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BEAM DYNAMICS IN g-2 STORAGE RING

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Abstract

The muon anomalous magnetic moment has played an important role in constraining physics beyond the Standard Model. The Fermilab Muon g-2 Experiment has a goal to measure it to unprecedented precision: 0.14 ppm. To achieve this goal, we must understand the beam dynamics systematic effects in the muon storage ring. We will present the muon beam dynamics and discuss two specific topics here: the beam resonance which is related to the muon loss and the fast rotation analysis to determine the muon momentum distribution.

INTRODUCTION

Charged elementary particles with half-integer intrinsic spin have a magnetic dipole moment aligned with their spin:

$$\vec{\mu} = g\frac{q}{2m}\vec{s} \tag{1}$$

where q is the electric charge and m is the mass. Dirac theory predicts the gyromagnetic ratio $g = 2$ [1,2], but hyperfine structure experiments conducted in the 1940's showed that $g \neq 2$ [3,4]. Schwinger introduced radiative corrections in 1948 to resolve this discrepancy [5,6]. His calculation agreed with the experimental results and motivated the exploration of more corrections to the magnetic dipole moment anomaly defined by $a \equiv (g-2)/2$.

Standard Model (SM) contributions of the muon anomaly a_μ come from the quantum electrodynamics, electroweak and quantum chromodynamics sectors. a_μ^{SM} has been calculated to a precision of around 0.42 parts per million (ppm) [7]. The most recent measurement of a_μ reported by the Brookhaven National Laboratory (BNL) E821 experiment achieved a precision of 0.54 ppm [8], which differs with SM prediction by about 3.6 standard deviation. This has motivated further theoretical and experimental investigation of a_μ.

The upcoming E989 experiment at Fermi National Accelerator Laboratory (Fermilab) has restored and will use the E821 muon storage ring to measure the a_μ with a goal of 0.14 ppm [9]. To achieve this goal, we must have a detailed understanding of the muon beam dynamics in the storage ring to lower the associated systematic uncertainties. In the following sections, we will present the principles of the experiment and the muon beam dynamics in the g-2 storage ring. We will discuss beam resonance and how a fast rotation analysis is used to determine the muon momentum distribution.

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PRINCIPLES OF THE EXPERIMENT

E989 will measure a_μ by using the spin precession resulting from the torque experienced by the muon magnetic moment when place in an external magnetic field. The rate of change of the component of spin $\vec{s}$ parallel to the velocity ($\vec{\beta} = \vec{v}/c$) is given by

$$\frac{d}{dt}(\hat{\beta}\cdot\vec{s}) = -\frac{e}{mc}\vec{s}_\perp \cdot [(\frac{g}{2}-1)\hat{\beta}\times\vec{B} + (\frac{g\beta}{2}-\frac{1}{\beta})\vec{E}] \tag{2}$$

where $\hat{\beta}$ is the unit vector in the direction of $\vec{\beta}$ and $\vec{s}_\perp$ is the component of $\vec{s}$ perpendicular to the velocity [10].

The experiment uses four Electrostatic Focusing Quadrupoles (ESQ) to provide the vertical focusing of the muon beam. For a constant and purely vertical $\vec{B}$, the anomalous precession frequency defined by the difference of the spin procession frequency and muon cyclotron rotation frequency is given by

$$\vec{\omega}_a = -\frac{q}{m}[a_\mu\vec{B} - a_\mu(\frac{\gamma}{\gamma+1})(\vec{\beta}\cdot\vec{B})\vec{\beta} - (a_\mu - \frac{1}{\gamma^2-1})\frac{\vec{\beta}\times\vec{E}}{c}] \tag{3}$$

The second term in Eq. (3) is related to pitch correction (if $\vec{\beta}\cdot\vec{B} \neq 0$) and the third term is related to electric field correction. The third term in Eq. (3) vanishes by choosing the "magic" momentum $p_{magic} = m/\sqrt{a_\mu} \simeq 3.09$GeV/c. However, the muon beam has a momentum spread, and so an electric field correction is still needed. If the muon has a magic momentum and its velocity is also perpendicular to the magnetic field, Eq. (3) then becomes $\vec{\omega}_a = -a_\mu q\vec{B}/m$.

The magnetic field is measured with nuclear magnetic resonance (NMR), where the calibrated Larmor precession frequency of a free proton is $\vec{\omega}_p = (g_p q\vec{B})/(2m_p)$. Combining $\vec{\omega}_a$ and $\vec{\omega}_p$ yields:

$$a_\mu = \frac{\omega_a/\omega_p}{\lambda - \omega_a/\omega_p} \tag{4}$$

where $\lambda = \mu_\mu/\mu_p$ is the muon-to-proton magnetic moment ratio externally determined from hyperfine splitting in muonium [11].

BEAM DYNAMICS

Polarized muons with positive charge are injected into the storage ring through the "Inflector" magnet. The "Kicker" magnet will kick muons onto the closed orbit. The muon storage ring uses ESQ for weak vertical focusing, of which the field index n with dipole magnetic field B and electric gradient $\partial E_R/\partial R$ is defined as:

$$n(s) = \frac{R}{\beta B}\frac{\partial E_R(s)}{\partial R}, \qquad \beta = v/c \tag{5}$$

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where s is the longitudinal coordinate with $\theta = s/R$ as the azimuthal angle. The muon equations of motion are then approximated by $\frac{d^2x}{d\theta^2} + (1-n)x = 0$ and $\frac{d^2y}{d\theta^2} + ny = 0$.

There are four symmetrically located quadrupole regions, where the electric potential map of the cross section is as shown in Fig. 1. As an approximation, the horizontal and vertical tunes are given by $\nu_x = \sqrt{1-n}$ and $\nu_y = \sqrt{n}$ for an ideal weak-focusing storage ring, where details about the lattice design of the g-2 storage ring and beam dynamics are discussed in Ref. [12].

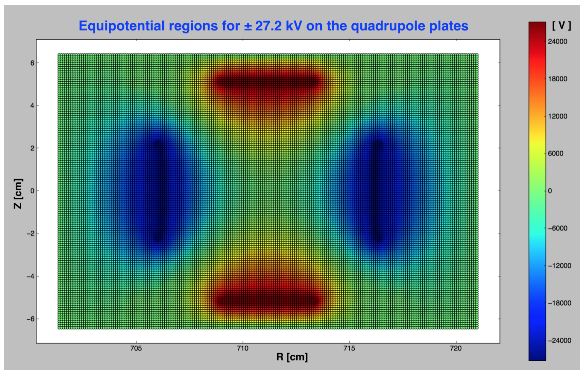

Figure 1: Electric equipotential map of quadrupoles from OPERA3D [13] with ±27.2 kV on the quadrupole plates and azimuthal average.

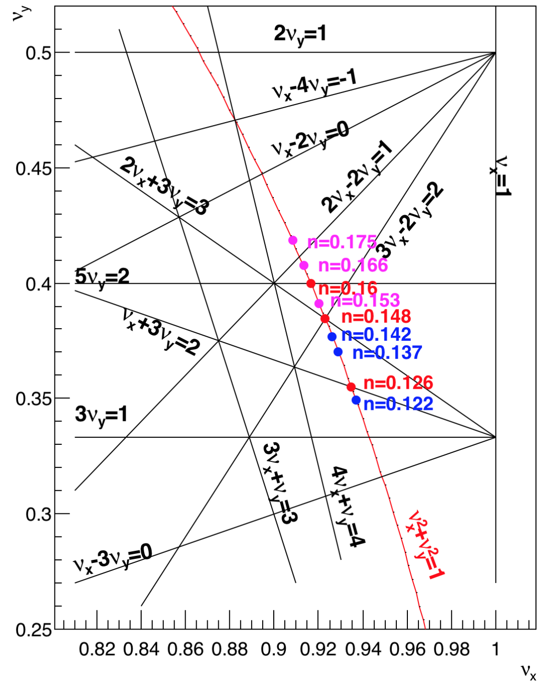

Figure 2: The tune plane with some resonance lines: E821 ran at $n = 0.122, 0.137$ and 0.142, and the possible n values for E989 are $0.142, 0.153, 0.166, 0.175$ [9].

Beam Resonance

Deviations from the electric quadrupole and magnetic dipole fields in the storage region will result in periodic forces that perturb the muon orbits. If the periodicity of the forces falls on some resonance, the horizontal or vertical oscillations increase and may cause a loss of muons. The beam resonances can be studied from the muon equations of motion. The condition for the resonance takes the form $L\nu_x + M\nu_y = N$, where L, M and N are integers.

The field index n increases for higher quadurpole voltage. The operating quadrupole voltage should be chosen to avoid the beam resonances, where a weak-focusing storage ring operates using the approximate condition $\nu_x^2 + \nu_y^2 = 1$. Therefore, the operating point should not intersect any of the resonance lines shown in Fig. 2.

Fast Rotation Analysis

Muons are injected into the storage ring as a bunch and not all of them sit on the magic momentum. Eq. (3) shows an electric term to ω_a and the muon momentum distribution is required to analyze and estimate the electric field correction to systematic errors.

The so called *Fast Rotation Analysis* (FRA) is a technique for measuring the muon radial momentum distribution that uses the time evolution of the bunch structure. Muons in a bunch with momentum $p > p_{magic}$ will naturally assume higher orbits ($r > r_{magic}$), which take longer to complete one cyclotron revolution. After some time (around 100 ~ 1000 revolutions), a muon with lower momentum will lap a muon with high momentum in the same bunch and the bunch will stretch. The stored muon momentum distribution as well as radial distribution can be determined by analyzing the debunching beam.

There are two techniques used to perform a fast rotation analysis: minimized χ^2 method [14] and Fourier Transform method [15]. Both methods were used in the BNL experiment [8], although only the minimized χ^2 method is described here.

The measurement of ω_a uses the decay position time spectra (e.g., Fig. 3) seen by the electromagnetic calorimeters sitting on the inside radius of the storage ring. Because of the parity violation in the weak decay $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$, there is a correlation between the muon spin and the direction of the high-energy daughter positron. After applying a high energy cut around 1.8 GeV, the number of daughter positrons can be modulated by the frequency ω_a (i.e., $N(t) = N_0 exp(-t/\gamma\tau_\mu)[1 - Acos(\omega_a t + \phi)]$) [8].

The expected decay positron count for the jth time bin in the minimized χ^2 method is given by

$$C_j = \sum_{i=1}^{I} f_i \beta_{ij} \quad (6)$$

where i is the radial momentum bin index, I is the total number of radial bins, f_i is the fraction of muons in the ith bin, and β_{ij} are the muon temporal-radial evolution coefficients. The f_i are the quantities being solved for, and the β_{ij} are muon beam bunch geometrical factors that can be calculated separately.

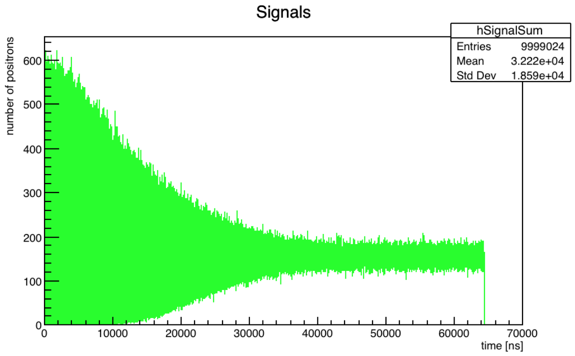

Figure 3: Simulated decay positron signal as a function of time from injection.

The minimized χ^2 method of fast rotation analysis yields maximum agreement between the observed counts (N_j) and the expected counts (C_j). Here,

$$\chi^2 = \sum_j \frac{(N_j - C_j)^2}{Z_j} = \sum_j \frac{(N_j - \sum_i f_i \beta_{ij})^2}{Z_j} \qquad (7)$$

Z_j are weighting factors that should be equal to the expected C_j. However, the C_j are initially unknown, so they are replaced by the N_j in the first pass. With the f_i obtained in the first pass, a second pass is carried out with $Z_j = \sum_{i=1}^{I} f_i \beta_{ij}$. Fig. 4 shows the resulting equilibrium radius distribution for the simulated signal shown in Fig. 3.

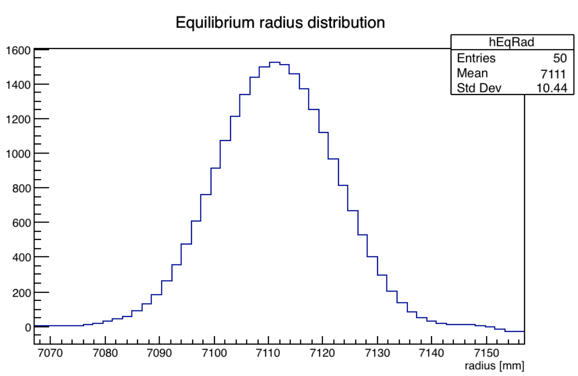

Figure 4: Equilibrium radius distribution determined by minimized χ^2 method of fast rotation analysis using the simulated decay positron signal in Fig. 3.

CONCLUSION

The Fermilab Muon g-2 Experiment is now in the commissioning run phase and should begin collecting physics data by the end of the year. The storage ring beam dynamics must be considered comprehensively to achieve the high precision measurement goal. The beam resonance and fast rotation analysis techniques discussed here are still under development. These techniques and the related systematic errors will be studied and tested on simulated and experimental data.

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