

## Stringy Newton gravity with $H$ -flux

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A symmetry principle has been shown to augment unambiguously the Einstein field equations, promoting the whole closed-string massless NS-NS sector to stringy graviton fields. Here we consider its weak field approximation, take a nonrelativistic limit, and derive the stringy augmentation of Newton gravity:  $\nabla^2\Phi = 4\pi G\rho + \mathbf{H} \cdot \mathbf{H}$ ,  $\nabla \cdot \mathbf{H} = 0$ ,  $\nabla \times \mathbf{H} = 4\pi G\mathbf{K}$ . Not only the mass density  $\rho$  but also the current density  $\mathbf{K}$  is intrinsic to matter. Sourcing  $\mathbf{H}$  which is of NS-NS  $H$ -flux origin,  $\mathbf{K}$  is nontrivial if the matter is “stringy.”  $\mathbf{H}$  contributes quadratically to the Newton potential, but otherwise is decoupled from the point particle dynamics, i.e.,  $\ddot{\mathbf{x}} = -\nabla\Phi$ . We define “stringization” analogous to magnetization and discuss regular as well as monopolelike singular solutions.

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### I. INTRODUCTION

One of the fundamental problems in physics today is the dark matter problem. Despite a variety of observational indications, e.g., galaxy rotation curves [1], the Bullet Cluster (1E0657-558) [2], and ghostly galaxies without dark matter [3], no single experiment has ever succeeded in the direct detection of dark matter particles. Alternative hypotheses, including notably MOND [4], pass over the equivalence principle and modify general relativity (GR), but only give partial explanations while being often accused of harming the mathematical beauty thereof.

Developments over the last decade in an area of string theory, now called double field theory (DFT), have gradually unveiled a new form of pure gravity [5–15]. It is based on the  $\mathbf{O}(D, D)$  symmetry principle, with  $D$  denoting the spacetime dimension. The symmetry can be broken only spontaneously but never explicitly, as the theory is constructed in terms of strictly  $\mathbf{O}(D, D)$ -covariant field variables, namely the DFT-dilaton  $d$  and DFT-metric  $\mathcal{H}_{AB}$  (or more powerfully DFT-vielbeins), forming the new pure gravity sector. The symmetry principle further fixes their coupling to generic matter contents which should also be in  $\mathbf{O}(D, D)$  representations. Examples include Yang–Mills [16,17], fermions [18] (cf. [19]), R-R sector [20–23], full-order supersymmetrizations [24,25], point

particles [26,27], fundamental strings [28–33], and the Standard Model itself [34]. Naturally, the Einstein field equations are augmented to an  $\mathbf{O}(D, D)$ -symmetric form [15] (cf. [35] for a short summary):

$$G_{AB} = \frac{8\pi G}{c^4} T_{AB}, \quad (1)$$

which carry  $\mathbf{O}(D, D)$  vector indices and unify the Euler–Lagrange equations of all the stringy graviton fields,  $\{\mathcal{H}_{AB}, d\}$ . In a parallel manner to GR,  $G_{AB}$  is the off-shell conserved  $\mathbf{O}(D, D)$ -symmetric Einstein curvature constructed out of  $\{\mathcal{H}_{AB}, d\}$  [14], while  $T_{AB}$  is the on-shell conserved energy-momentum tensor, defined through the variation of the matter Lagrangian with respect to  $\{\mathcal{H}_{AB}, d\}$  [15].

Remarkably, the perfectly  $\mathbf{O}(D, D)$ -symmetric vacua, satisfying  $G_{AB} = 0$ , turned out to be a topological phase which allows no moduli and no interpretation within Riemannian geometry, thus escaping beyond the realm of GR [30,36,37]. Only after a spontaneous symmetry breaking of  $\mathbf{O}(D, D)$ , the familiar string theory backgrounds characterized by the Riemannian metric  $g_{\mu\nu}$  and the Kalb–Ramond skew-symmetric two-form potential  $B_{\mu\nu}$  emerge: these component fields parametrize the DFT-metric while being identified as the Nambu–Goldstone bosons [38]. The master formula (1) then reduces to (cf. [39] for non-Riemannian cases)

$$\begin{aligned} \frac{1}{2}e^{2\phi}\nabla^\rho(e^{-2\phi}H_{\rho\mu\nu}) &= \frac{8\pi G}{c^4}K_{[\mu\nu]}, \\ R_{\mu\nu} + 2\nabla_\mu(\partial_\nu\phi) - \frac{1}{4}H_{\mu\rho\sigma}H_\nu{}^{\rho\sigma} &= \frac{8\pi G}{c^4}K_{(\mu\nu)}, \\ R + 4\nabla_\mu(\partial^\mu\phi) - 4\partial_\mu\phi\partial^\mu\phi - \frac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu} &= \frac{8\pi G}{c^4}T_{(0)}, \end{aligned} \quad (2)$$

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which imply a pair of conservation laws,

$$\begin{aligned} \nabla^\mu (e^{-2\phi} K_{[\mu\nu]}) &= 0, \\ \nabla^\mu K_{(\mu\nu)} - 2\partial^\mu \phi K_{(\mu\nu)} + \frac{1}{2} H_\nu{}^{\lambda\mu} K_{[\lambda\mu]} - \frac{1}{2} \partial_\nu T_{(0)} &= 0. \end{aligned} \quad (3)$$

The left and the right-hand sides of the equalities in (2) come from  $G_{AB}$  and  $T_{AB}$  in (1) separately. Schematically,  $K_{[\mu\nu]}$ ,  $K_{(\mu\nu)}$ , and  $T_{(0)}$  are the energy-momentum tensor components relevant to  $B_{\mu\nu}$ ,  $g_{\mu\nu}$ , and the string dilaton  $\phi = d + \frac{1}{4} \ln |g|$ , respectively. Having said that, since (1) is derived from the variations of the  $\mathbf{O}(D, D)$  covariant fields, in particular the DFT-dilaton  $d$  rather than the string dilaton  $\phi$ , the specific form of the Einstein curvature,  $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ , does not automatically appear in (2).

We stress that for each matter content, the  $\mathbf{O}(D, D)$  symmetry determines its coupling to the closed-string massless NS-NS sector of  $\{B_{\mu\nu}, g_{\mu\nu}, \phi\}$  and hence fixes  $K_{\mu\nu}, T_{(0)}$  completely [15]. For example, a point particle should couple to the string frame metric  $g_{\mu\nu}$  only—minimally in the standard way—resulting in  $K_{[\mu\nu]} = 0$ ,  $T_{(0)} = 0$ , and

$$K^{\mu\nu}(x) = \frac{mc}{2} \int d\tau \dot{y}^\mu(\tau) \dot{y}^\nu(\tau) \frac{\delta(x - y(\tau)) e^{2\phi}}{\sqrt{-g}}. \quad (4)$$

The particle follows geodesics defined in the string frame,

$$\ddot{x}^\lambda + \gamma_{\mu\nu}^\lambda \dot{x}^\mu \dot{x}^\nu = 0. \quad (5)$$

Rewriting this in the Einstein frame would involve the gradient of  $\phi$  and thus obscure the equivalence principle. That is to say, the  $\mathbf{O}(D, D)$  symmetry principle asserts that the equivalence principle for a point particle should hold in the string frame. On the other hand, fundamental strings, spinorial fermions, and the R-R sector couple to  $B_{\mu\nu}$  and  $g_{\mu\nu}$  but not to  $\phi$ , resulting in asymmetric  $K_{\mu\nu}$  and (still) trivial  $T_{(0)}$ . Further, in contrast, gauge bosons couple to  $g$  and  $\phi$ , but do not interact with the  $B$ -field (44) [34]. While the  $B$ -field does not interact with the electromagnetic force nor point particles, its electric  $H$ -flux nevertheless contributes to the mass formula [15],

$$M = \int e^{-2d} \left| 2K_0^0 + \frac{1}{16\pi G} H_{0\mu\nu} H^{0\mu\nu} \right|. \quad (6)$$

In fact, having a larger profile than the (visible) matter represented here by  $K_0^0$ , the electric  $H$ -flux can produce non-monotonic non-Keplerian rotation curves [26]. In this way,  $H$ -flux behaves like dark matter [40], cf. [41]. Note that its dual scalar is also known as a dark matter candidate, ‘‘an axion’’ [42–46].

It is the purpose of the present paper to consider the weak field approximation of (2), (3), (5) for  $D = 4$ , take a

consistent nonrelativistic limit, and faithfully derive the stringy augmentation of Newton gravity spelled out in the Abstract. We hope our work may deepen the physical understanding of the  $\mathbf{O}(D, D)$ -completed general relativity (1) and contribute to examining rigorously the prospect of  $H$ -flux as a dark matter candidate.

## II. LORENTZ SYMMETRIC WEAK FIELD APPROXIMATION

We start our weak field approximation of (2) by linearizing the metric,  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ ,  $g^{\mu\nu} \simeq \eta^{\mu\nu} - h^{\mu\nu}$ , around a flat Minkowskian background with trivial  $H$ -flux and dilaton  $\phi$ . The spacetime indices are raised or lowered by the constant metric  $\eta$ , e.g.,  $h^{\mu\nu} = \eta^{\mu\rho} \eta^{\nu\sigma} h_{\rho\sigma}$ . The linearized Riemann curvature,

$$R^\kappa{}_{\lambda\mu\nu} = \frac{1}{2} (\partial_\lambda \partial_\mu h^\kappa{}_\nu - \partial_\lambda \partial_\nu h^\kappa{}_\mu - \partial^\kappa \partial_\mu h_{\lambda\nu} + \partial^\kappa \partial_\nu h_{\lambda\mu}), \quad (7)$$

is invariant under the linearized diffeomorphisms,

$$\delta_\xi h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \quad \delta_\xi R^\kappa{}_{\lambda\mu\nu} = 0. \quad (8)$$

Using  $\delta_\xi (\partial_\rho h^\rho{}_\mu - \frac{1}{2} \partial_\mu h^\rho{}_\rho) = \partial_\rho \partial^\rho \xi_\mu$ , we fix the gauge,

$$\partial_\rho h^\rho{}_\mu - \frac{1}{2} \partial_\mu h^\rho{}_\rho + 2\partial_\mu \phi = 0. \quad (9)$$

It follows from (7) and (9),

$$\begin{aligned} R_{\mu\nu} &= -\frac{1}{2} \partial_\rho \partial^\rho h_{\mu\nu} - 2\partial_\mu \partial_\nu \phi, \\ \partial_\rho \partial^\rho \phi &= \frac{1}{4} \partial_\rho \partial^\rho h^\sigma{}_\sigma - \frac{1}{2} \partial_\rho \partial_\sigma h^{\rho\sigma}, \end{aligned} \quad (10)$$

and further, from the integrability of (9),

$$\partial_\mu \partial_\rho h^\rho{}_\nu = \partial_\nu \partial_\rho h^\rho{}_\mu. \quad (11)$$

Similarly, we choose a gauge for the  $B$ -field,

$$\delta_\lambda B_{\mu\nu} = \partial_\mu \lambda_\nu - \partial_\nu \lambda_\mu \Rightarrow \partial_\rho B^\rho{}_\mu = 0. \quad (12)$$

Now, assuming the following scales,

$$h_{\mu\nu} \sim K_{(\mu\nu)} \sim \phi \sim T_{(0)} \sim (H_{\lambda\mu\nu})^2 \sim (K_{[\mu\nu]})^2, \quad (13)$$

the consistent linearization of (2) can be achieved,

$$\begin{aligned} \partial^\rho H_{\rho\mu\nu} &= \partial_\rho \partial^\rho B_{\mu\nu} = \frac{16\pi G}{c^4} K_{[\mu\nu]}, \\ \partial_\rho \partial^\rho h_{\mu\nu} + \frac{1}{2} H_{\mu\rho\sigma} H_{\nu}{}^{\rho\sigma} &= -\frac{16\pi G}{c^4} K_{(\mu\nu)}, \\ \partial_\rho \partial_\sigma h^{\rho\sigma} + \frac{1}{12} H_{\rho\sigma\tau} H^{\rho\sigma\tau} &= -\frac{8\pi G}{c^4} T_{(0)}, \end{aligned} \quad (14)$$

which, with (11), imply the following linearized conservation equations,

$$\partial^\rho K_{[\rho\mu]} = 0, \quad \partial^\rho K_{(\rho\mu)} + \frac{1}{2} H_\mu{}^{\rho\sigma} K_{[\rho\sigma]} - \frac{1}{2} \partial_\mu T_{(0)} = 0. \quad (15)$$

With a well-known relation,

$$\nabla^2 \frac{1}{|\mathbf{x} - \mathbf{x}'|} = -\nabla \cdot \left( \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} \right) = -4\pi\delta(\mathbf{x} - \mathbf{x}'), \quad (16)$$

the first formula in (14) is solved by a ‘‘retarded potential,’’

$$B_{\mu\nu}(x) = -\frac{4G}{c^4} \int d^3x' \frac{K_{[\mu\nu]}(x_{\text{ret}}^0, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}, \quad (17)$$

where  $x_{\text{ret}}^0 := x^0 - |\mathbf{x} - \mathbf{x}'|$  is the retarded temporal coordinate. Thanks to  $\partial^\rho K_{[\rho\mu]} = 0$  (15), up to a surface integral, the gauge condition (12) is indeed fulfilled,

$$\partial^\mu B_{\mu\nu}(x) = -\frac{4G}{c^4} \int d^3x' \frac{\partial'^\mu K_{[\mu\nu]}(x')|_{x^0 \rightarrow x_{\text{ret}}^0}}{|\mathbf{x} - \mathbf{x}'|} = 0. \quad (18)$$

The expression for the  $H$ -flux follows

$$H_{\lambda\mu\nu}(x) = -\frac{4G}{c^4} \int d^3x' \frac{3\partial'^\lambda K_{[\mu\nu]}(x')|_{x^0 \rightarrow x_{\text{ret}}^0}}{|\mathbf{x} - \mathbf{x}'|}. \quad (19)$$

In a similar fashion, with some shorthand notations,

$$\begin{aligned} \hat{K}_{(\mu\nu)} &:= \frac{16\pi G}{c^4} K_{(\mu\nu)} + \frac{1}{2} H_{\mu\rho\sigma} H_\nu{}^{\rho\sigma}, \\ \hat{T} &:= \frac{8\pi G}{c^4} T_{(0)} + \frac{1}{12} H_{\rho\sigma\tau} H^{\rho\sigma\tau}, \end{aligned} \quad (20)$$

we solve the second formula in (14),

$$h_{\mu\nu}(x) = \frac{1}{4\pi} \int d^3x' \frac{\hat{K}_{(\mu\nu)}(x_{\text{ret}}^0, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}. \quad (21)$$

From (14), (15), and the  $H$ -flux Bianchi identity, we get

$$\partial^\mu \hat{K}_{(\mu\nu)} = \partial_\nu \hat{T}, \quad (22)$$

which gives, comparable to (18),

$$\partial_\rho h^\rho{}_\mu(x) = \partial_\mu \left[ \frac{1}{4\pi} \int d^3x' \frac{\hat{T}(x_{\text{ret}}^0, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} \right]. \quad (23)$$

This verifies for consistency that (21) indeed satisfies the integrability condition (11), while the third formula in (14) is automatically fulfilled as  $\partial_\rho \partial_\sigma h^{\rho\sigma}(x) = -\hat{T}(x)$ . Lastly, from (9), the string dilaton is fixed,

$$\phi(x) = \frac{1}{4\pi} \int d^3x' \frac{\frac{1}{4} \hat{K}_\mu{}^\mu(x') - \frac{1}{2} \hat{T}(x')}{|\mathbf{x} - \mathbf{x}'|} \Big|_{x^0 \rightarrow x_{\text{ret}}^0}. \quad (24)$$

To summarize, through (19), (21), and (24),  $\{K_{\mu\nu}, T_{(0)}\}$  (‘‘matter’’) determines  $\{H_{\lambda\mu\nu}, h_{\mu\nu}, \phi\}$  (‘‘geometry’’). While  $H_{\lambda\mu\nu}$  is given by a single volume integral, the other two,  $h_{\mu\nu}, \phi$ , involve triple volume integrals of matter. It is worth while to note that the quantity inside the bracket in (23) is related to the  $\mathbf{O}(D, D)$  singlet integral measure of DFT (scalar density with weight one),

$$\begin{aligned} e^{-2d} &= \sqrt{-g} e^{-2\phi} \simeq 1 + \frac{1}{2} h_\mu{}^\mu - 2\phi \\ &= 1 + \frac{1}{4\pi} \int d^3x' \frac{\hat{T}(x_{\text{ret}}^0, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}. \end{aligned} \quad (25)$$

The linearized geodesic equation assumes the form,

$$\ddot{x}^\lambda + \frac{1}{2} (\partial_\mu h^\lambda{}_\nu + \partial_\nu h^\lambda{}_\mu - \partial^\lambda h_{\mu\nu}) \dot{x}^\mu \dot{x}^\nu = 0. \quad (26)$$

### III. NONRELATIVISTIC LIMIT: STRINGY NEWTON

We proceed to take the nonrelativistic, large  $c$  limit of (14). With  $x^0 = ct$ , we let all the fields be functions of  $(t, \mathbf{x}) = (t, x^i)$ ,  $i = 1, 2, 3$ . We suppress  $\partial_0 = \frac{1}{c} \frac{\partial}{\partial t} = 0$  and put  $\partial_\rho \partial^\rho = \partial_i \partial^i = \nabla^2$ . Further, for the point particle of (4), (5), we set  $t = \tau$  (proper time) and  $|\dot{\mathbf{x}}| \ll c$ , such that the linearized geodesic equation (26) reduces to  $\ddot{x}^i = \frac{1}{2} \partial_i h_{00} c^2$  only. Combined with (9), (14), this fixes all the scales, which are compatible with (13):

$$\begin{aligned} h_{\mu\nu} &= c^{-2} \bar{h}_{\mu\nu}, & K_{(\mu\nu)} &= c^2 \bar{K}_{(\mu\nu)}, \\ \phi &= c^{-2} \bar{\phi}, & T_{(0)} &= c^2 \bar{T}_{(0)}, \\ B_{\mu\nu} &= c^{-1} \bar{B}_{\mu\nu}, & K_{[\mu\nu]} &= c^3 \bar{K}_{[\mu\nu]}, \\ H_{\lambda\mu\nu} &= c^{-1} \bar{H}_{\lambda\mu\nu}. \end{aligned} \quad (27)$$

This then straightforwardly produces the nonrelativistic limit of the previous weak field approximation: with  $\partial_0 = 0$  assumed, after removing the symbol  $c$ , replacing  $x_{\text{ret}}^0$  by  $t$ , and putting bars over each quantity, all the formulas from Sec. II survive to preserve their forms and set of relations. For example, (14) reduces to

$$\begin{aligned} \partial^i \bar{H}_{i\mu\nu} &= \nabla^2 \bar{B}_{\mu\nu} = 16\pi G \bar{K}_{[\mu\nu]}, \\ \nabla^2 \bar{h}_{\mu\nu} + \frac{1}{2} \bar{H}_{\mu\rho\sigma} \bar{H}_\nu{}^{\rho\sigma} &= -16\pi G \bar{K}_{(\mu\nu)}, \\ \partial_i \partial_j \bar{h}^{ij} + \frac{1}{12} \bar{H}_{\rho\sigma\tau} \bar{H}^{\rho\sigma\tau} &= -8\pi G \bar{T}_{(0)}, \end{aligned} \quad (28)$$

and (17) becomes

$$\bar{B}_{\mu\nu}(x) = -4G \int d^3x' \frac{\bar{K}_{[\mu\nu]}(t, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}. \quad (29)$$

Furthermore, the  $H$ -flux Bianchi identity reads now

$$\partial_{[i}\bar{H}_{jk]0} = 0. \quad (30)$$

Hereafter, we focus on the Newton potential which is the only quantity directly relevant to the particle dynamics,

$$\Phi := -\frac{1}{2}c^2 h_{00} = -\frac{1}{2}\bar{h}_{00}, \quad \ddot{\mathbf{x}} = -\nabla\Phi. \quad (31)$$

We then identify all the quantities which can affect the Newton potential: namely, the mass density  $\rho$ , the stringy current density  $\mathbf{K}$ , and  $B$ -field/ $H$ -flux vectors  $\mathbf{B}$ ,  $\mathbf{H}$ , as follows

$$\begin{aligned} \rho &:= 2\bar{K}_{00}, \\ \mathbf{K} &:= 2\sqrt{2}(\bar{K}_{[01]}, \bar{K}_{[02]}, \bar{K}_{[03]}), \\ \mathbf{B} &:= \frac{1}{\sqrt{2}}(\bar{B}_{10}, \bar{B}_{20}, \bar{B}_{30}), \\ \mathbf{H} &:= \nabla \times \mathbf{B} = \frac{1}{\sqrt{2}}(\bar{H}_{023}, \bar{H}_{031}, \bar{H}_{012}). \end{aligned} \quad (32)$$

Crucially,  $\{\rho, \mathbf{K}, \Phi, \mathbf{H}\}$  forms an ‘‘autonomy’’ of closed relations: from (28),

$$\nabla^2\Phi = 4\pi G\rho + \mathbf{H} \cdot \mathbf{H}, \quad \nabla \times \mathbf{H} = 4\pi G\mathbf{K}, \quad (33)$$

and, from (15), (30),  $\mathbf{K}$  and  $\mathbf{H}$  are both divergenceless,

$$\nabla \cdot \mathbf{K} = 0, \quad \nabla \cdot \mathbf{H} = 0. \quad (34)$$

That is to say, the Newton potential is fully determined by both the mass density and the stringy current density: directly so by  $\rho$ , and indirectly so by  $\mathbf{K}$  as mediated through  $\mathbf{H}$ ,

$$\begin{aligned} \Phi &= -G \int d^3x' \frac{\rho_{\text{eff}}(t, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}, \quad \rho_{\text{eff}} := \rho + \frac{1}{4\pi G} \mathbf{H} \cdot \mathbf{H}, \\ \mathbf{H} &= G \int d^3x' \mathbf{K}(t, \mathbf{x}') \times \frac{(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} = \nabla \times \mathbf{B}, \\ \mathbf{B} &= G \int d^3x' \frac{\mathbf{K}(t, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}, \quad \nabla \cdot \mathbf{B} = 0. \end{aligned} \quad (35)$$

Although  $\mathbf{H}$  originates from the electric components of the  $H$ -flux, its behavior is identical to the magnetic field in classical Magnetostatics such that a ‘‘Biot–Savart law’’ holds above. The vector potential  $\mathbf{B}$  has NS-NS  $B$ -field origin, and notation-wise should not be confused with the

magnetic field. It is instructive to note that  $\rho_{\text{eff}}$  is consistent with (6) at the linearized level (cf. [47]), and involves a double volume integral as

$$\begin{aligned} \mathbf{H} \cdot \mathbf{H} &= G^2 \int d^3x' \int d^3x'' \\ &\times \frac{\det \begin{bmatrix} (\mathbf{x} - \mathbf{x}') \cdot (\mathbf{x} - \mathbf{x}'') & (\mathbf{x} - \mathbf{x}') \cdot \mathbf{K}'' \\ (\mathbf{x} - \mathbf{x}'') \cdot \mathbf{K}' & \mathbf{K}' \cdot \mathbf{K}'' \end{bmatrix}}{|\mathbf{x} - \mathbf{x}'|^3 |\mathbf{x} - \mathbf{x}''|^3}, \end{aligned} \quad (36)$$

where  $\mathbf{K}' = \mathbf{K}(t, \mathbf{x}')$ ,  $\mathbf{K}'' = \mathbf{K}(t, \mathbf{x}'')$ .

In analogy to the magnetization in electrodynamics, we introduce the notion of *stringization* for the stringy current density  $\mathbf{K}$  which is divergence free,

$$\mathbf{K}(t, \mathbf{x}) = \nabla \times \mathbf{s}(t, \mathbf{x}). \quad (37)$$

The corresponding  $\mathbf{B}$ ,  $\mathbf{H}$  are, from (16), (35) (cf. [48]),

$$\begin{aligned} \mathbf{B} &= G \int d^3x' \frac{\mathbf{s}(t, \mathbf{x}') \times (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} + G \oint d\mathbf{A} \times \frac{\mathbf{s}(t, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}, \\ \mathbf{H} &= 4\pi G\mathbf{s}(t, \mathbf{x}) + G \int d^3x' \frac{3\hat{\mathbf{n}}'(\hat{\mathbf{n}}' \cdot \mathbf{s}(t, \mathbf{x}')) - \mathbf{s}(t, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} \\ &= 4\pi G\mathbf{s}(t, \mathbf{x}) - G\nabla\Phi_s(t, \mathbf{x}), \end{aligned} \quad (38)$$

in which  $\hat{\mathbf{n}}' = \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|}$  and  $\Phi_s$  is a *stringy scalar potential*,

$$\begin{aligned} \Phi_s(t, \mathbf{x}) &= \int d^3x' \frac{\mathbf{s}(t, \mathbf{x}') \cdot (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} \\ &= -\nabla \cdot \int d^3x' \frac{\mathbf{s}(t, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}. \end{aligned} \quad (39)$$

Clearly from (38),  $\nabla \times \mathbf{H} = 4\pi G\nabla \times \mathbf{s}$ . Far away from a localized source,  $|\mathbf{x}| \gg |\mathbf{x}'|$ , we observe a stringy dipole,

$$\mathbf{H} \simeq G \frac{3\hat{\mathbf{x}}(\hat{\mathbf{x}} \cdot \mathbf{S}(t)) - \mathbf{S}(t)}{|\mathbf{x}|^3}, \quad \mathbf{S}(t) = \int d^3x \mathbf{s}(t, \mathbf{x}). \quad (40)$$

As an example, we consider a uniformly ‘‘stringized’’ sphere of radius  $a$ , with constant  $\rho$  and  $\mathbf{s}$ , to get (cf. [48])

$$\begin{aligned} \Phi_s &= \frac{4\pi}{3} \mathbf{s} \cdot \mathbf{x}, \quad \mathbf{H} = \frac{8\pi G}{3} \mathbf{s} \quad \text{for } |\mathbf{x}| \leq a, \\ \Phi_s &= \frac{4\pi a^3}{3} \frac{\mathbf{s} \cdot \mathbf{x}}{|\mathbf{x}|^3}, \quad \mathbf{H} = \frac{4\pi G a^3}{3} \left( \frac{3\hat{\mathbf{x}}(\hat{\mathbf{x}} \cdot \mathbf{s}) - \mathbf{s}}{|\mathbf{x}|^3} \right) \quad \text{for } |\mathbf{x}| > a. \end{aligned} \quad (41)$$

Thus, the total effective mass density (35) reads

$$\rho_{\text{eff}}(t, \mathbf{x}) = \begin{cases} \rho + \frac{16\pi G}{9} |\mathbf{s}|^2 & \text{for } |\mathbf{x}| \leq a \\ \frac{4\pi G}{9} |\mathbf{s}|^2 a^6 \left( \frac{1+3\cos^2\theta}{|\mathbf{x}|^6} \right) & \text{for } |\mathbf{x}| > a, \end{cases} \quad (42)$$

where  $\theta$  is the angle between  $\mathbf{s}$  and  $\mathbf{x}$ . As anticipated from the general formula (36),  $\rho_{\text{eff}}$  has a  $|\mathbf{x}|^{-6}$  profile or ‘‘halo.’’

Another example is of Dirac monopole type (singular),

$$\mathbf{B} = Gq \int_{\mathbf{x}'=0}^{\infty} d\mathbf{x}' \times \frac{(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3}, \quad \mathbf{H} = Gq \frac{\mathbf{x}}{|\mathbf{x}|^3}, \quad (43)$$

where the path should not cross the point of  $\mathbf{x}$ . The profile is now thicker as  $\rho_{\text{eff}} = \frac{Gq^2}{4\pi} |\mathbf{x}|^{-4}$  which may be comparable to some known dark matter profiles [49,50]. In fact, this configuration corresponds to the linearization of a known exact spherical solution to (2) [51] which has been shown to feature a non-monotonic and hence non-Keplerian rotation curve [26].

#### IV. CONCLUSION

We have shown that the  $\mathbf{O}(D, D)$ -completed general relativity or Einstein field equations (1) reduce in the nonrelativistic limit to the stringy Newton gravity (33). Symmetry-wise, the  $\mathbf{O}(D, D)$  of (1) is broken spontaneously in (2) of which general covariance is reduced to Lorentz symmetry in (14) and further to Galilean symmetry in (33). It would be of interest to investigate whether general covariance can be recovered as in (stringy) Newton–Cartan gravity [52–54].

The final resulting formulas (33) resemble a hybrid of Newton gravity and magnetostatics. Out of  $D^2 + 1 = 17$  number of components of the stringy energy-momentum tensor,  $T_{AB}$  (1) or  $\{K_{\mu\nu}, T_{(0)}\}$  (2), only four, i.e., the mass density  $\rho$  and the stringy current density  $\mathbf{K}$ , participate in determining the Newton potential (35). Different types of

matter have different  $\rho$  and  $\mathbf{K}$  (or *stringization*  $\mathbf{s}$ ). If the matter is nonstringy particlelike,  $\mathbf{K}$  is trivial and we fully recover Newton gravity. On the other hand, in the presence of distinct kinds of matter, the center of  $\rho$  may not coincide with that of  $\mathbf{H} \cdot \mathbf{H}$ . These might explain the ghostly galaxies without dark matter (or without  $H$ -flux sourced by  $\mathbf{K}$ ) [3] and the Bullet Cluster [2] respectively. It would be of the utmost interest to test stringy Newton gravity more rigorously against observations. For this, one needs to also analyze light or electromagnetism (for a recent discussion see [55]). The  $\mathbf{O}(D, D)$  symmetry principle prescribes gauge bosons to couple to  $g$  and  $\phi$  but not to  $B$ -field [34],

$$S_{\text{photon}} = \int d^4x -\frac{1}{4} \sqrt{-g} e^{-2\phi} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma}. \quad (44)$$

This seems to imply that a photon would not merely follow a null geodesic, but would be also influenced by the dilaton  $\phi$ , e.g., [56]. Further analysis on the action (44), using methods such as eikonal approximation [57], is desirable.

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- [1] V. C. Rubin, N. Thonnard, and W. K. Ford, Jr., Rotational properties of 21 SC galaxies with a large range of luminosities and radii, from NGC 4605 ( $R = 4$  kpc) to UGC 2885 ( $R = 122$  kpc), *Astrophys. J.* **238**, 471 (1980).
  - [2] D. Clowe, M. Bradac, A. H. Gonzalez, M. Markevitch, S. W. Randall, C. Jones, and D. Zaritsky, A direct empirical proof of the existence of dark matter, *Astrophys. J.* **648**, L109 (2006).
  - [3] S. Danieli, P. van Dokkum, R. Abraham, C. Conroy, A. E. Dolphin, and A. J. Romanowsky, A tip of the red giant branch distance to the dark matter deficient Galaxy NGC 1052-DF4 from Deep Hubble Space Telescope Data, [arXiv: 1910.07529](https://arxiv.org/abs/1910.07529).
  - [4] M. Milgrom, A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis, *Astrophys. J.* **270**, 365 (1983).
  - [5] W. Siegel, Two-vierbein formalism for string-inspired axionic gravity, *Phys. Rev. D* **47**, 5453 (1993).
  - [6] W. Siegel, Superspace duality in low-energy superstrings, *Phys. Rev. D* **48**, 2826 (1993).
  - [7] C. Hull and B. Zwiebach, Double field theory, *J. High Energy Phys.* **09** (2009) 099.
  - [8] C. Hull and B. Zwiebach, The gauge algebra of double field theory and Courant brackets, *J. High Energy Phys.* **09** (2009) 090.
  - [9] O. Hohm, C. Hull, and B. Zwiebach, Background independent action for double field theory, *J. High Energy Phys.* **07** (2010) 016.
  - [10] O. Hohm, C. Hull, and B. Zwiebach, Generalized metric formulation of double field theory, *J. High Energy Phys.* **08** (2010) 008.
  - [11] I. Jeon, K. Lee, and J. H. Park, Differential geometry with a projection: Application to double field theory, *J. High Energy Phys.* **04** (2011) 014.
  - [12] I. Jeon, K. Lee, and J. H. Park, Stringy differential geometry, beyond Riemann, *Phys. Rev. D* **84**, 044022 (2011).

- [13] J. H. Park, Comments on double field theory and diffeomorphisms, *J. High Energy Phys.* **06** (2013) 098.
- [14] J. H. Park, S. J. Rey, W. Rim, and Y. Sakatani,  $O(D, D)$  covariant Noether currents and global charges in double field theory, *J. High Energy Phys.* **11** (2015) 131.
- [15] S. Angus, K. Cho, and J. H. Park, Einstein double field equations, *Eur. Phys. J. C* **78**, 500 (2018).
- [16] I. Jeon, K. Lee, and J. H. Park, Double field formulation of Yang–Mills theory, *Phys. Lett. B* **701**, 260 (2011).
- [17] O. Hohm and S. K. Kwak, Double field theory formulation of heterotic strings, *J. High Energy Phys.* **06** (2011) 096.
- [18] I. Jeon, K. Lee, and J. H. Park, Incorporation of fermions into double field theory, *J. High Energy Phys.* **11** (2011) 025.
- [19] A. Coimbra, C. Strickland-Constable, and D. Waldram, Supergravity as generalised geometry I: Type II theories, *J. High Energy Phys.* **11** (2011) 091.
- [20] A. Rocen and P. West, *E11, Generalised Space-Time and IIA String Theory: The R-R Sector* (World Scientific, Singapore, 2020).
- [21] O. Hohm, S. K. Kwak, and B. Zwiebach, Unification of Type II Strings and  $T$ -Duality, *Phys. Rev. Lett.* **107**, 171603 (2011).
- [22] O. Hohm, S. K. Kwak, and B. Zwiebach, Double field theory of type II strings, *J. High Energy Phys.* **09** (2011) 013.
- [23] I. Jeon, K. Lee, and J. H. Park, Ramond-Ramond cohomology and  $O(D, D)$  T-duality, *J. High Energy Phys.* **09** (2012) 079.
- [24] I. Jeon, K. Lee, and J. H. Park, Supersymmetric double field theory: A stringy reformulation of supergravity, *Phys. Rev. D* **85**, 081501(R) (2012); Erratum, *Phys. Rev. D* **86**, 089903 (2012).
- [25] I. Jeon, K. Lee, J. H. Park, and Y. Suh, Stringy unification of type IIA and IIB supergravities under  $\mathcal{N} = 2$   $D = 10$  supersymmetric double field theory, *Phys. Lett. B* **723**, 245 (2013).
- [26] S. M. Ko, J. H. Park, and M. Suh, The rotation curve of a point particle in stringy gravity, *J. Cosmol. Astropart. Phys.* **06** (2017) 002.
- [27] C. D. A. Blair, Particle actions and brane tensions from double and exceptional geometry, *J. High Energy Phys.* **10** (2017) 004.
- [28] C. M. Hull, Doubled geometry and T-folds, *J. High Energy Phys.* **07** (2007) 080.
- [29] C. M. Hull and R. A. Reid-Edwards, Non-geometric backgrounds, doubled geometry and generalised T-duality, *J. High Energy Phys.* **09** (2009) 014.
- [30] K. Lee and J. H. Park, Covariant action for a string in *doubled yet gauged* spacetime, *Nucl. Phys.* **B880**, 134 (2014).
- [31] C. D. A. Blair, E. Malek, and A. J. Routh, An  $O(D, D)$  invariant Hamiltonian action for the superstring, *Classical Quantum Gravity* **31**, 205011 (2014).
- [32] J. H. Park, Green-Schwarz superstring on doubled-yet-gauged spacetime, *J. High Energy Phys.* **11** (2016) 005.
- [33] C. D. A. Blair, A worldsheet supersymmetric Newton-Cartan string, *J. High Energy Phys.* **10** (2019) 266.
- [34] K. S. Choi and J. H. Park, Standard Model as a Double Field Theory, *Phys. Rev. Lett.* **115**, 171603 (2015).
- [35] J. H. Park,  $O(D, D)$  completion of the Einstein field equations, *Proc. Sci., CORFU2018* (2019) 145. [arXiv:1904.04705].
- [36] K. Morand and J. H. Park, Classification of non-Riemannian doubled-yet-gauged spacetime, *Eur. Phys. J. C* **77**, 685 (2017); , Erratum, *Eur. Phys. J. C* **78**, 901 (2018).
- [37] K. Cho, K. Morand, and J. H. Park, Kaluza–Klein reduction on a maximally non-Riemannian space is moduli-free, *Phys. Lett. B* **793**, 65 (2019).
- [38] D. S. Berman, C. D. A. Blair, and R. Otsuki, Non-Riemannian geometry of M-theory, *J. High Energy Phys.* **07** (2019) 175.
- [39] K. Cho and J. H. Park, Remarks on the non-Riemannian sector in Double Field Theory, arXiv:1909.10711.
- [40] J. H. Park, Stringy gravity: Solving the dark problems at ‘short’ distance, *EPJ Web Conf.* **168**, 01010 (2018).
- [41] Y. K. E. Cheung and F. Xu, Fitting the Galaxy rotation curves: Strings versus NFW profile, arXiv:0810.2382.
- [42] J. E. Kim, Weak Interaction Singlet and Strong  $CP$  Invariance, *Phys. Rev. Lett.* **43**, 103 (1979).
- [43] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Can confinement ensure natural  $CP$  invariance of strong interactions? *Nucl. Phys.* **B166**, 493 (1980).
- [44] M. Dine, W. Fischler, and M. Srednicki, A simple solution to the strong  $CP$  problem with a harmless axion, *Phys. Lett. B* **104**, 199 (1981).
- [45] A. R. Zhitnitsky, On possible suppression of the axion Hadron interactions, *Yad. Fiz.* **31**, 497 (1980) [*Sov. J. Nucl. Phys.* **31**, 260 (1980)].
- [46] P. Svrcek and E. Witten, Axions in string theory, *J. High Energy Phys.* **06** (2006) 051.
- [47] S. Angus, K. Cho, G. Franzmann, S. Mukohyama, and J. H. Park,  $O(D, D)$  completion of the Friedmann equations, arXiv:1905.03620 [Phys. Rev. D (to be published)].
- [48] J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (Wiley, New York, 1998), p. 832.
- [49] L. Hernquist, An analytical model for spherical galaxies and bulges, *Astrophys. J.* **356**, 359 (1990).
- [50] J. F. Navarro, C. S. Frenk, and S. D. M. White, The structure of cold dark matter halos, *Astrophys. J.* **462**, 563 (1996).
- [51] C. P. Burgess, R. C. Myers, and F. Quevedo, On spherically symmetric string solutions in four dimensions, *Nucl. Phys.* **B442**, 75 (1995).
- [52] R. Andringa, E. Bergshoeff, J. Gomis, and M. de Roo, ‘Stringy’ Newton-Cartan gravity, *Classical Quantum Gravity* **29**, 235020 (2012).
- [53] D. Hansen, J. Hartong, and N. A. Obers, Action Principle for Newtonian Gravity, *Phys. Rev. Lett.* **122**, 061106 (2019).
- [54] E. A. Bergshoeff, J. Gomis, J. Rosseel, C. Simsek, and Z. Yan, String theory and string Newton–Cartan geometry, *J. Phys. A* **53**, 014001 (2020).
- [55] D. Hansen, J. Hartong, and N. A. Obers, Gravity between Newton and Einstein, *Int. J. Mod. Phys. D* **28**, 1944010 (2019).
- [56] T. Basile, E. Joung, and J. H. Park, A note on Faddeev–Popov action for doubled-yet-gauged particle and graded Poisson geometry, arXiv:1910.13120.
- [57] E. J. Post, *Formal Structure of Electromagnetics* (Dover Publications, New York, 1997), p. 224.