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# $\alpha$-cluster structure in $^{62,64}$Ge described by the local potential approach

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**Abstract.** The unstable nuclei $^{62}$Ge and $^{64}$Ge are analyzed in terms of the $\alpha$ + core structure applying a nuclear potential with (1 + Gaussian)×(W.S. + W.S.$^3$) shape. The ground state bands of $^{62}$Ge and $^{64}$Ge and first negative parity band of $^{64}$Ge are calculated and compared with experimental data. The calculated $^{64}$Ge ground state band gives a good account of the experimental energies from $0^+$ to $8^+$ state. The rms intercluster separations and $B(E2)$ transition rates are obtained for the $^{64}$Ge ground state band and a discussion is presented.

## 1. Introduction

The $\alpha$-cluster model has been applied successfully to nuclei near the doubly closed shells assuming an $\alpha$ + core configuration [1,2]. Previous works [3–5] indicate that the $\alpha$-cluster model with local potential approach is also adequate to describe the spectroscopic properties of nuclei farther from double shell closures. However, there are many nuclei not studied in this viewpoint yet, particularly in the region between $^{40}$Ca and $^{90}$Zr. The recent work of Souza and Miyake [4] discusses the $\alpha$ + core structure in $^{46}$Cr and $^{54}$Cr using the (1 + Gaussian)×(W.S. + W.S.$^3$) nuclear potential shape and a good account of the respective ground state bands and $B(E2)$ transition rates is obtained. The present work aims to apply the same potential shape in Ge isotopes to test its efficiency in different isotopic chains of this mass region.

## 2. Preferential Ge isotopes for $\alpha$-clustering

The preferential nuclei for $\alpha$-clustering were selected applying the same criterium used in previous work [3] focusing the intermediate mass region. The variation of binding energy per nucleon was revealed to be an appropriate quantity to pick the preferential $\alpha$ + core configuration among different nuclei. This value is given by $Q_\alpha/A_T$, where $Q_\alpha$ is the $Q$-value for $\alpha$-separation and $A_T$ is the mass number of the total nucleus. The chain of even-even Ge isotopes is shown

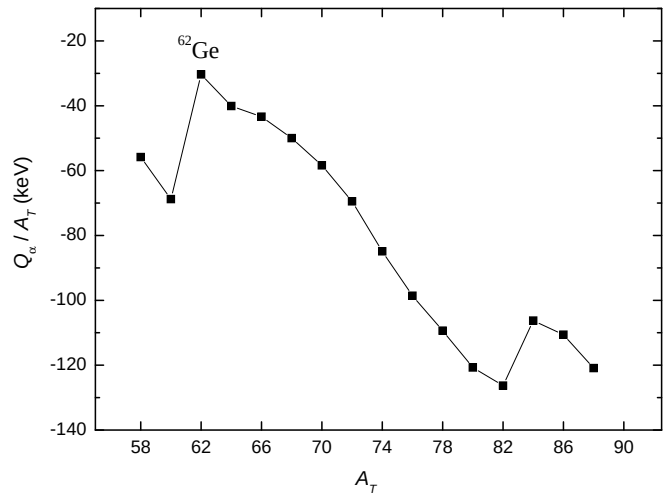


**Figure 1.** $Q_\alpha/A_T$ values obtained for the $\alpha$ + core decomposition of even-even Ge isotopes as a function of the mass number $A_T$. The $Q_\alpha/A_T$ maximum corresponding to $^{62}$Ge is indicated.

in figure 1 pointing that $^{62}$Ge is the preferential nucleus for $\alpha$-clustering in this set followed by $^{64}$Ge, with the second highest $Q_\alpha/A_T$ value.

## 3. The Model

The $\alpha$-cluster model regards the total nucleus as an $\alpha$-particle orbiting an inert core. The $^{62}$Ge and $^{64}$Ge nuclei are assumed as $\alpha$+$^{58}$Zn and $\alpha$+$^{60}$Zn systems, respectively. The $\alpha$ + core potential is the sum

$$V(r) = V_N(r) + V_C(r) \tag{1}$$

of nuclear and Coulomb terms. The Coulomb potential is described by an $\alpha$-particle interacting with an uniformly charged spherical core of radius $R$. The nuclear potential is expressed by:

$$V_N(r) = -V_0 \left[1 + \lambda \exp\left(-\frac{r^2}{\sigma^2}\right)\right] \left\{\frac{b}{1+\exp[(r-R)/a]} + \frac{1-b}{\{1+\exp[(r-R)/3a]\}^3}\right\}, \tag{2}$$

where $V_0$, $\lambda$, $a$, and $b$ are fixed parameters, and $R$ and $\sigma$ are variable parameters. The description of the ground state bands of the two nuclei is obtained with the fixed values $V_0 = 220$ MeV, $a = 0.65$ fm, $b = 0.3$, and $\lambda = 0.14$, while $R$ and $\sigma$ are fitted for each nucleus. The employed $R$ and $\sigma$ values are: $R = 4.621$ fm and $\sigma = 0.366$ fm for $^{62}$Ge, and $R = 4.647$ fm and $\sigma = 0.278$ fm for $^{64}$Ge. The values of $V_0$, $\lambda$, $a$, and $b$ were adjusted previously to describe the ground state bands of the nuclei $^{20}$Ne, $^{44}$Ti, $^{94}$Mo, and $^{212}$Po, in which the $\alpha$-cluster structure is recognized in preceding studies [1–3]. The depth $V_0$ is increased to 241 MeV only for the calculation of the $^{64}$Ge negative parity band.

The nucleons of the $\alpha$-cluster, according to Pauli principle, occupy shell-model orbitals out of the core. This restriction is defined by the global quantum number $G = 2N + L$, where $N$ is the number of internal nodes in the radial wave function and $L$ is the orbital angular momentum. In the case of the $\alpha$+$^{58}$Zn and $\alpha$+$^{60}$Zn systems we have $G \geq 12$, where $G = 12$ corresponds to the ground state band. This value is obtained from the Wildermuth condition [6] and yields an appropriate description of the $^{62}$Ge and $^{64}$Ge ground state bands. The two $\alpha$ + core systems are solved numerically to obtain the properties of the bound and quasi-bound states.

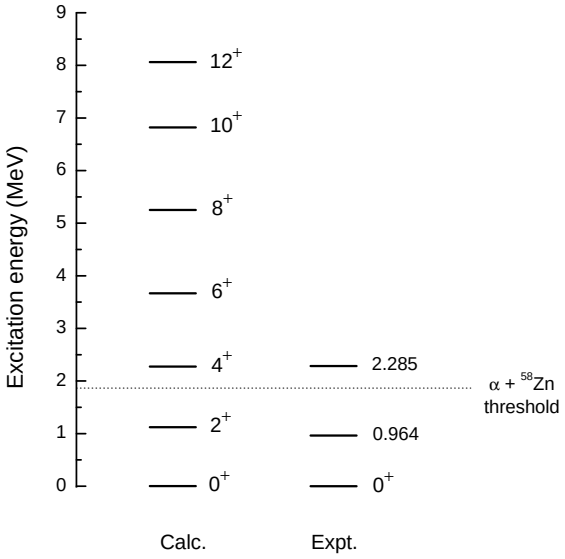


**Figure 2.** Calculated energy levels for the ground state band ($G = 12$) of the $\alpha$+$^{58}$Zn system in comparison with experimental energies of $^{62}$Ge [7]. A correspondence of the experimental energy levels 0.964 MeV and 2.285 MeV with the $2^+$ and $4^+$ states, respectively, is suggested.

| $J^\pi$ | $\langle R^2\rangle^{1/2}$ (fm) | $B(E2; J \to J-2)$ (W.u.) |
|---|---|---|
| $0^{+\,a}$ | 4.350 | |
| $2^{+\,a}$ | 4.354 | 6.574 |
| $4^{+\,a}$ | 4.312 | 8.897 |
| $6^{+\,a}$ | 4.234 | 8.542 |
| $8^{+\,a}$ | 4.136 | 7.071 |
| $10^{+\,b}$ | 4.016 | |
| $12^{+\,b}$ | 3.921 | |

[a] Properties calculated with use of the experimental energy levels.

[b] Properties calculated with use of the theoretical energy levels.

**Table 1.** Calculated rms intercluster separations ($\langle R^2\rangle^{1/2}$) and $B(E2)$ transition rates for the ground state band of $^{64}$Ge. The calculated $B(E2)$ values have been obtained without effective charges.

## 4. Results and conclusions

The $\alpha$-cluster model was applied to $^{62}$Ge and $^{64}$Ge; the energy levels of the ground state band ($G = 12$), in comparison with experimental data [7, 8], are presented in figures 2 and 3. The results show a good description of the $^{64}$Ge ground state band from $0^+$ to $8^+$ state. One should take into account that the fixed parameters $V_0$, $\lambda$, $a$, and $b$ are the same applied for nuclear spectra description of different mass regions [1, 3, 4]. The results for the negative parity band of $^{64}$Ge are not conclusive, since the respective experimental levels have undefined assignments. Unfortunately there are few experimental data on $^{62}$Ge for a consistent comparison; therefore we suggest a correspondence of the experimental energy levels 0.964 MeV and 2.285 MeV with the $2^+$ and $4^+$ states, respectively.

The calculated $B(E2)$ values for the $^{64}$Ge ground state band are presented in table 1. The formulae used to determine the $B(E2)$ values are detailed in Ref. [3]. Ref. [9] presents the experimental value $B(E2) = 27(4)$ W.u. for the $2_1^+ \to 0_1^+$ transition, which implies an effective charge $\delta e \approx 0.5\, e$ so that the model reproduces the experimental value. However, more $B(E2)$ experimental data are needed for a complete evaluation of the theoretical values.

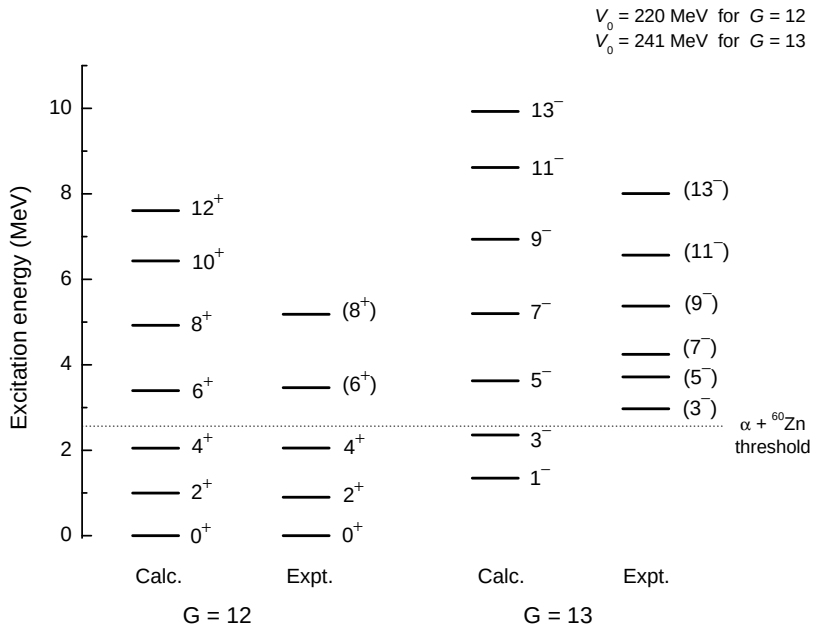


**Figure 3.** Calculated energy levels for the ground state band ($G = 12$) and negative parity band ($G = 13$) of the $\alpha$+$^{60}$Zn system in comparison with experimental energies of $^{64}$Ge [8].

The calculated intercluster rms radii for $^{64}$Ge (see table 1) indicate that the $\alpha$-cluster character is stronger for the first members of the ground state band. Such behavior (antistretching effect) has already been observed in the $^{46}$Cr and $^{54}$Cr ground state bands with the same $\alpha$ + core potential shape [4].

The present work, in addition to our previous article on Cr isotopes [4], indicates that the (1 + Gaussian)×(W.S. + W.S.$^3$) potential shape is suitable for a global description of the $\alpha$-cluster structure in the nuclei of transition region. New experimental data, mainly from $\alpha$-transfer reactions, will be welcome for comparison with our results.

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