (\rho-p)\ddot{H})]^{-1}. \tag{3.27}$$

The EoS for DE is found to be as, $\omega_{DE} = \frac{p_{DE}}{\rho_{DE}}$

$$\omega_{DE} = [6H^4(-p+\rho) + 2H^3(\dot{p} + 5\dot{\rho}) + 4(-\dot{p} + \dot{\rho})\ddot{H} + 2H(9\dot{H}(\dot{\rho} - \dot{p})$$
$$+2\ddot{H}(-2p + 3\rho)) + \dot{H}(p(\rho + p) + (-11p + 7\rho)\dot{H} - 2\ddot{p} + 2\ddot{\rho})H^2(3p(p$$
$$+\rho) - (p - 25\rho)\dot{H} - 5\ddot{p} + 3\ddot{\rho}) + 2(-p + \rho)H^3][-18H^4(\rho + p) + 3H^2(\rho$$
$$(\rho + p) + \dot{H}(5p - 7\rho)) + \dot{H}(\rho(\rho + p) + 3(\rho - p)\dot{H}) + 3H^3(5\dot{p} - 3\dot{\rho})$$
$$+6H((\dot{p} - \dot{\rho})\dot{H} + \ddot{H}(p - \rho))]^{-1}. \tag{3.28}$$

and conservation equation (2.28) takes the following form for this model

$$\dot{\rho} + 3H(\rho + p) = \frac{9\alpha(2H^2 + \dot{H})(3H^2 + \dot{H})(2H(p + \rho) + \dot{p} + \dot{\rho})}{1 + 18\alpha(2H^2 + \dot{H})^2}. \tag{3.29}$$

In this scenario, by using the equation (3.29) and converting it into redshift by using the relation $a(t) = \frac{1}{1+z}$ and $\frac{d}{dt} = -(1+z)H\frac{d}{dz}$ whereas $p = p(z)$. Energy density $\rho(z)$ is found to be as

$$e^{(1+\omega)(3\log(1+z) - \frac{m(-1+3m)(1+3\omega)\log[-m^2 + 9H_0^4(-1+2m)(1+z)^{\frac{4}{m}}\alpha(1-\omega+m(-1+3\omega))]}{4-4\omega+4m(-1+3\omega)})}c, \tag{3.30}$$

where c stands for constant of integration. As energy density is found to be in an exponential form so it will remain positive for all values of unknowns parameters like α, ω, m, z. It will only depend on constant of integration c when we take negative value of c then energy density will be negative or less than zero otherwise for all positive values of c energy density will remain positive.

One can also get the relation between time and redshift as

$$t = \left(\frac{1}{1+z}\right)^{\frac{1}{m}}. \tag{3.31}$$

Using the value of $\rho(z)$, one can get ρ_{eff} and p_{eff} in terms of redshift as and we take $c = 10$ and $\omega = 1$

$$\rho_{eff} = [10(1+z)^6(m^2 + 180H_0^4m^2(1+z)^{\frac{4}{m}}\alpha - 972H_0^8(-1+2m)(1+$$
$$z)^{\frac{8}{m}}\alpha^2)][m(-m + 18H_0^4(-1+2m)(1+z)^{\frac{4}{m}}\alpha)^{3m} + 60H_0^2(1+z)^{6+\frac{2}{m}}\alpha m$$
$$(m - 3m^2 + 18H_0^4(1+m)(-5+6m))(1+z)^{\frac{4}{m}}\alpha]^{-1}, \tag{3.32}$$

$$p_{eff} = [10(1+z)^6(m^4 + 12H_0^4 m^2(-1+6m(2+5m))(1+z)^{\frac{4}{m}}\alpha + 108$$
$$H_0^8 m(-1+2m)(-16-51m+78m^2)(1+z)^{\frac{8}{m}}\alpha^2 - 34992H_0^12(1-2m)^2$$
$$(-1+m)(1+z)^{\frac{12}{m}}\alpha^3)][m(m-18H_0^4(-1+2m)(1+z)^{\frac{4}{m}}\alpha)^2(60H_0^2(-1$$
$$+3m)(1+z)^{6+\frac{2}{m}}\alpha + m(m(-m+18H_0^4(-1+2m)(1+z)^{\frac{4}{m}}\alpha))^{-1+3m})]^{-1},$$

and effective EoS in term of redshift can be written as

$$\omega_{eff} = -2 + \frac{2}{m} - \frac{4m}{-m+18H_0^4(-1+2m)(1+z)^{\frac{4}{m}}\alpha} - [m(2+m)+6$$
$$H_0^4(2+9m(3+2m))(1+z)^{\frac{4}{m}}\alpha][m^2+180H_0^4m^2(1+z)^{\frac{4}{m}}\alpha - 972H_0^8(-1$$
$$+2m)(1+z)^{\frac{8}{m}}\alpha^2]^{-1}. \tag{3.33}$$

Graphical representation of effective components EoS ω_{eff} are shown in Fig. **3.4**

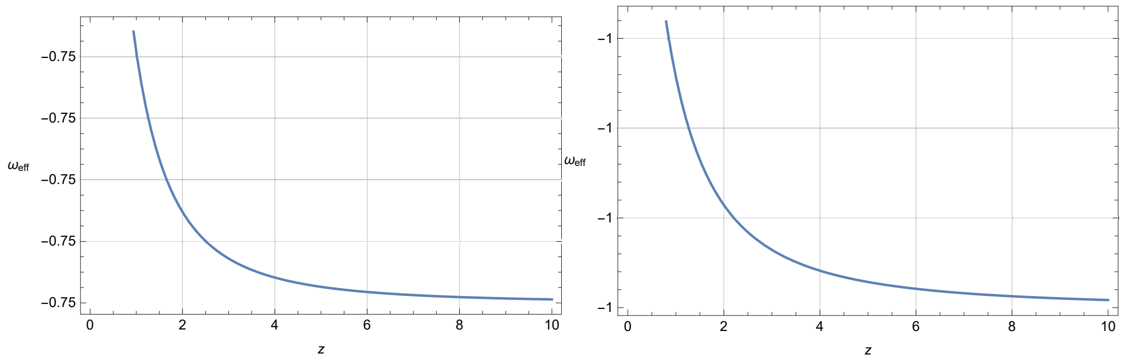

Figure 3.4: The LHS shows the evolution of ω_{eff} for $m = 1.6$ whereas the RHS shows the behavior ω_{eff} for $m = 2$. Herein, we set $\alpha = 10$ and $H_0 = 67.3$ [7].

for two different values of m. In this discussion, we choose the following values of unknown parameters $\alpha = 10$, $m = 1.6$, and $m = 2$. For the value of $m = 2$, deceleration parameter is -0.5 which favors the expanding behavior of cosmos. It can be seen that ρ_{eff} is positive and increasing function as shown on right plot and ω_{eff} and ω_{DE} are exactly equal to -1, shown in Fig. **3.5**, representing the ΛCDM epoch in accordance with recent observations from Planck's data [7].

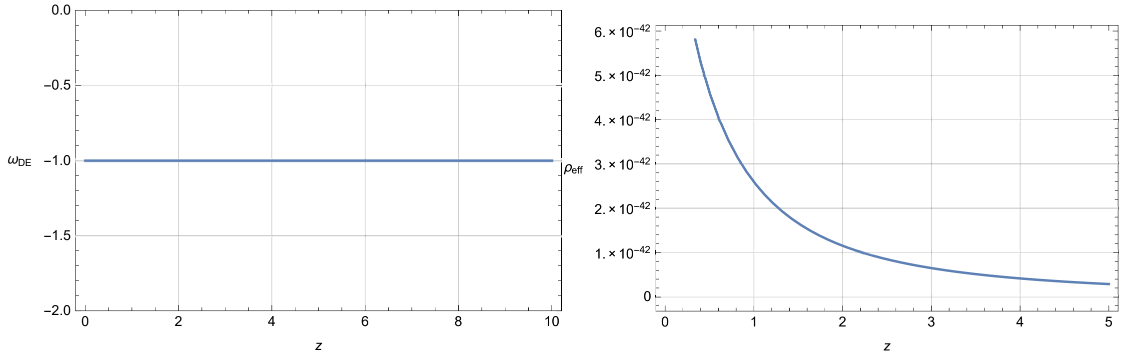

Figure 3.5: The LHS shows the Evolution of ω_{DE} whereas the RHS shows the behavior of ρ_{eff}. Herein, we set $\alpha = 10$, $m = 2$, and $H_0 = 67.3$ [7].

3.4 Energy conditions for the model

$$f(R, T, Q) = R(1 + \alpha Q)$$

Now, we will discus the energy conditions for our second model of $f(R, T, Q)$ gravity which is $R(1 + \alpha Q)$ by considering FLRW metric. The validity of above mentioned inequalities (3.14) is totaly model dependent. In this model these depend on two parameters "α" coupling parameter, and "m" the power law exponent. Here, we fixed $H_0 = 67.3$, $c = 10$. $\rho_{eff} \geq 0$ is valid for all positive values of α, i.e., $\alpha > 0$ and $m > 1$. $\rho_{eff} + p_{eff} \geq 0$ is valid for $\alpha > 0$, and m is restrict in this model between $1 < m < 2$, this inequality is not valid for greater values of m. NEC is also valid for the positive values of α and m should be between $1 < m < 2$. We showed the valid regions of NEC and WEC in Fig. **3.6** and Fig. **3.7**. respectively.

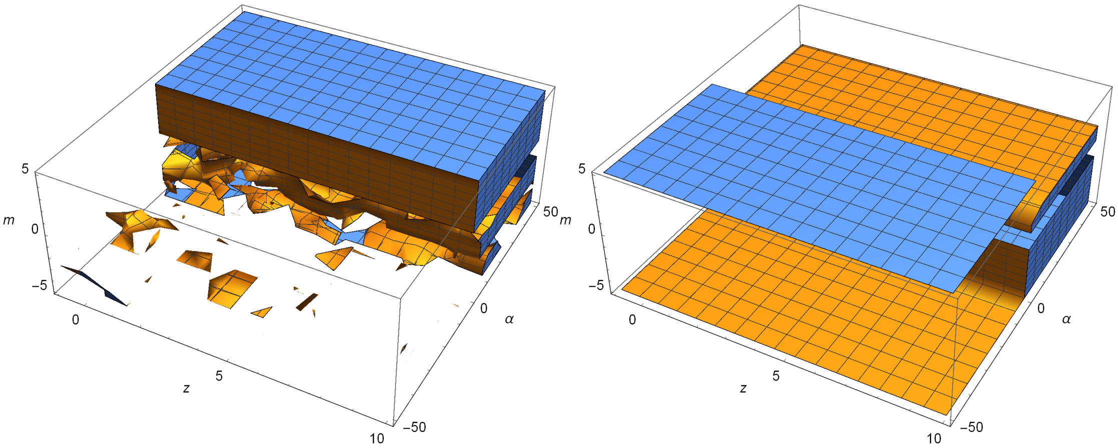

Figure 3.6: The LHS shows the validity region for $\rho_{eff} \geq 0$ whereas the RHS shows the validity region for $\rho_{eff} + p_{eff} \geq 0$. Herein, we set $H_0 = 67.3$.

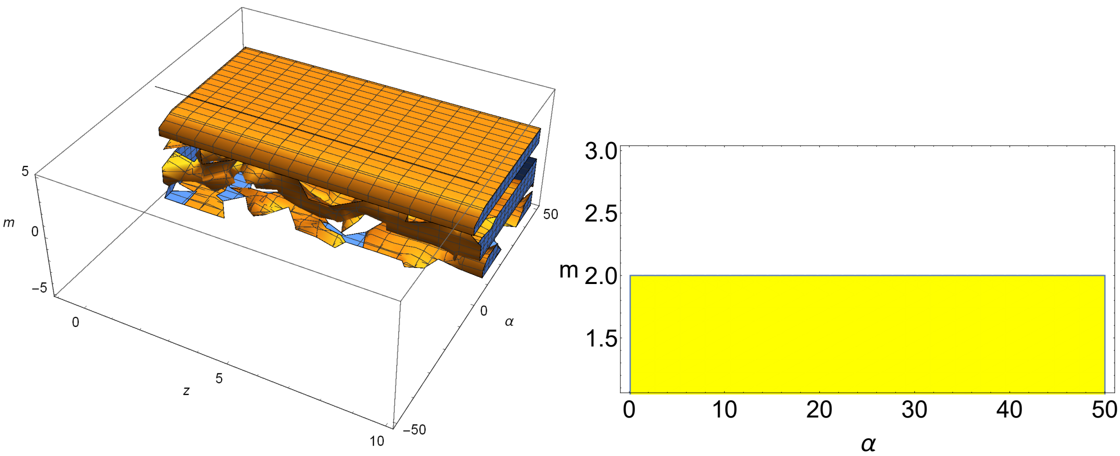

Figure 3.7: The LHS shows the validity region for WEC ($\rho_{eff} \geq 0$, $\rho_{eff}+p_{eff} \geq 0$) in $3D$ whereas the RHS also shows the validity region for WEC in $2D$. Herein, we set $H_0 = 67.3$, and for $2D$ plot we set $z = 0$, $m = 10$.

Chapter 4

Late time cosmic evolution with collisional matter in $f(R,T)$ gravity

In this chapter, we discuss the cosmic evolution of non-minimally coupled $f(R,T)$ gravity in the presence of CM plus radiation. We find the cosmic evolution in the background of CM and compare the results with NCM and well known ΛCDM model. In this chapter, we also consider the FLRW space-time and formulate the dynamic equations. Here, we choose the two significant non-minimal models and discuss the cosmological parameters, the effective EoS, and deceleration parameter by considering CM plus radiations. In graphical representation, we show the comparison between CM without radiation, NCM, CM with radiations and ΛCDM model.

4.1 Collisional Matter within $f(R,T)$ Theory

The analytical results of high energy particle detectors, like WMAP, PAMELA and ATIC show that the production of the electron-positron in the universe is greater than to that observed by cosmic ray collisions and Supernovae SNeIa explosions [69]. This issue helps us to find the total destruction of WIMP's which are the candidate for DM and this process is collisional [70]. The consideration of collisional DM is the solution for the problem of recent cosmic picture which shows the greater amount of energy than its total matter contents. Now, our discussion topic is late times dynamics of the cosmos by considering CM. Kleidis and spyrou [71] discover the cosmic dynamics exclusively by considering CM in the Einstein gravity and realized that deceleration parameter q is not dependent of redshift and remains constant. So eventually, we are not able to discus the transition phase from decelerated phase to accelerated one. The late time dynamics in the presence of CM is discussed by Oikonomou, V.K. et al. [72, 73] and they came with the result that, the CM is not the only one that can not describe the late-time cosmic acceleration but it can modifies the cosmic acceleration in $f(R)$ gravity. This discovery motivated us to the future evolution of cosmic parameters in $f(R,T)$ gravity. Simply, DE EoS parameter ω_{DE} is able to cross the phantom divide line even in the presence of CM. We can see that firstly CM was introduced and studied in GR and $f(R)$ gravity. Mostly, self interacting CM model was discussed with perfect fluid in which total mass-energy density is basic assumption, denoted by ϵ_m which depends on two terms and defined as

$$\epsilon_m = \rho_m + \rho_m \Pi, \tag{4.1}$$

Because of the fact of checking the ordinary matter content relating $\rho_m \Pi$, the invariant part referred by ρ_m here. The energy density part of the EMT related to thermodynamical content of CM is represented by $\rho_m \Pi$. Although fluid is not dust like, but it has positive pressure and satisfies the EoS:

$$P_m = \omega \rho_m, \tag{4.2}$$

where EoS parameter is represented by ω and $0 < \omega < 1$. And it is assumed that the potential energy density is

$$\Pi = \Pi_0 + \omega ln(\frac{\rho_m}{\rho_{m_0}}) \tag{4.3}$$

with ρ_{m_0} and Π_0 are present day values. Using (4.1) and (4.3) we obtain the following relation of total energy density of the Universe as follows:

$$\epsilon_m = \rho_m(1 + \Pi_0 + \omega ln(\frac{\rho_m}{\rho_{m_0}})). \tag{4.4}$$

In the conserved medium the continuity equation for the motion of volume element is stated as:

$$\nabla^{\nu} T_{\mu\nu} = 0. \tag{4.5}$$

And the EMT reduces as

$$T_{\mu\nu} = (\epsilon_m + P_m) u_\mu u_\nu - P_m g_{\mu\nu}, \tag{4.6}$$

where $u_\mu = \frac{dx_\mu}{ds}$ is the four velocity and satisfies the equation $u_\mu u_\nu = 1$. It is noted that $P_m = p_m$ because of the negligence of the pressure of the ordinary matter. Using FLRW line element, conservation equation given in (4.5) becomes

$$\dot{\epsilon_m} + 3\frac{\dot{a}}{a}(\epsilon_m + P_m) = 0. \tag{4.7}$$

Combining (4.2) and(4.4), gives the following result

$$\rho_m = \rho_{m_0}\left(\frac{a_0}{a}\right)^3. \tag{4.8}$$

with a_0 the current scale factor. Here we can see that CM can be described by (4.4) and (4.8). Π_0 has the following form and $\Omega_m = 0.3183$ by Planck's data [7]

$$\Pi_0 = \left(\frac{1}{\Omega_M} - 1\right) \tag{4.9}$$

and its numerical value is $\Pi_0 = 2.14169$.

There is an important case need to be discused that our universe is filled with both CM and relativistic matter which is commonly known as radiations. To discus this significant case ρ_{matt} is defined as following

$$\rho_{matt} = \epsilon_m + \rho_{r0}a^{-4}, \tag{4.10}$$

where ρ_{r0} represent the present energy density for radiation. The pressure for this case is given by

$$P_{matt} = p_m + p_r, \tag{4.11}$$

where p_m is the pressure for CM and p_r is the pressure from radiation. By using equation (4.4) and (4.8) we can get equation (4.10) in the following form

$$\rho_{matt} = \rho_{m0}a^{-3}(1 + \Pi_0 + 3\omega ln(a)) + \rho_{r0}a^{-4}. \tag{4.12}$$

We can also rewrite above equation (4.12) as

$$\rho_{matt} = \rho_{m0}(g(a) + \chi a^{-4}), \tag{4.13}$$

where χ is defined as $\chi = \frac{\rho_{r0}}{\rho_{m0}}$, its numerical value is $\chi = 3.1 \times 10^{-4}$. $g(a)$ represent the nature of CM and it is defined as following

$$g(a) = a^{-3}(1 + \Pi_0 - 3\omega ln(a)). \tag{4.14}$$

Keep in mind that, if we will take $\omega = 0$ and $\Pi_0 = 0$ in this formalism then we will get $g(a) = a^{-3}$ and its represent the NCM which is considered to be dust.

4.2 Late time Cosmological Evolution in $f(R,T)$ Theory

We will check the behavior and characteristic of deceleration parameter $q(z)$ and EoS parameter ω_{eff} in $f(R,T)$ gravity. We are assuming that the cosmos is not only filled with ordinary matter and DE but also is filled with self interacting CM. So rewriting field equation in these conditions as:

$$3H^2 = \frac{1+f_T}{f_R}\epsilon_m + \frac{1}{f_R}[\frac{1}{2}(f - Rf_R) - 3\dot{R}Hf_{RR} + P_m f_T], \tag{4.15}$$

$$-2\dot{H} - 3H^2 = \frac{1+f_T}{f_R}P_m + \frac{1}{f_R}[2\dot{R}Hf_{RR} + \ddot{R}f_{RR} + \dot{R}^2 f_{RRR} - \frac{1}{2}(f - Rf_R) - P_m f_T]. \tag{4.16}$$

Partial derivative with respect to time is represented by dot. We rewrite the RHS of (4.15) and (4.16) in terms of effective energy density ρ_{eff} and pressure P_{eff} as follows

$$3H^2 = k^2{}_{eff}\rho_{eff}, \tag{4.17}$$

$$-2\dot{H} - 3H^2 = k^2{}_{eff}P_{eff}. \tag{4.18}$$

Here $k^2{}_{eff} = \frac{1+f_T}{f_R}$, and ρ_{eff}, P_{eff} has the following form

$$\rho_{eff} = \epsilon_m + \frac{1}{1+f_T}[\frac{1}{2}(f - Rf_R) - 3\dot{R}Hf_{RR} + P_m f_T], \tag{4.19}$$

$$P_{eff} = P_m + \frac{1}{1+f_T}[2\dot{R}Hf_{RR} + \ddot{R}f_{RR} + \dot{R}^2 f_{RRR} - \frac{1}{2}(f - Rf_R) - P_m f_T]. \tag{4.20}$$

With the energy density $\rho_{DE} = \rho_{eff} - \epsilon_m$ and the pressure $P_{DE} = P_{eff} - P_m$. Conservation Law with the effective energy density is found to be

$$\frac{d(k^2_{eff}\rho_{eff})}{dt} + 3Hk^2_{eff}(\rho_{eff} + P_{eff}) = 0. \tag{4.21}$$

Using (4.17) and (4.18) the above equation takes the form

$$18\frac{f_{RR}}{f_R}H(\ddot{H} + 4H\dot{H}) + 3(\dot{H} + H^2) + \frac{1+f_T}{f_R}\epsilon_m + P_m\frac{f_T}{f_R} + \frac{f}{2f_R} = 0. \tag{4.22}$$

Using redshift $z=\frac{1}{a}-1$ above equation can be written as

$$\frac{d^2H}{dz^2}=\frac{3}{z+1}\frac{dH}{dz}-\frac{1}{H}(\frac{dH}{dz})^2-[3f_R(H^2-(1+z)H\frac{dH}{dz})+\frac{f}{2}+P_mf_T$$
$$+(1+f_T)\epsilon_m][18H^3f_{RR}(1+z)^2]^{-1}. \tag{4.23}$$

In following subsection, we will describe the late time cosmology by considering the CM and radiations. We numerically solve the differential equation (4.23) for $H(z)$ and used this interpolating function $H(z)$ for further graphical analysis to check the correspondence of models [1] with existing literature in the presence of self interacting CM and radiations. We will discus two different types of non-minimal models in this theory. We have the following parameters for all plots. ($\omega=0$ for NCM), ($\omega=0.5$ for CM) and ($\omega=0.8$ for CM+radiations) and the observational value for present Hubble parameter is taken $H_0=68.3$, and fractional energy density $\Omega_{m0}=0.3183$.

4.2.1 $f(R,T)=\alpha_1R^{\gamma_1}T^{\gamma_2}+\alpha_2T$

In first place we consider pure non-minimally coupled model in the background of $f(R,T)$ gravity of the form $f(R,T)=\alpha_1R^{\gamma_1}T^{\gamma_2}+\alpha_2T$. In [1], authors reconstructed this model and check the instability of such model against density matter perturbations and Dolgov Kawasaki instability criterion.

The condition for the viability of this model requires that

$$f_{RR}=\alpha_1\gamma_1(\gamma_1-1)R^{\gamma_1-2}T^{\gamma_2}\geq 0, \tag{4.24}$$

so that we need to have $\gamma_1>1$, and $\alpha_1>0$. Herein, in graphical analysis, we choose the following parameters $\alpha_1=20,\alpha_2=15$, $H_0=68.3$, and $\Omega_m=0.3183$. We employ the numerical approach to find $H(z)$ by solving Eq. (4.23). We discuss the deceleration parameter, ω_{eff} and ω_{DE} for three cases namely, non-CM, CM, and CM plus radiations. We present two different cases depending on

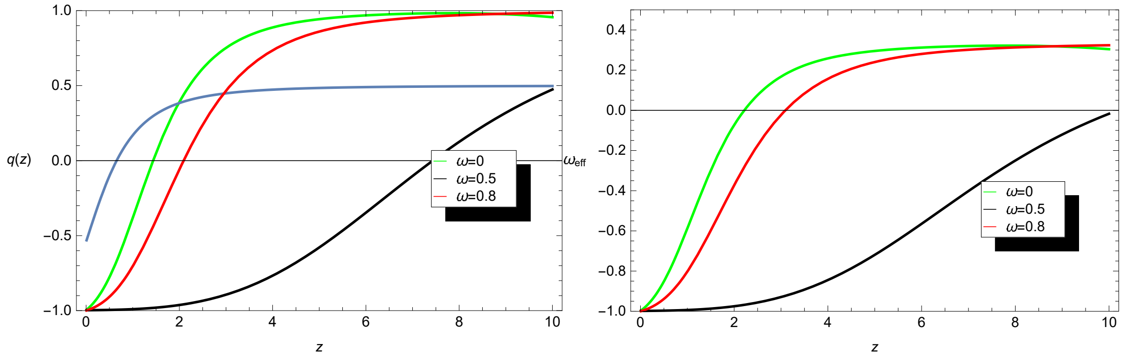

Figure 4.1: The LHS plot shows the evolution of deceleration parameter whereas the RHS plot shows the evolution of effective EoS. Herein we set, $\alpha_1 = 20, \alpha_2 = 15, \gamma_1 = 10$, and $\gamma_2 = -0.7$.

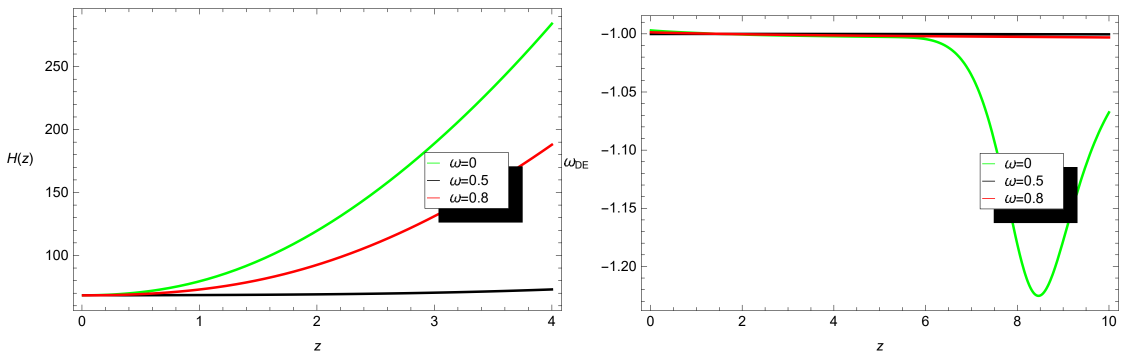

Figure 4.2: The LHS plot shows the evolution of $H(z)$ whereas the RHS plot shows the evolution of EoS for dark energy. Herein we set, $\alpha_1 = 20, \alpha_2 = 15, \gamma_1 = 10$, and $\gamma_2 = -0.7$.

the powers of R and T in above model. For the first model, we set $\gamma_1 = 10$ and $\gamma_2 = -0.7$, so that $f(R, T)$ takes the form

- $\alpha_1 R^{10} T^{-4/5} + \alpha_2 T$

In Fig. **4.1** the LHS represent the evolution of deceleration parameter for four different cases in term of redshift. Blue curve represent the standard ΛCDM model, green curve is for NCM, black curve is for CM, red curve is for combine CM plus radiations. We can clearly see that from Fig. **4.1** that the behavior of the green curve and red curve is almost same as blue (ΛCDM) curve but the black curve is lower than the blue curve and changes its behavior. We can also observe the transition phase in LHS of Fig. **4.1** for NCM, CM, and CM plus radiation. In the case of NCM, CM plus radiation transition phase is almost equal to ΛCDM model, equal to $z_t = 1.9$ and for CM case shows the transition phase at $z_t = 7.6$ and this is the larger value as compare to observations. In the RHS of the Fig. **4.1**, we represents the evolution of effective EoS for NCM, CM, and for radiations plus CM case and the color scheme is same as for deceleration parameter. The graphs shows that in all cases ω_{eff} approaches to -1 and black line is not crossing the phantom divide line [74] but green and red line crossed the phantom divide line. The LHS of Fig. **4.2** represents the evolution of $H(z)$ which is numerically calculated for this model and we found the current value of $H(z)$ for this model at $z = 0$ is equal to 68.4 for all cases NCM, CM, and CM plus radiation consistent with the recent Planck's data [7]. The RHS of Fig. **4.2** represents the evolution of EoS for DE. DE is approaches to -1 for CM, for NCM and CM plus radiation case its greater than -1.

For the second case we set $\gamma_1 = 10$ and $\gamma_2 = -0.5$, so that $f(R, T)$ takes the form

- $\alpha_1 R^{10} T^{-1/2} + \alpha_2 T$

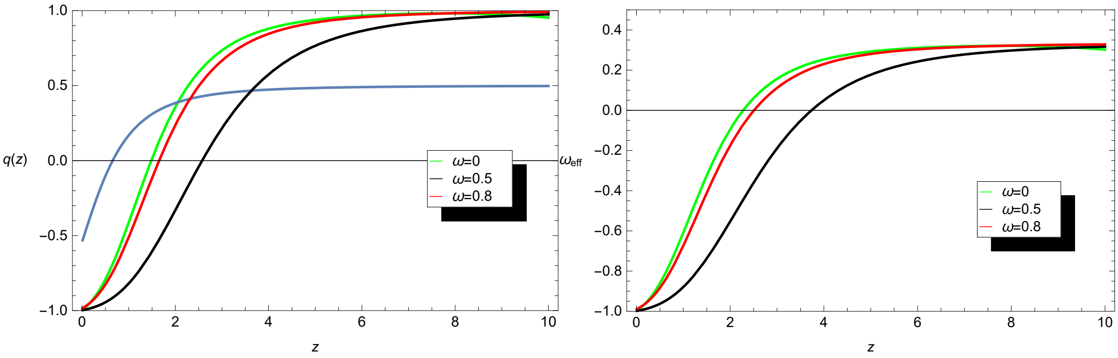

Figure 4.3: The LHS plot shows the evolution of deceleration parameter whereas the RHS plot shows the evolution of effective EoS. Herein we set, $\alpha_1 = 20, \alpha_2 = 15, \gamma_1 = 10$ and $\gamma_2 = -0.5$.

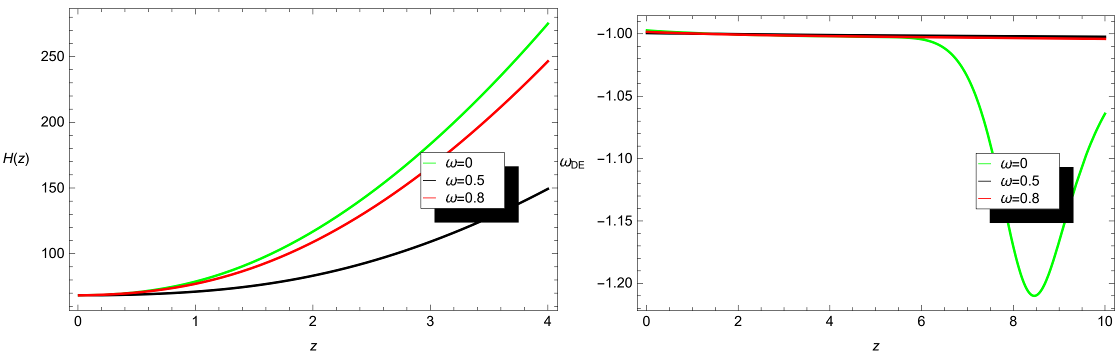

Figure 4.4: The LHS plot shows the evolution of $H(z)$ whereas the RHS plot shows the evolution of EoS for dark energy. Herein we set, $\alpha_1 = 20, \alpha_2 = 15, \gamma_1 = 10$ and $\gamma_2 = -0.5$.

Again we use the numerical approach to discuss this $f(R,T)$ model by using equation of motion (4.23). In Fig. **4.3** the LHS represent the evolution of deceleration parameter for four different cases in term of redshift. Blue curve represent the standard ΛCDM model, green curve is for NCM, black curve is for CM, red curve is for combine CM plus radiations. We can clearly see that from Fig. **4.3** that the behavior of the green curve, black curve and red curve is almost same as blue (ΛCDM) curve for this model. We can also observe the transition phase in LHS of Fig. **4.3** for NCM, CM, and CM plus radiation. In the case of NCM transition phase is almost equal to $z_t = 2$, CM plus radiation transition phase is almost equal to $z_t = 1.9$ and for CM case shows the transition phase at $z_t = 1.8$. In the RHS of the Fig. **4.3**, we represents the evolution of effective EoS for NCM, CM, and for radiations plus CM case and the color scheme is same as for deceleration parameter. The graphs shows that in all cases ω_{eff} approaches to -1 and all the lines, green, black, red crosses the phantom line [74] for this model. The LHS of Fig. **4.4** represents the evolution of $H(z)$ which is numerically calculated for this model and we found the current value of $H(z)$ for this model at $z = 0$ is equal to 68.4 for all cases NCM, CM, and CM plus radiation in accordance with [7]. The RHS of Fig. **4.4** represents the evolution of EoS for DE. In this model DE for CM, for NCM and CM plus radiation case is greater than -1.

4.2.2 $f(R,T) = \beta_1 R^{\mu} + \beta_2 R^{\nu} + \frac{2k^2}{-1+3\omega} T + \beta_3 T^{\frac{1}{2} - \frac{\sqrt{-1+\omega(3+\omega-3\omega^2)}}{(1+\omega)\sqrt{-2+6\omega}}} + \beta_4 T^{\frac{1}{2} + \frac{\sqrt{-1+\omega(3+\omega-3\omega^2)}}{(1+\omega)\sqrt{-2+6\omega}}}$

This model represents a generic minimal coupling model of the form $f(R,T) = f(R) + f(T)$. The second derivative of above model is given by

$$f_{RR} = \beta_1 \mu(\mu-1)R^{\mu-2} + \beta_2 \nu(\nu-1)R^{\nu-2} \geq 0. \tag{4.25}$$

The validity of this model also depends on second derivative f_{RR} which should be greater than zero and it would be valid when $\mu > 1$, $\nu > 1$, $\beta_1 > 0$ and

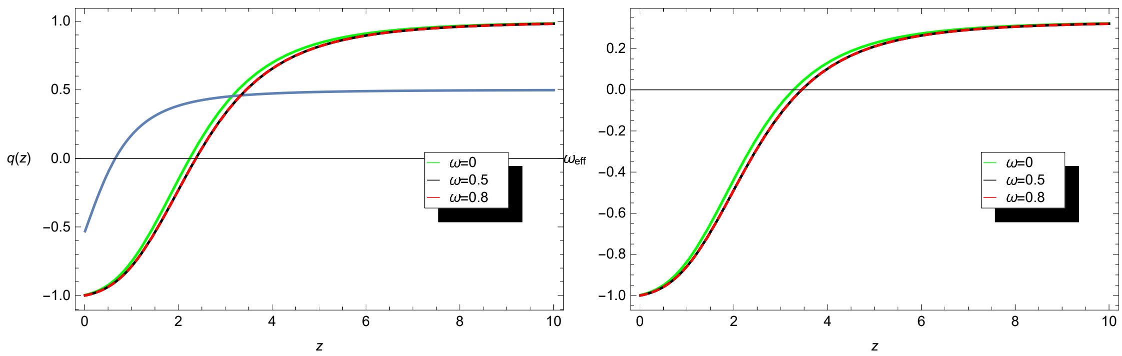

Figure 4.5: The LHS plot shows the evolution of deceleration parameter whereas the RHS plot shows the evolution of effective EoS. Herein we set, $\beta_1 = 5, \beta_2 = 10, \beta_3 = 15$, $\beta_4 = 20$, $\mu = 25$ and $\nu = 30$.

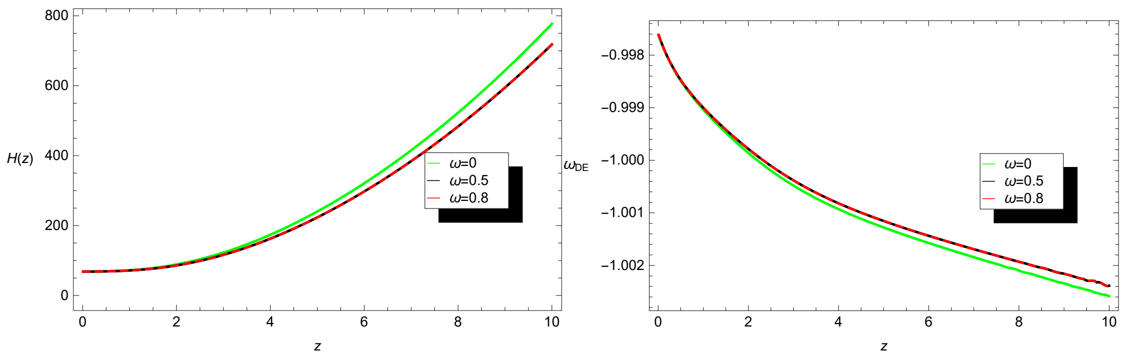

Figure 4.6: The LHS plot shows the evolution of $H(z)$ whereas the RHS plot shows the evolution of EoS for dark energy. Herein we set, $\beta_1 = 5, \beta_2 = 10, \beta_3 = 15$, $\beta_4 = 20$, $\mu = 25$ and $\nu = 30$.

$\beta_2 > 0$. We choose the following parameters, $\beta_1 = 5, \beta_2 = 10, \beta_3 = 15$, $\beta_4 = 20$, $\mu = 25, \nu = 30$, and $\Omega_m = 0.3183$ for NCM, CM and combine case for CM plus radiation. We will numerically discuss this $f(R,T)$ model by using equation of motion (4.23). In Fig. **4.5** the LHS represent the evolution of deceleration parameter for four different cases in term of redshift. Blue curve represent the standard ΛCDM model, green curve is for NCM, black curve is for CM, red curve is for combine CM plus radiations. We can clearly see from Fig. **4.5** that the red curve and black curve overlapped each other, here we distinguished them with black and red dashed line. Green line is slightly varies from red and black line. We can also observe the transition phase in LHS of Fig. **4.5** for CM, and CM plus radiation is also same at $z_t = 2.2$. In the case of NCM transition phase is almost equal to $z_t = 2.1$. In the RHS of the Fig. **4.5**, we represents the evolution of effective EoS for NCM, CM, and for radiations plus CM case and the color scheme is same as for deceleration parameter. The graphs shows that in all cases ω_{eff} approaches to -1 and all the lines, green, black, red crossed the phantom divide line [74] for this model. The LHS of Fig. **4.6** represents the evolution of $H(z)$ which is numerically calculated for this model and we found the current value of $H(z)$ for this model at $z = 0$ is equal to 68.4 for all cases NCM, CM, and CM plus radiation. The RHS of Fig. **4.6** represents the evolution of EoS for DE. In this model DE for CM, for NCM and CM plus radiation case is greater than -1.

Chapter 5

Summary and Discussion

Modified theories become the appropriate candidates to discus the accelerated cosmic expansion. $f(R, T, Q)$ is a generalized MTG based on the coupling of matter and curvature. This complicated theory involve the contraction of $T^{\mu\nu}$ and $R_{\mu\nu}$ and it is the extended form of $f(R, T)$ theory [24]. Although, $f(R, T, Q)$ gravity is the extension of $f(R, T)$ theory but there exists a notable difference while taking the contraction term $R_{\mu\nu}$. For example, if we involve the role of radiation dominated fluid, then filed equations turn down to $f(R)$ gravity and effect of non-minimal coupling would be vanished in $f(R, T)$ but this is not the case for $f(R, T, Q)$ due to the involvement of contraction term Q. Q is the inclusive term which involve the strong non-minimal coupling as compared to other MTG. So it is strongly motivational to test these models with non-minimal coupling and explore their results in cosmology.

In this thesis, we have constructed a cosmological scenario from the complicated non-minimal coupling of matter and geometry in the $f(R, T, Q)$ gravity. We consider two simple cases of non-minimal coupling in this modified theory in the form of models $f(R, T, Q) = R + \alpha Q + \beta T$ and $f(R, T, Q) = R(1 + \alpha Q)$. Dynamical equations are presented in chapter **III**, where we consider the power law cosmology to find an expression for energy density ρ. Using Eq. (3.30), it is obvious to find the expressions of effective EMT and its components. In power law cosmology, one can represent the cosmic history depending on the choice of parameter m. Here, we set parameter m according to the evolution of q as per recent observational data. In Fig. **2.22**, we set $m = 1.0666580$ with $q = -0.0624924$ to see the evolution of ρ_{eff} and ω_{eff}, it is found that WEC is satisfied and $\omega_{eff} \to -1$ validating the current cosmic epoch [7]. It is to be noted that we set the choice of parameters α and β as per validity ranges expressed in Table. **3.1**, where we develop the constraints on these parameters for different values of m satisfying WEC and NEC. Evolution of WEC and NEC versus redshift z is presented in Fig. **2.23-2.25**.

$Data$	q	$H_0(kms^{-1}Mpc^{-1})$	m	ω_{eff}
$H(z)(14points)$[76]	$-0.18^{+0.12}_{-0.12}$	$68.43^{+2.84}_{-2.80}$	1.221	-1.31601
$SN(Union2)$[76]	$-0.38^{+0.05}_{-0.05}$	$69.18^{+0.55}_{-0.54}$	1.613	-1.84677
$H(z)+SN(Union2)$[76]	$-0.34^{+0.05}_{-0.05}$	$68.93^{+0.53}_{-0.52}$	1.516	-1.74099
$H(z)(29points)$[77]	$-0.0451^{+0.0614}_{-0.0625}$	$65.2299^{+2.4862}_{-2.4607}$	1.0473	-0.953795
$SN(Union2.1)$[77]	$-0.3077^{+0.1045}_{-0.1036}$	$68.7702^{+1.4052}_{-1.3754}$	1.4445	-1.65392

Table 5.1: Observational data results for power law exponent m and ω_{eff} for the model $f(R,T,Q) = R + \alpha Q + \beta T$.

In literature, observational constraints have been developed on the choice of power law exponent m, cosmological parameters q and H_0. Kaeonikhom et al. [75] explored the phantom power law cosmology using cosmological observations from CMBR, BAO and observational Hubble data, they found the best fit value of power law exponent as $m \approx -6.51^{+0.24}_{-0.25}$. In [76], Kumar found the constraints on Hubble and deceleration parameters from the latest $H(z)$ and SNeIa data as $q = -0.18^{+0.12}_{-0.12}$, $H_0 = 68.43^{+2.84}_{-2.80}$kms-1Mpc-1 and $q = -0.38^{+0.05}_{-0.05}$, $H_0 = 69.18^{+0.55}_{-0.54}$ kms-1Mpc-1 respectively. The combination of $H(z)$ and SNeIa data yields the constraints $q = -0.34^{+0.05}_{-0.05}$, $H_0 = 69.18^{+0.55}_{-0.54}$kms-1Mpc-1. The consistent observational constraints on both of the parameters q and H_0 according to latest 28 points of $H(z)$ are found as $q = -0.0451^{+0.0.0614}_{-0.0625}$, $H_0 = 65.2299^{+2.4862}_{-2.4607}$, in case of Union2.1 SN data, these parameters take the values $q = -0.3077^{+0.1045}_{-0.1036}$, $H_0 = 68.7702^{+1.4052}_{-1.3754}$ [77]. Using the data set of Kumar [76] and Rani et al. [77], we choose the parameter m and develop the ranges of ω_{eff} as shown in Table. **5.1** for the model $f(R,T,Q) = R + \alpha Q + \beta T$. For $m = 1.221$, ω_{eff} is found to be -1.31601 which agrees with the observational results of Planck+WMAP+H_0 [7]. Also, for the choice of $m = 1.0473$ and $m = 1.4445$, results of ω_{eff} are consistent with the observational data of 95%(WMAP5+BAO+SN) [78] and WMAP9 [43].

$Data$	q	$H_0(kms^{-1}Mpc^{-1})$	m	ω_{eff}
$H(z)(14points)$[76]	$-0.18^{+0.12}_{-0.12}$	$68.43^{+2.84}_{-2.80}$	1.221	-0.361998
$SN(Union2)$[76]	$-0.38^{+0.05}_{-0.05}$	$69.18^{+0.55}_{-0.54}$	1.613	-0.760074
$H(z)+SN(Union2)$[76]	$-0.34^{+0.05}_{-0.05}$	$68.93^{+0.53}_{-0.52}$	1.516	-0.680739
$H(z)(29points)$[77]	$-0.0451^{+0.0614}_{-0.0625}$	$65.2299^{+2.4862}_{-2.4607}$	1.0473	-0.0903275
$SN(Union2.1)$[77]	$-0.3077^{+0.1045}_{-0.1036}$	$68.7702^{+1.4052}_{-1.3754}$	1.4445	-0.615438

Table 5.2: Data results for power law exponent m and ω_{eff} for the model $R(1+\alpha Q)$.

We consider a second model of non-minimal coupling in the $f(R,T,Q)$ modified theory in the form of model $f(R,T,Q) = R(1+\alpha Q)$. Dynamical equations are also presented in section **III**, where we consider the power law cosmology to find an expression for energy density ρ. Using Eq.(3.30), it is obvious to find the expressions of effective energy momentum tensor and its components. In power law cosmology, one can represent the cosmic history depending on the choice of parameter m. Here, we set parameter m according to the evolution of q as per recent observational data. It is to be noted that we set the choice of parameters α and m as per validity ranges expressed in Table **3.1**, where we develop the constraints on coupling parameter α for different values of m satisfying WEC and NEC. In Fig. **2.22**, we set $m = 2$ with $q = -0.5$ to see the evolution of ρ_{eff} and ω_{eff}, it is found that WEC is satisfied and $\omega_{eff} = -1$ validating the current cosmic epoch [7].

For the second model $f(R,T,Q) = R(1+\alpha Q)$ we also use the data set of Kumar [76] and Rani et al. [77], we choose the parameter m and develop the ranges of ω_{eff} as shown in Table. **5.2** for the second model $f(R,T,Q) = R(1+\alpha Q)$. For $m = 2$, ω_{eff} is found to be -0.5 which agrees with the observational results of Planck+WMAP+H_0 [7]. Also, for the choice of $m = 1.0473$ and $m = 1.4445$, results of ω_{eff} are consistent with the observational data of

95%(WMAP5+BAO+SN) [78] and WMAP9 [43].

In chapter **IV**, we have discussed the cosmological features of $f(R,T)$ models which involve both minimal and non-minimal matter geometry coupling in the presence of CM plus radiations, and compared our results NCM, and CM without radiations. Usually, in theories of gravity, we only discuss the interaction of ordinary matter and DE with zero pressure but both of them are also self-interacting. In this study we assume the new form of matter expect ordinary matter and DE, which is self interacting CM and having a positive pressure with EoS ω satisfying the relation $0 < \omega < 1$. The observational results are also supporting the concept of self-interacting CM so, its absolutely reasonable to discuss such kind of CM and its effects on cosmological evolution of the universe. Here, we focus on the transition of EoS parameter from decelerated phase to the accelerated phase and explore the crossing of phantom divide line for the considered $f(R,T)$ models. The crossing of phantom divide line is also feasible from observational data [74].

In [1], authors reconstructed nonminimally coupled $f(R,T)$ gravities in the scenario of cosmological evolution including ΛCDM, phantom or non-phantom eras and possible phase transition from decelerating to accelerating. Here, the power law reconstructed models of $f(R,T)$ gravity from [1], and discussed their cosmological evolution in the framework of NCM, CM and CM plus radiations. Our main focus is to explore the cosmological evolution of the effective EoS ω_{eff} and deceleration parameter $q(z)$ in terms of redshift. In first case we consider the $f(R,T)$ model of the form $f(R,T) = \alpha_1 R^{\gamma_1} T^{\gamma_2} + \alpha_2 T$. keeping in view the Dolgov Kawasaki instability criterion, we find $\gamma_1 > 1$ and $\alpha_1 > 0$. We took values of powers of R and T of the form (i) $\gamma_1 = 10$, $\gamma_2 = -0.7$ and (ii) $\gamma_1 = 10$, $\gamma_2 = -0.5$. For the first case $\alpha_1 R^{10} T^{-4/5} + \alpha_2 T$, we find the transition of decelerated to acceleration phase in the evolution of EoS parameter ω_{eff}, and it approaches to -1 in accordance with the recent observational data. Moreover, the behavior of deceleration parameter and ω_{DE} supports the ΛCDM model. For $\alpha_1 R^{10} T^{-1/2} + \alpha_2 T$ model, we find similar sort of behavior of parameters. For the second model of the form $f(R,T) = f(R) + f(T) = \beta_1 R^{\mu} + \beta_2 R^{\nu} + \frac{2k^2}{-1+3\omega} T + \beta_3 T^{\frac{1}{2} - \frac{\sqrt{-1+\omega(3+\omega-3\omega^2)}}{(1+\omega)\sqrt{-2+6\omega}}} + \beta_4 T^{\frac{1}{2} + \frac{\sqrt{-1+\omega(3+\omega-3\omega^2)}}{(1+\omega)\sqrt{-2+6\omega}}}$, we find that it is all the curves merge for NCM, CM and CM plus radiations. In order to settle the Dolgov Kawasaki instability we set the parameters satisfying the conditions $\mu > 1$, $\nu > 1$, $\beta_1 > 0$ and $\beta_2 > 0$. It is observed that time of transition in all the cases is almost similar. In our discussion we find the present day value of Hubble parameter $H(z)$ as 68.4 in accordance with recent Planck's results [7].

We compared our results in $f(R,T)$ with ΛCDM model and pressure less matter. We found that the results are strongly depends on model. The transition phases also varies with respect to models and cases as well like for CM, NCM, Radiations we get the transition phases at different points. It would be nice to reconstruct more consistent model as per the recent observational data.

List of Publications

- Zubair, M. Zeeshan, M. *Cosmic evolution in the background of non-minimal coupling in* $f(R, T, R_{\mu\nu}T^{\mu\nu})$ *Gravity*,
  submitted for publication. arXiv:1805.03958

- Zubair, M. Zeeshan, M. and Syed Sibet Hasan, *Late time cosmic evolution with collisional matter in* $f(R, T)$ *Gravity*,
  submitted for publication.

- Zubair, M. Zeeshan, M. and Saira Waheed, *Cosmic evolution in the background of* $R(1 + \alpha Q)$ *Gravity*,
  submitted for publication.

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