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# Some considerations on formalism for the spin-dependent WIMP-nucleus elastic scattering

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**Abstract.** We review the formalism for spin-dependent WIMP-nucleus elastic scattering discussing the simplification obtained using the normalized spin structure functions and some applications.

## 1. Introduction

During the past year some of the numerous direct detection experiments have reported excess of events that can be indication of the interaction in the detectors of the weakly interacting massive particles (WIMP) that should form the halo of the galaxy.

In particular, the CoGENT collaboration [1] reported evidence for the annual modulation in the event rate thus confirming the long-standing evidence claimed in the last ten years by the DAMA collaboration [2]. The CRESST collaboration [3] reported an unexplained excess of events in their detector. On the other hand, the best fit areas of the three experiments in the plane $(m_\chi, \sigma^{\rm SI})$ –WIMP's mass, spin-independent (SI) WIMP–nucleon cross section– are not consistent between themselves and other experiments, CDMS [4], XENON100 [5], have reported upper limits that exclude their favoured parameter space. At present the experimental evidences are thus unclear and controversial.

Other experiments with light nuclei, for example COUPP [6], SIMPLE [7], are more sensitive to the spin-dependent (SD) scattering.

Here we review the simplification of the SD formalism obtained using the normalized spin structure functions and discuss its application in various aspects of the SD scattering [8, 9].

## 2. Simplifying the structure functions

The differential WIMP-nucleus cross section, as function of the recoil energy of the nucleus $E_R = q^2/2m_A$ being $q$ the modulus of the momentum transfer, is given by

$$\frac{d\sigma_A^{\rm SD}}{dE_R} = \frac{m_A}{2\mu_A^2 v^2}\sigma_A(0)\Phi^{\rm SD}(E_R). \tag{1}$$

$m_A$ is the mass of the nucleus with mass number $A$, $\mu_A$ the neutralino-nucleus reduced mass and $v$ the relative velocity. The total cross section at $q = 0$ reads

$$\sigma_A^{\rm SD}(0) = \frac{\mu_A^2}{\pi}\, 4\, \frac{J+1}{J}\, |a_p\langle \mathbf{S}_p\rangle + a_n\langle \mathbf{S}_n\rangle|^2. \tag{2}$$

$\langle \mathbf{S}_{p,n} \rangle$ are the spin matrix elements of the proton and neutron groups calculated in a nuclear state with maximal projection of the ground state angular momentum, $\langle \mathbf{S}_{p,n} \rangle \equiv \langle J, M_J = J | S^z_{p,n} | J, M_J = J \rangle$. The function $\Phi^{\mathrm{SD}}(E_R)$ contains the spin structure functions that depend on the momentum transfer and is normalized as $\Phi^{\mathrm{SD}}(0) = 1$.

In the standard formalism introduced by Engel in Refs. [10, 11] we have

$$\Phi^{SD}_E(E_R) = \frac{S(E_R)}{S(0)}, \tag{3}$$

$$S(E_R) = a_0^2 S_{00}(E_R) + a_0 a_1 S_{01}(E_R) + a_1^2 S_{11}(E_R), \tag{4}$$

$$S(0) = \frac{2J+1}{\pi}\frac{J+1}{J}(a_p\langle \mathbf{S}_p\rangle + a_n \langle \mathbf{S}_n\rangle)^2 \tag{5}$$

Here $a_{0,1}$ are the neutralino-proton/neutron scattering amplitudes in the isospin representation and are related to the proton-neutron representation by $a_{p,n} = (a_0 \pm a_1)/2$. At $q = 0$ $S_{ij}(0) \neq 1$, the functions are not normalized to one and the function $S_{01}$ for some nuclei can be negative. Furthermore $\Phi^{SD}_E(E_R)$ depends not only from nuclear physics but also on particle physics. This situation is different from the spin-independent cross section where $\Phi^{SI}_E(q) = F^2(q^2)$ is the nuclear form-factor.

An alternative formalism was introduced by Vergados and collaborators [12]. In this framework we can write

$$\Phi^{SD}_V(E_R) = \frac{\mathcal{F}(E_R)}{\mathcal{F}(0)}, \tag{6}$$

$$\mathcal{F}(E_R) = a_0^2 F_{00}(E_R) + 2a_0 a_1 F_{01}(E_R) + a_1^2 F_{11}(E_R), \tag{7}$$

$$\mathcal{F}(0) = a_0^2 + 2a_0 a_1 + a_1^2. \tag{8}$$

In this case $F_{ij}(0) = 1$ by construction.

In Refs. [8, 9] it is shown that the two formalisms are equivalent and connected by

$$F_{ij}(E_R) = \frac{S_{ij}(E_R)}{S_{ij}(0)}, \tag{9}$$

thus if the $S_{ij}$ are known also the $F_{ij}$ are known and vice versa.

In Fig. 1, upper panels, from left to right, we show the functions $F_{ij}(E_R)$ for one light nucleus, $^{19}$F, one medium-heavy, $^{73}$Ge, and one heavy nucleus, $^{127}$I, all of them employed in numerous experiments. In the abscissas we use the dimensionless variable $y = (qb/2)^2$ where $b = 1\,\mathrm{fm}\,A^{1/6}$ is the oscillator size parameter appearing in shell-model calculations using harmonic oscillator wave functions. The functional form of $S_{ij}$ and $F_{ij}$ is typically a polynomial times an exponential in $y$. The function $F_{ij}$ for $^{19}$F are taken from Ref. [12], the functions for $^{73}$Ge are from Ref. [13], for $^{127}$I from Ref. [14] (set calculated with the Bonn A potential). The functions are practically identical in the recoil energy interval of interest for experiments,

$$F_{00}(E_R) \simeq F_{01}(E_R) \simeq F_{11}(E_R). \tag{10}$$

Thanks to Eq. (10), Eq. (8) reduces to

$$\Phi^{SD}_V(E_R) = F_{11}(E_R). \tag{11}$$

Hence the SD "form factor" is determined by only one SSF. It does not depend on particles physics.

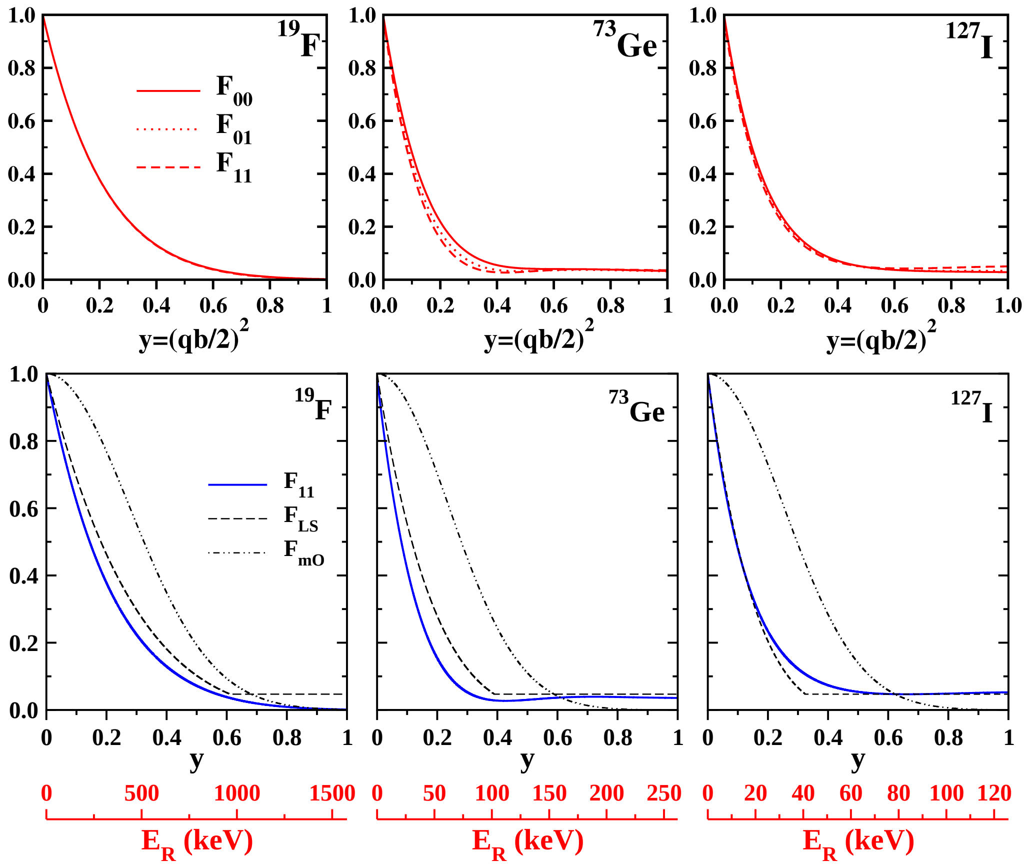


**Figure 1.** Top panels, from left to right: the momentum transfer dependent spin structure functions $F_{ij}$ for the nuclei $^{19}$F, $^{73}$Ge, $^{127}$I. Bottom panels, from left to right: blue line the normalized structure function $F_{11} = S_{11}(q)/S_{11}(0)$, for the nuclei $^{19}$F, $^{73}$Ge and $^{127}$I. The dashed line refers to the parametrization of Eq. (12) and the dashed-dotted line to the parametrization of Eq. (13). See text for details.

This simplification of the formalism is largely overlooked in literature. In some cases phenomenological parametrizations are used. One is the parametrization of Lewin and Smith given in [15, 16]

$$F_{\rm LS}(qr_n) = \begin{cases} \left(\frac{\sin(qr_n)}{qr_n}\right)^2 & qr_n < 2.55,\ qr_n > 4.5, \\ 0.047 & 2.55 \leq qr_n \leq 4.5, \end{cases} \tag{12}$$

with the nuclear radius $r_n \simeq 1.0A^{1/3}$ fm, or Gaussian parametrizations like the one used in the code micrOMEGAs [17] for the case of nuclei for which the $S_{ij}$ are not available:

$$F_{\rm mO} = \frac{S_{ij}(q)}{S_{ij}(0)} = \exp\left(-\frac{q^2 R_A^2}{4}\right), \tag{13}$$

where $R_A = 1.7A^{1/3} - 0.28 - 0.78(A^{1/3} - 3.8 + [(A^{1/3} - 3.8)^2 + 0.2]^{1/2})$. A different Gaussian parametrization is given for example in Ref. [18]. Figure 1, bottom panels, shows the normalized SSF $F_{11}$, $F_{\rm LS}$ and $F_{\rm mO}$. The approximation furnished by $F_{\rm LS}$ is reasonable both at low recoil energies and at higher energies in the region of the plateau, especially for the heavy nucleus, while the approximation furnished by $F_{\rm mO}$ is much worse in all the cases.

## 3. The problem of setting model independent upper limits

In supersymmetric models where the WIMP is the lightest neutralino the SI proton and neutron cross sections are to a very good approximation equal and the WIMP-nucleus cross section is determined only by one WIMP-nucleon cross section $\sigma^{\rm SI}$. Setting constrains on the SD couplings is complicated by the fact that the two elementary cross sections, WIMP-proton and WIMP-neutron, can differ up to 30-40% and in principle both can contribute.

The origin of the problems is the fact that in the standrd formula for the neutralino-nucleus cross section, Eqs. (6-8) the particle physics degrees of freedom $a_{0,1}$ are not factorized from the momentum dependent spin structure functions $S_{ij}$, thus when setting an upper limit one is forced to fix the neutralino "composition" by the ratio of the couplings $a_0/a_1$ or equivalently $a_p/a_n$. A way to avoid this problem was discussed in Ref. [19]. These assumptions and the method were anyway criticized in in Refs. [21, 22]. With the formalism of Section 2 we can gain some new insight.

The total cross section for a single nucleon is $\sigma^{SD}_{p,n} = 3\frac{\mu_p^2}{\pi}|a_{p,n}|^2$, thus

$$\sigma^{\rm SD}_A(0) = \left(\frac{\mu_A}{\mu_p}\right)^2 \frac{4}{3}\frac{J+1}{J}\left(\langle \mathbf{S}_p\rangle\sqrt{\sigma^{\rm SD}_p} + \varrho\langle \mathbf{S}_n\rangle\sqrt{\sigma^{\rm SD}_n}\right)^2. \tag{14}$$

In general both the SD WIMP-nucleon scattering amplitudes $a_p$ and $a_n$ and the spin matrix elements can have opposite sign, hence $\varrho = \pm 1$ is the relative sign between $|a_p\langle \mathbf{S}_p\rangle|$ and $|a_n\langle \mathbf{S}_n\rangle|$. The case of more general phases is treated in Ref. [20]. We introduce the factors

$$\phi_A = \frac{\rho_0 v_0}{m_\chi m_A}, \quad (\mathcal{C}^{p,n}_A)^2 = \left(\frac{\mu_A}{\mu_p}\right)^2 \frac{4}{3}\frac{J+1}{J}\langle \mathbf{S}_{p,n}\rangle, \quad t^{\rm SD}_A = \int \frac{dE_R}{\epsilon_0}F_{11}(E_R)\int \frac{dv}{v}f(v), \tag{15}$$

with $\rho_0$ the local dark matter density, $v_0$ the circular velocity of the Sun and $t^{\rm SD}_A$ a suitably normalized dimensionless integral over the velocity distribution and the recoil energy, see for details Ref. [9]. The expression for the total event rate thus becomes

$$R^{\rm SD} = \phi_A\left(\mathcal{C}^p_A\sqrt{\sigma^{\rm SD}_p} \pm \mathcal{C}^n_A\sqrt{\sigma^{\rm SD}_n}\right)^2 t^{\rm SD}_A. \tag{16}$$

If an experiment with exposure $\mathcal{E}_A$ set an upper limit at some confidence level on the number of events, $N^{UL}$, then this is converted in an upper limit on the cross section requiring $R_A \times \mathcal{E}_A < N^{UL}$,

$$\left(\mathcal{C}^p_A\sqrt{\sigma^{\rm SD}_p} \pm \mathcal{C}^n_A\sqrt{\sigma^{\rm SD}_n}\right)^2 < \frac{N^{UL}}{\phi_A t^{\rm SD}_A \mathcal{E}_A}. \tag{17}$$

The right-hand side of (17) is by definition the experimental upper limit on the neutralino-nucleus SD cross section. As in Ref. [19], we call it $\sigma^{\rm lim}_A$ and introduce the rescaled limts per nucleon $\sigma^{\rm lim(A)}_{p,n}$

$$\sigma^{\rm lim}_A \equiv \frac{N^{UL}}{\phi_A t^{\rm SD}_A \mathcal{E}_A}, \tag{18}$$

$$\sigma^{\rm lim(A)}_{p,n} \equiv \frac{\sigma^{\rm lim}_A}{(\mathcal{C}^{p,n}_A)^2}. \tag{19}$$

Dividing both members of (17) by (18) and using the quantities (19) we arrive at

$$\left(\frac{\sqrt{\sigma^{\rm SD}_p}}{\sqrt{\sigma^{\rm lim(A)}_p}} \pm \frac{\sqrt{\sigma^{\rm SD}_n}}{\sqrt{\sigma^{\rm lim(A)}_n}}\right)^2 < 1, \tag{20}$$

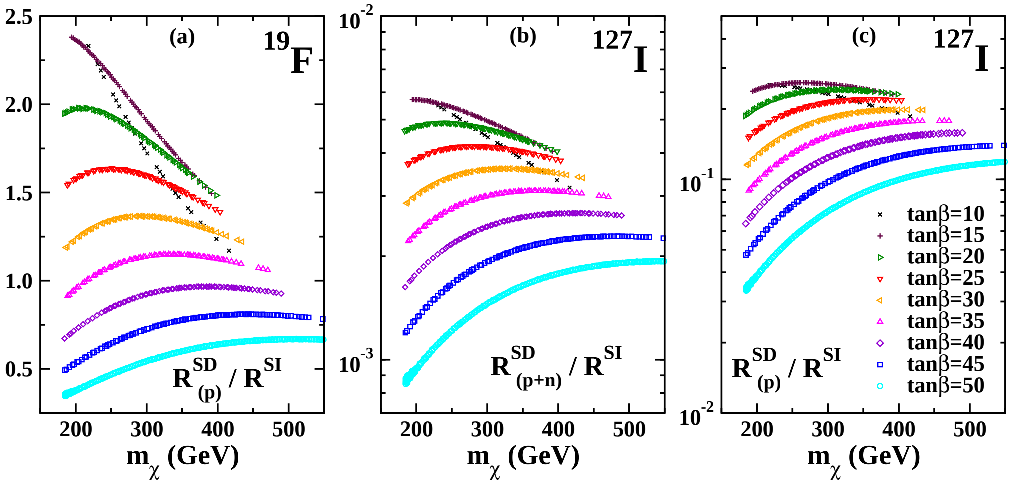


**Figure 2.** Ratio of the spin-dependent total event rate over the spin-independent rate $R^{\rm SD}/R^{\rm SI}$ varying $\tan\beta$ in the stau co-annihilation region of the CMSSM with $A_0 = 0$ and $\mu > 0$. In panel (a) for $^{19}$F. In panel (b), the ratio is plotted for $^{127}$Itaking into account both the protons and neutrons contribution in the spin-dependent rate; in panel (c) only the proton contribution is included.

that is exactly Eq. (13) of Ref. [19] in the case of the allowed region in $(\sigma_p, \sigma_n)$ plane.

In Ref. [19] the nucleon cross section limits in Eq. (19) are defined as basic quantities that then are combined to give Eq. (20). To do this it is necessary to assume that for a given nucleus it is possible to set separately limits on the SD-proton and SD-neutron cross sections even in the case that one contribution is clearly sub-dominant. In reality our derivation shows that such hypothesis are unnecessary and that the full justification of Eq. (20) only relies on the fact that as the spin-dependent form factor one can take the $F_{11}$ instead of Eq. (8). This function, included in the factor $t^{\rm SD}$, also provide the correct momentum dependent behaviour of the upper limit $\sigma^{\rm lim(A)}$.

### *3.1. The meaning of the limits on the nucleon cross sections*

At this stage the "upper limits" on the single proton or neutron cross sections, Eq. (19), must be considered only as useful quantities introduced to write Eq. (17) in the compact form (20). They become the actual experimental upper limits if, for the nucleus from which these are determined and in a specific WIMP model, one can prove that the protons contribution is dominant over the neutrons contribution or vice-versa (given the dominance of the SD rate over the SI rate).

To appreciate this point let us consider the following framework: a light nucleus, $^{19}$F, (employed in COUPP, PICASSO, SIMPLE for example) a heavy nucleus, $^{127}$I, (COUPP, DAMA, KIMS) and the stau co-annihilation region of the constrained minimal supersymmetric standard model [9].

The first nuclear shell-model calculation for $^{19}$F [23] found $\langle \mathbf{S}_p \rangle = 0.441$, $\langle \mathbf{S}_n \rangle = -0.109$. The successive calculation of Ref. [12] using a more realistic interaction, found $\langle \mathbf{S}_p \rangle = 0.4751$ and $\langle \mathbf{S}_n \rangle = -0.0087$. The protons contribution is thus similar but the neutrons contribution is clearly negligible. In the case of $^{127}$I, although proton favouring, the neutrons group contribution to the nuclear spin is of the same order of magnitude. If the neutralino couplings to the proton and neutron are similar, the neutrons contribution to the nuclear spin must be considered.

We show the ratio $R^{\rm SD}/R^{\rm SI}$ for fluorine in Fig. 2(a). The SD rate is bigger by a factor up more than 2 at low and medium $\tan\beta$ but it is smaller than the SI rate at large $\tan\beta$; in any

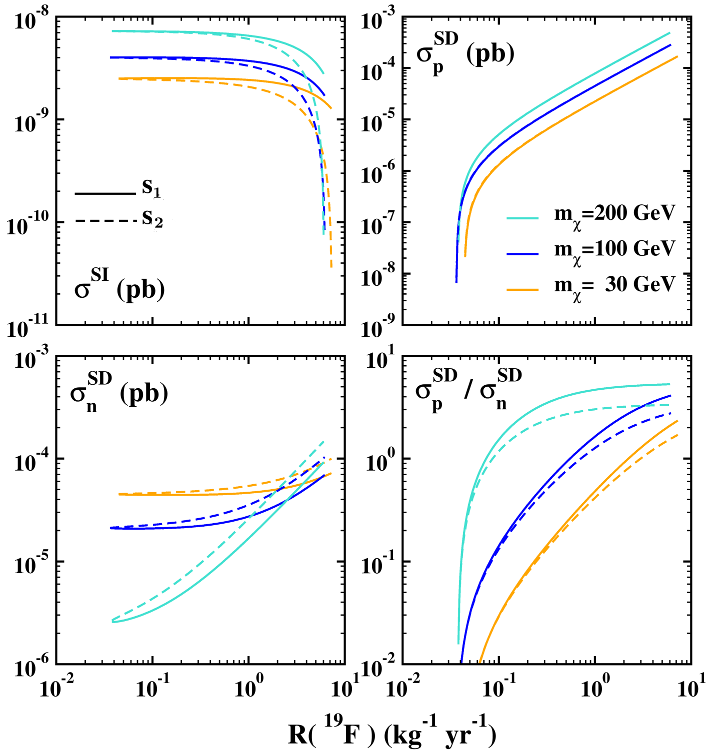


**Figure 3.** The solutions of the system Eq. (21) for the nuclei $^{127}$I, $^{73}$Ge, $^{19}$F as a function of $R(^{19}\mathrm{F})$: $s_+$, solid lines, and $s_-$, dashed lines. We fix $R(^{127}\mathrm{I}) = R(^{73}\mathrm{Ge}) = 1$ kg$^{-1}$ yr$^{-1}$ and three values of the neutralino mass.

case the two rates are always of the same order of magnitude. The SI rate cannot be completely neglected at high $\tan\beta$ and for lower $\tan\beta$, neglecting it, one underestimates the total rate.

Fig. 2(c) shows the ratio of $R^{\mathrm{SD}}/R^{\mathrm{SI}}$ in $^{127}$Ionly considering the proton contribution: due to the $A^2$ proportionality, the SI rate always dominate by a factor from 4 to 25. On the other hand in Fig. 2(b) both are included: given that $a_p < 0$ and $a_n > 0$, a cancellation in the SD rate is expected because the products $a_p\langle\mathbf{S}_p\rangle$ and $a_n\langle\mathbf{S}_n\rangle$ are of the same order and have opposite sign. This makes the SD rate from 2 to 3 orders of magnitude smaller than the SI.

In the case of the specific WIMP model examined, hence, iodine can only constrain the SI interaction, while the exclusion plots in the $(m_\chi,\, \sigma_p^{\mathrm{SD}})$ from fluorine are inaccurate In this case one has to draw an exclusion plot in the $(\sigma_p^{\mathrm{SD}},\, \sigma^{SI})$ plane for each fixed mass, the so-called mixed coupling approach [24].

## 4. Extracting the cross sections from total events rate

The simplified formalism allows to analytically determine the three elementary cross sections and connect the solutions to the relative sign between the proton and the neutron spin scattering amplitudes once the measurements of total event rate from three appropriate targets become available.

Consider again the three nuclei already discussed. Neglecting the neutron contribution in $^{19}$F and the proton contribution in $^{127}$I, and keeping both for $^{127}$I, we can write a system of three equations using the expression for the total rate in the following form:

$$\begin{cases} \sigma^{SI} + \mathcal{R}_{\rm I}\left(\Omega_p^{\rm I}\sqrt{\sigma_p^{SD}} + \varrho\Omega_n^{\rm I}\sqrt{\sigma_n^{SD}}\right)^2 - \mathcal{S}_{\rm I} = 0 \\ \sigma^{SI} + \mathcal{R}_F(\Omega_p^{\rm F})^2\sigma_p^{SD} - \mathcal{S}_{\rm F} = 0 \\ \sigma^{SI} + \mathcal{R}_{\rm Ge}(\Omega_n^{\rm Ge})^2\sigma_n^{SD} - \mathcal{S}_{\rm Ge} = 0. \end{cases} \tag{21}$$

We refer the reader to Ref. [8] for meaning and the definitions of the various factors in these expressions. Here it is sufficient to note that system is non-linear and that there exist two sets of solutions, indicated as $s_1$ and $s_2$, each set being associated with one sign of $\varrho$. We find numerically that, for the chosen nuclei $s_1$ ($s_2$) is the set of solutions associated to $\varrho = -1$ ($\varrho = +1$). As discussed in Section 3, in the stau coannihilation region, $a_p$ is negative and $a_n$ is positive, the fundamental cross sections are thus provided by the set $s_1$. An example is given in Fig. 3 where we fix $R(^{127}\mathrm{I}) = R(^{73}\mathrm{Ge}) = 1$ kg$^{-1}$ yr$^{-1}$ and three values of the neutralino mass and plot the extracted cross sections as a function of the rate in fluorine.

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