Occupancies of proton and neutron orbits of nuclei in the mass range A=90–150 participating in $0\nu\beta^-\beta^-$ decay

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Introduction

The sixteen rare modes of nuclear double beta ($\beta\beta$) decay, namely double-electron emission ($\beta^-\beta^-$), double-positron emission ($\beta^+\beta^+$), electron-positron conversion ($\varepsilon\beta^+$) and double-electron capture ($\varepsilon\varepsilon$) with the emission of two neutrinos, no neutrinos, single Majoron and double Majorons provide fundamental information on mass and nature of neutrinos as well as nuclear structure aspects. Among all these modes, the $0\nu\beta\beta$ decay, which violates the conservation of lepton number by two units, is of paramount importance as its observation will immediately imply the Majorana nature of neutrinos with finite mass. The $0\nu\beta\beta$ decay has not been observed yet and major projects are under the way to detect this peculiar decay mode [1,2].

In any gauge theory allowing the violation of lepton number conservation, the inverse half-life of $0\nu\beta\beta$ decay is a product of gauge theoretical parameter of the underlying theory, appropriate phase space factors and nuclear transition matrix elements (NTMEs) $M^{0\nu}$. The phase space factors are accurately calculable [3–5]. However, the calculation of NTMEs is highly dependent on the nuclear models employed and the extraction of various lepton number violating gauge parameters depends on the reliability of NTMEs calculated. On the other hand there exist noticeable uncertainties in the value of $M^{0\nu}$ calculated in different models [6].

Prior to the calculation of $M^{0\nu}$, the reliability of the wave functions can be checked by reproducing the experimentally observed values, namely yrast energies, reduced transition probabilities, quadrupole moments, deformation parameters and NTMEs $M_{2\nu}$ of $2\nu\beta^-\beta^-$ decay. Recently, the occupation of valence orbits by nucleons have been studied experimentally [7-10]. These experimental values can provide a further check on the reliability of wave functions by comparing them with calculated values. In the present work we have calculated the occupation numbers of various orbits for protons and neutrons within the projected Hartree-Fock Bogoliubov (PHFB) model.

Theoretical framework

The sub-shell occupation numbers η_J in a yrast state J is given by

$$\eta_J = \frac{\langle \Phi_0 | \left(\sum_m C^+_{jm} C_{jm} \right) P^J_{00} | \Phi_0 \rangle}{\langle \Phi_0 | P^J_{00} | \Phi_0 \rangle} = \frac{\int_0^\pi p(\theta) d^J_{00}(\theta) \sin\theta \, d\theta}{\int_0^\pi n(\theta) d^J_{00}(\theta) \sin\theta \, d\theta} \quad (1)$$

where

$$p(\theta) = n(\theta) \left[\sum_m \left(\frac{M}{1+M} \right)_{jm,jm} \right] \quad (2)$$

An approximate estimate of the subshell occupation numbers can be easily obtained in terms of the expectation value of the operator η_J with respect to the intrinsic state $|\Phi_0\rangle$

$$\eta_J^{\text{intrinsic}} = \langle \Phi_0 | \hat{\eta}_J | \Phi_0 \rangle = \sum_{i,m} |c^m_{Ji}|^2 \left(V_i^m \right)^2 \quad (3)$$

Results and discussions

In the present work we use four parametrizations of pairing plus multipolar effective two-body interaction, namely *PQQ*1, *PQQHH*1, *PQQ*2 and *PQQHH*2. The model

space and details about these parametrizations and method to fix them have been provided in our earlier work [11]. We have calculated the values of occupation numbers of orbits for protons (p) and neutrons (n) in the ground states of ^{94,96}Zr, ^{94,96,100}Mo, ^{100}Ru, ^{110}Pd, ^{110}Cd, ^{128,130}Te, ^{128,130}Xe, ^{150}Nd and ^{150}Sm nuclei for all the four parametrizations of effective two-body interaction. In Table 1 and 2, we present the results for ^{100}Mo, ^{100}Ru, ^{130}Te and ^{130}Xe nuclei for *PQQ*1 and *PQQHH*1 parametrizations.

Table 1: The calculated values of the occupation numbers of orbits for protons and neutrons in the ground states of ^{100}Mo and ^{100}Ru nuclei for (a) *PQQ*1 and (b) *PQQHH*1 parametrizations.

Orbits		^{100}Mo		^{100}Ru	
		p	n	p	n
$2s_{1/2}$	a	0.042	0.705	0.044	0.635
	b	0.042	0.691	0.044	0.628
$1p_{1/2}$	a	0.114	1.994	1.035	1.998
	b	1.144	1.994	1.042	1.199
$1d_{3/2}$	a	0.020	1.298	0.028	1.190
	b	0.020	1.262	0.029	1.172
$1d_{5/2}$	a	0.485	3.233	0.456	3.036
	b	0.461	3.258	0.454	3.031
$0g_{7/2}$	a	0.021	1.419	0.037	0.994
	b	0.021	1.373	0.037	1.005
$0g_{9/2}$	a	3.278	9.843	4.329	9.879
	b	3.255	9.854	4.316	9.880
$0h_{11/2}$	a	0.037	1.505	0.689	0.267
	b	0.055	1.565	0.075	0.284

Table 2: The calculated values of the occupation numbers of orbits for protons and neutrons in the ground states of ^{130}Te and ^{130}Xe nuclei for (a) *PQQ*1 and (b) *PQQHH*1 parametrizations.

Orbits		^{130}Te		^{130}Xe	
		p	n	p	n
$2s_{1/2}$	a	0.477	1.970	0.552	1.916
	b	0.546	1.923	0.546	1.923
$1d_{3/2}$	a	0.236	3.917	0.973	3.726
	b	0.933	3.750	0.933	3.750
$1d_{5/2}$	a	1.258	5.948	1.891	5.907
	b	1.996	5.910	1.996	5.910
$1f_{7/2}$	a	0.000	0.422	0.004	0.823
	b	0.004	0.758	0.004	0.758
$0g_{7/2}$	a	0.028	7.597	0.515	6.582
	b	0.456	6.683	0.456	6.683
$0g_{9/2}$	a	0.000	0.186	0.003	0.242
	b	0.003	0.226	0.003	0.226
$0h_{11/2}$	a	0.000	7.956	0.059	6.800
	b	0.060	6.746	0.060	6.746

Conclusions

To summarize, we have calculated the occupation numbers of orbits for protons and neutrons for ^{94,96}Zr, ^{94,96,100}Mo, ^{100}Ru, ^{110}Pd, ^{110}Cd, ^{128,130}Te, ^{128,130}Xe, ^{150}Nd and ^{150}Sm nuclei employing four sets of wave functions generated through PHFB model. Complete results will be presented in the symposium.

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