COMPUTER CODES FOR CALCULATION OF ELECTROMAGNETIC RADIATION GENERATED IN MAGNETIC FIELDS

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Abstract

Software for simulation of different characteristic of spontaneous electromagnetic radiation, generated by a relativistic electron beam in external magnetic field, is presented. It consists of different computer codes, which are in consistent with each other. These codes calculate spatial distributions of electromagnetic radiation spectral densities, spatial distributions of radiation power, electron beam emittance effects. The set of input parameters allows including the real geometry of experiment into simulations. For example, the near-field effects are taken into account. The electron beam emittance is simulated by numerical convolution of the Gaussian electron distribution in a beam with the spatial distribution of the electromagnetic radiation, generated by one electron.

INTRODUCTION

It can be assumed that the undulator has a perfect sinusoidal magnetic field, such as the planar or helical [1], elliptical [2], two-harmonic [3] and figura-8 [4] undulators. In this case, the spectral-angular density of undulator radiation in the far-field region involves a series of Bessel functions. This method is used in the computer codes SMUT [5], URGENT [6], and US, which is incorporated into the software toolkit XOP [7]. This approach has a high speed of computation as their main advantage. At the same time, they cannot work with imperfect magnetic fields, near-field effects also cannot be taken into account.

Some computer codes are able to use a real magnetic field map and compute the Fourier transformation of the radiation field numerically. One of three ways can be used. The spectral integrals may be evaluated by the saddle point method [8]. This approach uses the formulae for standard synchrotron radiation and is applicable to insertion devices with strong magnetic fields [9, 10]. This method is used in the computer code RADID [9, 11].

The second method involves the calculation of the radiation field in the time domain followed by the Fourier analysis in order to find the radiation spectrum. Such method is used in the codes B2E [12], UR [13], YAUP [14], and SPECTRA [15] and is also integrated in the code RADID [9, 11, 16, 17]. It gives direct insight into the physics of radiation emitting process and enables the calculation of radiation spectra in a wide range simultaneously, while requiring huge amount of memory for angular distribution computation [14].

The Lienard-Wiechert retarded potentials can be integrated in the frequency domain. This method is used in the codes SpontLight [18], SRW [19], WAVE [20], SPUR [21] and in a number of others programs [12, 22-26]. The main problem of this approach comes from the fast oscillating factor in the integrand, requiring the use of extreme care in integration routines. Comparative studies of these algorithms can be found in [9, 27, 28].

Emittance effects are usually simulated by the Monte Carlo method or by using the off-axis approximation [5], also known as the shift-invariant property of the radiation pattern [23]. The Monte Carlo method, generally considered to be the most accurate, is employed in a number of computer codes, such as RADID [9, 11, 16, 17], UR [13], SpontLight [18], see also papers [22, 25]. The problem with this approach is that the large number of individual computations for single-electron radiation can sometimes be too time consuming and impractical [17, 22, 29].

The off-axis approximation method is based on the concept that spatial distributions of radiation from different electrons are identical in form and are related to each other by the angle shifts arising due to angular and spatial electrons spread. If this is so, there is no need to compute the radiation for each electron separately and the computational task is simplified considerably. As we know, this idea was first proposed in the paper [30]. Owing to computational speed, this method is widely used in a number of computer codes such as SMUT [5], URGENT [6], B2E [12], YAUP [14], US [7], SRW [19], SPECTRA [15], see also [23, 24, 26]. It was believed for a long time that the off-axis approximation is valid in the far-field region only [5, 23], since the equations controlling the pattern of radiation in the near-field region are rather cumbersome. It was rigorously proven in [31] that the electromagnetic radiation, generated by a relativistic electron in an external magnetic field, has the shift-scale invariance property. This proves the applicability of the off-axis approximation in the near-field region.

This paper describes the development of a set of computer codes for modeling of spontaneous electromagnetic radiation, generated by a relativistic electron beam in external three-dimensional magnetic field. It consists of three main computer programs. The first program computes the distributions of spontaneous radiation spectral density from one electron. The second program computes the distributions of spontaneous electromagnetic radiation power. The last program simulates the electron beam emittance effects.

ANALYTICAL EXPRESSIONS

For definitenesss, we will consider here the case of undulator magnetic field. It is assumed that the electron does not lose any kinetic energy due to emission of radiation in the magnetic field. Let the *X*-axis of the right-hand Cartesian coordinate system is directed horizontally, the *Y*-axis is directed upwards and the *Z*-axis is aligned with the undulator axis. The magnetic field is presumed to be

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homogeneous in the transversal (XOY) plane and the longitudinal component of the field is also dropped, so that the undulator focusing properties are ignored. At the same time, there are no additional assumptions about the field profile so that its transversal components may include the correction fields and magnetic field errors, if necessary. In the vicinity of the undulator axis, the magnetic field can be described as $B_{y,x}(z) = \{B_x(z), B_y(z), 0\}$. The equations of motion give the following Eqs. (1) and (2) for the electron trajectory:

$$\gamma\beta_{x,y}(z) = \gamma\beta_{x,y}(0) \mp \frac{e}{mc^2}\int_0^z B_{y,x}(z')dz' , \qquad (1)$$

$$r_{x,y}(z) = r_{x,y}(0) + \int_0^z \beta_{x,y}(z')dz' , \qquad (2)$$

where c is the speed of light, e, m, $c\beta$, γ>>1, $\vec{r}$ are the electron charge, mass, velocity, reduced energy and trajectory, respectively. Equation (1) is exact, while Eq. (2) implies the paraxial approximation, i.e. the angles of the electron deflection are small. The electron's trajectory is calculated using Eqs. (1) and (2) step by step along the undulator axis. We use cubic spline interpolation of magnetic field data, which should be described in the input file. Since at every interval the magnetic field is described by the cubic polynomial, we can integrate this polynomial analytically. This method increases considerably the calculation speed.

The electrical component of the radiation field $\vec{E}(\tau)$ is given by the following exact expression:

$$\vec{E}(\tau) = \frac{e}{cR}\frac{\vec{n}\times[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{(1-\vec{n}\cdot\vec{\beta})^3} + \frac{e}{R^2}\frac{(\vec{n}-\vec{\beta})}{\gamma^2(1-\vec{n}\cdot\vec{\beta})^3} , \qquad (3)$$

where $\vec{R}(t) = \vec{X}_0 - \vec{r}(t)$ with absolute value $R(t)$ represents the distance between the emission point and the observation point $\vec{X}_0$, the unit vector $\vec{n}(t) = \vec{R}(t)/R(t)$ points from the instantaneous position of the electron to the observer. The quantities $\vec{n}(t)$, $\vec{\beta}(t)$, $\dot{\vec{\beta}}(t)$ and $R(t)$ on the right-hand side of Eq. (3) are to be evaluated at the retarded time t which must obey the equation:

$$c\tau = ct + R(t) . \qquad (4)$$

The spectral density of radiation intensity with wavelength λ is proportional to the square of the Fourier transform of the field:

$$\tilde{E}_{x,y}(\lambda) = \int_{-\infty}^{\infty} \exp(i\frac{2\pi c}{\lambda}\tau)E_{x,y}(\tau)d\tau . \qquad (5)$$

By using the relationship $d\tau = [1-(\vec{n}(t)\cdot\vec{\beta}(t))]dt$ and substituting Eq. (4) into Eq. (5), we can change the integration in Eq. (5) with respect to retarded time t. More detailed theoretical description of the calculation method used in the computer code can be found in [31].

The computer code calculates the spectrum of radiation into a rectangular aperture. The aperture is defined by the user giving the longitudinal distance from the beginning of the undulator field to the aperture, its horizontal and vertical sizes in two transverse directions and the number of points within the aperture to be calculated. Clearly, it is possible to calculate the spectrum at single observation point. The spectral range and the number of spectral points are also defined by the user.

The spatial distribution of radiation power is proportional to the square of the electric component of the emitted wave. It can be easily deduced from Eq. (3) that:

$$\left|\vec{E}(\tau)\right|^2 = \frac{e^2}{c^2\gamma^2R^2}\frac{\gamma^2(1-\vec{n}\cdot\vec{\beta})^2(\dot{\vec{\beta}})^2-(\vec{n}\cdot\dot{\vec{\beta}})^2}{(1-\vec{n}\cdot\vec{\beta})^6} \qquad (6)$$

The reduced acceleration $\dot{\vec{\beta}}(t)$ is defined by the equations of electron motion in the magnetic field. The computer code for simulation of radiation power distribution integrates numerically Eq. (6) over the undulator length. Electron trajectory is computed by using Eqs. (1) and (2). The geometry for simulation is similar to the previous case. The power distribution is calculated into a rectangular aperture, which is defined by the user, giving the longitudinal distance from the beginning of the undulator field to the aperture, its sizes in two transverse directions and the number of points within the aperture.

The shift – scale invariance of electromagnetic radiation was proven in [31]. This property establishes that it is suffice to compute the relevant single-electron radiation distribution for a number of different wavelengths λ and at different observer points with coordinates X_0 and Y_0. With knowledge of this single electron distribution, the electron beam emittance effects, its energy spread and finite width the spectral device may be included into simulation via the numerical convolution. To include all these effects, we should perform the three-dimensional integration as a maximum. This method was realized numerically in the computer code, which evaluate through a convolution the electron beaml emittance effects. The electron beam is assumed to have Gaussian distributions in angular and spatial spreads.

NUMERICAL SIMULATIONS

Electromagnetic edge radiation (ER) is produced by a relativistic charged particle in its passage through the fringe fields at the bending magnet edges. In long-wavelength spectral range (at radiation wavelengths much longer than synchrotron radiation critical wavelength) its intensity is much higher than corresponding intensity of synchrotron radiation from uniform magnetic field of the same bending magnet.

The pole of each bending magnet of Siberia-2 storage ring is divided into two parts: the long one with the main field $B = 1.7$ T (bending radii of 490.54 cm) and a short-

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er one with a quarter field *B/4*=0.425 T. The shorter part of the magnetic pole with quarter field adjoins to the long straight section. The distance between the down- and upstream edges of the bending magnets is 5340 mm. Synchrotron radiation with 7.2 keV critical energy from the homogeneous 1.7 T field is extracted by 10x10 $mrad^2$ beam lines. The radiation distributions were calculated at the following beam parameters: electron beam energy 2.5 GeV, electron beam current 100 mA, $\sigma_x = 0.72$ mm, $\sigma_{x'} = 0.11$ mrad, $\sigma_z = 0.014$ mm, $\sigma_{z'} = 0.056$ mrad. Since 0° port has a mask with entrance aperture 44 mm hor. × 16 mm vert. which is installed at 1580 mm downstream from the straight section, ER distributions were calculated in the plane of this mask.

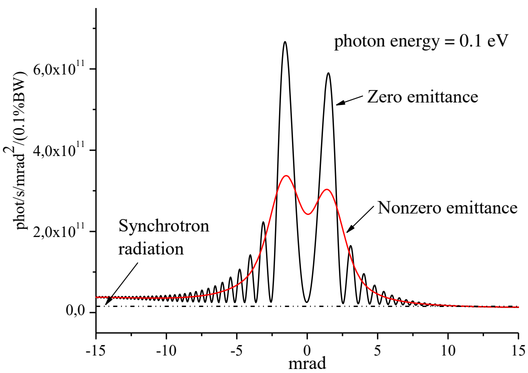

Figure 1: Horizontal distributions of edge radiation.

Figure 1 shows the computed flux density in the Siberia-2 median plane 1580 mm downstream of a straight section. The flux density with 0.1 eV photon energy is plotted versus horizontal angle. The calculations were carried out for the electron beam with zero and nonzero electron beam emittance. One can readily see that the nonzero emittance effects smooth out the fine interference oscillations. The distributions are substantially asymmetric about the straight section axis because of the relatively short distance from the screen to the straight section. The radiation distribution tends to the correspondent SR intensity as the distance from the straight section axis in the median plane increases.

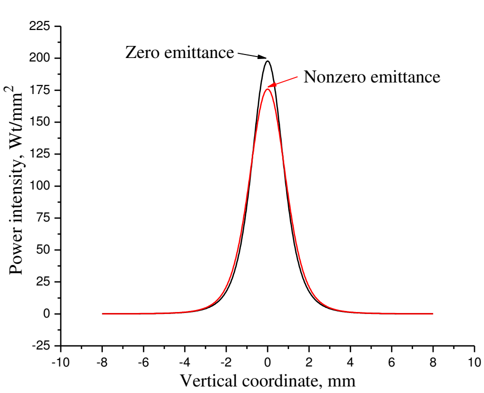

Figure 2: Wiggler radiation power distribution.

Figure 2 shows vertical distribution of radiation power (Watt/mm^2), generated by 7.5 Tesla superconducting wiggler. This wiggler is installed at Siberia-2 storage ring. It has 19 main poles (9.5 periods) and 2 half-field poles. Maximum magnetic field amplitude is equal to 7.5 Tesla. Electron beam energy 2.5 GeV, electron beam current 300 mA, distance to the screen 6816 mm.

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