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GAUGE INVARIANT MODELS FOR TWO-BODY SCATTERING WITH THE BETHE-SALPETER LADDER APPROXIMATION*

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ABSTRACT

A natural and selfconsistent method is given in the context of the ladder approximation of the Bethe-Salpeter equation for the construction of gauge invariant models for two-body scattering involving photons. The vertices in these models are assumed to have structure and to be described by Bethe-Salpeter equations. As examples, models for the following reactions are given:

$$\gamma N \to \gamma N, \quad \gamma N \to \pi^{\overset{\pm}{0}} N, \quad \rho^{\overset{\pm}{0}} N,$$

$$\gamma \pi \to \gamma \pi, \quad \gamma \pi \to \rho^{\overset{\pm}{0}} \pi .$$

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I. INTRODUCTION

In order to explain particular experimentally observed properties of scattering amplitudes, such as the existence of fixed j-plane poles, scaling and forward peaks, it is useful to have several possible models that describe the process, so that the observed property can be traced back to features inherent in a limited class of such models. For example, the peaking of differential cross sections near the forward direction is associated with models in which cross channel exchanges are important. For reactions involving the photon, the constraint that the models be gauge invariant makes it difficult to propose test models. Although it is usually possible to propose a set of Born diagrams whose sum is gauge-invariant, any attempt to introduce particle structure by the insertion of form factors in these diagrams invariably destroys the gauge-invariance. In order, then, to restore gauge-invariance, it becomes necessary to add contact terms or contributions unmotivated by basic diagrams.[1] The prescription for finding such gauge-invariance restoring terms is often nonunique.

Within the framework of the Bethe-Salpeter equation in the ladder approximation it is possible to propose gauge-invariant models in which all contributions are motivated by scattering diagrams. In this framework structure for the particles is introduced by assuming that vertex functions satisfy Bethe-Salpeter equations. The covariant potential responsible for the structure of the particles then naturally leads to a consideration of appropriate diagrams whose inclusion in the model ensures gauge-invariance.

In the following we discuss several models beginning with the standard reaction $\gamma N \rightarrow \pi N$ which illustrates this technique. Models of the type discussed here are most appropriate for considering the existence of fixed j-plane poles in Compton scattering[2] and scaling of structure functions in deep inelastic scattering. In fact one of the models we discuss is that used by Drell and T. D. Lee.[3]

II. GAUGE-INVARIANT MODELS

To illustrate the technique for constructing gauge-invariant models with the framework of the Bethe-Salpeter equation in the ladder approximation we consider first pion photoproduction of nucleons, $\gamma_q N_p \to \pi_{q'} N_{p'}$, where the indices represent the respective four-momenta. The normal gauge-invariant model for structureless particles is given by the sum of the Born diagrams in Fig. 1. Denoting the charge of the incoming and outgoing nucleons by Q and Q', respectively, the gauge-invariant amplitude is just

$$T_\mu = \bar{u}(p')\Big[Q\gamma_5 P(p+q)\gamma_\mu + Q'\gamma_\mu P(p'-q)\gamma_5 - (Q-Q')\gamma_5\ \pi(q-q')(-2q'+q)_\mu\Big]u(p) \tag{1}$$

where $P(x)$ and $\pi(x)$ are the nucleon and pion propagators given by

$$P^{-1}(x) = -(ix\cdot\gamma + m - i\epsilon)$$

$$\pi^{-1}(x) = x^2 + p^2 - i\epsilon$$

Using the relations

$$\begin{aligned} P(p+q)\ q\cdot\gamma\ u(p) &= i\ u(p) \\ \bar{u}(p')\, q\cdot\gamma\ P(p'-q) &= -i\ \bar{u}(p') \end{aligned} \tag{2}$$

and the fact that the external particles are on their mass shells, we obtain

$$q^\mu T_\mu = i\ \bar{u}(p')\ \gamma_5\ u(p)\ [Q - Q' - (Q-Q')] = 0 \quad .$$

Structure can be introduced by assuming that the vertex functions describing the coupling of vector particles and pions to nucleons satisfy Bethe-Salpeter equations as illustrated in Fig. 2. The algebraic form of these equations is seen to be

$$\Gamma_\mu^{NVN}(p, q, p+q) \equiv \Gamma_\mu^{NVN}(p)$$

$$= Z^V \gamma_\mu + \int d^4x\, W_N(x)\; P(p+q+x)\; \Gamma_\mu^{NVN}(p+x)\; P(p+x) \quad (3)$$

and

$$\Gamma^\pi(p, q, p+q) \equiv \Gamma^\pi(p)$$

$$= \int d^4x\; W_N(x)\; P(p+x)\; \Gamma^\pi(p+x)\; P(p+q+x) \quad (4)$$

where $W_N(x)$ is a potential and Z^γ the $N\gamma N$ coupling constant. We assume that the potential is sufficiently well behaved so that the integrals (3) and (4) exist. In this model it is clear that the potential W_N coupling the nucleons should be the same for both photons as for pions. We have made the tacit assumption that this potential describes the exchange of scalar mesons. This is, of course, not necessary, for Dirac matrices, i.e., γ_5, γ_μ, etc., could be inserted before the first propagator and after the last propagator to enable the potential to describe the exchange of other types of mesons, i.e., pseudo-scalar, vector, etc. We have also made the physically reasonable assumption that the pion, π_R, is a bound state of two nucleons (i.e., of a $N\bar{N}$ pair) or two quarks and thus that it lies on a Regge trajectory. Since it is impossible to conceive of a Regge trajectory for photons, we have written an inhomogeneous Bethe-Salpeter equation for its vertex function.

Clearly the potential W_N, which couples nucleons to give structure to the vertices should also cause exchanges between the initial and final nucleons.

This leads us to consider the model shown in Fig. 3. Denoting the contributions of the individual direct channel diagrams of Fig. 3a by D^p_μ, D^n_μ, $n=1,2,3,\ldots$ and those of the crossed channel diagrams of Fig. 3b by C^p_μ, C^n_μ, $n=1,2,3,\ldots$, we write

$$D_\mu = D^p_\mu + \sum_{n=1}^{\infty} D^n_\mu \quad ,$$
$$C_\mu = C^p_\mu + \sum_{u=1}^{\infty} C^n_\mu \tag{5}$$

The photoproduction amplitude T_μ is then given by

$$T_\mu = QD_\mu + Q'C_\mu \tag{6}$$

It is interesting to notice that the assumption that the pion is a bound state of an $N\bar{N}$ pair means that the pion will appear as a bound state in the scattering process $N\bar{N} \to N\bar{N}$, and thus that the ladder diagrams shown in Fig. 3 simulate the exchange of a Reggeized pion as indicated in Fig. 4. This is not an unexpected feature since the exchange of an elementary pion was necessary to ensure gauge-invariance for structureless scattering as depicted in Fig. 1. With the help of the generalized Ward identity for $\Gamma^{N\gamma N}$,

$$i\, q^\mu \Gamma^{N\gamma N}_\mu (p, p+q, q) = P^{-1}(p+q) - P^{-1}(p) = i\, q \cdot \gamma \tag{7}$$

and the Bethe-Salpeter equation for Γ^π it can be shown that the model given in Fig. 3 is gauge-invariant. In particular, the generalized Ward identity results in each diagram giving two terms, one containing no propagators involving q and the other with just one propagator containing q. If the Bethe-Salpeter equation[4] for Γ^π is used to reexpress Γ^π in the term whose propagators are independent of q, the resulting expression will just cancel the term having one propagator containing q in the next higher diagram. As an illustration of this cancellation we consider D^p and D^1.

$$q^\mu D^p_\mu = \bar{u}(p') \, \Gamma^\pi(p') \, P(p+q) \, q^\mu \, \Gamma^\gamma_\mu(p) \, u(p)$$

$$= -i \, \bar{u}(p') \, \Gamma^\pi(p') \, P(p+q) \left\{ P^{-1}(p+q) - P^{-1}(p) \right\} u(p)$$

$$= i \, \bar{u}(p') \, \Gamma^\pi(p') \, P(p+q) \, P^{-1}(p) \, u(p)$$

$$-i \;\; \bar{u}(p') \int d^4x \; W_N(x) \; P(p'+x) \; \Gamma^\pi(p'+x) \; P(p'+q'+x) \; u(p)$$

$$= 0 - i \, \bar{u}(p') \int d^4x \; W_N(x) \; P(p'+x) \; \Gamma^\pi(p'+x) \; P(p'+q'+x) \; u(p) \tag{8}$$

Similarly

$$q^\mu D^1_\mu = \bar{u}(p') \int d^4x_1 \; W_N(x_1) \; P(p'+x_1) \; \Gamma^\pi(p'+x_1) \; P(p+q+x_1) \cdot$$

$$\cdot \; q^\mu \, \Gamma^\gamma_\mu(p+x_1) \; P(p+x_1) \; u(p)$$

$$= i \, \bar{u}(p') \int d^4x \; W_N(x) \; P(p'+x) \; \Gamma^\pi(p'+x) \; P(p+q+x) \; u(p)$$

$$-i \, \bar{u}(p') \int d^4x \; W_N(x) \; P(p'+x) \; \Gamma^\pi(p'+x) \; P(p+q) \; u(p) \quad . \tag{9}$$

Clearly, the term left over from $q^\mu D^p_\mu$ just cancels the first term in $q^\mu D^1_\mu$, since $q+p = q'+p'$. Similarly the second term in $q^\mu D^1_\mu$ cancels the first term in $q^\mu D^2_\mu$ — again with the help of the Bethe-Salpeter equation for Γ^π. This cancellation scheme is shown diagramatically in Table I. Since D_μ and C_μ are separately gauge-invariant, the model described by Fig. 3 can be used to describe charged pion as well as neutral pion photoproduction.

In the proof of gauge-invariance the fact that Γ^π described the coupling of a bound state pion to the nucleons was not essential. In fact, Γ^π could be replaced by any vertex function describing the coupling of some particle to the nucleons. For example, if Γ^π is replaced by Γ^V_ν, where Γ^V_ν describes the coupling of a vector meson to the nucleons as illustrated in Fig. 2a, the model is then a gauge-invariant description for reactions of the type

$$\gamma N \to \gamma N\,, \qquad \gamma N^{+} \to \rho^{\substack{+\\0}} N^{\substack{0\\+}}\,, \text{ etc.}$$

Since it is reasonable to consider the ρ-meson as a Regge pole and thus a bound state, Z^V for the ρ-meson would be zero. The appropriate primary diagrams and the cancellation scheme are given in Table I. The q-independent terms containing Z^V which result from the replacement of Γ_μ^V by its Bethe-Salpeter equation cancel between D^n and C^n.

It is also easy to include models where the incoming and outgoing nucleons are considered as bound states. For example, we can consider the nucleon as satisfying a homogeneous Bethe-Salpeter equation. The diagramatic form of this equation is given in Fig. 5. We then have the following equations

$$\begin{aligned} \phi(p+x) &\equiv \phi_p(x) = \int d^4x'\ V(x')\ P(p+x+x')\ \phi_p(x+x')\ \pi(x+x') \\ \bar{\phi}(p+x) &\equiv \bar{\phi}_p(x) = \int d^4x'\ V(x')\ \pi(x+x')\ \bar{\phi}_p(x+x')\ P(p+x+x') \end{aligned} \tag{10}$$

where π is the appropriate propagator of the bare meson x and P the propagator of the bare nucleon N. The potential V describes the coupling of the nonidentical particles x and N. We assume that the potential V is sufficiently well behaved so that the integrals (10) exist. Previously we assumed the existence of a potential W_N between two bare nucleons N which gives structure to the πNN, VNN, etc., vertices. Similarly there should exist a potential W_x to describe structure at πxx, Vxx, etc., vertices. The potentials V and W_N lead to basic or primary diagrams for the reactions

$$\gamma N_R \to \pi_R N_R\,, \qquad \gamma N_R \to V_z N_R\,, \qquad \gamma \pi_R \to V_z \pi_R$$

(π_R a bound state of an $N\bar{N}$ pair) as shown in Table I. We also show in Table I the ladder diagrams which have to be added in order to make the model gauge-invariant. The gauge-invariance cancellation scheme is again shown in the last column. Here D^{n0} is a diagram containing n horizontal lines each denoting the potential W_N between bare nucleons; D^{0n} is a corresponding diagram containing n vertical lines with each denoting the potential V between a bare nucleon N and a bare meson X. As an example we consider the cancellation of D^{00} by contributions from D^{10} and D^{01}. The contribution of the primary direct diagram D^{00} multiplied by q^μ is

$$\int d^4x\ \bar{\phi}_{p'}(x)\ P(p'+x)\ \Gamma^\pi(p'+x)\ P(p+q+x)\ q^\mu\,\Gamma_\mu^{N\gamma N}(p+x)\ P(p+x)\ \phi_p(x)\ \pi(x)$$

Replacing $q^\mu\Gamma_\mu^{N\gamma N}(p+x)$ by $iP^{-1}(p+x) - iP^{-1}(p+q+x)$ and using (4) and (10), this expression may be written

$$i\int d^4x\ \bar{\phi}_{p'}(x)\ P(p'+x)\ \Gamma^\pi(p'+x)\ P(p+q+x)\left\{\int d^4x_1\, V(x_1)\ P(p+x+x_1)\ \phi_p(x+x_1)\ \pi(x+x_1)\right\}\pi(x)$$
$$-i\int d^4x\ \bar{\phi}_{p'}(x)\ P(p'+x)\left\{\int d^4x_1 W_N(x_1)\ P(p'+x+x_1)\ \Gamma^\pi(p'+x+x_1)\ P(p'+q'+x+x_1)\right\}P(p+x)$$
$$\cdot\ \phi_p(x)\ \pi(x) \qquad (11)$$

The first of these two terms is cancelled by a contribution from D^{01}. This diagram multiplied by q^μ is seen to give on using (7)

$$i\int d^4x\, d^4x_1\ \bar{\phi}_{p'}(x)\ P(p'+x)\,\Gamma^\pi(p'+x)\,P(p+q+x)\,P(p+q+x+x_1)\,\phi(x+x_1)\ \pi(x+x_1)\,\pi(x)\ V(x_1)$$
$$-i\int d^4x\, d^4x_1\ \bar{\phi}_{p'}(x)\,P(p'+x)\,\Gamma^\pi(p'+x)\,P(p+q+x)\,P(p+x+x_1)\,\phi(x+x_1)\ \pi(x+x_1)\ \pi(x)\ V(x_1)$$

Similarly D^{10} yields on using (7)

$$i \int d^4x\, d^4x_1\, \bar{\phi}_{p'}(x)\, P(p'+x)\, P(p'+x+x_1)\, \Gamma^{\pi}(p'+x+x_1)\, P(p+q+x+x_1)\, P(p+x)\, \phi_p(x)\, \pi(x)\, W_N(x_1)$$

$$-i \int d^4x\, d^4x_1\, \bar{\phi}_{p'}(x)\, P(p'+x)\, P(p'+x+x_1)\, \Gamma^{\pi}(p'+x+x_1)\, P(p+x+x_1)\, P(p+x)\, \phi_p(x)\, \pi(x)\, W_N(x_1)$$

Clearly the first of these terms cancels the second term in (11). The cancellation of the remaining terms proceeds in the manner indicated in Table I, row 3.

If we replace the pion vertex by a vector meson vertex we obtain — in an analogous manner — the gauge-invariance cancellation scheme shown in row 4 of Table I. Unless the vector meson is composite, i.e., a ρ meson, the sum of the "D" (direct) diagrams and the sum of the "C" (crossed) diagrams are not individually gauge-invariant. The primary diagrams for reactions of the type we have just been considering are box diagrams. The additional diagrams necessary for gauge-invariance are all planar exchanges possible within the boxes, i.e., the sum of all diagrams with n horizontal exchanges W_N and of all diagrams with n vertical exchanges V_N for n=1 to infinity. If we let $W \to 0$ in the case of Compton scattering, the resulting model is that of Drell and Lee.[3]

As another example we can consider the case when the meson X carries an electric charge and couples to the photon. In this case the structure of vertices of the form $\pi XX, VXX$ is described by a potential W_x, and we can consider models for the reactions $\gamma N_R \to \pi_R N_R$ and $\gamma N_R \to V_z N_R$ where π_R and $V_R(Z=0)$ would be considered as bound states of two X-particles. In the case of Compton scattering, in addition to the primary diagrams D^p and C^p a primary seagull diagram S^p must be added, and gauge-invariance requires the addition — to planar ladder diagrams obtained from D^p and C^p by inserting W_x and V rungs — of a corresponding ladder of seagull diagrams obtained by inserting any number of rings W_x between the meson lines. This model can describe the following reactions in a gauge-invariant form:

$$\gamma N_R^{\overset{+}{o}} \rightarrow (\rho_R^o, \gamma)\, N_R^{\overset{+}{o}}, \qquad \gamma N_R^+ \rightarrow \rho_R^+ N_R^o, \qquad \gamma N_R^o \rightarrow \rho_R^- N_R^+ \tag{12}$$

Since the proof of the gauge-invariance of these models is not at all trivial, we indicate the main steps. Consider the diagram for D^{0n} (see Table I) multiplied by q_λ. Its contribution to the amplitude may be written

$$\begin{aligned} D_\mu^{0n} &= -\bar{V}_n \left[\pi(\epsilon_n - p)\, q^\mu \Gamma_\mu^{X\gamma X}(q, p-\epsilon_n)\, \pi(p+q-\epsilon_n)\right] \phi_p(\epsilon_n - p) \\ &= -\bar{V}_n \;\; \pi(\epsilon_n - p)\;\; \phi_p(\epsilon_n - p) + \bar{V}_{n+1}\, \pi(\epsilon_{n+1} - p)\;\; \phi_p(\epsilon_{n+1} - p) \end{aligned} \tag{13}$$

where

$$\epsilon_n = x + \sum_{i=1}^{n} x_i$$

and

$$\begin{aligned} \bar{V}_n &= \bar{V}_{n-1} \int dx_n^4\, V(x_n)\, P(\epsilon_n)\, \pi(\epsilon_{n-1} - p - q) \quad , \\ \bar{V}_0 &= \int dx^4 \Gamma_\mu(x-p')\;\; \pi(x-p')\;\; \bar{\phi}_{p'}(x-p')\;\; P(x) \end{aligned} \tag{14}$$

In obtaining (13) we have used (10) and the generalized Ward identity

$$q^\mu\, \Gamma_\mu^{X\gamma X}(q, p) = \pi^{-1}(p+q) - \pi^{-1}(p) \tag{15}$$

where in the case of the bare point interaction $q^\mu \Gamma_\mu^{X\gamma X}(q, p) = q \cdot (2p+q)$. From (13) it follows that

$$\begin{aligned} \sum_{n=0}^{\infty} D_\mu^{0n} = -\int dx^4 \Gamma_\mu(x-p')\;\; \pi(x-p')\;\; \bar{\phi}_{p'}(x-p')\;\; P(x)\, \cdot \\ \cdot\; \phi_p(x-p)\;\; \pi(x-p) \end{aligned} \tag{16}$$

Proceeding in a similar manner we find for the contributions of the crossed terms

$$\sum_{n=0}^{\infty} C_\mu^{0n} = \int dx^4 \Gamma_\mu(x-p+q')\;\; \pi(x-p)\, \phi_p(x-p)\, P(x)\, \pi(x-p)\, \bar{\phi}_{p'}(x-p') \tag{17}$$

Next we consider the terms containing horizontal rungs W_x. The contribution of the direct channel diagram containing n rungs is

$$D_\mu^{n0} = -\overline{W}_n \ \pi(\epsilon_n-p') \Gamma_\mu(\epsilon_n-p') \pi(\epsilon_n-p) q^\mu \Gamma_\mu^{X\gamma X}(q, p-\epsilon_n) \pi(\epsilon_n-p-q) \quad (18)$$

where

$$\begin{aligned} \overline{W}_n &= \overline{W}_{n-1} \int dx_n^4 W(x_n) \pi(\epsilon_{n-1}-p) \ \pi(\epsilon_{n-1}-p') \quad , \\ \overline{W}_0 &= \int dx^4 \phi_p(x-p) \ P(x) \ \bar{\phi}_{p'}(x-p') \end{aligned} \quad (19)$$

Proceeding as before but using now the first of the vector vertex equations

$$\begin{aligned} \Gamma_\mu(x-p') &= \int dx'^4 \pi(x'+x-p') \pi(x'+x-p-q) \Gamma_\mu(x'+x-p') \ W(x') + Z(2x-2p'-q') \\ \Gamma_\mu(x-p+q') &= \int dx'^4 \pi(x'+x-p) \pi(x'+x-p+q') \ \Gamma_\mu(x'+x-p+q') W(x') + Z(2x-2p+q') \end{aligned}$$

(20)

we find that (18) may be written

$$\begin{aligned} D_\mu^{n0} = {} & \overline{W}_n \ \pi(\epsilon_n-p') \ \Gamma_\mu(\epsilon_n-p') \pi(\epsilon_n-p-q) \\ & - Z\pi(\epsilon_n-p') \ \overline{W}_n(2\epsilon_n-2p'-q')_\mu \ \pi(\epsilon_n-p) \\ & - \Gamma_\mu(\epsilon_{n+1}-p') \ \overline{W}_{n+1} \ \pi(\epsilon_{n+1}-p') \ \pi(\epsilon_{n+1}-p-q) \end{aligned} \quad (21)$$

The contribution of the corresponding crossed term is similarly found to be

$$\begin{aligned} C_\mu^{n0} = {} & -\overline{W}_n \ \pi(\epsilon_n-p) \ \Gamma_\mu(\epsilon_n-p+q') \ \pi(\epsilon_n-p'+q) \\ & + Z\pi(\epsilon_n-p) \ \overline{W}_n(2\epsilon_n-2p+q')_\mu \ \pi(\epsilon_n-p') \\ & + \Gamma_\mu(\epsilon_{n+1}-p+q') \ \overline{W}_{n+1} \ \pi(\epsilon_{n+1}-p') \ \pi(\epsilon_{n+1}-p+q') \end{aligned} \quad (22)$$

The appropriate contribution of the seagull diagram containing n horizontal rungs W_x is

$$S_\mu^n = -2q_\mu \ \overline{W}_n \ Z\pi(\epsilon_n-p) \ \pi(\epsilon_n-p') \quad (23)$$

Using (19) and (20) one now finds that

$$\sum_{n=1}^{\infty}\left(D_{\mu}^{n0}+S_{\mu}^{n}+C_{\mu}^{n0}\right) = -S_{\mu}^{0} - \sum_{n=0}^{\infty}\left(D_{\mu}^{0n}+C_{\mu}^{0n}\right)$$

which proves our claim.

Finally it is also possible to replace the incoming and outgoing nucleons by pions. This results in gauge-invariant models for $\gamma\pi \rightarrow V_Z\pi$ (Table I, row 6) and $\gamma\pi_R \rightarrow V_Z\pi_R$ (Table I, row 7), where in the last reaction the vector particle if it is a ρ and the pion are considered as bound states of two nucleons. The vertex functions for the two external pions may be written

$$\begin{aligned}\Gamma(p+x) &= \int d^4x_1\, W(x_1)\; P(p+x+x_1)\; \Gamma(p+x+x_1)\; P(x+x_1)\\ \bar{\Gamma}(p+x) &= \int d^4x_1\; W(x_1)\; P(x+x_1)\; \Gamma(p+x+x_1)\; P(p+x+x_1)\end{aligned} \qquad (24)$$

The vertex function for the external vector particle is, of course, given by (3). The model describes in a gauge-invariant manner the reactions

$$\begin{aligned}\gamma\pi_R^{\pm} \rightarrow (\gamma, \rho_R^{o})\, \pi_R^{\pm}\,, \qquad \gamma\pi_R^{\pm} \rightarrow \rho_R^{\pm}\pi_R^{o}\,,\\ \gamma\pi_R^{o} \rightarrow \rho_R^{\pm}\pi_R^{\mp}\end{aligned} \qquad (25)$$

III. CONCLUSION

It has been possible to construct within the framework of the Bethe-Salpeter equation in the ladder approximation gauge-invariant models containing structured vertices for reactions of the form $\gamma N \rightarrow \pi_R N$, $V_Z N$ where the nucleon N could be considered either elementary or as a bound state and $\gamma\pi \rightarrow V_Z\, \pi$ where the pion π could be considered either elementary or as a bound state. Clearly a bound state pion model will provide a gauge-invariant description of $\gamma\pi_R \rightarrow \pi_R\pi_R$. Thus we have presented a method of constructing gauge-invariant models for an apparently unlimited variety of photonic two-body reactions that is natural and self-consistent. The models are useful for the investigation of numerous properties of photonic reactions such as, for instance, their behavior in the deep Regge region in analogy to the investigations of Blankenbecler et al.[4] We remark finally that two special cases of the models discussed here are implicit in the work of Brodsky et al.[5] and Scott.[6]

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TABLE CAPTION

I. Gauge invariance cancellation schemes.

FIGURE CAPTIONS

1. Primary diagrams for $\gamma N \rightarrow \pi N$ (structureless particles).
2. Bethe-Salpeter equations for photon and pion vertices assuming the pion to be a bound state of two nucleons.
3. Diagrams necessary to give a gauge-invariant model for $\gamma N \rightarrow \pi_R N$.
4. Simulation of a Reggeized pion.
5. Bethe-Salpeter equation for the nucleon N_R considered as a bound state of elementary particles N and X.

TABLE I

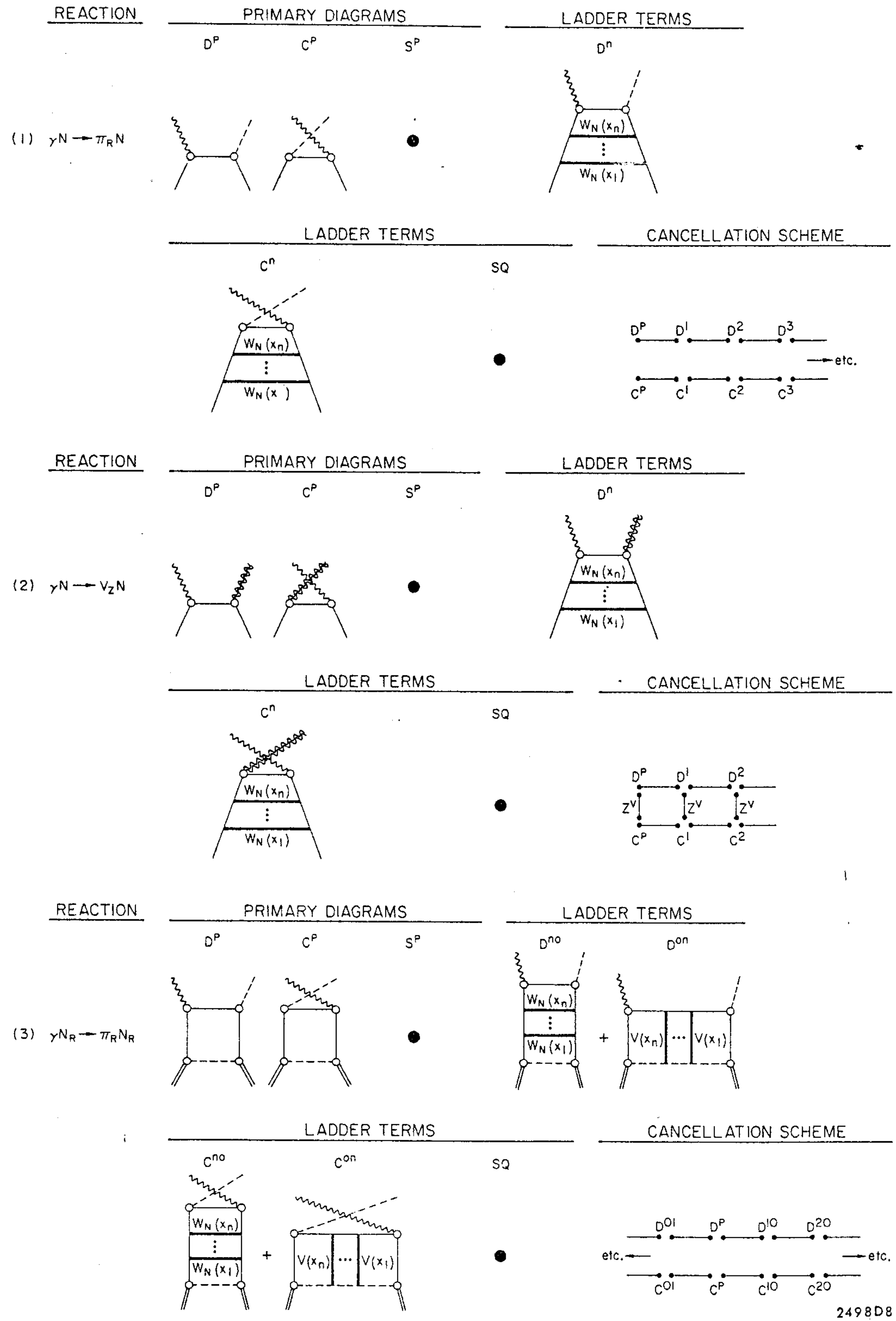
REACTION
PRIMARY DIAGRAMS
LADDER TERMS
D^P C^P S^P
D^n
(1) γN → π_R N
W_N (x_n)
W_N (x_1)
LADDER TERMS
CANCELLATION SCHEME
C^n
SQ
W_N (x_n)
W_N (x)
D^P D^1 D^2 D^3
etc.
C^P C^1 C^2 C^3
REACTION
PRIMARY DIAGRAMS
LADDER TERMS
D^P C^P S^P
D^n
(2) γN → V_Z N
W_N (x_n)
W_N (x_1)
LADDER TERMS
CANCELLATION SCHEME
C^n
SQ
W_N (x_n)
W_N (x_1)
D^P D^1 D^2
Z^V Z^V Z^V
C^P C^1 C^2
REACTION
PRIMARY DIAGRAMS
LADDER TERMS
D^P C^P S^P
D^no D^on
(3) γN_R → π_R N_R
W_N (x_n)
W_N (x_1)
+
V(x_n) ··· V(x_1)
LADDER TERMS
CANCELLATION SCHEME
C^no C^on
SQ
W_N (x_n)
W_N (x_1)
+
V(x_n) ··· V(x_1)
D^01 D^P D^10 D^20
etc. etc.
C^01 C^P C^10 C^20
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Table I (cont'd) - 2

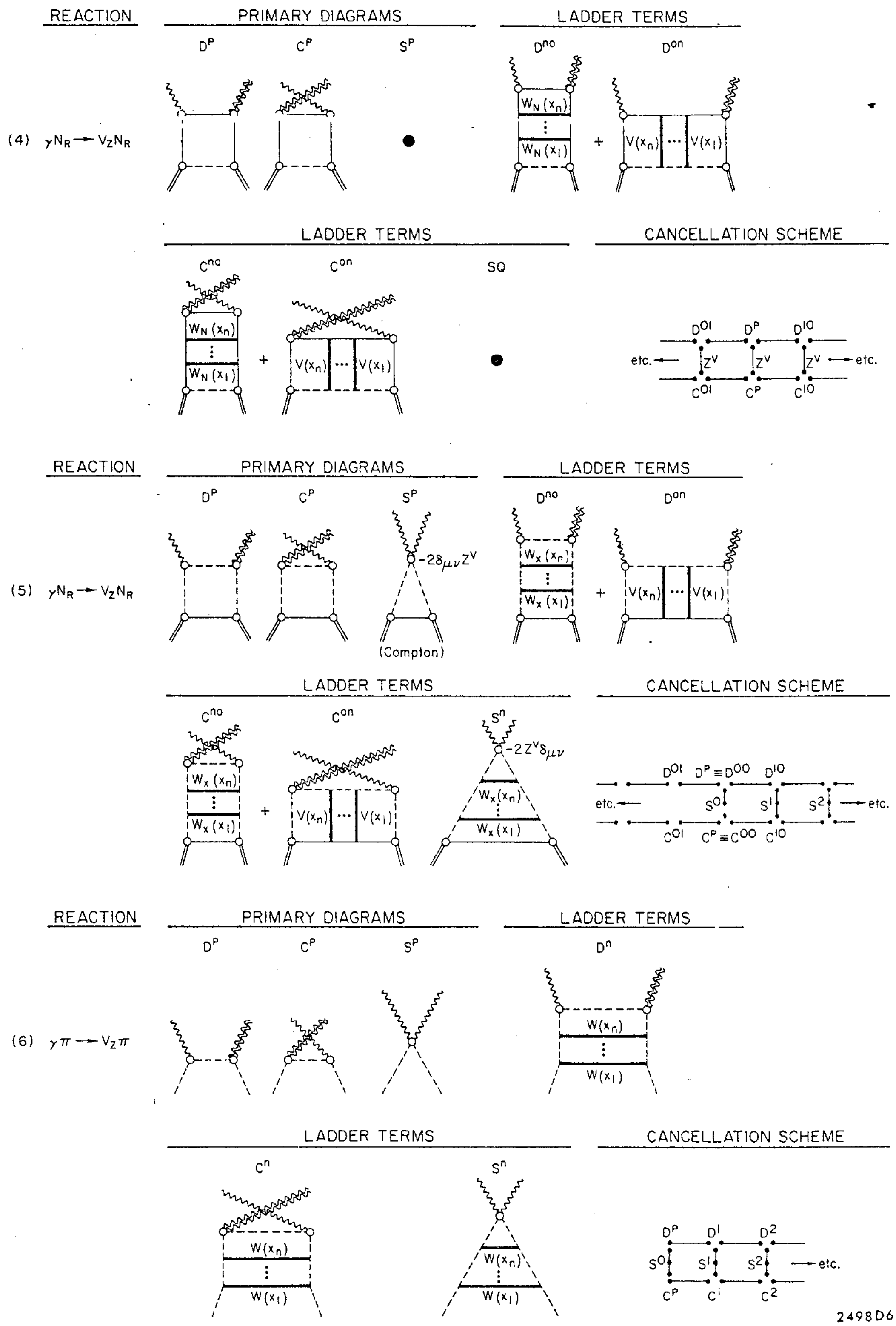
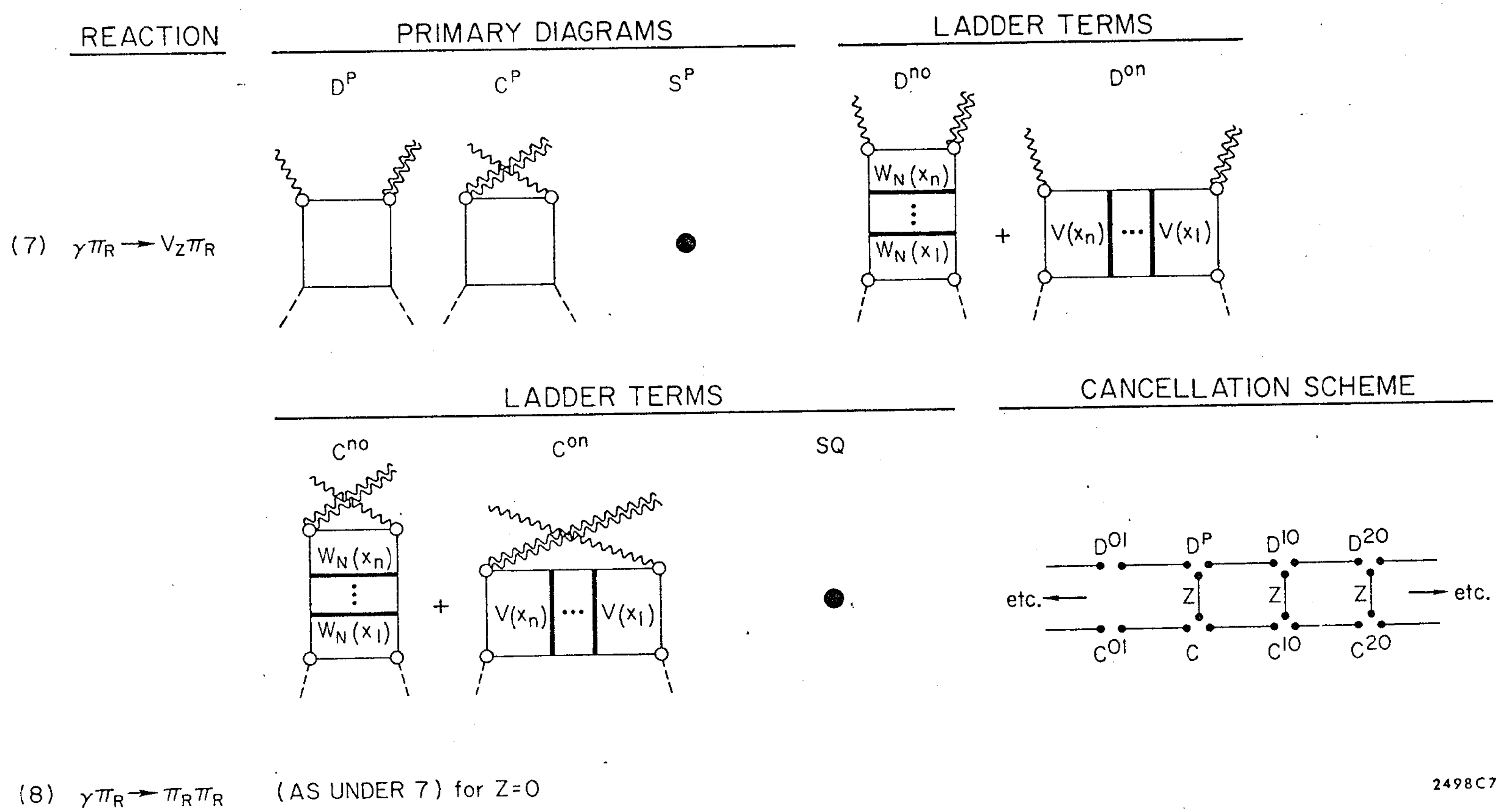


Table I (cont'd) - 3



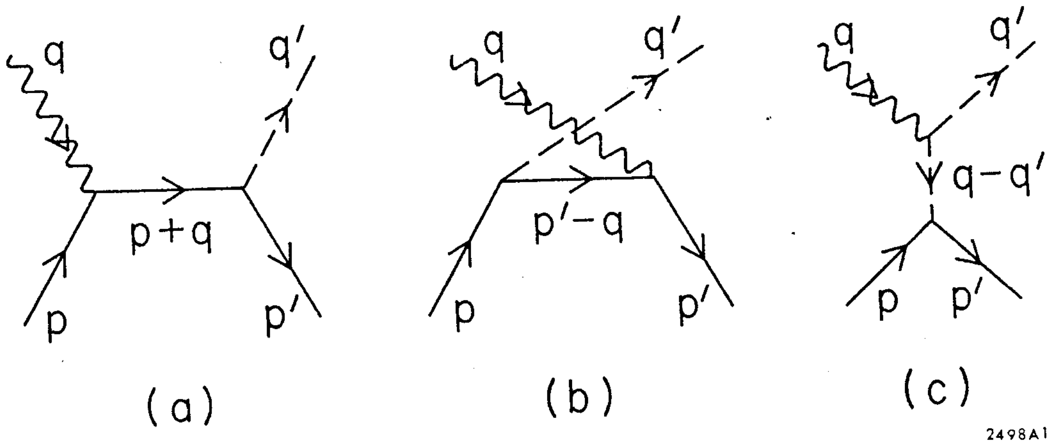


Fig. 1

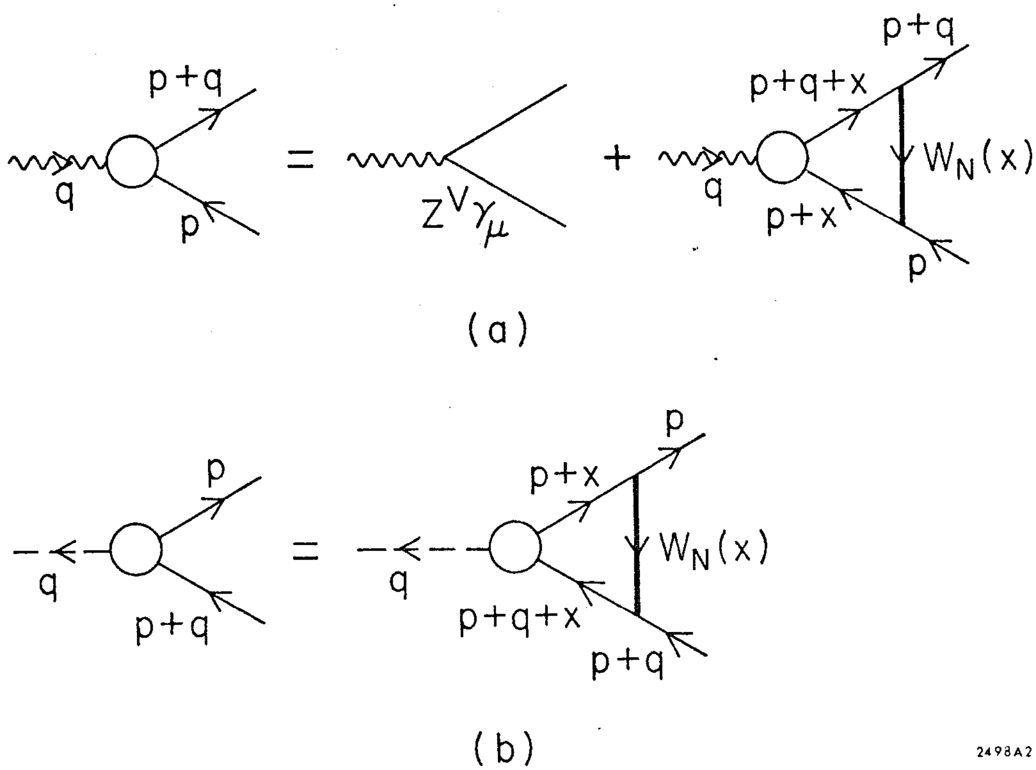


Fig. 2

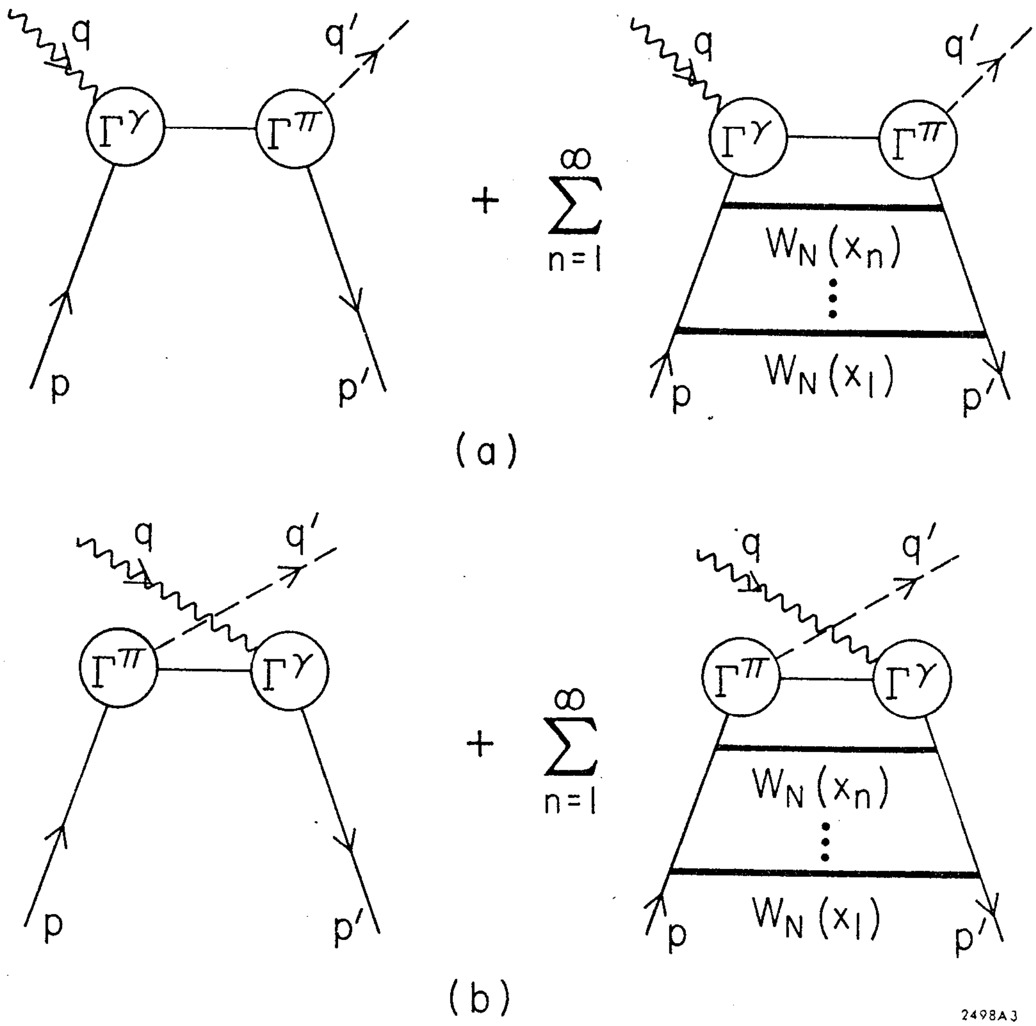


Fig. 3

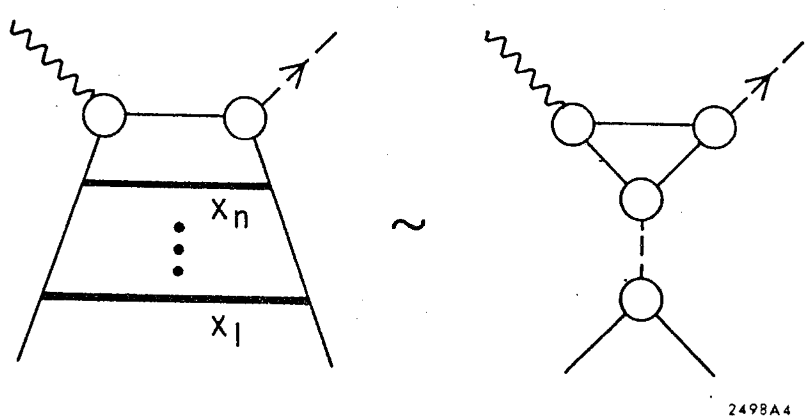


Fig. 4

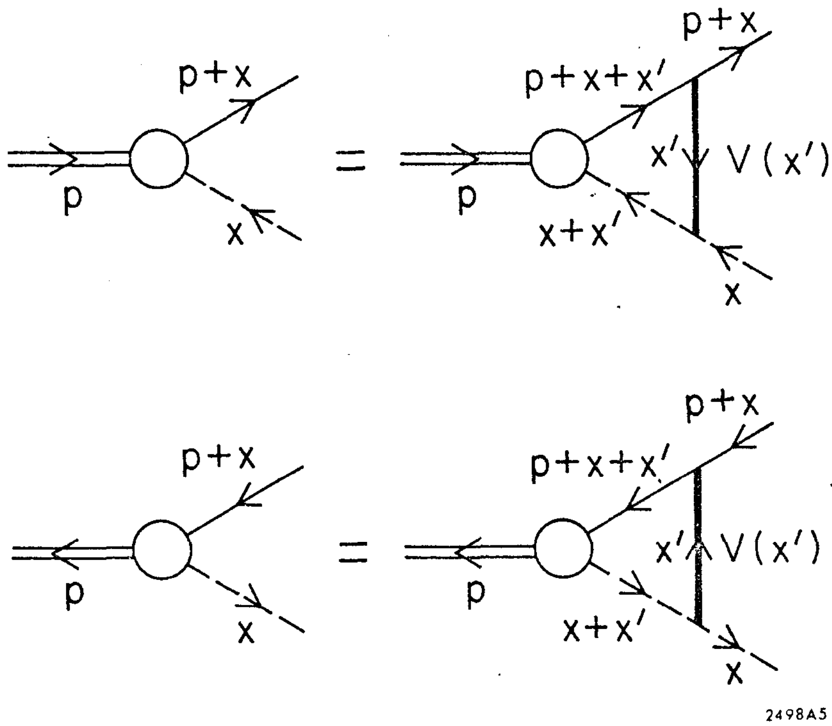


Fig. 5