CONNECTING NEUTRINO MASSES, MIXING AND RELATED COSMOLOGY BEYOND STANDARD MODEL

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To my Parents

and

Grand parents

Abstract

The discovery of the 125 GeV neutral boson in 2012 at the LHC is off course the most awaited event of this decade in the area of Particle Physics. This milestone discovery declares the Standard Model (SM) as the most successful theory till date with respect to many experimental evidences which meet the predictions made by it. But at the same time, the SM has some inadequacies such as the explanation for origin of neutrino mass, dark matter and matter-antimatter asymmetry. Such phenomena build the primary motivation to look for other avenues beyond the SM (BSM). It will be compelling enough if these three problems can be addressed within a same framework. The BSM scenarios are generally constructed with the extension of SM particle sector, scalar and/or fermions. Inclusion of fermions and/or Higgs triplet have become an essential criteria in order to explain neutrino mass via the various seesaw mechanisms. Apart offering neutrino mass these seesaw mechanisms also have some role to play in modern cosmology due to the presence of these additional particles. As pointed out by Fukugita and Yanagida these right handed neutrinos can play a vital role in leptogenesis through the CP violating decay of the singlet fermion. This fact allows us to study leptogenesis through which baryogenesis can be realized. On the other hand extension of the scalar sector permits us to explore the possibility of establishing one of the extra scalars as a potential particle dark matter (DM) candidate. This thesis has been dedicated for a motivation addressing the above mentioned issues.

As discovery of the neutrino oscillation confirms about the mass of the neutrinos, study of neutrino mass and mixing have become a very contextual subject in the culture of particle physics. The existence of this particle *neutrino* was first proposed by an Austrian physicist Wolfgang Pauli in 1930, in order to preserve

energy-momentum conservation in nuclear β decay. It is good to start with some properties of this new particle called neutrino e.g. it is an electrically neutral fermion, its mass was long thought to be zero, but later on the neutrino oscillation phenomenon along with robust experimental evidences have confirmed the existence of their tiny but nonzero mass. The oscillation phenomenon is realized in the form of oscillation probability which is a function of several parameters termed as neutrino oscillation parameters. These parameters play the key role in the phenomenon of neutrino oscillation. There has been several exercises extensively performed in order to be precisely familiar with these oscillation parameters. With this motivation several BSM scenarios have been proposed which although explain the existence of nonzero neutrino mass, but could not address many long sought queries regarding the oscillation parameters. Among the oscillation parameters are three mixing angles, one phase, two mass squared differences. Depending on various possible values of mixing angles there have been a class of mixing patterns, proposed till date. Three mixing angles constitute, solar, atmospheric and reactor mixing angle, the third of which was thought to be zero till 2011. Then some dedicated experiments e.g. CHOOZ, Daya Bay and RENO revealed that the reactor angle is very small but non-vanishing. Then, some of the queries, that any model in the neutrino sector requires to answer are the exact values of the three phases, which hierarchy the neutrino mass follow, the octant of the atmospheric mixing angle etc. It has now become a tradition to realize the neutrino mass and its mixing considering the discrete flavor symmetry groups due to the fact that the underlying symmetry and their product rules can beautifully offer the existing neutrino mixing patterns. Keeping this in mind we also exercise model building paradigms considering the discrete flavor symmetry groups, specially S_4 and A_4 in our work and within the same frame we try to address some cosmological consequences of the same. We are particularly interested in finding a common platform for exploring the neutrino phenomenology, origin of matter-antimatter asymmetry and Dark Matter. Here, this thesis, therefore is an attempt in this direction.

In **Chapter 1** we first aim at presenting a literature survey of the present updates on neutrino oscillation parameters, what we have with how much accuracy. Then we briefly discuss the Standard Model of particle physics and its inadequacy in realizing some observed phenomena. Here we also discuss the neutrino oscillation phenomena and the class of seesaw scenarios in short with the motivation of going beyond the Standard Model for explaining light neutrino mass via the inclusion of heavy right handed neutrinos. The seesaw models considered for this task correspond to high energy scale and some other, relatively low energy scale. We keep a section for detailed discussion on matter-antimatter asymmetry of the universe. We also have dedicated one section for Dark Matter history. Finally we end up with a section discussing the non-Abelian discrete flavor symmetries like S_4 and A_4 which have extensively been used in model building purpose in this thesis.

In **Chapter 2** we present a TeV scale seesaw mechanism for exploring the dark matter and neutrino phenomenology in the light of recent neutrino and cosmology data. A different realization of the Inverse seesaw (ISS) mechanism with A_4 flavor symmetry is being implemented as a leading contribution to the light neutrino mass matrix which usually gives rise to vanishing reactor mixing angle θ_{13}. Using a non-diagonal form of Dirac neutrino mass matrix and 3σ values of mass square differences we parameterize the neutrino mass matrix in terms of Dirac Yukawa coupling "y". We then use type II seesaw as a perturbation which turns out to be active to have a non-vanishing reactor mixing angle without much disturbing the other neutrino oscillation parameters. Then we constrain a common parameter space satisfying the non-zero θ_{13}, Yukawa coupling and the relic abundance of dark matter. Contributions of neutrinoless double beta decay are also included for standard interaction.

In **Chapter 3** we study an inverse seesaw model of neutrino mass within the framework of S_4 flavour symmetry from the requirement of generating non-zero reactor mixing angle θ_{13} along with correct dark matter relic abundance. The leading order S_4 model gives rise to tri-bimaximal type leptonic mixing resulting in $\theta_{13} = 0$. Non-zero θ_{13} is generated at one loop level by extending the model with additional scalar and fermion fields which take part in the loop correction. The particles going inside the loop are odd under an in-built Z_2^{Dark} symmetry such that the lightest Z_2^{Dark} odd particle can be a dark matter candidate. Correct neutrino and dark matter phenomenology can be achieved for such one loop corrections either to the light neutrino mass matrix or to the charged lepton mass matrix although the latter case is found to be more predictive. The predictions for neutrinoless double beta decay is also discussed and inverted hierarchy in the charged lepton correction case is found to be disfavoured by the latest KamLAND-Zen data.

In **Chapter 4** we study the possibility of generating non-zero reactor mixing angle θ_{13} and baryon asymmetry of the Universe within the framework of an A_4 flavour symmetric model. Using the conventional type I seesaw mechanism, we construct the Dirac and Majorana mass matrices which give rise to the correct light neutrino mass matrix. Keeping the right handed neutrino mass matrix structure trivial so that it gives rise to a (quasi) degenerate spectrum of heavy neutrinos suitable for resonant leptogenesis at TeV scale, we generate the non-trivial structure of Dirac neutrino mass matrix that can lead to the light neutrino mixing through type I seesaw formula. Interestingly, such a setup naturally leads to non-zero θ_{13} due to the existence of anti-symmetric contraction of the product of two triplet representations of A_4. Such antisymmetric part of triplet products usually vanish for right handed neutrino Majorana mass terms, leading to $\mu - \tau$ symmetric scenarios in the most economical setups. We constrain the model parameters from the requirement of producing the correct neutrino data as well as baryon asymmetry of the Universe for right handed neutrino mass scale around TeV. The A_4 symmetry is augmented by additional $Z_3 \times Z_2$ symmetry to make sure that the splitting between right handed neutrinos required for resonant leptogenesis is generated only by next to leading order terms, making it naturally small. We find that the inverted hierarchical light neutrino masses give more allowed parameter space consistent with neutrino and baryon asymmetry data.

In **Chapter 5** we have exercised an Inverse seesaw model based on the S_4 flavor symmetry with an adaptation of type II seesaw mechanism. The leading order neutrino mass is explained under the scheme of ISS, which is later on accompanied by the type II seesaw mechanism in order to reproduce non-zero reactor mixing angle. The type II seesaw perturbation at the same time yields the other oscillation parameters undeviated from their correct 3σ range. A detailed analysis has been performed by varying the Dirac Yukawa coupling and type II seesaw strength which together play a crucial role in obtaining the oscillation parameters in agreement with the recent experiments. We calculate the contribution to the effective mass governing $0\nu\beta\beta$ decay assuming it to take place through the exchange of light neutrinos.

In **Chapter 6** we discuss the overall conclusions and summary of the work carried out in this thesis. Finally, we end up with the future plan of the research in the field of neutrino physics.

DECLARATION BY THE CANDIDATE

I, Ananya Mukherjee, hereby declare that the subject matter in this thesis entitled, "**Connecting neutrino masses, mixing and related cosmology beyond standard model**", is a presentation of my original research work. Although contributions of others are involved, every effort is made to indicate this clearly, with due reference to the literature and acknowledgement of collaborative research and discussions.

The work is original and has not been submitted earlier as a whole or in part for a degree or diploma at this or any other institute or university.

This thesis is being submitted to the Tezpur University for the degree of Doctor of Philosophy in Physics.

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CERTIFICATE OF THE SUPERVISOR

This is to certify that the thesis entitled "**Connecting neutrino masses, mixing and related cosmology beyond standard model**" submitted to the School of Sciences Tezpur University in partial fulfillment of the requirements for the award of the degree of **Doctor of Philosophy** in **Physics** is a record of original research work carried out by **Ms. Ananya Mukherjee** under my supervision and guidance.

All help received by her from various sources have been duly acknowledged.

No part of this thesis has been submitted elsewhere for award of any other degree.

Place : Tezpur

(Mrinal Kumar Das)

Date :

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Date : *Ananya Mukherjee*

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List of Abbreviations

SM	Standard Model
BSM	Beyond Standard Model
EWSB	Electro Weak Symmetry Breaking
SUSY	Supersymmetry
LHC	Large Hadron Collider
LEP	Large Electron Positron
CERN	The European Organization for Nuclear Research
CC	Charged Current
NC	Neutral Current
PMNS	Pontecorvo-Maki-Nakagawa-Sakata
CKM	Cabibbo-Kobayashi-Masakawa
GUT	Grand Unified Theory
CP	Charge-Parity
RENO	Reactor Experiment for Neutrino Oscillations
NDBD	Neutrinoless Double Beta Decay
BM	Bi-Maximal
TBM	Tri-bimaximal
HM	Hexagonal Mixing
GRM	Golden Ratio Mixing
NH	Normal Hierarchy
IH	Inverted Hierarchy
ISS	Inverse seesaw
eV	Electron Volt

GeV	Giga Electron Volt
TeV	Tera Electron Volt
RH	Right Handed Neutrino
bfp	Best fit point
LH	Left handed
B+L	Baryon + Lepton
LN	Lepton Number
VEV	Vacuum Expectation Value
BBN	Big Bang Nucleosynthesis
CMBR	Cosmic Microwave Background Radiation
WMAP	Wilkinson Mass Anisotropy Probe
DM	Dark Matter

"Never theorize before you have data. Invariably, you end up twisting facts to suit theories instead of theories to suit facts"

Sherlock Holmes in "A Scandal In Bohemia"

1

Introduction

In this introductory chapter we first aim at presenting a literature survey of the present updates on neutrino oscillation parameters, what we have with how much accuracy. Then we briefly discuss the Standard Model of particle physics and its inadequacy in realizing some observed phenomena. Here we also discuss the neutrino oscillation phenomena and the class of seesaw scenarios in short with the motivation of going beyond the Standard Model for explaining light neutrino mass via the inclusion of heavy right handed heavy neutrinos. The seesaw models considered for this task correspond to high energy scale and some other, relatively low energy scale. We keep a section for detailed discussion on matter-antimatter asymmetry of the universe. We also have dedicated one section for dark matter history. Finally we end up with a section discussing the non-Abelian discrete flavour symmetries like S_4 and A_4 which have extensively been used in model building purpose in this thesis.

Study of Neutrinos and its associate observables continue to intrigue. An Austrian physicist Wolfgang Pauli [1–4] in 1930, proposed the existence of a neutral particle called *neutrino* (as a mathematical trick) in order to preserve energy-momentum conservation in nuclear β decay. This proposal of Pauli opened up a new avenue in the particle physics ball park. Thus to start with, neutrinos are electrically neutral fermions. Their mass was long thought to be zero, although neutrino oscillation experiments have confirmed the tiny mass that they possess. However Pauli had also supposed that nobody would ever be able to detect this new particle due to the fact that they interact feebly with matter. Then in the year 1956, Clyde Cowan and Fred Reins [5] had gone through an observation of anti-neutrinos emitted by a nuclear reactor at Savannah River at South Carolina, USA. It was later found that the observed neutrino was an electron neutrino which is a partner of an electron. The SM is unable to accommodate neutrino mass as there is no right handed counter part of neutrino in the SM. And this fact calls for some BSM frameworks, by the inclusion of right handed (RH) Majorana neutrinos to the SM fermion sector. Neutrinos being electrically neutral are allowed to possess Majorana masses. For, a Majorana neutrino mass can not arise from the neutrino analogue of the SM coupling that gives quarks and charged lepton their masses. That analogue would be a Yukawa coupling of the form $H_{SM}\bar{\nu}_R\nu_L$, where H_{SM} is the SM Higgs field. Rather, Majorana masses must come from couplings such as $H_{SM}H_{SM}\bar{\nu_L^c}\nu_L$ or $H_{I_W=1}\bar{\nu_L^c}\nu_L$, the first of which implies non-renormalizability and therefore outside the scope of the SM but the second involves a Higgs Boson with weak isospin $I_W = 1$, which the SM does not accommodate. In this way within the SM, neutrinos remain mass less. Although this theoretical prediction was consistent with the experiments till 1960 due to lack of evidence of neutrino mass but this fact had gained much interest due to the fact that results from solar neutrino experiment and atmospheric neutrinos indicated towards a massive neutrino.

1.1 Present status of neutrino parameters

In the year 1968, an American physicist Raymond Davids Jr. while detecting solar neutrinos [6, 7] for the first time from a deep underground experiment, observed that the number of electron neutrino measured was one third of the actual number that was expected to come from Sun, the phenomenon later on named as Solar neutrino problem. In the same way, was found a discrepancy [8, 9] while measuring muon neutrino flux coming from earth atmosphere and this is familiar as atmospheric neutrino problem. In this context Mikheyev, Smirnov along with Wolfenstein told that only electron neutrinos are emitted by the Sun and they could be converting into muon and tau neutrino which were not being detected on earth. Such a scenario of inter-conversion from one kind to another is termed as neutrino oscillation [10]. Theoretical justification of neutrino oscillation beautifully fix the puzzle created from solar and atmospheric neutrino fluxes. Then in the year 1998, Super Kamiokande [11, 12] experiment, piloted by Takaaki Kajita from Japan, evinced that there was a deficit in the number of muon neutrinos reaching from earth when cosmic rays strike with earth's atmosphere. This experiment was able to detect only half of the muon neutrinos actually expected. Then in the year 2001/2002, Arthur B. McDonald in Canada guided the Sudbury Neutrino observatory (SNO) collaboration, and did a detailed measurement of the fluxes of both the neutrinos along with total flux of all the three types of neutrinos. Interestingly, the result found in SNO collaboration was consistent with the theoretically predicted result for electron neutrinos coming from the Sun. This experiment confirmed the conversion of electron neutrino to the other two kinds i.e., muon and tau neutrino. This phenomenon of oscillation from one particular kind of neutrino to the other two is termed as neutrino flavor oscillation, where the term flavor is used to mean the three kinds of neutrinos namely electron (ν_e), muon (ν_μ) and tau neutrino (ν_τ). For the above extensive and nontrivial study led by Takaaki Kajita and Arthur B. McDonald, they shared the 2015 Nobel Prize in Physics. Later on many experiments such as KamLAND [13, 14] nuclear reactor in Japan, K2K [15] long base line experiments also in Japan, Fermilab-MINOS [16] in U.S. put concrete evidence of the phenomenon called neutrino oscillation.

After the discovery of neutrino oscillation there is no doubt that neutrino possess masses, however, tiny. That time the particle physics community did not remain silent only with this discovery and started to think about the other properties associated with this oscillation phenomenon. Some of them are what are their mixing angles, what is their absolute mass scale, then which flavored neutrino is the heaviest (and which is the lightest). Therefore the above mentioned queries are also extensively exercised both theoretically from some neutrino mass models and practically in some neutrino oscillation experiments. T2K [17], Double Chooz [18], Daya-bay [19] and RENO [20] are some of the experiments which provided us with information about the neutrino mass squared splittings and mixing angles with a very strong precision. These experiments gave bounds on the mass squared splittings of order $\Delta m^2_{\rm sol} \approx 10^{-5} eV^2$ and $\Delta m^2_{\rm atm} = 10^{-3} eV^2$. Such a small mass splitting not only hints towards the tiny magnitude of neutrino mass but also shows a 10^{12} order of mass difference between the neutrino and top quark mass. These experiments only could measure two mass squared splittings rather than the individual masses possessed by three flavor of them. Moreover, the leptonic mixing angles also are under huge discussion. There are three mixing angles in the neutrino sector: solar (θ_{12}), atmospheric (θ_{23}) and reactor (θ_{13}). Earlier it was believed that the value of reactor mixing angle is zero. But later on some dedicated neutrino oscillation experiments confirmed that the reactor mixing angle is non-zero although tiny as compared to the other two. In support of these neutrino data, there have been found several mixing schemes namely bimaximal (BM), Tri-bimaximal (TBM), hexagonal (HM) and Golden ratio mixing (GRM). Among them TBM has gained more popularity as, the mixing angles predicted by this mixing pattern is very much consistent with the angles observed in experiments. In TBM scenario we find $sin^2\theta_{12} = 0.33$, $sin^2\theta_{23} = 0.5$ with $sin^2\theta_{13} = 0$. However TBM has also lost the favor as the latest data ruled out a zero value for the reactor angle. In order to address a non-vanishing reactor angle, thus one needs to break the above mentioned mixing patterns, since all of them accommodate a zero value for reactor angle. Now if we look at the Pontecorvo Maki Nakagawa Sakata (PMNS) mixing matrix we find that a CP violating phase delta is associated with the reactor angle in the third column of it. The value of which is still unknown and hence is kept in the

"To find" list.

As already said that within the SM we do not have neutrino mass and at the same time the existence of neutrino mass is also a truth, thus one needs to go beyond the SM to validate the above two facts. And this task is carried out by an extension of the SM particle content by the inclusion of the missing RH neutrinos. Seesaw mechanisms are such methods to implement the consequences of adding two or more right handed heavy neutrinos to the SM fermion sector and generating the neutrino mass via some higher dimension terms. Below we discuss a brief overview on the SM and its drawbacks.

1.2 Standard model

It was in the year 1960, G. Glashow, S. Weinberg and P. Salam proposed the Standard Model [21–23] which is a quantum field theory particularly a spontaneously broken Yang-Mills theory, that takes all the three fundamental forces (strong, weak and electromagnetic) into account except the gravity. The SM gauge group is given by

$$G_{SM} \subset SU(3)_C \times SU(2)_L \times U(1)_Y \tag{1.2.1}$$

The symmetry group $SU(3)_C$ is the group of "color" that comes from quantum chromodynamics, having eight generators, particle representatives of which are gluons, the carrier of the strong force. The weak isospin group is named as $SU(2)_L$ which has three generators. The SM gauge group has four gauge bosons, three of which, namely $W^{\pm}$ and Z^0 are mediators of weak interaction and the particle representatives of $SU(2)$ group generators as, $SU(2)$ has three generators. Likewise $U(1)_Y$ is the group of hypercharge, the generator of the group corresponding to the massless boson: photon, which is the mediator of electro-magnetic interaction. The SM provides a concrete platform to describe the particles and their interactions that constitute the model itself. Now, on the basis of some physical properties of the particles, they are categorized as scalars, fermions and gauge bosons. Among them left handed fermions of the SM transform as $SU(2)$ doublets. The SM fermions are categorized into three generations. The scalar boson Higgs is also a doublet under $SU(2)$ and singlet of $SU(3)$ symmetry group. The entire particle content and their charges under each symmetry groups are listed in Table 1.1. In addition the newly discovered

		I	I_3	Y	Q
Lepton Doublet	$\binom{\nu_e}{e}_L$	1/2	1/2 -1/2	-1 -1	0 -1
Lepton Singlet	e_R	0	0	-2	-1
Quark doublet	$\binom{u}{d}_L$	1/2	1/2 -1/2	1/3 1/3	2/3 -1/3
Quark Singlets	U_R d_R	0	0 0	4/3 -2/3	2/3 -1/3
Higgs Doublets	$\binom{\phi^+}{\phi^0}$	$\frac{1}{2}$	1/2 -1/2	1 1	1 0

Table 1.1: Charges of the SM particles and the Higgs boson under isospin(I), third component of isospin(I_3), Hypercharge(Y) and electric charge(Q)

$L_L(1,2,-\frac{1}{2})$	$Q_L(3,2,\frac{1}{6})$	$E_R(1,1,-1)$	$U_R(3,1,\frac{2}{3})$	$D_R(3,1,-\frac{1}{3})$
$\binom{\nu_e}{e}_L$	$\binom{u}{d}_L$	e_R	u_R	d_R
$\binom{\nu_\mu}{\mu}_L$	$\binom{c}{s}_L$	μ_R	c_R	s_R
$\binom{\nu_\tau}{\tau}_L$	$\binom{t}{b}_L$	τ_R	t_R	b_R

Table 1.2: Charge assignments of SM particle contents [24]

Higgs field gets the charges under SM gauge group as,

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \sim (1, 2, \frac{1}{2}) \tag{1.2.2}$$

The vacuum expectation value of the Higgs field breaks the gauge symmetry,

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \implies G_{SM} \to SU(3)_C \times U(1)_{em} \tag{1.2.3}$$

Therefore, the SM has only three active neutrinos with their charge conjugate partners. Charged lepton mass eigenstates are denoted as e, μ and τ with their $SU(2)_L$ partners ν_e, ν_μ and ν_τ respectively. The active neutrinos undergo weak charged current (CC) interaction in the following manner

$$-\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \sum_{l} \bar{\nu}_L \gamma_\mu l_L^- W_\mu^+ + H.C. \tag{1.2.4}$$

Moreover the three active neutrinos undergo neutral current (NC) interactions,

$$-\mathcal{L}_{NC} = \frac{g}{2cos\theta_W} \sum_{l} \bar{\nu}_{Ll} \gamma_\mu \nu_{Ll} Z_\mu^0 + H.C. \tag{1.2.5}$$

where θ_W is termed as Weinberg or weak mixing angle. All the interaction by SM neutrinos are described by the above two equations. The SM also follows an accidental global symmetry

$$G_{\text{SM}}^{\text{global}} = U(1)_B \times U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau} \tag{1.2.6}$$

where $U(1)_B$ is the baryon number and $U(1)_{L_{(e,\mu,\tau)}}$ are the symmetries of the three lepton flavor with total lepton number $L = \sum_i L_i$ where i represents three flavors of lepton: e, μ and τ. The Lepton Number (LN) is said to be an accidental symmetry as it is not an imposed symmetry rather generated as a result of the gauge symmetry. The fermions and gauge bosons get their masses from Higgs mechanism via the spontaneous symmetry breaking, that we discuss in the following section. But only the neutrinos remain massless. Fermions in the SM gets masses from the Yukawa interactions of a left handed doublet with its right handed counter part and SM Higgs field. The complete Yukawa Lagrangian of the SM is given by

$$-\mathcal{L}_{\text{Yukawa}} = Y_{ij}^d \bar{Q}_{Li} H D_{Rj} + Y_{ij}^u \bar{Q}_{Li} \tilde{H} U_{Rj} + Y_{ij}^l \bar{L}_{Li} H E_{Rj} + \text{H.C.} \tag{1.2.7}$$

with $\tilde{H} = i\sigma_2 H^*$, the isospin conjugate of the Higgs doublet with σ_2 as the Pauli's spin matrix and also one of the generators of the weak isospin group $SU(2)$. The SM enlightened on the existence of three massive gauge bosons, later on the existence of which got verified in LEP experiment at CERN, Geneva. In addition it also predicts nine massless gauge bosons and existence of massive fermions.

1.3 Spontaneous symmetry breaking and Higgs mechanism

Spontaneous Symmetry breaking is a scenario where, the symmetry of the Lagrangian is not the symmetry of the vacuum state or the minimum energy state. If the vacuum state takes a nonzero value v of any field H ($< H >= v$), then any physical field can be written as $H^{\rm phys} = H - v$. Where we call v as the vacuum expectation value (VEV) of the field ϕ. For a scalar particle the Lagrangian is written as,

$$\mathcal{L} \equiv T - V = \frac{1}{2}(\partial_\mu H)^2 - (\frac{1}{2}\mu^2 H^2 + \frac{1}{2}\lambda H^4), \quad (1.3.1)$$

with positive definite λ, provided that, the Lagrangian remains the same under the interchange of H by $-H$. Depending on the sign of the μ^2 term, the minimum of the potential implies the following conditions:

$$< H^2 >= 0, \mu^2 > 0 \quad (1.3.2)$$

$$< H^2 >= v^2 = -\frac{\mu^2}{\lambda} > 0, \mu^2 < 0 \quad (1.3.3)$$

Now the extremum $H = 0$ does not interpret the minimum energy state which we are looking for. Whereas $H = \pm v$ with $v = \sqrt{-\mu^2/\lambda}$ represents the spontaneous breaking of the symmetry as the ground state of the system corresponds to a nonvanishing value of H. When the vacuum takes a value $< H >= v$, this is called the vacuum expectation value of H.

Higgs mechanism is the mass generation mechanism of all fermions and gauge bosons within the SM except neutrinos. Higgs is a complex scalar transforming as an $SU(2)$ doublet which has a hypercharge quantum number 1. H^0 is the neutral component of the scalar field which acquires a VEV and break the EW symmetry $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$ at that scale making fermions and gauge bosons massive [25]. But, the gluon and photon remain massless as the $SU(3)_C$ and $U(1)_{em}$ symmetry are protected.

The relevant part of the Lagrangian particular in purpose of Higgs mechanism is give by

$$\mathcal{L}_{higgs} = (D_\mu H)^\dagger (D_\mu H) - V(H) \tag{1.3.4}$$

being D_μ as the covariant derivative has the following form,

$$D_\mu = (\partial_\mu - \frac{i}{2} g W_\mu^j \tau^j - \frac{iY_H}{2} g' B_\mu) \tag{1.3.5}$$

In the above expression for the covariant derivative we define τ^j as the Pauli spin matrices, Y_H is the hypercharge of the SM Higgs, g is the coupling constant for $SU(2)_L$ group and g' for $U(1)_Y$ gauge group. The scalar potential of the Higgs field has the following form

$$V(H) = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2 \tag{1.3.6}$$

Minimization of this potential gives the solution for H, and is obtained as

$$< H > = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \text{ where}, v = \sqrt{\frac{-\mu^2}{\lambda}}. \tag{1.3.7}$$

The masses of the vector gauge bosons obtained from the Lagrangian (1.3.4) are written by,

$$\mathcal{L} = M_W^2 W_\mu^+ W_\mu^- + \frac{1}{2} M_Z^2 Z_\mu Z^\mu \tag{1.3.8}$$

where, $W^+ = \frac{W_\mu^1 - iW_\mu^2}{\sqrt{2}}, W^- = \frac{W_\mu^1 + iW_\mu^2}{\sqrt{2}}, Z_\mu = cos\theta_W W_\mu^3 - sin\theta_W B_W$. Masses of the vector boson thus can be written as; $M_W = \frac{gv}{2}$ and $M_Z = \frac{gv}{2cos\theta_W}$. Experiments gave a bound on mass of W boson equal to 80 GeV and Z boson of 90 GeV. However, the photon field remains massless as the $U(1)_{em}$ is preserved in the end. One can express the photon field in terms of the W_μ^3 and B_μ field as

$$A_\mu = cos\theta_W W_\mu^3 + sin\theta_W B_\mu. \tag{1.3.9}$$

The process of spontaneous symmetry breaking along with the Higgs mechanism together make this job of generating the masses of fermions (except neutrinos) and bosons easy in the SM. The fermion masses generated after the EWSB are given by

$$m_l = \frac{Y_{ij}^l}{\sqrt{2}} v,\ m_u = \frac{Y_{ij}^u}{\sqrt{2}} v,\ m_d = \frac{Y_{ij}^d}{\sqrt{2}} v. \tag{1.3.10}$$

where, Y_{ij}^l,Y_{ij}^u, Y_{ij}^d are the Yukawa couplings of charged leptons, up-type quark and down-type quark respectively, v is the SM Higgs VEV.

1.4 Drawbacks of the standard model

Notwithstanding, there are some conclusive experimental evidences, such as neutrino masses, dark matter and the matter-antimatter asymmetry, along with theoretical issues, like the hierarchy problem, the strong-CP problem or the flavor puzzle, which are not addressed or explained within the SM, thereby inviting us to a journey towards new physics beyond the SM (BSM). It is in general believed that there exists new physics (NP) beyond the SM at a higher energy scale above the electroweak symmetry breaking scale ($\Lambda_{EW} \sim 100$GeV). Even the SM can not address physics at Planck scale (10^{19} GeV). Now within these two scales there lies some new physics and origin of whom are interrelated. It has become essential to list some relevant problems of the SM from experiment and observation point of view.

- Experimental evidences (the dedicated neutrino oscillation experiments and also the issue of solar and atmospheric neutrino problems) of massive neutrinos contradicts the facts that is apprised by the SM about the neutrino mass.

- Then some observational confirmations of NP consists of the Cosmic Microwave Background Radiation (CMBR) and the standard Big Bang Nucleosynthesis (BBN) scenario which push us to think seriously about the biggest mystery of the Universe– *the matter-antimatter asymmetry.* Now, in order to generate such asymmetry, adequate amount of CP violation is required which the SM interaction schemes are unable to produce. This again leads us to find a new source of CP violation which can account for the observed amount of baryon asymmetry set by CMBR and BBN data. The detail of this issue we address in Section 1.7.12.

- Another significant drawbacks of the SM is that, it does not enlighten us on the existence of dark matter, whose abundance is nearly the 26% of the total density of the Universe. The detail of dark matter observation we keep in Section 1.7.2.

- The unification of four gauge couplings of the strong, weak, electromagnetic and gravitational interactions is one of the chief concern in particle physics. The SM places only the electromagnetic and weak couplings in a single frame and unify them keeping gravity completely aside. Unification of electroweak and gravitational force is a difficult job to pursue within the SM, as the SM does not provide any quantum description of gravity. Strong and electroweak force unification too is not accommodated within the SM. String theory can unify all the four fundamental forces, but for that again one needs to go beyond the SM.

Moreover, there are some other limitations also. *Naturalness* is one of the most serious ones: which says there are small parameters in the SM and it demands supernatural fine-tuning to explain them.

- Loop corrections to the Higgs mass are commonly quadratic in the mass of the heaviest particle present in the loop, which is also a property of the hierarchy problem. It is thought that the heavy particle which plays a role in making the neutrino mass, can also couple to the Higgs boson, which can result in making an impermissible contribution to the Higgs mass. And for this reason there is always a concern for the upper limit on the mass of the heavy particle responsible for the generation of neutrino mass. To be precise, the Higgs mass is unstable against quantum corrections [26, 27] and is not protected by any symmetry. If we impose a one loop correction to Higgs mass, it is proportional to $\Lambda_{\rm UV}^2$, where Λ_{UV} is the cutoff scale where NP is awaited. Difficulty arises if Λ becomes of the order of Planck Mass ($M_{\rm Pl}$), then value of the quantum correction turns out to exceed the required value of the Higgs Boson mass. Adjusting the Higgs mass to be around 100 GeV, one needs a tremendous fine-tuning. Supersymmetry (SUSY) can solve this fine-tuning problem, stabilizing the ratio $\Lambda_{\rm EW}/M_{\rm Pl}$ [28–31].

- The Yukawa couplings are quite small as compared to the top Yukawa coupling and thus hierarchical. The same fact holds good for masses of the fermions as well. For example, the electron mass is 0.5 MeV whereas the top quark mass is nearly 175 GeV which shows a 10^6 order of magnitude difference. There is no explanation of such vast hierarchy within the SM.

Keeping the above mentioned agendas in mind we look for a theory beyond the Standard Model which possibly will be able to shade light on these phenomena. Since the SM does not accommodate neutrino mass, a chief job will be to build a model which can easily make the neutrino mass non-zero however tiny. For that Stephen Weinberg introduced dimension 5 operator through the implementation of seesaw mechanisms. To implement seesaw mechanism one needs to incorporate the missing RH neutrinos to the SM fermion sector.

1.5 Neutrino mass beyond the SM

Although the SM does not offer the explanation for neutrino mass, but neutrino oscillation phenomena established the fact that neutrinos have tiny but nonzero mass. Now this fact needs a theoretical justification too. The justification for the solar and atmospheric neutrino anomaly reveals that neutrinos from one flavor oscillate to another flavor after traveling through a considerably large distance; the phenomenon known as neutrino flavor oscillation which is the observational evidence of neutrinos being massive. On the other hand, KamLAND and some recent experiments involving solar, atmospheric, reactor and accelerator neutrinos have confirmed the neutrino oscillation (please see [32] for a review).

Now, when a neutrino is produced, it is in a specific flavor state, which is expressed as a superposition of the mass eigenstates. Had the neutrinos been massless or degenerate in mass, all the mass eigenstates would have the same time evolution and, thereafter, the initial flavor state would remain unchanged. Here, in this section we briefly summarize the mathematical expressions showing the oscillation probability and mixing of flavor and mass eigenstates. The flavor and mass eigenstate have different bases. We denote the flavor state as ν_α for $\alpha = e, \mu, \tau$ and mass state as ν_i for $i = 1, 2, 3$. The flavor and mass eigenstate are related with each other by a unitary matrix of order three, popularly known as the PMNS mixing matrix. The name arises after Pontecorvo, who proposed neutrino oscillations, and from Maki-Nakagawa-Sakata, who introduced the mixing matrix. This is analogous to the CKM mixing matrix in the quark sector. The order three implies the number of three generations of neutrinos. One can write the following equation showing the relation between the neutrino mass and flavor eigenstate.

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U_{\alpha i}|\nu_i\rangle \tag{1.5.1}$$

The PMNS mixing matrix is parameterized in terms of three mixing angles and six phases, which are popularly known as CP-phases. Since all the phases are not physical and hence three of them gets removed by phase redefinition and rest three remains. Now the Dirac nature of neutrino leads to the removal of more two phases, thus we are left with only one physical phase δ popular as Dirac CP-phase. In the same context if we consider a Majorana type neutrino then we have two more phases α and β. All the above mentioned parameters are called neutrino mixing parameter which altogether construct the unitary matrix as the following

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} U_{\rm Maj} \tag{1.5.2}$$

where $c_{ij} = \cos\theta_{ij},\ s_{ij} = \sin\theta_{ij}$. The diagonal matrix $U_{\rm Maj} = {\rm diag}(1, e^{i\alpha}, e^{i(\zeta+\delta)})$ contains the Majorana CP phases α, ζ. The oscillation probability for a neutrino going from a flavor α to β is given by

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} {\rm Re}[U_{\beta i}U^*_{\alpha i}U^*_{\beta j}U_{\alpha j}]\sin^2\Big(\frac{\Delta m^2_{ij}L}{4E}\Big) \tag{1.5.3}$$

$$- 2\sum_{i>j} {\rm Im}[U_{\beta i}U^*_{\alpha i}U^*_{\beta j}U_{\alpha j}]\sin^2\Big(\frac{\Delta m^2_{ij}L}{2E}\Big) \tag{1.5.4}$$

Under the interchange of $U \to U^*$, the first two terms in the Lagrangian remain same, which reveals the conservation of CP, whereas the last term alters the sign implying the difference between neutrino and antineutrino oscillation probability, which we can express analytically as,

$$P(\bar{\nu_\alpha} \to \bar{\nu_\beta}) - P(\nu_\alpha \to \nu_\beta) = 4\sum_{i>j} \mathrm{Im}[U_{\beta i}U^*_{\alpha i}U^*_{\beta j}U_{\alpha j}]\mathrm{sin}^2\Big(\frac{\Delta m^2_{ij}L}{2E}\Big) \qquad (1.5.5)$$

where, $U = U_{PMNS}$ in short, $E \sim |p|$ is the neutrino energy, L is the distance between the source and the detector, and $\Delta m^2_{ij} = m^2_i - m^2_j$ is the mass squared difference. Where, the condition $\alpha = \beta$ makes the RHS of the equation vanish, resulting into a zero CP asymmetry. Thus we need at least two generations or flavors of light neutrinos to have an estimation of CP asymmetry. For having a nonzero probability for flavor oscillation the mass squared difference is needed to be non-zero, which fact thereafter proves the existence of neutrino mass.

Various neutrino mixing parameters are under observation and study, e.g., KamLAND experiment has evinced a considerably large solar mixing angle and confirmed the solar neutrino oscillation. Very recently the value of reactor angle has been found to be tiny but non-zero as declared by Double CHOOZ [33–35], Daya Bay [19] and RENO [20]. Now there are series of questions after proven the existence of neutrino mass and nonzero reactor angle, which are worrying the neutrino physics community to a grater extent. Some of them are, (i) which hierarchy of mass pattern, does the neutrino mass follow? (ii) what is the absolute neutrino mass scale?, as we only have two mass squared splitting (solar and atmosphere) and the sum over absolute masses (iii) why there is a deviation of the atmospheric mixing angle from the maximal value and in which octane does it belong? Very recently some dedicated experiment groups planned to study all the above mentioned issues: Daya Bay, T2K, RENO, NOνA and Double Chooz are such examples. The information on sum over absolute neutrino masses come from cosmological observation: WMAP analysis set an upper bound on $\sum m_i$ and is found to be $\sum m_i \leq 0.17$ eV. The figure 1.1 evinces the possible hierarchy pattern, which the neutrino mass may follow. There are two possible, as such, namely inverted and normal. Even if we are sure about the neutrino mass, however it is still under observation and analysis, asking, what mass hierarchy the neutrino mass exactly does follow. Well, till date we do not have any answer for

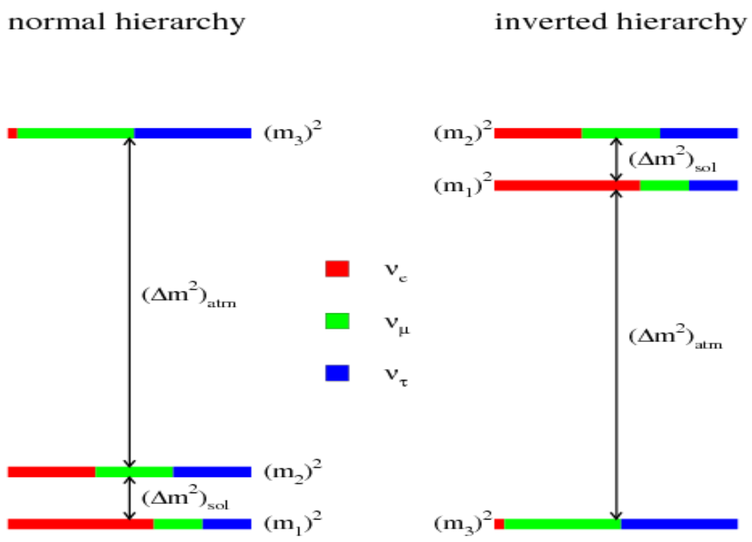

Figure 1.1: Possible hierarchy pattern of neutrino mass, we call them normal (left) and inverted (right) hierarchies [36]. The colors represent the flavor composition of each of the physical neutrinos: red for ν_e, green for ν_μ and blue for ν_τ

that although there have been several theoretical models which rule out the either mass ordering. The oscillation formula evince that the probability depends on the mass squared splitting, with no information on the absolute neutrino mass. The oscillation experiments always gave a positive value for the solar mass splitting which clearly implies $m_2 > m_1$, however they do not say about the sign of the atmospheric mass splitting Δm_{31}^2. Thus we can have two possible mass orderings [37–39] depending on the sign of Δm_{3l}^2.

- Normal mass hierarchy, which follow $m_3 > m_2 > m_1$ and
- Inverted mass hierarchy, which follow $m_2 > m_1 > m_3$

The list of queries does not end here! It is a long standing mystery asking whether neutrinos are Dirac or Majorana particle? Well for both the cases, the mass term can be generated via gauge invariant Yukawa like interaction followed by

$$\mathcal{L}_{\text{Dirac}} = \bar{\nu_R} m_\nu \nu_L + \text{H.C.} \tag{1.5.6}$$

$$\mathcal{L}_{\text{Majorana}} = \nu_L^C m_\nu \nu_L + \text{H.C.} \tag{1.5.7}$$

In essence, these two mass terms are different from each other in the point that for a Dirac type neutrino mass there is no violation of lepton number unlike the case for a Majorana neutrino mass term, which violates lepton number by two units. For both the scenario one has to look for a BSM scenario where there is an introduction of a gauge singlet RH neutrino. The RH neutrinos transform under the SM gauge group following a charge assignment like (1,1,0) under the respective symmetry groups. The Dirac mass term arises from a Yukawa coupling of the SM Higgs with the left handed neutral lepton of the lepton doublet in presence of the RH neutrino whose Yukawa Lagrangian reads

$$- \mathcal{L}_{\rm Yuk} = Y_\nu \bar{N_R} \tilde{H} L + {\rm H.C.} \tag{1.5.8}$$

with $\tilde{H} = i\sigma_2 H^*$, ϕ being the SM Higgs field. After the EWSB, the neutral component of the Higgs acquires VEV of around 174 GeV and neutrino gets a Dirac mass as $\bar{N_R} m_\nu L$, where $m_\nu = Y_\nu v$.

As the neutrinos are only electrically neutral fermions in the SM, they could be Majorana particles, which by definition could be their own antiparticles. This would be in contrast to the rest of the SM Dirac fermions, for which their antiparticle is a different state. This hypothetical Majorana character of the neutrinos, although very common in theoretical models (as we will see later), does not have any impact on neutrino oscillations and, therefore, new observables to distinguish between Majorana and Dirac fermions need to be considered. The fact that a lepton can be its own antiparticle is directly related to the total lepton number (LN) violation, since a Majorana mass terms breaks LN symmetry by two units. Consequently, LN violating processes are usually considered as the smoking gun signatures for Majorana neutrinos, like neutrinoless double beta decay. Unfortunately, no experimental evidence has been found yet for any LN violating processes. Thus, knowing, if neutrinos are Majorana or Dirac fermions still an unprecedented job. Majorana mass comes from the seesaw mechanism where we introduce the RH heavy gauge singlet fermion. Seesaw models generate neutrino mass via the dimension-5 Weinberg operator, which we discuss in the following section. After the EWSB the Higgs doublet takes VEV and generates the following Majorana mass term

$$m_\nu = \frac{Y_\nu v^2}{4\Lambda_L} \tag{1.5.9}$$

For a coupling strength Y_ν of the order 1, in order to get a sub-eV light neutrino mass scale, Λ_L has to fall around $10^{14} - 10^{16}$ GeV where lepton number violation takes place.

1.6 Seesaw mechanism

Seesaw mechanisms play a non-trivial role in making the neutrinos massive. In essence this mechanism accounts for lepton number violation via the implementation of non-renormalizable dimension-5 Weinberg operator. Seesaw mechanism necessitates the extension of the SM by the incorporation of some extra fermions or scalars. Depending on the class of particle we add to the SM, different seesaws are named such as: type I seesaw, where right handed heavy neutrinos N_R (gauge singlets) are introduced; type II seesaw requires the inclusion of a scalar $SU(2)$ triplet Δ; and type III seesaw, which demands the introduction of fermion triplet (under $SU(2)$) field Σ. There is another kind of seesaw mechanism, termed as Inverse seesaw mechanism, which is a low scale seesaw scenario, we explain that in the end of this section.
Neutrino, being a member of the $SU(2)$ doublet representation, only one possible Weinberg operator is there to contribute to the Lagrangian, the expression for which one may write as

$$\delta\mathcal{L}_{d=5} = \frac{1}{2}\frac{c_{ij}}{\Lambda}(\bar{L_i^C}\tilde{H}^*)(\tilde{H}^\dagger L_j), \tag{1.6.1}$$

with c_{ij} as the dimensionless complex coefficient and $i, j = 1, 2, 3$. This operator yields a Majorana mass term for the neutrinos after the EWSB as,

$$\delta\mathcal{L}_{d=5} \to \frac{v^2 c_{ij}}{2\Lambda}(\bar{\nu_i^C}\nu_j + H.C.). \tag{1.6.2}$$

The higher the scale Λ, the smaller is the neutrino mass which naturally seems a seesaw scenario. And thus it is called so. The structure of the Weinberg operator decides the property of the new particles of a particular seesaw model. From the

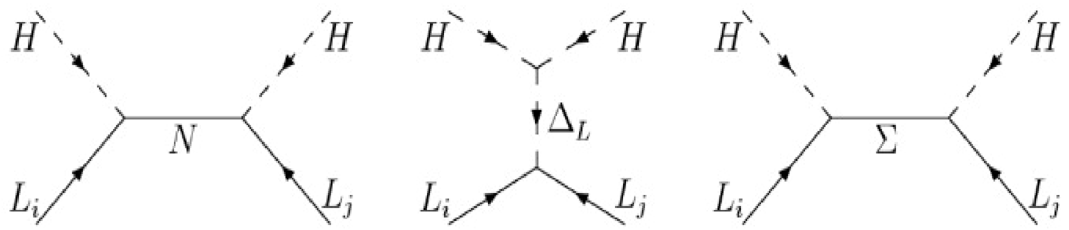

Figure 1.2: Schematic representation of type I, type II and type III seesaw mechanism

requirement of gauge invariance the new particle can be an $SU(2)$ singlet or triplet as it is supposed to couple to two $SU(2)$ doublets. despite this fact, the new particle can be either scalar or fermion depending on which we have a class of three chief seesaw models, pictorial representation of which we show in figure 1.2. In the next subsections all the above mentioned seesaw scenarios are shortly introduced.

1.6.1 Type I seesaw

As already discussed that the implementation of type I seesaw needs the inclusion of RH neutrinos [40–43], this new field offers a possible Yukawa coupling between the SM neutrino and the Higgs field in addition having a Majorana mass for the new field itself. The relevant Lagrangian responsible for type I seesaw is given by

$$-\mathcal{L}_{\rm TypeI} = Y_\nu \bar{N}_R \tilde{H}^\dagger L + \frac{1}{2} M_R \bar{N}_R N_R^C + \text{H.C.} \tag{1.6.3}$$

where, the second term in the above equation violates lepton number by two units. Y_ν is a complex 3×3 non-symmetric mass matrix and M_R is a symmetric matrix with order 3. The above Lagrangian can be written in terms of the column vector of the left handed field as

$$\mathcal{L}^{\rm mass}_{\rm TypeI} = \frac{1}{2}\Big(\bar{\nu}_L^C \bar{N}_R\Big) \begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R \end{pmatrix} \tag{1.6.4}$$

from the above neutrino mass Lagrangian, we can display the Majorana neutrino mass matrix as

$$M_\nu^{\rm TypeI} = \begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix} \tag{1.6.5}$$

which is typically a 6×6 matrix for three RH neutrinos added to the SM particle spectrum. It is better to have one RH neutrino per generation. Block diagonalizing this matrix, results into following two eigenvalues,

$$m_\nu \approx -\frac{m_D^2}{M_R} = -\frac{v^2 Y_\nu^2}{M_R}, m_N \approx M_R \tag{1.6.6}$$

M_R is that mass scale for the RH neutrino, where Lepton number violation took place whereas m_ν is the mass scale for the SM neutrino. Thus one can conclude that SM neutrino mass is a ratio of the smaller Dirac mass scale with the large Majorana mass scale. Now, m_ν can be written as $m_\nu \sim m_D^T M_R^{-1} m_D = U^*_{PMNS} m_\nu^{\text{diag}} U^\dagger_{PMNS}$. We call this mechanism as the Type I seesaw mechanism.

1.6.2 Type II seesaw

Addition of the scalar triplet Higgs to the SM, allows us to generate the neutrino mass (for detail you may see references [44–46]) via the following Lagrangian,

$$\mathcal{L}_{\text{Type II}} = -\frac{1}{2} Y_\Delta^{ij} \bar{L_i^C} \tilde{\Delta} L_j - \mu H^T i\sigma_2 \Delta^\dagger H - \frac{1}{2} M_\Delta^2 Tr(\Delta^\dagger \Delta) + \text{H.C.} \tag{1.6.7}$$

where the scalar triplet is denoted by Δ, in terms of three complex scalars $\Delta^0, \Delta^+, \Delta^{++}$ and having the following form

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix} \tag{1.6.8}$$

The first term of Eq. (1.6.7) represents the Yukawa interaction between the scalar triplet with the SM lepton doublet, with a coupling Y_Δ with $\tilde{\Delta} = i\sigma_2 \Delta^*$. σ_2 is Pauli spin matrix. Under the SM gauge group the scalar triplet transforms as (1,3,+1). M_Δ is the mass of the Higgs triplet with μ as its coupling with two Higgs doublets. When the neutral component of the Higgs doublet generates a nonzero VEV it induces a tadpole term for the scalar triplet via the second term of Eq. (1.6.7) giving an induced VEV to the scalar triplet Δ, and thus neutrino mass is generated.

$$m_\nu = \frac{Y_\Delta v_\delta}{\sqrt{2}}, v_\Delta \approx \mu \frac{v^2}{M_\Delta^2}. \tag{1.6.9}$$

1.6.3 Type III seesaw

Type III seesaw is realized via the inclusion of a fermion triplet [47, 48], denoted by Σ, to the SM particle content. The fermion triplet field couples to the LH neutrinos and the SM Higgs doublet by the following Lagrangian,

$$\mathcal{L}_{\text{TypeIII}} = -Y_{\Sigma}^{ij}\bar{L}_i\tilde{H}\Sigma - \frac{1}{2}M_{\Sigma}^{ij}Tr(\bar{\Sigma_i^C}\Sigma_j) + \text{H.C.} \tag{1.6.10}$$

Here Y_ν is a 3×3 dimensionless Yukawa coupling matrix. The third panel of the figure 1.2 represents the type III seesaw model. The $SU(2)_L$ triplet fermion has a definition in terms of three components (η_1, η_2, η_3) with the following $SU(2)_L$ representation.

$$\Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & \Sigma^0/\sqrt{2} \end{pmatrix}. \tag{1.6.11}$$

Where, the neutral component of the triplet fermion plays the role similar to that played by the RH neutrino in case of type I seesaw. In this seesaw model the neutrino mass is generated similarly as the type I seesaw, by the following formula

$$-m_\nu = m_D M_\Sigma m_D^T. \tag{1.6.12}$$

with $m_D = \frac{Y_\Sigma v}{\sqrt{2}}$. Interestingly, here also Lepton number is violated as the simultaneous appearance of Y_Σ and M_Σ does not assign any Lepton charges to Σ.

In this Thesis, we are also interested in the phenomenology involving a TeV scale right-handed neutrinos which is natural in inverse seesaw (ISS) models. Of special importance is the fact that these TeV scale RH neutrinos have better sensitivity of being accessed in the future colliders. The canonical type I seesaw also can accommodate TeV scale RH neutrino but with a very small Yukawa coupling of the order of 10^{-6}. But the inverse seesaw mechanism naturally accommodates a low scale RH neutrino with a larger value of Yukawa coupling, which is the driving cause of taking the ISS for explaining light neutrino mass. A bit detailed study of ISS is provided in the following subsection.

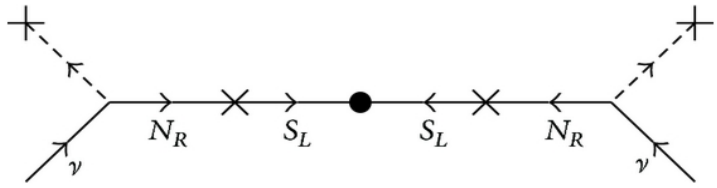

Figure 1.3: Schematic representation of inverse seesaw mechanism

1.6.4 Inverse seesaw

As mentioned earlier the ISS model offers the neutrino mass at the sub-eV scale at the cost of proposing a TeV scale RH neutrino [49–51]. To realize this scenario one needs to consider another RH fermion singlet (S) in addition to that (N_R), already taken for type I seesaw. This scenario is realized by making use of some extra symmetries, e.g., via the global lepton number symmetry. Essentially, the new fermion singlet is assigned a lepton number $L = -1$ which is opposite to that for N_R ($L = 1$). Now if the LN is conserved then the light neutrino mass matrix that this model yields has two degenerate eigenvalues, one Dirac neutrino and one massless neutrino. Since the LN symmetry is responsible for the generation of massless neutrinos we need to include a LN breaking parameter in order to generate non-zero neutrino masses. For small breaking, the neutrino masses will be small, which establish a relation between the smallness of neutrino masses with the scale where LN symmetry is broken. The figure 1.3 represents the ISS mechanism. For a three generations picture where three pairs of fermion singlets ν_R, S are added to the SM, the ISS Lagrangian in this case is given by

$$\mathcal{L} = -Y_\nu^{ij}\bar{L}_i\tilde{H}\nu_{Rj} - M_R^{ij}\bar{\nu_{Ri}^C}S_j - \frac{1}{2}\mu_R^{ij}\bar{\nu_{Ri}^C}\nu_{Rj} - \frac{1}{2}\mu_S^{ij}\bar{S_i^C}S_j + \text{H.C.} \tag{1.6.13}$$

where, Y_ν is the 3×3 neutrino Yukawa coupling matrix, M_R is a lepton number conserving complex 3×3 mass matrix, and μ_R and μ_S are Majorana complex 3×3 symmetric mass matrices that violate LN conservation by two units. After the EWSB, we obtain the complete 9×9 neutrino mass matrix as given by the following structure

$$m_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & \mu_R & M_R \\ 0 & M_R^T & \mu_S \end{pmatrix}. \tag{1.6.14}$$

It is to note that μ_R does not generate light neutrino masses at the tree level. We will set μ_R to zero for the rest of this thesis and consider a small μ_S as the only lepton number violating parameter leading to the light neutrino masses. In the mass range of our interest with μ_S, m_D, M_R, the mass matrix m_ν can be block diagonalized which leads to the following light neutrino mass formula under the ISS scheme.

$$m_{\text{light}} \approx m_D (M_R^T)^{-1} \mu_S M_R^{-1} m_D^T \tag{1.6.15}$$

1.7 Cosmological consequences of BSM physics

Neutrinos are supposed to be very tiny creatures in the sub atomic world, however they have big impact in the study of cosmos. Two foremost puzzles in modern cosmology are origin of dark matter and baryon asymmetry of the universe. It is indeed a delight to address these two issues along with the explanation of neutrino mass, within a single framework, although it a matter of choice only. And it will be even more delightful if the mechanism through which existence of neutrino mass is addressed, can also be a viable cause for the origin of the two above mentioned puzzles of cosmology. In this subsection we will briefly discuss such possibilities. As already mentioned in the earlier subsection that the presence of right handed neutrinos (RHN) are essential for generating neutrino mass beyond the standard model, thus one can say that there might be some phenomena associated with this RHN which knocks the door to cosmology.

1.7.1 Baryogenesis via leptogenesis

Cosmological and astronomical observations indicate that there is a tiny excess of matter over antimatter that is the present number of matter and antimatter are unequal. And there are strong evidences also which confirms this fact of matter excess over antimatter. This presently small but nonzero asymmetry in the amount of matter and antimatter is familiar as baryon asymmetry of the universe (BAU). The large scale structures such as galaxies, galaxy clusters, stars predominantly consist of matter rather than antimatter in appreciable measure. This fact takes us to a journey manifest for finding the precise cause behind it. Previously there was baryon symmetric universe at the epoch as suggested by many considerations, due to the fact that the early universe was radiation dominated, thus the photons always decayed to one matter-antimatter pair, hereby bringing the equality in their number. The evolution of this baryon asymmetric era from a previously baryon symmetric universe through the generation of a tiny but non-zero amount of baryon asymmetry is termed as baryogenesis. Even if we think that the present universe consists of equal numbers of matter and antimatter, there must be some annihilation process like $\mathrm{M} + \bar{\mathrm{M}} = 2\gamma$ one would expect. Unfortunately, till date we did not observe any process as such. Therefore it is claimed that the Universe is baryon asymmetric. Now the question is whether the SM of particle physics can explain the origin of this asymmetry or not! Well, the answer is NO. Although we have all the ingredients that are necessary to generate this asymmetry dynamically in an initial baryon-symmetric universe, yet it is unable to explain an observed amount of asymmetry [52]. There are two problems with SM baryogenesis. The Higgs is too heavy for the electroweak phase transition, which is to be of first order to account for a successful baryogenesis and which is of second order within the SM. Along with it, the amount of CP violated within the SM is too small to yield the observed BAU. Thereby we need to call for a Beyond Standard Model scenario for the study of baryogenesis. Andrew Sakharov in the year 1967 put forward a theory postulating the key ingredients which particle interactions and the cosmological evolution have to satisfy for having a successful baryogenesis. The criterion are as follows:

- There must be baryon number violation. A system must evolve from an initial state with $Y_{\Delta B} = 0$ to a state with $Y_{\Delta B} \neq 0$.

- There must be C and CP violation. In principle, the number of left-handed particles generated in any process would be different from the number of right-handed antiparticles (which are the CP conjugates of the left-handed particles): which is possible only when CP is violated. Moreover, C violation is also essential, as the generation of the left-handed particles should not compensate the generation of the left-handed antiparticles (which are the conjugates of the right-handed particles).

- Depart significantly from thermal equilibrium. The departure from equilibrium is realized when the above mentioned B-violating interaction rate is slower than the expansion rate of the universe, this fact generally does not escort the distribution of baryons and antibaryons of the universe into equilibrium. In essence, as the heavy particle decays, the decay product will move apart before it could participate in the inverse decay, causing a departure from equilibrium. In other words, before the chemical potentials of the two states become equal, they move apart from each other. Analytically one can write the out-of-equilibrium condition as $\Gamma(T) < H(T) = 1.66\sqrt{g_*}\frac{T^2}{M_{\rm Pl}}$ where Γ is the baryon-number violating interaction rate under discussion, g_* is the effective number of degrees of freedom available at temperature T and $M_{\rm Pl}$ is the Planck mass.

There are several mechanisms through which baryogenesis can be realized, viz., **GUT baryogenesis** [53–61]: where the out-of-equilibrium decay of heavy bosons create the baryon asymmetry in Grand Unified Theories; **Leptogenesis** [62]: the most popular mechanism of realizing the baryogenesis is leptogenesis, where the presence of singlet RHNs, as an ingredient of the seesaw mechanism (in particular type I and inverse) makes it possible to go through a decay, hereby creating an adequate lepton asymmetry which later on converts into baryon asymmetry through electro-weak sphaleron process. The rate of this decay process should be less than the expansion rate of the universe, to satisfy Sakharov's 3rd condition as discussed above; **Electroweak baryogenesis** [63, 64] is the scenario where departure from thermal equilibrium brought by electroweak phase transition; and finally the **Affleck-Dine mechanism** [65, 66] where the asymmetry arises in a classical scalar field which later on decays to particles. Among all these scenarios baryogenesis via Leptogenesis has gained popularity which has been picked for our work in this thesis.

The amount of baryon asymmetry has been confirmed by various cosmological observations. Big-Bang-nucleosynthesis(BBN) is one of them. In BBN observation, abundance of light elements like D, ^{3}He, ^{4}He, ^{7}Li has been predicted. The crucial time for premordial nucleosynthesis is when the thermal bath temperature falls below T $\leq$ 1 MeV. And this prediction depends on a single parameter η. One can find the BAU in two different ways given by the following equations where the difference between the number of baryons and antibaryons is normalized to the number of photons.

$$\eta = \left.\frac{n_B - n_{\bar{B}}}{n_\gamma}\right|_0 = (6.21 \pm 0.16) \times 10^{-10} \tag{1.7.1}$$

$$y_{\Delta B} = \left.\frac{\eta_B - \eta_{\bar{B}}}{s}\right|_0 = (8.75 \pm 0.23) \times 10^{-11} \tag{1.7.2}$$

where, n_B, $n_{\bar{B}}$, n_γ and s are the number densities of respectively, baryons, antibaryons, photons and entropy. The entropy density is also a function of temperature, given by $s = g_*(2\pi^2/45)T^3$ which is conserved during the expansion of the Universe. The subscript "0" means the observation of these ratios at present time. The primordial abundances of the above elements are confirmed by several observations. From those observations a range of η is found which is in agreement with all the four abundances and that in turn favors the standard hot big bang cosmology. The range can be shown as (with 95% CL)

$$4.7 \times 10^{-10} \leq \eta \leq 6.5 \times 10^{-10}, \quad 0.017 \leq \Omega_B h^2 \leq 0.024 \tag{1.7.3}$$

The other impressive choice to determine Ω_B is from measurements of the cosmic microwave background (CMB) anisotropies, the detail of which can be found in [67]. From a very recent observation such as WMAP5 data only, gives (at 68% CL) [68]

$$0.02149 < \Omega h^2 < 0.02397 \tag{1.7.4}$$

Now it is better to measure the baryon asymmetry by the Eq. (1.7.2), as in this equation the difference between numbers of baryons and antibaryons is normalized to the entropy density, since the entropy density is conserved during the expansion of the universe. From Eq. (1.7.3) and Eq. (1.7.4), one can write the BAU in terms of Y_B at 3σ level as

$$Y_{\Delta B}^{BBN} = (8.10 \pm 0.85) \times 10^{-11}, \;\; Y_{\Delta B}^{CMB} = (8.79 \pm 0.44) \times 10^{-11} \tag{1.7.5}$$

Baryogenesis via leptogenesis is a simple mechanism to explain the BAU as suggested by Fukugita and Yanagida [62]. A lepton asymmetry is dynamically generated in the lepton sector first, then it gets converted into baryon asymmetry by (B+L) violating sphaleron interactions [69] which exist in the SM. A platform to implement this mechanism can be a class of seesaw models (in particular type I in this thesis), where the presence of RH neutrinos brings out the scenario of leptogenesis via the CP-violating decay of the RH neutrinos themselves. With the growing interest of taking leptogenesis as the process of explaining the BAU several BSM frame works have shown anticipating role. Therefore leptogenesis is a mechanism of generating lepton asymmetry before the electroweak phase transition, which later on gets converted into baryon asymmetry after reprocessing by electroweak sphalerons. The relation between baryon asymmetry and lepton asymmetry is given by

$$Y_B = -\Big(\frac{8n_G + 4n_H}{14n_G + 9n_H}\Big) Y_L \tag{1.7.6}$$

with n_H as the number of Higgs doublets and n_G the number of fermion generations (in thermal equilibrium). The CP asymmetry generated by the decay of the lightest RHN is given by

$$\epsilon_1 = \frac{\Sigma_\alpha [\Gamma(N_1 \to H l_\alpha)] - [\Gamma(N_1 \to \bar{H}\bar{l}_\alpha)]}{\Sigma_\alpha [\Gamma(N_1 \to H l_\alpha) + [\Gamma(N_1 \to \bar{H}\bar{l}_\alpha)]]} \tag{1.7.7}$$

Now the process of leptogenesis belongs to two distinct scales, high scale and low scale. By high scale leptogenesis we mean when the RHN mass is of order 10^{12} GeV or more which naturally comes from the generation of light neutrino mass by the canonical type I seesaw, whereas the second kind rules over a lower mass regime of RH neutrinos e.g., when M_R falls around a TeV. For the explanation of smallness of neutrino masses seesaw mechanisms demands the inclusion of heavy RHN. The mass of these heavy RHN needs to fall around 10^{12} GeV if the seesaw is canonical type I as already said. Now the RHN mass can stay in a lower mass regime (as around a TeV) if inverse seesaw explains the tiny neutrino mass. Thus depending on various seesaw scenarios the RHN mass scale varies. These RHNs transform as singlets under $SU(2)_L$ symmetry group. In a basis where charged lepton Yukawa couplings are diagonal, we can write the SM Lagrangian with the newly added RHN as,

$$\mathcal{L} = \mathcal{L}_{SM} + \Big(\frac{M_i}{2}N_i^2 + \lambda_{i\alpha}N_i l_\alpha H + h_\alpha H + h_\alpha H^c \bar{e}_{R\alpha} l_\alpha + h.c.\Big). \tag{1.7.8}$$

with l_α and $e_{R\alpha}$ as lepton doublet and singlet of flavor $(\alpha = e, \mu, \tau)$. The newly added RHN s undergo out-of-equilibrium decay to SM leptons and Higgs via their complex Yukawa couplings, which later on play as a new source of CP violation in order to yield a nonzero lepton asymmetry in the lepton sector. The RHNs are Majorana in nature. The Majorana mass term of these RHNs indicate lepton number violation. During the decoupling of the very heavy RHN s from thermal bath, they decay to create leptons and antileptons via the Yukawa coupling

$$\mathcal{L} \subset \lambda \bar{N}_i L H + \lambda^\dagger N_i \bar{L} H^* \tag{1.7.9}$$

where λ is a 3×3 matrix containing the Yukawa coupling governing the decay of RHNs.

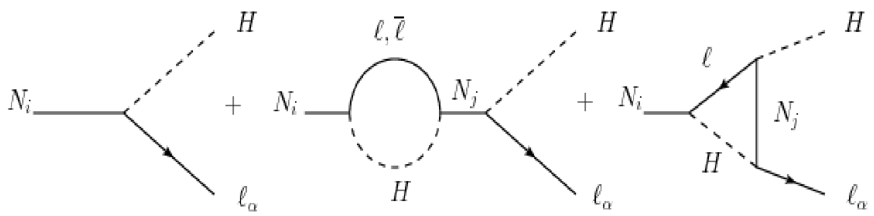

Figure 1.4: Decay modes of right handed neutrinos taking part in leptogenesis

Resonant leptogenesis

Apart from the tree level diagram as shown in figure 1.4 the two one-loop diagrams also contribute to the CP-violating lepton asymmetry. In principle, the interference of the tree-level decay amplitude with the absorptive parts of the one-loop self-energy and vertex diagrams violates CP and hence gives rise to a considerable amount of lepton asymmetry. The amount of CP violation that comes from the self-energy plot can be relatively larger than that comes from the vertex graph (see for e.g., [70–72]) through the mixing of two nearly degenerate heavy Majorana neutrinos. Even the lepton asymmetry can attain a value of order unity if two of the heavy Majorana neutrinos have a mass difference comparable to their decay widths. Generally the self energy diagram holds good when the heavy Majorana neutrino mass falls around TeV [73]. In other words, the heavy neutrino self energy effects on the CP asymmetry become dominant and hence gets resonantly enhanced. Because of this resonant enhancement of the asymmetry, this scenario of leptogenesis is termed as resonant leptogenesis. The larger the amount of lepton asymmetry, the smaller the lower bound on RHN mass.

It is said that leptogenesis is a consequence of seesaw mechanisms due to the presence of heavy Majorana fields. Now for a common search for the origin of neutrino mass and baryogenesis via leptogenesis, seesaw mechanisms take a strong hold. Resonant leptogenesis can be regarded as one of the consequences of that motivation. For resonant leptogenesis to occur, some sufficient and necessary conditions are to be satisfied, which even results into a tremendous enhancement of the leptonic asymmetry up to order unity [73]. For a pair of Majorana neutrino, one can write the conditions as

$$m_{N_i} - m_{N_j} \sim \frac{\Gamma_{N_{i,j}}}{2}, \quad \frac{|Im(Y_\nu^\dagger Y_\nu)^2_{i,j}|}{(Y_\nu^\dagger Y_\nu)_{i,i}(Y_\nu^\dagger Y_\nu)^2_{j,j}} \sim 1 \tag{1.7.10}$$

where, Γ_{N_i} are the N_i decay widths. The lepton asymmetry can be found from the following formula taken from [74, 75]

$$\epsilon_{il}^{\text{mix}} = \sum_{j\neq i} \frac{\text{Im}[Y_{\nu_{il}} Y^*_{\nu_{jl}} (Y_\nu Y_\nu^\dagger)_{ij}] + \frac{M_i}{M_j}\text{Im}[(Y_{\nu_{il}} Y^*_{\nu_{jl}} (Y_\nu Y_\nu^\dagger)_{ji}]}{(Y_\nu Y_\nu^\dagger)_{ii}(Y_\nu Y_\nu^\dagger)_{jj}} f_{ij}^{mix} \tag{1.7.11}$$

with the regulator given by,

$$f_{ij}^{mix} = \frac{(M_i^2 - M_j^2)M_i\Gamma_j}{(M_i^2 - M_j^2)^2 + M_i^2\Gamma_j^2}$$

with $\Gamma_i = \frac{M_i}{8\pi}(Y_\nu Y_\nu^\dagger)_{ii}$ as the tree level heavy-neutrino decay width. Now, there is a similar contribution ϵ_{il}^{osc} to the CP asymmetry from RH neutrino oscillation

[76, 77]. Its form is given by Eq. (1.7.11) with the replacement $f_{ij}^{mix} \rightarrow f_{ij}^{osc}$, where

$$f_{ij}^{osc} = \frac{(M_i^2 - M_j^2)M_i\Gamma_j}{(M_i^2 - M_j^2)^2 + (M_i\Gamma_i + M_j\Gamma_j)^2\frac{\det[\mathrm{Re}(Y_\nu Y_\nu^\dagger)]}{(Y_\nu Y_\nu^\dagger)_{ii}(Y_\nu Y_\nu^\dagger)_{ii}}}$$

The total CP asymmetry therefore can be written as $\epsilon_{il} = \epsilon_{il}^{mix} + \epsilon_{il}^{osc}$. One can write the final BAU as,

$$\eta_B \simeq -3 \times 10^{-2} \sum_{l,i} \frac{\epsilon_{il}}{K_l^{\mathrm{eff}} \mathrm{min}(z_c, z_l)} \tag{1.7.12}$$

where, $z_c = \frac{M_N}{T_c}$, $T_c \sim 149$ GeV, the critical temperature below which the sphaleron transition processes freeze-out, $z_l \simeq 1.25 \ln(25 K_l^{\mathrm{eff}})$ and $K_l^{\mathrm{eff}} = \kappa_l \sum_i K_i B_{il}$, with $K_i = \Gamma_i / H_N$ is the wash out factor. H_N is $1.66\sqrt{g^*}M_N^2/M_{\mathrm{Pl}}$ is the Hubble expansion rate at temperature $\sim M_N$ and $g^* \simeq 106.75$. B_{il}'s are the branching ratios of the N_i decay to leptons of lth flavor: $B_{il} = \frac{|Y_{\nu_{il}}|^2}{(Y_\nu Y_\nu^\dagger)_{ii}}$. Including the RIS(Real Intermediate State) subtracted collision terms one can write the factor κ as,

$$\kappa_l = 2 \sum_{i,jj\neq i} \frac{\mathrm{Re}[Y_{\nu il}Y_{\nu jl}^*(YY^\dagger)_{ij}] + \mathrm{Im}[(Y_{\nu il}Y_{\nu jl}^*)^2]}{\mathrm{Re}[(Y^\dagger Y)_{ll}\{(YY^\dagger)_{ii} + (YY^\dagger)_{jj}\}]} \left(1 - 2i\frac{M_i - M_j}{\Gamma_i + \Gamma_j}\right)^{-1} \tag{1.7.13}$$

where, Y_ν is the Dirac Yukawa coupling matrix in a basis where RH neutrino mass is diagonal. As seen from the expression Eq. 1.7.11. It is worth noting that, during the calculation of RIS contribution since only the diagonal terms are considered in the sum, κ_l can take its maximum value and hence we can have $\kappa_l = 1 + O(\delta_l^2)$.

1.7.2 Dark matter

Starting from some astrophysical observations, such as rotation curve of spiral galaxies around the cluster by Fritz Zwicky [78, 79], inhomogeneity in cosmic microwave background radiation (CMBR) [80], or more recent observations in Bullet cluster [81] to the latest cosmology data provided by the Planck satellite [52], hint towards the existence of dark matter (DM) in the universe.

One of the main evidences for DM comes from measuring the rotation speed of galaxies. One can compute the mass of a galaxy by finding the velocity of the stars as they orbit the center of the galaxies. Had the galaxies composed of visible matter only, major portion of their masses would be concentrated in the center. But the Kepler's law says that the orbital velocities of the stars decreases as one goes to the outer edges of the galaxy, because there would be less mass. But intriguingly, astronomers observed that the orbital velocities of the stars around the center remain constant and do not decrease even if we go to a larger distance from the center of the galaxy, where there are fewer stars. And this fact implies that there must be an unseen mass in the galaxies even beyond the area containing majority of the stars.

The study of galaxy clusters gives another important observation for DM evidence. "Gravitational lensing" is one of the methods for measuring the mass of galaxy clusters. Einstein's theory of relativity tells that a massive object can bend the light which is coming from a distant source towards us, that way the object behaves like a gravitational lens. By measuring the distortion of the light, the total mass of the galaxy cluster is estimated. From this method it is found that a major portion of the galaxy clusters are composed of dark matter.

Then among the direct observational evidences, Bullet cluster gives strong confirmation regarding the existence of DM. Bullet cluster is composed of two galaxy clusters passing through each other. Now, interestingly when the two galaxies pass each other, the visible matter portions collide and slow down while the dark matter components pass each other without interacting and slowing down. This fact creates a separation between the dark and visible matter of each cluster. This separation was detected by comparing X-ray images of the luminous taken with the Chandra X-ray observatory. The dark matter components were found moving away from the center with high speeds, however the two narrower region of the ordinary matter were moving with less speeds behind them. As this evidence does not obey Newtonian mechanics, thus it is announced as a direct evidence for dark matter.

A particle description of DM is much sought after as the SM fails to provide a particle DM candidate that can satisfy all the criteria of a good DM candidate [82] and lot of exercises are performed (for a brief review, please refer to [83, 84]) to accommodate DM in extensions of SM. With the motivation of accessing an experimental verification of DM, a plethora of BSM frameworks are constructed assuming the DM to be a scalar, fermion or a vector boson and which can give rise to the correct DM phenomenology along with the possibility to be tested at several different experiments. Among them, thermal freeze-out of the weakly interacting massive particle (WIMP) [85] paradigm is the most popular BSM scenario as the correct DM relic abundance can be achieved for such a particle as it has interaction strength similar to weak interactions. This coincidence is also referred to as the *WIMP Miracle*. In terms of density parameter and $h =$ (Hubble Parameter)/100, the present dark matter abundance is conventionally reported as [52]

$$\Omega_{\rm DM} h^2 = 0.1187 \pm 0.0017 \tag{1.7.14}$$

Using the measured value of Hubble parameter, this announces that, approximately 26% of the total energy density of the present Universe being made up of DM.

1.8 Discrete flavour symmetry

Particle physics community shall ever remain indebted to Symmetry, as it plays a nontrivial role in addressing many observable phenomena associated with this ball park of particles and forces. Starting from continuous symmetries such as Lorentz, Poincare and gauge symmetries, we see that they are essential to understand several particle physics phenomena like strong, weak and electromagnetic interactions among particles. Along with these, there are discrete symmetries such as Charge conjugation(C), Parity(P) and Time reversal(T), which are also of special importance. To realize them particle physicists of different decades put forward many models.

Several continuous symmetry groups such as SU(N) and SO(N) are found to play vital role in explaining the masses of elementary particles. The non Abelian continuous symmetry groups are also termed as the lie groups of particle physics. The Standard Model is a collection of $SU(3)_c \times SU(2)_L \times U(1)_Y$ Lie groups, which is popularly known as Glashow-Weinberg-Salam model. It does not accommodate neutrino mass as the SM Higgs can not give mass to the neutrinos due to the absence of the right handed neutrinos within it. There are several extensions of the SM, where the SM gauge groups are augmented with one or more symmetry groups. In essence, those symmetry groups beautifully accommodate some extra right handed neutrinos in particular, and holds a concrete theory explaining the existence of massive neutrinos. In this context some non-Abelian discrete symmetry groups (please see e.g., [86] for a detailed analysis) took a strong hold of the entire scenario, with the introduction of the RHNs along with some additional scalar fields. In this thesis we build a few new models and modify some of earlier in the light of some non-Abelian discrete flavour symmetry groups. We extensively use S_4 and A_4 symmetry groups to explain the neutrino phenomenology in this thesis. On the other hand some sub groups such as Z_N of the bigger groups e.g., S_N makes it possible to control the desired and permitted Yukawa couplings in model building beyond the standard model. A class of Z_N groups are in extensive use in this context. This class of Z_N groups are even helpful to shade light on the dark sector, specially in stabilizing a potential dark matter candidate in a particular model. In this regard the non-Abelian groups are of special importance as they can simultaneously accommodate neutrino mass and a stable dark matter candidate under a proper charge assignment to the particle contents of a model. In the following subsections we will briefly discuss the properties of S_4 and A_4 symmetry groups.

1.8.1 The group S_4 and its properties

S_N is a group of permutation of N objects, that is all possible kinds of permutation among these N number of objects form a group called S_N. Order of S_N goes as N!. The S_N group has two one-dimensional representations, one is trivial singlet, which is by definition invariant under all the elements (symmetric representation), the other is pseudo singlet, that is, symmetric under the even permutation-elements but anti-symmetric under the odd permutation-elements. Depending on the number N, there are variants of groups starting from S_1, S_2 and so on. According to the definition of S_N, there is only one element in the group S_1. Similarly S_2 has got two elements formed by the permutation of two objects, which is nothing but the group Z_2 and Abelian. Therefore we can start with the immediate bigger group formed by the permutation of three objects familiar as S_3 with order $3! = 6$. S_3 is isomorphic to the symmetry group of a equilateral triangle. Following the same definition for S_N, S_4 consists of all permutations among four objects, x_1, x_2, x_3, x_4. The generalized transformation among the positions of four objects, one can write as

$$x_1, x_3, x_2, x_4 \to x_i, x_j, x_k, x_l$$

The order of S_4 is equal to $4! = 24$. There are two generators of S_4 familiar as S and T which satisfy

$$T^4 = S^3 = e,\ TS^2T = S \tag{1.8.1}$$

All of the S_4 elements can be written as products of these two generators. There are five in-equivalent irreducible representations of S_4, among which there are two singlets 1 and $1'$, one doublet 2 and two triplets 3 and $3'$. S and T have different structures, depending on which irreducible representation we are considering singlet, doublet or triplet. The representations are given as follows:

$$a, b \sim 1_1, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \sim 2, \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \sim 3, \begin{pmatrix} a'_1 \\ a'_2 \\ a'_3 \end{pmatrix}, \begin{pmatrix} b'_1 \\ b'_2 \\ b'_3 \end{pmatrix} \sim 3'.$$

$$(A)_3 \times (B)_3 = (A \cdot B)_1 + \begin{pmatrix} A \cdot \Sigma \cdot B \\ A \cdot \Sigma^* \cdot B \end{pmatrix}_2 + \begin{pmatrix} \{A_y B_z\} \\ \{A_z B_x\} \\ \{A_x B_y\} \end{pmatrix}_3 + \begin{pmatrix} [A_y B_z] \\ [A_z B_x] \\ [A_x B_y] \end{pmatrix}_{3'} . \tag{1.8.2}$$

$$
\begin{aligned}
A\cdot B &= A_xB_x + A_yB_y + A_zB_z \\
\{A_iB_j\} &= A_iB_j + B_jA_i \\
[A_iB_j] &= A_iB_j - A_jB_j \\
A\cdot\Sigma\cdot B &= A_xB_x + \omega A_yB_y + \omega^2 A_zB_z \\
A\cdot\Sigma^*\cdot B &= A_xB_x + \omega^2 A_yB_y + \omega A_zB_z.
\end{aligned} \tag{1.8.3}
$$

Later on for simplicity, we can replace $3 \to 3_1$, $3' \to 3_2$, $1 \to 1_1$, $1' \to 1_2$. The tensor products of S_4 that has been used in the present analysis are given below .

$$3 \times 1_1 = 3, 3 \times 1_2 = 3_2, 3_2 \times 1_2 = 3, 2 \times 1_2 = 2.$$

$$
\begin{aligned}
2 \otimes 2 &= 1_1 \oplus 1_2 \oplus 2, \\
3_1 \otimes 3_1 &= 1_1 \oplus 2 \oplus 3_1 \oplus 3_2.
\end{aligned}
$$

The Clebsch-Gordon coefficients for the product of two triplets can be written from [86] as follows.

$$
\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{3_1} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3_1} = (a_1b_1 + a_2b_2 + a_3b_3)_{1_1} \oplus \begin{pmatrix} 1/\sqrt{2}(a_2b_2 - a_3b_3) \\ 1/\sqrt{6}(-2a_1b_1 + a_2b_2 + a_3b_3) \end{pmatrix}_2 \oplus
$$
$$
\begin{pmatrix} a_2b_3 + a_3b_2 \\ a_1b_3 + a_3b_1 \\ a_1b_2 + a_2b_1 \end{pmatrix}_{3_1} \oplus \begin{pmatrix} a_3b_2 - a_2b_3 \\ a_1b_3 - a_3b_1 \\ a_2b_1 - a_1b_2 \end{pmatrix}_{3_2} .
$$

$$
\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{3_2} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3_2} = (a_1b_1 + a_2b_2 + a_3b_3)_{1_1} \oplus \begin{pmatrix} 1/\sqrt{2}(a_2b_2 - a_3b_3) \\ 1/\sqrt{6}(-2a_1b_1 + a_2b_2 + a_3b_3) \end{pmatrix}_2 \oplus
$$
$$
\begin{pmatrix} a_2b_3 + a_3b_2 \\ a_1b_3 + a_3b_1 \\ a_1b_2 + a_2b_1 \end{pmatrix}_{3_1} \oplus \begin{pmatrix} a_3b_2 - a_2b_3 \\ a_1b_3 - a_3b_1 \\ a_2b_1 - a_1b_2 \end{pmatrix}_{3_2} .
$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{3_1} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3_2} = (a_1b_1 + a_2b_2 + a_3b_3)_{1_2} \oplus \begin{pmatrix} 1/\sqrt{6}(2a_1b_1 - a_2b_2 - a_3b_3) \\ 1/\sqrt{2}(a_2b_2 - a_3b_3) \end{pmatrix}_2 \oplus$$
$$\begin{pmatrix} a_3b_2 - a_2b_3 \\ a_1b_3 - a_3b_1 \\ a_2b_1 - a_1b_2 \end{pmatrix}_{3_1} \oplus \begin{pmatrix} a_2b_3 + a_3b_2 \\ a_1b_3 + a_3b_1 \\ a_1b_2 + a_2b_1 \end{pmatrix}_{3_2} .$$

1.8.2 The group A_4 and its properties

A_4 consists of all even permutations of S_4 with order equal to $4!/2 = 12$. The A_4 group is the symmetry of a tetrahedron. Thus, the A_4 group is often denoted as T. A_4 has four conjugacy classes and hence four irreducible representations, among which there are three singlets and one triplet. The group has got two generators namely **S** and **T**. The triplet multiplication rules of A_4 that has been used in this thesis are given below. There are two sets of Clebsch Gordan coefficients involved in the triplet product rules. One has been prepared by the **S**-diagonal basis, i.e. when the generator **S** is diagonal, and another is built from T-diagonal basis, i.e. when the generator **T** is diagonal (for more details see [87–89]).
The representations are given as follows

$$a, b \sim 1, \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \sim 3.$$

$$\mathbf{1} \otimes \mathbf{1} = \mathbf{1}, \quad \mathbf{1}' \otimes \mathbf{1}' = \mathbf{1}'', \quad \mathbf{1}' \otimes \mathbf{1}'' = \mathbf{1}$$

$$\mathbf{1}'' \otimes \mathbf{1}'' = \mathbf{1}', \quad \mathbf{1}' \otimes \mathbf{3} = \mathbf{3}, \quad \mathbf{1}'' \otimes \mathbf{3} = \mathbf{3}$$

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \otimes \mathbf{1}' \otimes \mathbf{1}'' \otimes \mathbf{3_a} \otimes \mathbf{3_s}$$

where a and s in the subscript corresponds to anti-symmetric and symmetric parts respectively. Denoting two triplets as (a_1, a_2, a_3) and (b_1, b_2, b_3) respectively, their direct product can be decomposed into the direct sum mentioned above as

$$\mathbf{1} \backsim \mathbf{a_1b_1 + a_2b_3 + a_3b_2}$$

$$\mathbf{1'} \backsim \mathbf{a_3b_3 + a_1b_2 + a_2b_1}$$

$$\mathbf{1''} \backsim \mathbf{a_2b_2 + a_1b_3 + a_3b_1}$$

$$\mathbf{3_s} \backsim (\mathbf{2a_1b_1 - a_2b_3 - a_3b_2, 2a_3b_3 - a_1b_2 - a_2b_1, 2a_2b_2 - a_1b_3 - a_3b_1})$$

$$\mathbf{3_a} \backsim (\mathbf{a_2b_3 - a_3b_2, a_1b_2 - a_2b_1, a_1b_3 - a_3b_1})$$

The above product rules are built by considering the triplet representation in a basis where the generator $\mathbf{T}$ is diagonal. Moreover we also have another set of Clebsch Gordan coefficients for the triplet product rule, considering the triplets in a basis where $\mathbf{S}$ is diagonal instead $\mathbf{T}$. They are as follows.

$$\mathbf{1} \backsim \mathbf{a_1b_1 + b_2b_2 + a_3b_3}$$

$$\mathbf{1'} \backsim \mathbf{a_1b_1} + \omega^{\mathbf{2}}\mathbf{a_2b_2} + \omega\mathbf{a_3b_3}$$

$$\mathbf{1''} \backsim \mathbf{a_1b_1} + \omega\mathbf{a_2b_2} + \omega^{\mathbf{2}}\mathbf{a_3b_3}$$

$$\mathbf{3} \backsim (\mathbf{a_2b_3, a_3b_1, a_1b_2})$$

$$\mathbf{3} \backsim (\mathbf{a_3b_2, a_1b_3, a_2b_1})$$

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"The Dark Arts are many, varied, ever-changing, and eternal. Fighting them is like fighting a many-headed monster, which, each time a neck is severed, sprouts a head even fiercer and cleverer than before. You are fighting that which is unfixed, mutating,indestructible."

Severus Snape in "Harry Potter and the Half-Blood Prince"

2

Neutrino phenomenology and scalar dark matter with inverse and type II seesaw

In the second chapter we present a TeV scale seesaw mechanism for exploring the dark matter and neutrino phenomenology in the light of recent neutrino and cosmology data. A unique realization of the Inverse seesaw (ISS) mechanism with A_4 flavor symmetry is being implemented as a leading contribution to the light neutrino mass matrix which usually yields vanishing reactor angle θ_{13}. Making use of a non-diagonal structure of Dirac neutrino mass matrix and 3σ values of mass square differences the neutrino mass matrix is parameterized in terms of Dirac Yukawa coupling "y". We then use type II seesaw mechanism as a correction which turns out to be active to have a non-vanishing reactor mixing angle without much disturbing the other neutrino oscillation parameters. Then we constrain a common parameter space satisfying the non-zero θ_{13}, Yukawa coupling and the relic abundance of dark matter. Contributions of neutrinoless double beta decay are also included for standard light neutrino interaction. This study may have relevance in future neutrino and Dark Matter experiments.

2.1 Introduction

The link between neutrino oscillation and modern cosmology needs an elucidation since both of them infer physics beyond Standard Model (BSM). Several theories have been deciphered to bridge between these two separate sectors of particle physics and cosmology [1]. There is now a plethora of evidences in support of the existence of dark matter (DM) which constructs approximately one-fourth of the energy density of the universe [2–5]. Despite a number of recent studies of simplified DM models their nature remains rather elusive. Even the most successful Standard Model (SM) also does not furnish any signature of dark matter candidates and their properties. This is one of the pressing problems in both high energy physics and cosmology. Therefore, searching for a concrete realization to provide a hint towards physics BSM will be of utmost interest. It will be more fascinating if the discovery of neutrino oscillation and the existence of DM can be framed within a single particle physics model.

Even though astrophysical and cosmological observations, strongly suggest about the Presence of DM in the universe, the exact particle nature of DM is still unidentified. Planck 2013 data [5] says that, DM composes 26.8% of the energy density of the present universe, which predicts the present abundance (familiar as relic abundance) of DM as

$$\Omega_{DM}h^2 = 0.1187 \pm 0.0017, \tag{2.1.1}$$

where Ω implies the density parameter, Hubble parameter/100= is denoted as $h =$ [6]. Authors in [7] proposed a ten-point test that new particle has to satisfy so that it can be regarded as a potential DM candidates. The existence of dark matter is universally accepted, its nature remains elusive. It is usually assumed to be a single particle, but it may also be more than one. In specific models, it is often considered to be a fermion, scalar or vector [8]. Among the requirements the potential DM candidate must meet, the stability is protected by invoking some parity symmetry like Z_2 which is supposed to appear as a residual of a discrete flavor symmetry. There have been extensive studies in this field adopting various flavor symmetry groups [9–11]. We have plenty of examples where different kinds of DM were extensively studied with their stability in several ways. Recently connection between neutrino and the DM, using various flavor symmetries is drawing more attention in particle physics and cosmology. Here also we present a picture to construct a bridge between these two different sectors of particle physics adopting the A_4 based ISS realization. The most peculiar signatures of the ISS scenario are the additional decay channels of the Higgs boson into a heavy and ordinary neutrino, which confirms the SM particles to be a gateway to the scalar DM. In order for the SM particles being a portal to the dark sector, there must be at least two particles, one fermion and one boson in the dark sector. Here in our model Higgs boson, is considered as a DM candidate, couples with SM neutrino through a right handed neutrino. Two neutral components of this Higgs which is a triplet under A_4 is responsible in making correlation with neutrino mass and dark matter. A remnant Z_2 symmetry can explain the stability issue of the potential dark matter. This Z_2 symmetry also prevents the interaction of other particle contents of the model with the DM. Apart from the stability issue one more important test it must pass is to satisfy the observed relic density given by Eq. (2.1.1). For getting the correct relic abundance we require to take the DM mass from 50 GeV onwards. The Yukawa, which is responsible in making correlation between neutrino mass and DM coupling also needs to be fixed in such a way that the potential DM candidate gives rise to correct relic abundance.

Several seesaw mechanisms have shown a promising role in explaining neutrino mass and mixing. The inverse Seesaw (ISS) has been found to be an entirely different realization, which delicately attempts for the generation of a tiny neutrino mass at the cost of proposing the RH neutrino masses at the TeV scale which may have a better collider accessibility in near future. The essence of the ISS lies in the fact that, the double appearance of the mass scale associated with M in the denominator of the inverse seesaw formula allows it (M) to take a mass scale, which is much lighter than the one associated with the type I seesaw mechanism. Which in turn renders us with sub-eV scale SM neutrinos, at the cost of electroweak scale m_D, TeV scale M and keV scale μ, as explained in [12]. This

RH neutrino mass at TeV scale helps us to get the required mediator mass in order to obtain the appropriate relic abundance of relics. In addition to the ISS we are working with the Type II seesaw mechanism which turns out to be instrumental to have the non-vanishing reactor mixing angle. Both the inverse and type II seesaw are realized adopting the A_4 flavor symmetry. Then we have also studied the effective mass prediction to neutrinoless double beta decay (NDBD) for standard contribution.

We organize this chapter as follows. In section. 2.2 we present our model. Section 2.3 provides the stability issue of DM. Non-zero reactor angle is explained in the section 2.4. Section 2.5 has been presented with the analysis on Neutrinoless double beta decay. Section 2.6 offers the observation of the Relic abundance of DM in the background of the presented model. We have kept the numerical analysis in section 2.7. Finally, in section 2.8 we end up with our conclusion.

2.2 Neutrino mass model with various seesaw scenarios

2.2.1 Inverse seesaw mechanism

In our work we focus on the simplest ISS mechanism which is able to open up a new window to look for a comparatively lower right handed neutrino mass scale than the one present in type I seesaw [12–19]. The fulfillment of the ISS scheme requires the SM fermion sector to be extended by the inclusion of three RH neutrinos N and three additional neutral fermion singlets S_{iL}, where $i = 1, 2, 3$. It is worth stating that, the implementation of the ISS allows us to make use of extra symmetries in order to provide the neutrinos the following bilinear terms,

$$\mathcal{L} = -\bar{\nu}_L m_D N - \bar{S}_L M N - \frac{1}{2}\bar{S}_L \mu S_L^C + H.C., \tag{2.2.1}$$

The above Lagrangian implies a 9×9 leptonic mass matrix,

$$M_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix}. \tag{2.2.2}$$

In spite of its many phenomenological successes the ISS has a drawback that the RH mass term in the $M_{\nu_{22}}$ entry of M_ν is allowed by symmetries. This is a typical problem of inverse seesaw models. But it is prevented here by using Z_3 symmetry. After block diagonalization of the Eq. (2.2.2) we get the lightest neutrino mass eigenvalue as ,

$$m_\nu^I = m_D (M^T)^{-1} \mu M^{-1} m_D^T, \tag{2.2.3}$$

which is considered as the leading contribution to the light neutrino mass. Unlike the GUT scale seesaw mechanism, the ISS still needs an appropriate ground where the six new neutrinos could find their places in the elemental particle content and normally can get a mass term.

Non Abelian discrete flavor symmetries have played an important role in particle physics since long. In particular the symmetry group A_4 have been immensely found of utmost operation [20–24]. In this work we have analyzed the model presented by the authors in [9], extended with additional flavons with inverse and type II seesaw. We summarize the A_4 based ISS model by assigning the matter fields as shown in Table 2.1. We introduce four RH neutrinos, three of which $N = (N_1, N_2, N_3)$ are supposed to be a triplet of A_4 and the rest as a singlet N_4. We assign the SM type Higgs η to the A_4 triplet, which is considered as a DM candidate in the present analysis. We have four additional SM fermion singlets among which 'S' is transforming as A_4 triplet and S_4 as A_4 singlet. To get a desired neutrino mass matrix structure we are extending the Higgs sector by introducing six more Higgs fields, boosted by two additional symmetries Z_2 and Z_3 whose quantum numbers are given in Table 2.1. We construct the ISS mass matrices using the multiplication rules of A_4 as given in Section 1.8 of Chapter 1.

2.2.2 Type II seesaw with triplet Higgs

To implement the type II seesaw mechanism, the SM is extended by the addition of a new $SU(2)_L$ triplet scalar field Δ whose 2×2 matrix representation is given as

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & \Delta^+/\sqrt{2} \end{pmatrix}, \tag{2.2.4}$$

The VEV of the SM Higgs $< \phi_0 >= v/\sqrt{2}$, the trilinear mass term $\mu_{\phi\Delta}$ generate an induced VEV for the Higgs triplet as $\Delta^0 = v_\Delta\sqrt{2}$ where, $v_\Delta \simeq \mu_{\phi\Delta}v^2/\sqrt{2}M_\Delta^2$ [25]. The light neutrino mass is contributed by the type II seesaw mechanism in the following manner

$$m_{LL}^{II} = f_\nu v_\Delta, \tag{2.2.5}$$

where the analytic formula for induced VEV for the neutral component of the Higgs scalar triplet, derived by minimizing the scalar potential [25], is

$$v_\Delta \equiv \langle \Delta^0 \rangle = \frac{\mu_{\phi\Delta}v^2}{\sqrt{2}M_\Delta^2} \tag{2.2.6}$$

In the low scale type II seesaw which is dynamic at the TeV scale, we can consider a very small value of the trilinear mass parameter to be

$$\mu_{\phi\Delta} \simeq 10^{-8}GeV.$$

The sub-eV scale neutrino mass coming from type II seesaw mechanism constrains the corresponding Majorana Yukawa coupling as

$$f_\nu^2 < 1.4 \times 10^{-5}(\frac{M_\Delta}{1TeV})$$

Within the reasonable value of $f_\nu \simeq 10^{-2}$, the triplet Higgs scalar VEV is $v_\Delta \simeq 10^{-7}GeV$ which is in agreement with oscillation data. It is worth to note here that the tiny trilinear mass parameter $\mu_{\phi\Delta}$ controls the neutrino overall mass scale, but does not play any role in the couplings with the fermions. The structure of the matrix m_{LL}^{II}, with $w = f_\nu v_\Delta$ is explained in Section. 2.4.

2.3 Stabilizing the dark matter

An elegant way to establish the DM stability is by invoking a parity symmetry like Z_2. Here is an attempt to search for a theory which is responsible for explaining neutrino phenomenology and dark matter stability as well. The $A_4 \times Z_2 \times Z_3$

symmetry here only allows the coupling of the η with the singlet RH neutrinos rather than with charged fermions or quarks. It is worth noting that the alignment $\langle\eta\rangle \sim (1,0,0)$ breaks $A_4 \times Z_2 \times Z_3$ to Z_2 since $(1,0,0)$ remains manifestly invariant under one of the generators of the group A_4. In this manner spontaneously breaking of the symmetry, obeyed by the bigger group $A_4 \times Z_2 \times Z_3$ to Z_2 confirms the DM stability. The stability of the DM candidate is guaranteed by this remnant symmetry. The Z_2 residual symmetry is defined by

$$N_2 \rightarrow -N_2, S_2 \rightarrow -S_2, \eta_2 \rightarrow -\eta_2$$
$$N_3 \rightarrow -N_3, S_3 \rightarrow -S_3, \eta_3 \rightarrow -\eta_3$$

The leading order Yukawa Lagrangian for the neutrino part is given by the following equation.

$$\begin{aligned}\mathcal{L}_\nu^I = {} & y_1^\nu L_e(N\eta)_1 + y_2^\nu L_\mu(N\eta)_{1''} + y_3^\nu L_\tau(N\eta)_{1'} + y_4^\nu L_e N_4 h \\ & + y_s(SS)\phi_s + y_s' S_4 S_4 \phi_s + y_R(NS)\phi_R + y_R' N_4 S_4 \phi_R.\end{aligned} \tag{2.3.1}$$

	L_e	L_μ	L_τ	l_e^c	l_μ^c	l_τ^c	N	N_4	h	η	S_4	S	ϕ_R	ϕ_s	ζ	ξ	Δ
$SU(2)_L$	2	2	2	1	1	1	1	1	2	2	1	1	1	1	1	1	3
A_4	1	$1'$	$1''$	1	$1''$	$1'$	3	1	1	3	1	3	1	1	$1'$	$1''$	1
Z_2	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	1	1	1	1
Z_3	ω	ω	ω	ω^2	ω^2	ω^2	ω^2	ω^2	1	1	ω	ω	1	ω	1	1	ω

Table 2.1: Particles and their quantum numbers under $SU(2)_L$ symmetry, and A_4, Z_2, Z_3 flavour symmetry groups

The following flavon alignments help us to get a desired neutrino mass matrix.

$$\langle\Phi_R\rangle = v_R, \langle\Phi_s\rangle = v_s, \langle h\rangle = v_h, \langle\eta\rangle = v_\eta(1,0,0).$$

It is clear from the Eq. (2.3.2) and Eq. (2.3.3) that, m_D is related to v_η and v_h, M is determined by the VEV v_R. From this, the order of magnitude involved in the Eq. (2.2.3) is so, that $m_\nu \propto \frac{(v_\eta+v_h)^2}{v_R^2}\mu$. Here v_η and v_h are of the order of electroweak breaking, v_R is of the order of TeV scale. Therefore, to get m_ν in sub-eV, μ which is coming from the VEV of Φ_S should be of the order of keV. The two components of η are not generating the VEV [9], considered potential DM candidate. Decomposition of the following terms present in the Eq. (2.3.1) has been shown as follows

$$y_s(SS)\phi_s = y_s(S_1S_1 + S_2S_2 + S_3S_3)\phi_s,$$
$$y_R(NS)\phi_R = y_R(N_1S_1 + N_2S_2 + N_3S_3)\phi_R.$$

The chosen flavon alignments and the A_4 product rules allow us to have the Yukawa coupling matrices as follows

$$m_D = \begin{pmatrix} y_1^\nu\langle\eta\rangle & 0 & 0 & y_4^\nu\langle h\rangle \\ y_2^\nu\langle\eta\rangle & 0 & 0 & 0 \\ y_3^\nu\langle\eta\rangle & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} x_1a & 0 & 0 & y_1b \\ x_2a & 0 & 0 & 0 \\ x_3a & 0 & 0 & 0 \end{pmatrix}, \tag{2.3.2}$$

$$M = \begin{pmatrix} y_R\langle\phi_R\rangle & 0 & 0 & 0 \\ 0 & y_R\langle\phi_R\rangle & 0 & 0 \\ 0 & 0 & y_R\langle\phi_R\rangle & 0 \\ 0 & 0 & 0 & y_R'\langle\phi_R\rangle \end{pmatrix} = \begin{pmatrix} M_1 & 0 & 0 & 0 \\ 0 & M_1 & 0 & 0 \\ 0 & 0 & M_1 & 0 \\ 0 & 0 & 0 & M_2 \end{pmatrix}, \tag{2.3.3}$$

$$\mu_s = \begin{pmatrix} y_s\langle\phi_s\rangle & 0 & 0 & 0 \\ 0 & y_s\langle\phi_s\rangle & 0 & 0 \\ 0 & 0 & y_s\langle\phi_s\rangle & 0 \\ 0 & 0 & 0 & y_s'\langle\phi_s\rangle \end{pmatrix} = \begin{pmatrix} \mu_1 & 0 & 0 & 0 \\ 0 & \mu_1 & 0 & 0 \\ 0 & 0 & \mu_1 & 0 \\ 0 & 0 & 0 & \mu_2 \end{pmatrix}, \tag{2.3.4}$$

The light neutrino mass matrix as produced by the above three matrices under the ISS framework is given by

$$m_\nu = \begin{pmatrix} \frac{y_1^2b^2\mu_2}{M_2^2} + \frac{a^2x_1^2\mu_1}{M_1^2} & \frac{a^2x_1x_2\mu_1}{M_1^2} & \frac{a^2x_1x_3\mu_1}{M_1^2} \\ \frac{a^2x_1x_2\mu_1}{M_1^2} & \frac{a^2x_2^2\mu_1}{M_1^2} & \frac{a^2x_2x_3\mu_1}{M_1^2} \\ \frac{a^2x_1x_3\mu_1}{M_1^2} & \frac{a^2x_2x_3\mu_1}{M_1^2} & \frac{a^2x_3^2\mu_1}{M_1^2} \end{pmatrix}. \tag{2.3.5}$$

The assigned A_4 charge of this Higgs triplet η restricts the interaction of η with the charged leptons. In this model the charged leptons gain mass from the

Lagrangian give by

$$\mathcal{L}_l^I = y_e L_e l_e^c h + y_\mu L_\mu l_\mu^c h + y_\tau L_\tau l_\tau^c h \tag{2.3.6}$$

Following is the mass matrix for charged leptons.

$$m_l = \begin{pmatrix} y_e\langle h\rangle & 0 & 0 \\ 0 & y_\mu\langle h\rangle & 0 \\ 0 & 0 & y_\tau\langle h\rangle \end{pmatrix} \tag{2.3.7}$$

2.4 The reactor mixing angle

It is needless to say that there is a menagerie of theories, put forward in establishing the θ_{13} as having a nonzero value. Here also we are trying to present such a picture by including a perturbation called type II perturbation to the Lagrangian given by Eq. (2.3.1) which is realized within the type II seesaw mechanism [25–30]. The type II seesaw Lagrangian is followed by this term

$$\mathcal{L}^{II} = f_\nu(L_eL_\tau + L_\mu L_\mu + L_\tau L_e)\zeta\frac{\Delta}{\Lambda} + f_\nu(L_eL_\mu + L_\mu L_e + L_\tau L_\tau)\xi\frac{\Delta}{\Lambda} \tag{2.4.1}$$

Where, Λ represents the cutoff scale. With the type II perturbation the Lagrangian takes the following form,

$$\begin{aligned}\mathcal{L} = y_e L_e l_e^c h + y_\mu L_\mu l_\mu^c h + y_\tau L_\tau l_\tau^c h + y_1^\nu L_e(N\eta)_1 + y_2^\nu L_\mu(N\eta)_{1''} + y_3^\nu L_\tau(N\eta)_{1'} \\ + y_4^\nu L_e N_4 h + y_s(SS)\phi_s + y_s' S_4S_4\phi_s + y_R(NS)\phi_R + y_R' N_4S_4\phi_R \\ \mathcal{L}^{II} = f_\nu(L_eL_\tau + L_\mu L_\mu + L_\tau L_e)\zeta\frac{\Delta}{\Lambda} + f_\nu(L_eL_\mu + L_\mu L_e + L_\tau L_\tau)\xi\frac{\Delta}{\Lambda}.\end{aligned} \tag{2.4.2}$$

The last two terms represent the perturbation to the leading order terms in the above Lagrangian giving rise to non-zero θ_{13}. Here we have implemented the A_4 group to explain the structure of the type II seesaw neutrino mass matrix given by Eq. (2.4.3). The triplet Higgs field Δ_L is supposed to be an A_4 singlet. Two more flavon fields ζ and ξ have been introduced which are assumed to transform as A_4 singlets as summarized in the Table. 2.1. The flavon alignments which help in constructing the m_{LL}^{II} matrix are as follows

$$\langle\Delta\rangle \sim v_\Delta, \quad \langle\zeta\rangle \sim v_\zeta, \quad \langle\xi\rangle \sim v_\xi$$

. ζ and ξ are assumed to take the VEV in the same scale $v_\zeta = v_\xi = \Lambda$. With these flavon alignments the structure of mass matrix m_{LL}^{II} will take the form

$$m_{LL}^{II} = \begin{pmatrix} 0 & -w & w \\ -w & w & 0 \\ w & 0 & -w \end{pmatrix}. \quad (2.4.3)$$

2.5 Neutrinoless double beta decay

The time period for neutrinoless double beta ($0\nu\beta\beta$) decay rate is exactly proportional to the effective neutrino mass square $|m_\nu^{ee}|^2$ (for a detail please see [31–33]). Which implies that in determining the time period for NDBD, the effective mass plays a non-trivial role in the scenario of three generations of neutrinos. The effective neutrino mass can be given by

$$|m_\nu^{ee}| = |U_{ei}^2 m_i|, \quad (2.5.1)$$

In addition to this, following non-standard contributions become transparent in

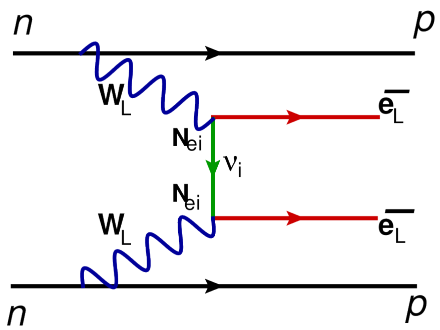

Figure 2.1: Feynman diagram representing $0\nu\beta\beta$ Decay because of light neutrino exchanges.

the present model.

- Two separate contributions due to light and heavy neutrino exchanges to $0\nu\beta\beta$ come into play. And this event is established by writing the flavor eigenstates as a linear combination of light and heavy mass eigenstates. The only contribution that becomes effective in the ISS regime comes from the contribution due to light neutrino exchanges.

$$\nu_\alpha = N_{\alpha i}\nu_i + U_{\alpha j}\xi_j, \quad (2.5.2)$$

where, $N_{\alpha i}$ and $U_{\alpha j}$ are the mixing matrices for light and heavy neutrino respectively. $|m_\nu^{ee}|$ takes distinct values depending on the framework (quasi degenerate or normal/inverted hierarchies), the neutrino mass states are in. Now considering the light neutrino contribution (the only contribution for ISS in this model), the key formula which determines the effective neutrino mass is

$$m_{\nu,LL}^{ee} \simeq U_{e1}^2 m_1 + U_{e2}^2 e^{2i\alpha} m_2 + U_{e3}^2 e^{2i\beta} m_3. \tag{2.5.3}$$

- The triplet Higgs contribution from the type II seesaw is of the order of $10^{-13} m_i$ which is much smaller as compared to the leading contributions.

Of special importance is the fact that, the chosen value of Yukawa coupling giving rise to the observed relic abundance of our DM candidate, constrains the lightest neutrino mass significantly in the presented forum. The fine tuned Yukawa couplings $(0.994 - 1)$ is noticed to play a vital role in achieving the lightest neutrino mass and in turn to get the effective neutrino mass prediction within the GERDA bound $(0.5eV)$. The type II perturbation strength is found to play some role in giving $m_{lightest}$ within the PLANK bound (0.065 eV for IH). The introduced model also evinces the role of $U_{\rm PMNS}$ matrix elements and the lightest neutrino mass as $|m_\nu^{ee}|$ is dependent upon them.

2.6 Relic density of dark matter

The relic abundance of a DM particle χ is given by the Boltzmann equation [34–37]

$$\frac{dn_\chi}{dt} + 3Hn_\chi = - < \sigma v > (n_\chi^2 - (n_\chi^{eqb})^2), \tag{2.6.1}$$

where n_χ is the dark matter (χ) number density with n_χ^{eqb} as the equilibrium number density of χ, in thermal equilibrium. The Hubble rate is denoted as H and $< \sigma v >$ is the thermally averaged annihilation cross-section of the DM χ. Numerical solution of the Boltzmann equation is given by [35]

$$\Omega_\chi h^2 \approx \frac{1.04 \times 10^9 x_F}{M_{pl}\sqrt{g_*}(a + 3b/x_F)}, \tag{2.6.2}$$

where $x_F = \frac{m_\chi}{T_F}$ with T_F as the freeze-out temperature, g_* denotes the number of effective relativistic degrees of freedom at the time of freeze-out. DM particles with electroweak scale mass and couplings freeze out at temperatures in the range $x_F \approx 20 - 30$. This in turn simplifies to, as shown by the authors in [38],

$$\Omega_\chi h^2 \approx \frac{3 \times 10^{-27} cm^3 s^{-1}}{< \sigma v >}. \tag{2.6.3}$$

For complex scalar DM, the annihilation rate is given by Eq. (2.6.4). The relic abundance is related to the cross section of the DM-DM interaction. The terms in Eq. (2.3.1) evinces the interaction shown by figure 2.2. While finding the allowed parameter space satisfying the correct relic abundance and neutrino oscillation parameters we vary the Relic mass and the Majorana fermion mass(the right handed neutrino) both of which are involved in the cross section formula as shown in [39] reads as

$$(\sigma v)^{\chi\chi\dagger}_{complexscalar} = \frac{v^2 y^4 m_\chi^2}{48\pi (m_\chi^2 + m_\psi^2)^2}. \tag{2.6.4}$$

With v = relative velocity of the two relic particles and is typically $0.3c$ at the freeze out temperature, χ is the relic particle (DM), y is the Yukawa coupling, m_χ the mass of the relic, m_ψ is the mass of the mediator particle. The dark

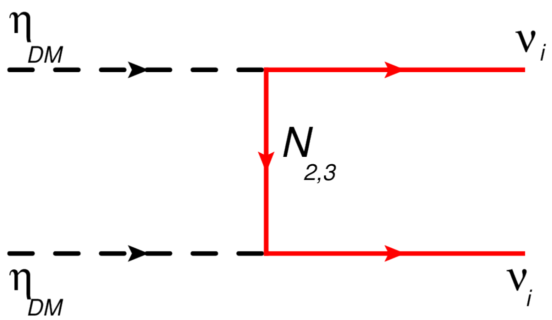

Figure 2.2: Feynman diagram showing the scattering of η_2 and η_3.

matter relic abundance may get affected by some kind of annihilation processes which might have taken place between the two neutral scalars depending on their mass difference $\Delta m = m_{\eta_2} - m_{\eta_3}$. If the mass splitting has the same order with the freeze-out temperature, the co-annihilation between the two neutral scalars play a significant role in finding the dark matter relic abundance. But if Δm is

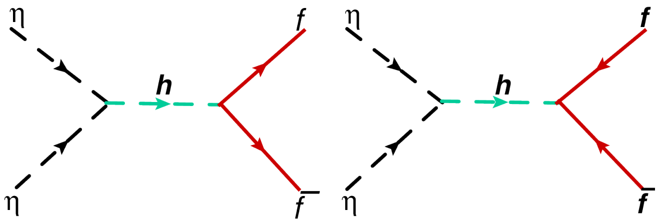

Figure 2.3: Self annihilation of η_2 and η_3 into SM fermions (conventions are followed from [41]).

larger than the freeze-out temperature, then the immediate heavier neutral scalar affects the dark matter relic density notably. The self annihilation between dark matter and immediately heavier component of the scalar triplet η contribute to the dark matter annihilation cross section. Many authors in [34, 36, 40] explored this kind of self annihilation consequences on dark matter relic abundance. To compute the effective annihilation cross section we are following the analysis done by the authors in [34]. The relevant annihilation channels and interactions can be given by figure 2.3. For low mass scheme ($m_{DM} < M_W$), the self annihilation of either η_2 or η_3 into SM particles takes place via the SM Higgs, which is depicted in figure 2.3. The according annihilation cross section [36, 40] is followed by Eq. (2.6.5).

$$\sigma_{xx} = \frac{|Y_f|^2|\lambda_x|^2}{16\pi s}\frac{(s-4m_f^2)^{3/2}}{\sqrt{s-4m_x^2}((s-m_h^2)^2+m_h^2\Gamma_h^2)}, \tag{2.6.5}$$

where $x \to \eta_{2,3}$, the coupling of x with SM Higgs h is denoted by λ_x and Y_f implies the fermion Yukawa coupling, which has been estimated to be 0.32 albeit the full possible range of values is $\lambda_f = 0.26 - 0.63$ [6]. $\Gamma_h = 4.15MeV$ is the decay width of the SM Higgs, m_h is 126 GeV. s is the thermally averaged center of mass squared energy given by

$$s = 4m^2 + m^2v^2. \tag{2.6.6}$$

where, v is the relative velocity and m is the mass of the relic. In order to yield the correct relic abundance we need to constrain the Yukawa coupling along with the relic mass and the mediator mass. Similar to the works done in [42, 43] here also we suppose the neutral component of the scalar triplet as the DM candidate. We choose the relic mass as lighter than the W boson mass $m_{DM} \leq M_W$. And interestingly for the relic mass is kept in a comparatively smaller mass scale which is around 50 GeV. The mediator mass here in our case, i.e., the Majorana neutrino mass is required to vary from 153 GeV to 154 Gev to obtain the observed relic density. This type of findings have been extensively studied in the literature [39, 44]. For a light DM with a mass below 10 GeV, the LHC searches have a better awareness for complex scalar DM cases. Moreover, the LHC has a better reach than direct detection experiments with DM masses up to around 500 GeV for the complex scalar DM case.

2.7 Numerical analysis

The latest global fit [45] value with their best fit point (bfp) for 3σ range of neutrino oscillation parameters used to study neutrino phenomenology are given in Table 2.2 and Table 2.3: Cosmological constraint says that,

Oscillation parameters	bfp	3σ Cl
$\Delta m^2_{21}[10^{-5}eV^2]$	7.5	$(7.02, 8.07)$
$\Delta m^2_{31}[10^{-3}eV^2]$	2.457	$(2.317, 2.607)$
$\sin^2\theta_{12}$	0.304	$(0.270, 0.344)$
$\sin^2\theta_{13}$	0.0218	$(0.0186, 0.0250)$
$\sin^2\theta_{23}$	–	0.381-0.643

Table 2.2: Neutrino Oscillation data for Normal mass Ordering

Oscillation parameters	bfp	3σ Cl
$\Delta m^2_{21}[10^{-5}eV^2]$	7.5	$(7.02, 8.07)$
$\Delta m^2_{23}[10^{-3}eV^2]$	-2.449	$-2.590, -2.307$
$\sin^2\theta_{12}$	0.304	$0.270, 0.34$
$\sin^2\theta_{13}$	0.0219	$0.0188, 0.0251$
$\sin^2\theta_{23}$	–	$0.388, 0.644$

Table 2.3: Neutrino Oscillation data for Inverted mass Ordering

$$m_1 + m_2 + m_3 \leq 0.23eV.$$

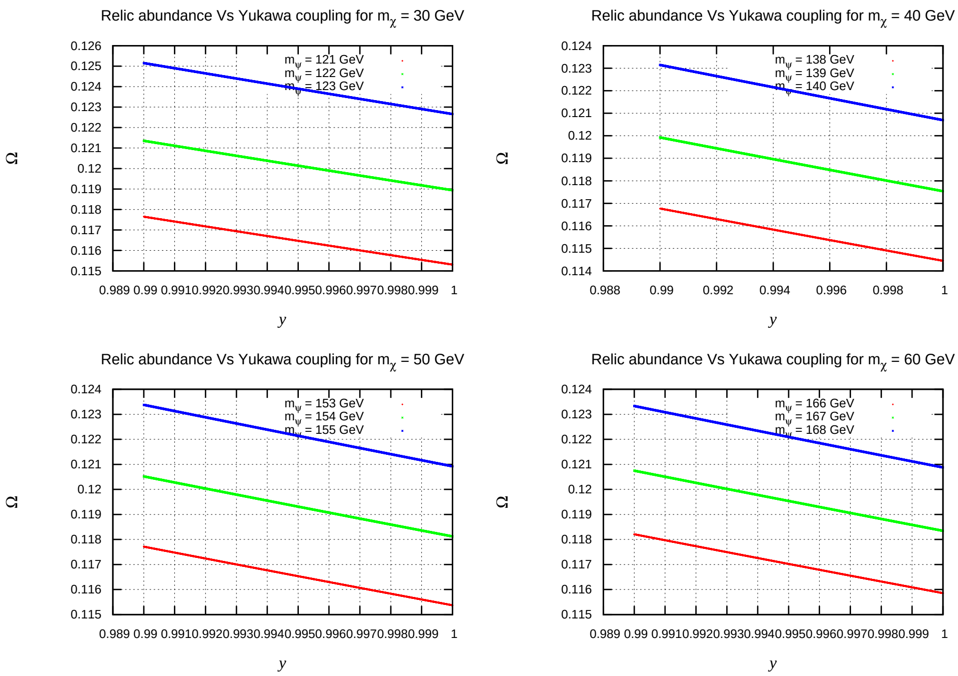

Figure 2.4: Variation of relic abundance with Yukawa coupling.

The Yukawa coupling governing the interaction is present in the established mathematical expression which computes the scattering cross section of this interaction in turn the relic abundance of the potential DM. As a proper choice of Yukawa coupling, the mediator mass along with the complex scalar mass allows us to achieve the observed relic abundance we need to put constraints on them. In our work we first fix the above mentioned parameters to get the relic abundance which is reported by PLANCK 2013 data. Fixing the relic mass around 50 GeV and varying the mediator mass from 153 to 154 GeV we get the idea of Yukawa coupling yielding the correct relic abundance. Since the required relic abundance for the potential DM candidate desires a mediator mass at a much lower scale (around 153 GeV), the ISS realization helps us in this regard (which is here, the mediator particle governing the t-channel scattering as shown in figure 2.2). The Yukawa coupling needs to fall between 0.99 to 1 to have a better reach of the relic abundance as shown in figure 2.4. We redefine the parameters of the matrix shown by the Eq. (2.3.5) in terms of p, q and r. Where, $p = \frac{ax_1\sqrt{\mu_1}}{M_1}$, $q = \frac{ax_2\sqrt{\mu_1}}{M_1}$ and $r = \frac{ax_3\sqrt{\mu_1}}{M_1}$. From the requirement of bringing the light neutrino mass matrix into TBM form we equate the 11-element of m_ν to $2q^2 - pq$ [9]. This is done in accordance with adjusting the Yukawa couplings and the associated VEVs. Along with this redefenition we also make $q = r$ by $x_2 = x_3$ for numerical analysis. This structure of light neutrino mass matrix leads to a neutrino mass spectrum which is of inverted hierarchical type and a zero eigenvalue with $m_3 = 0$. For numerical analysis we take another couple of definitions for the Yukawa couplings $x_1 = x$ and $x_2 = x_3 = y$. We have kept $x = 1$ and varied y for computing the oscillation parameters and m_ν^{ee}, however there is no significant changes observed by keeping y fixed and varying x. Each value of y gives rise to various sets of the neutrino mass matrix parameters p, q. We parameterize the light neutrino mass matrix obtained from the ISS realization with the help of recent neutrino oscillation data given in Table 2.2 and Table 2.3. Along with the redefined parameters of the light neutrino mass matrix and using Eq. (2.3.2), Eq. (2.3.3) and Eq. (2.3.4) the new light neutrino mass matrix is found to be of TBM type given by Eq. (2.7.1)

$$m_\nu = \begin{pmatrix} 2q^2 - pq & pq & pq \\ pq & q^2 & q^2 \\ pq & q^2 & q^2 \end{pmatrix}. \tag{2.7.1}$$

We have analyzed the model only for IH case as the light neutrino mass matrix structure only allows us to have the inverted hierarchy mass pattern. After diagonalizing the complete mass matrix the mass eigenvalues are found to be $m_1 = -2(pq - q^2)$, $m_2 = q(p + 2q)$ and $m_3 = 0$. Then we parametrize the mass matrix keeping $x = 1$ while at the same time varying y between a range around $0.994 - 1$. Choosing each set of p, q values which have been found different for different "y" values, we get several light neutrino mass matrices. The same Yukawa coupling y is being varied in the dark matter sector too for showing its contribution to obtain the correct relic abundance. For the generation of non-zero reactor mixing angle, we include type II correction [25] to the leading order neutrino mass matrix as explained in Section 2.4. This perturbation brings out non-zero θ_{13} in 3σ range along with $m_3 \neq 0$ leaving the light neutrino masses with IH nature only. The numerical value of the perturbation term $w = f_\nu v_\Delta$ critically depends upon the Majorana coupling f_ν, trilinear mass parameter $\mu\phi\Delta$, and M. Accordingly, we vary the type II seesaw strength from 10^{-6} to 0.01 to produce non-zero θ_{13}. It is observed from the figure 2.5 that, the type II seesaw strength of 10^{-3} eV is generating the non-zero θ_{13} in the 3σ range in all cases. The perturbation matrix takes the following structure.

$$m_\nu^{II} = \begin{pmatrix} 0 & -w & w \\ -w & w & 0 \\ w & 0 & -w \end{pmatrix},$$

After adding the perturbation we get the neutrino mass matrix as follows.

$$m_\nu = m_\nu^I + m_\nu^{II}.$$

Now the elements of these diagonalized matrices are associated with the parameters of the model and the type II perturbation term. The set of p, q values obtained for each y value and chosen for analysis are listed in Table 2.4, Table 2.5 and Table 2.6. In addition p, q corresponds to some complex sets of solution too. Taking them under consideration, no significant changes in the numerical analysis have been noticed.

A comparison among the various sets of results obtained in the DM phenomenology part has been made in Table 2.7 and neutrino phenomenology has been shown in the Table 2.8. The light neutrino mass matrix (2.7.1) is having only

Parameters	$y = 0.994$	$y = 0.996$	$y = 0.998$	$y = 1$
p	0.366138	0.366146	0.366154	0.357719
q	0.0899502	0.089768	0.0895865	0.091516

Table 2.4: Values of p, q obtained by solving for IH case with best fit central value of 3σ deviations

two unknown parameters, solution for which demands two equations. Two mass squared differences which we get from neutrino oscillation data, lead to those two parameters. Then using the solutions for p and q the light neutrino mass matrix is obtained. Then we fix the mass eigenvalues from that light neutrino mass matrix.

Parameters	$y = 0.994$	$y = 0.996$	$y = 0.998$	$y = 1$
p	0.371351	0.371359	0.371367	0.362663
q	0.0911924	0.0910077	0.0908236	0.0928181

Table 2.5: Values of p, q obtained by solving for IH case with a upper bound of 3σ deviations

Parameters	$y = 0.994$	$y = 0.996$	$y = 0.998$	$y = 1$
p	0.360693	0.3607	0.360708	0.352452
q	0.088626	0.0884465	0.0882677	0.0901551

Table 2.6: Values of p, q obtained by solving for IH case with an lower bound of 3σ deviations

Using the best fit central values from the oscillation data, we numerically fit the leading order neutrino mass matrix. A thorough analysis has been carried out to check whether the oscillation parameters are near to reach or not by taking the upper and lower bound of 3σ deviation as well. Here we try to exhibit an unexplored parameter space satisfying both the DM relic abundance and neutrino phenomenology.

The scattering cross section of the decay channel described by figure 2.3 to various SM fermions have been calculated. They are found to have an order of 10^{-60}cm^2 / 10^{-42}GeV^{-2} which is much smaller than the cross section which has been achieved for the t-channel contribution (of the order of 10^{-44}cm^2). They will have little contribution (can be neglected therefore) to the relic abundance of the potential DM candidate. We have already noticed that for obtaining the observed Ω we need to fix the Yukawa coupling. Fixing the Yukawa coupling as varying from 0.99 to 1, varying m_{DM} from 30 to 60 GeV and varying M_R from 120 to 167 GeV, we study the order of relic abundance. We fit the values of oscillation parameters using recent cosmological constraints for inverted mass ordering. We compute all the oscillation parameters also by varying the type II seesaw strength. Variation of type II seesaw strength with the non-vanishing θ_{13}, has been shown in figure 2.5 and figure 2.6. The production of other oscillation parameters, e.g. the two mixing angles and two mass squared splitting as a function of nonzero θ_{13} has been shown in the figure 2.7, figure 2.8 and figure 2.9 for different values of Yukawa coupling. The sum of absolute masses has also been calculated to see whether it satisfies the Planck upper bound or not. Seeing that, the sum of absolute neutrino masses can give some clue on neutrinoless double beta decay, a little study has been performed to check whether the presented model is able to contribute to the $0\nu\beta\beta$ physics. In figure 2.10 we plot for the contribution of the effective mass to $0\nu\beta\beta$ decay due to light neutrino exchanges for standard contribution showing the variation of effective mass with the type II seesaw strength. Figure 2.11 displays the variation of m_ν^{ee} with the lightest neutrino mass, in our model m_3. In figure 2.12 we present the variation of effective mass with $m_{lightest}$ and type II seesaw strength taking the upper and lower bound of 3σ deviation. Since the presented model only present a hierarchy of inverted kind the lowest mass range has been selected which is resulted from the perturbation. The variation in m_ν^{ee} for non-standard contribution with different y values have been checked and found to be in agreement with the experimental bounds. The effective mass for non-standard contribution has been obtained around 0.0489 almost for all the values of Yukawa couplings chosen for the analysis. It is worth noting that the variation in Yukawa coupling leaves trivial impacts on m_ν^{ee} for non-standard contribution. For showing the variation of m_ν^{ee} with m_3, we choose those values of m_3 obtained as a result of adding the type II seesaw strength.

The following observations have been made from the results and analysis.

- The relic abundance has been found to match the value shown by PLANCK 2013 data, for a choice of Yukawa coupling ranging from 0.99 to 1 provided the Relic mass is fixed at 50 GeV keeping the mediator mass at a range from 153 to 154 GeV. A detailed analysis of the choice of Yukawa coupling, the Relic mass (m_χ) and the mediator mass (m_ψ) for this particular model has been presented in the Table 2.7.

- The oscillation parameters are near to reach only when the Yukawa coupling is varied from 0.994 to 1 and as a further increase/decrease of the Yukawa coupling does not yield good neutrino phenomenology we have considered

m_χ	m_ψ	y	Ω
30 GeV	(121 – 122) GeV	(0.99 – 1)	✓
40 GeV	139 GeV	(0.99 – 1)	✓
50 GeV	(153 – 154) GeV	(0.99 – 1)	✓
60 GeV	(166 – 167) GeV	(0.99 – 1)	✓

Table 2.7: Comparison of relic abundance Ω with various choices of Yukawa couplings, DM mass, RH neutrino mass

3σ ranges	θ_{13}	θ_{12}	θ_{23}	Δm^2_{21}	Δm^2_{23}	Σ mod m_i
bfp	✓	✓	✓	✓	✓	✓
lower bound	✓	✓	✓	✓	✓	✓
upper bound	✓	✓	✓	✓	×	✓

Table 2.8: Summary of results obtained from various allowed mass schemes.

those corresponding values of relic abundance obtained for Yukawa coupling ranging from 0.994 to 1.

- It has been noticed that the proposed model evidences correct neutrino phenomenology using the best fit and lower 3σ bound in case of inverted hierarchy mass pattern only. All the oscillation parameters have seen to come inside the frame while taking the the best fit and lower 3σ bound.

- The non-zero value of θ_{13} has been found to be consistent with the variation of type II seesaw strength.

- Both the standard and new physics contribution to $0\nu\beta\beta$ decay in the allowed hierarchy is obtained in the vicinity of experimental results [46].

2.8 Conclusion

An A_4 based IH neutrino mass model originating from both inverse and type II seesaw have been studied. Here ISS is implemented as a leading order contribution to the light neutrino mass matrix yielding zero reactor mixing and

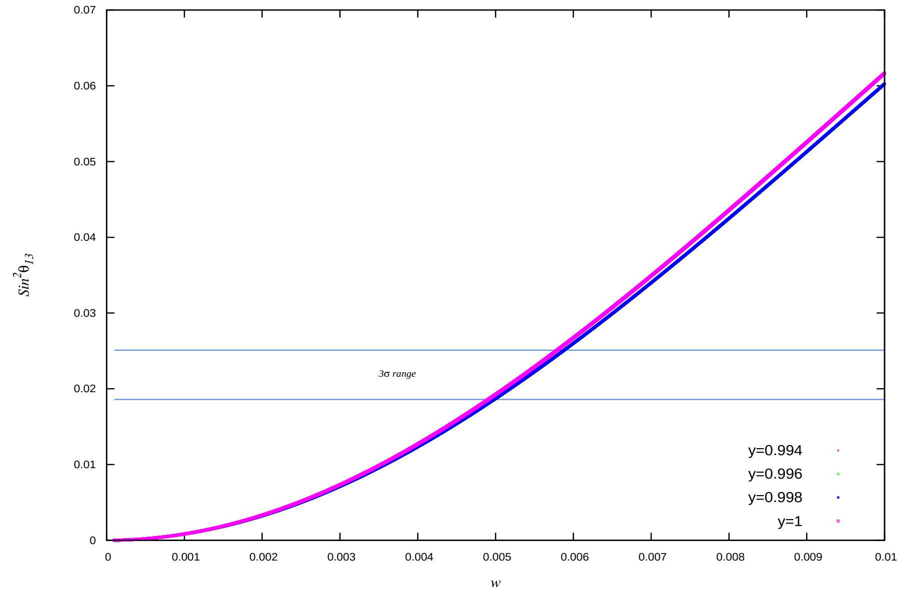

Figure 2.5: Generation of non-zero $sin^2\theta_{13}$ varying the type II strength for best fit values.

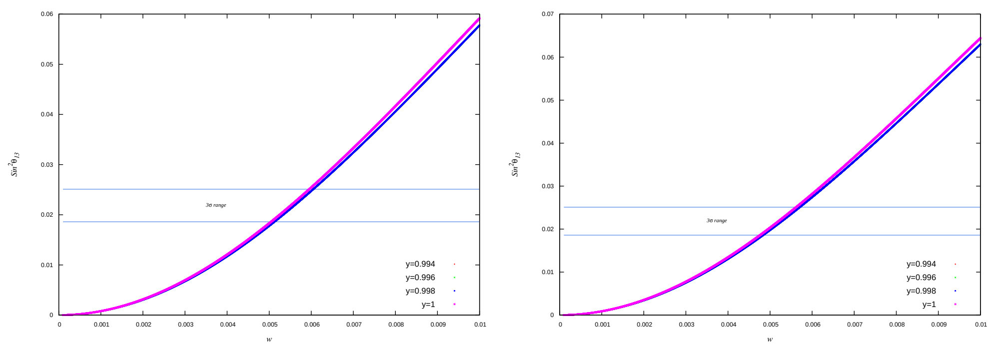

Figure 2.6: Generation of non-zero $sin^2\theta_{13}$, varying the type II strength using upper bound (left panel) and lower bound (right panel) of 3σ deviations.

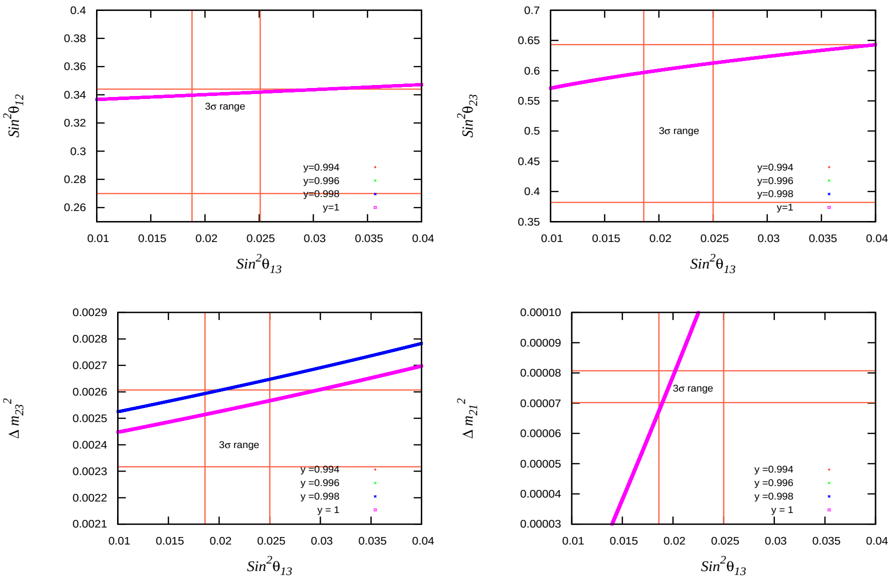

Figure 2.7: Variation of $sin^2\theta_{12}$, $sin^2\theta_{23}$,Δm_{23}^2 and Δm_{21}^2 with $sin^2\theta_{13}$ with best fit value.

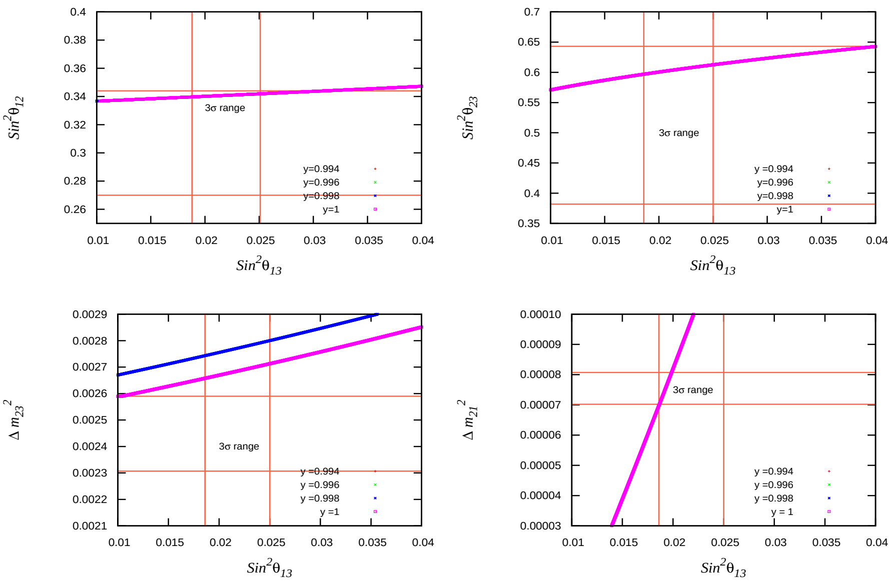

Figure 2.8: Variation of $sin^2\theta_{12}$, $sin^2\theta_{23}$,Δm_{23}^2 and Δm_{21}^2 with $sin^2\theta_{13}$ with upper bound of 3σ deviation.

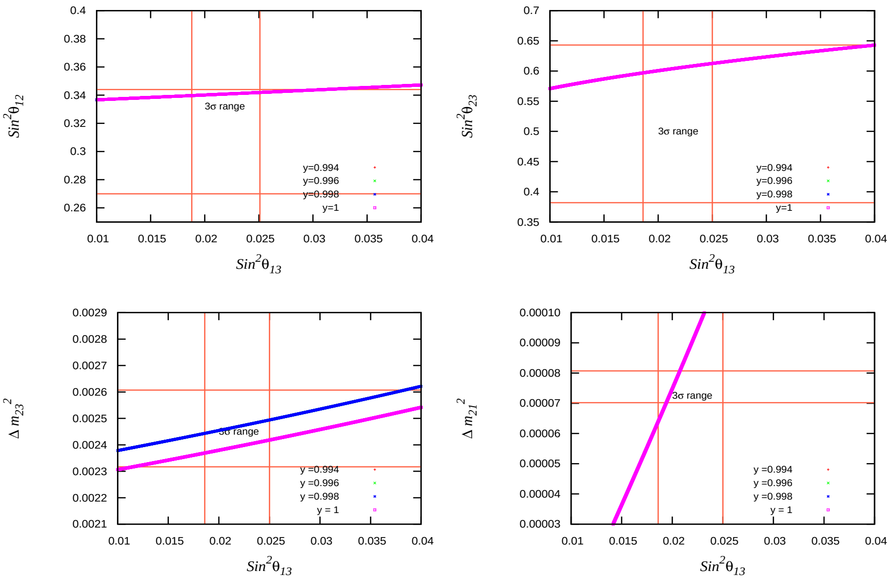

Figure 2.9: Variation of $sin^2\theta_{12}$, $sin^2\theta_{23}$,Δm^2_{23} and Δm^2_{21} with $sin^2\theta_{13}$ with lower bound of 3σ deviation.

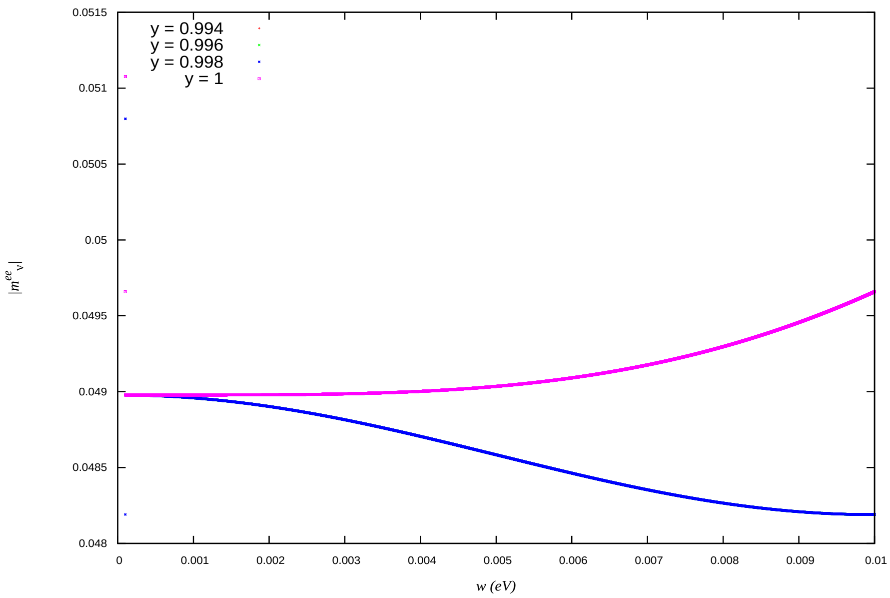

Figure 2.10: Variation of effective mass m_ν^{ee} (in eV) with type II seesaw strength using bfp.

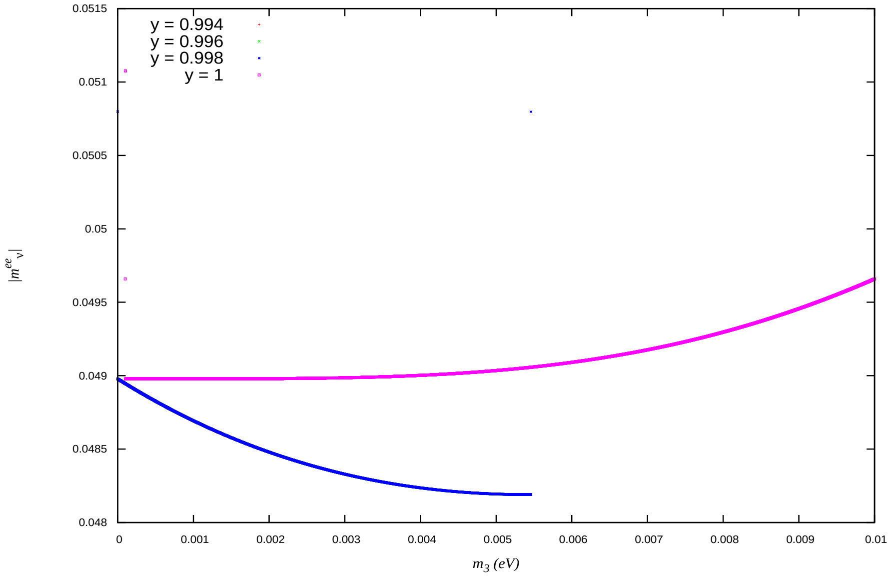

Figure 2.11: Variation of effective mass m_ν^{ee} (in eV) with the lightest neutrino mass using bfp.

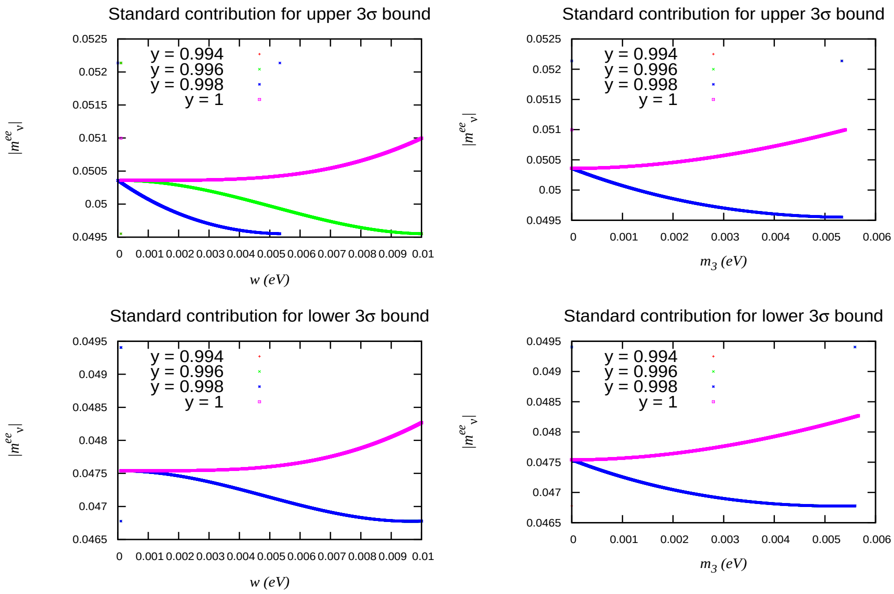

Figure 2.12: Variation of effective mass m_ν^{ee} (in eV) with type II seesaw strength and the m_3 for upper and lower 3σ bounds.

$m_3 = 0$. Then the type II seesaw has been used in order to produce non-Zero reactor mixing angle, which later on produces $m_3 \neq 0$ keeping the hierarchy as inverted only. We have studied the possibility of having a common parameter space where both the Neutrino oscillation parameters in the 3σ range and DM relic abundance has a better reach. With a proper choice of Yukawa coupling(y), right handed neutrino (mediator particle) mass (m_ψ) , and complex scalar (potential DM candidate) mass (m_χ) the variation in relic abundance as a function of Yukawa coupling has been shown. For a choice of Yukawa coupling between 0.994 to 0.9964, m_{DM} around 50 GeV, the mediator mass needs to fall around 153 GeV to match the correct relic abundance. The same Yukawa coupling has got a key role in generating the Neutrino oscillation parameters as well. We have studied the prospect of producing non-zero θ_{13} by introducing a perturbation to the light neutrino mass matrix using type II seesaw within the A_4 model. We have also determined the strength of the type II seesaw term which is responsible for the generation of non-zero θ_{13} in the correct 3σ range. We have also checked whether the proposed model can project about neutrinoless double beta decay or not. In context to the presented model we have found a wide range of parameter space where one may have a better reach for both neutrino and dark matter sector as well. This model may have relevance in studying baryon asymmetry of the universe, which we leave for future study.

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"Symmetry is what we see at a glance; based on the fact that there is no reason for any difference . . ."

Blaise Pascal

3

Non-zero θ_{13} and dark matter in an S_4 flavour symmetric model with inverse seesaw

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In this chapter we study an inverse seesaw model of neutrino mass within the framework of S_4 flavour symmetry from the requirement of generating non-zero reactor mixing angle θ_{13} along with correct dark matter relic abundance. The leading order S_4 model gives rise to tri-bimaximal type leptonic mixing resulting in $\theta_{13} = 0$. Non-zero θ_{13} is generated at one loop level by extending the model with additional scalar and fermion fields which take part in the loop correction. The particles going inside the loop are odd under an in-built Z_2^{Dark} symmetry such that the lightest Z_2^{Dark} odd particle can be a dark matter candidate. Correct neutrino and dark matter phenomenology can be achieved for such one loop corrections either to the light neutrino mass matrix or to the charged lepton mass matrix although the latter case is found to be more predictive. The predictions for neutrinoless double beta decay is also discussed and inverted hierarchy in the charged lepton correction case is found to be disfavoured by the latest KamLAND-Zen data.

3.1 Introduction

The Standard Model (SM) of particle physics surmises on the minimal choice that a single Higgs doublet provides masses to all particles. Some questions however remain unanswered, including the origins of neutrino mass and dark matter (DM), keeping other avenues open for physics beyond the Standard Model (BSM). There have been several conclusive evidences in the last two decades which validate the existence of non-zero neutrino masses and large leptonic mixing [1–9]. The SM can not address this observed phenomena simply because the neutrinos remain massless in the model, because the SM does not accomodate any RH neutrino. If the right handed neutrinos are included by hand, one needs the Yukawa couplings to be heavily fine tuned to around 10^{-12} for the production of sub-eV neutrino masses from the same Higgs field of the SM. One can generate a tiny Majorana mass for the neutrinos from the same Higgs field of the SM at non-renormalisable level through the dimension five Weinberg operator [10]. The realisation of this dimension five operator within renormalisable theories are also available in the literature, popularly known as the seesaw mechanism [11–13]. Even if the tiny neutrino masses are produced dynamically within such seesaw frameworks, understanding the origin of the large leptonic mixing is another puzzle. Since the quark sector mixing is observed to be small, it also indicates that there may be some new dynamics operating in the leptonic sector that generates the large mixing. As we see from the global fit data, among the three mixing angles, the solar and atmospheric angles are reasonably large but the reactor angle is comparatively small. In fact, before the discovery of non-zero reactor mixing angle θ_{13} in 2012, the neutrino data were consistent with a class of neutrino mass matrices respecting $\mu-\tau$ symmetry (For a recent review, we refer [14]). This class of models predicts $\theta_{13}=0, \theta_{23}=\frac{\pi}{4}$ whereas the value of θ_{12} depends upon the particular model. Out of different $\mu-\tau$ symmetric neutrino mass models, the Tri-Bimaximal (TBM) mixing [15–17] received lots of attention within several neutrino mass models. The TBM mixing predicts $\theta_{12} = 35.3^o$. Such a mixing can be easily accommodated within popular discrete flavour symmetry models [18–20]. Since the measured value of θ_{13} is small, such $\mu - \tau$ symmetric models can still be considered to be valid at leading order, while the small but non-zero θ_{13} can be generated by perturbations to either the charged lepton or the neutrino sector, as studied in several works in the literature including [21–29].

On the other hand, the SM also fails to provide a particle DM candidate that can satisfy all the criteria of a good DM candidate [30]. Although there have been sufficient evidences [31–33] from astrophysics and cosmology offering the existence of DM the particle nature of DM is not yet known. This has driven the particle physics community to explore different possible BSM frameworks which can give rise to the correct DM phenomenology and can also be tested at several different experiments. Amidst them, the most popular BSM scenario is the weakly interacting massive particle (WIMP) paradigm, as the correct DM relic abundance can be achieved for such a particle if it has interaction strength similar to weak interactions. This coincidence is also referred to as the *WIMP Miracle*. The estimation on present dark matter abundance as a function of density parameter and $h =$ (Hubble Parameter)/100, is reported as [34]

$$\Omega_{\rm DM} h^2 = 0.1187 \pm 0.0017 \tag{3.1.1}$$

Using the measured value of Hubble parameter, this yields approximately 26% of the total energy density of the present Universe being composed of DM. The same Planck experiment also puts a bound on the sum of absolute neutrino masses $\sum_i |m_i| < 0.17$ eV [34]. Although the fundamental origin of DM may not be related to the origin of neutrino mass as well as leptonic mixing, it is pretty exciting to look for a common platform that can explain both the phenomena. In spite of keeping the BSM physics minimal, this also permits for its probe in a much larger range of experiments. We find two such frameworks very appealing: one where neutrino masses originate at one loop level with DM particles going in the loop [35] and the other where the same discrete flavour symmetry responsible for generating large leptonic mixing also guarantees a stable DM candidate [36].

More detailed phenomenology of similar models can be found in several works including [37–42]. Another recent proposal to connect dark mater with non-zero θ_{13} can be found in [43].

Motivated by this, here also we consider an inverse seesaw model [44–46] based on S_4 discrete flavour symmetry that gives rise to TBM type neutrino mixing at leading order. Unlike canonical seesaw models, the inverse seesaw can be a low scale framework where the singlet heavy neutrinos can be at or below the TeV scale without any fine tuning of Yukawa couplings. This is possible due to softly broken global lepton number symmetry by the singlet mass term as we discuss later. The existence of sterile neutrinos around TeV scale with sizeable Yukawa couplings in these models makes these models testable at planned future particle colliders [47]. Another motivation to study this particular model is the neutrino mass sum rules it predicts, which relates the three light neutrino masses [48]. This predicts the lightest neutrino mass, once the experimental data of two mass squared differences are given as input and hence can be examined at experiments perceptive to the lightest neutrino mass say, neutrinoless double beta decay (NDBD)[1]. Since the model gives rise to TBM mixing, disallowed by latest neutrino data, we extend the model in order to reproduce non-zero θ_{13} in such a way that automatically takes DM into account. For this we make use of the scotogenic mechanism [35] mentioned above where DM particles going in loop can generate tiny neutrino mass. We implement this idea in two different ways. First we add a one loop correction to the leading order light neutrino mass matrix from inverse seesaw and secondly we give a similar correction to the charged lepton mass matrix. In both the cases, the correct neutrino and DM phenomenology can be reproduced. However, the charged lepton correction is found to have advantage over the former due the fact that it does not disturb the mass sum rule prediction of the leading order model. Also, one requires less fine-tuning to generate correction to charged lepton masses due to which the lepton portal limit of inert scalar DM can be achieved, which can give different DM phenomenology compared to the well studied Higgs portal DM scenario, as

[1] For a review, please see [49]

we discuss later.

This work is organised in the following manner. In section 3.2 we summarize the S_4 based inverse seesaw model at leading order along with its predictions. In section 3.3 we explain the origin of non-zero reactor mixing angle and Dark Matter by extending the leading order model. In section 3.4 we briefly discuss DM phenomenology of the model and then briefly comment upon neutrinoless double beta decay prediction in the context of the present model in section 3.5. We discuss our results in section 3.6 and then write the conclusion in section 3.7.

3.2 Inverse Seesaw Model with S_4 Symmetry

In this section we shortly review the inverse seesaw (ISS) model and its S_4 realisation. The ISS model is an extension of the SM by two different types of singlet neutral fermions N_R, S_L three copies each. The Lagrangian reads

$$-\mathcal{L} = Y\bar{L}HN_R + M\bar{S}_L N_R + \frac{1}{2}\mu S_L S_L + \text{h.c.} \tag{3.2.1}$$

Here H represents the SM Higgs doublet and L is the lepton doublet. The presence of some additional symmetries is assumed which prevents the Majorana mass term of N_R. This Lagrangian gives rise to the following 9×9 mass matrix in the (ν_L, N_R, S_L) basis

$$M_\nu = \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M^T \\ 0 & M & \mu \end{pmatrix} \tag{3.2.2}$$

where $m_D = Y\langle h^0\rangle$ is the Dirac neutrino mass generated by the VEV of the neutral component of the SM Higgs. Block diagonalisation of the above mass matrix results in the effective light neutrino mass matrix as ,

$$m_\nu = m_D^T (M^T)^{-1} \mu M^{-1} m_D \tag{3.2.3}$$

Unlike canonical seesaw where the light neutrino mass is inversely proportional to the lepton number violating Majorana mass term of singlet neutrinos, here the light neutrino mass is directly proportional to the singlet mass term μ. The heavy neutrino masses are proportional to M. Here, even if $M \sim 1$ TeV, correct neutrino masses can be generated for $m_D \sim 10$ GeV, say if $\mu \sim 1$ keV. Such small μ term is natural as $\mu \to 0$ helps in recovering the global lepton number symmetry $U(1)_L$ of the model. Thus, inverse seesaw is a natural TeV scale seesaw model where the heavy neutrinos can remain as light as a TeV and Dirac mass can be as large as the charged lepton masses and can still be consistent with sub-eV light neutrino masses.

In general, the inverse seesaw formula for light neutrino mass can generate a very general structure of neutrino mass matrix. Since the leptonic mixing is found to have some specific structure with large mixing angles, one can look for possible flavour symmetry origin of it. In this context, non Abelian discrete flavour symmetries have gained lots of attention in the last few decades. For reviews and related references, please see [50, 51]. For the purpose of the present work, we are particularly interested in the inverse seesaw model proposed by [48] where the non Abelian discrete flavour symmetry is S_4 (For detail please see Section 1.8 of Chapter 1. The field content of the S_4 based inverse seesaw model is shown in Table 3.1. The additional discrete symmetry $Z_2 \times Z_3$ as well as the global $U(1)_L$ symmetry is chosen in order to generate the desired inverse seesaw mass matrix along with TBM type leptonic mixing. The lepton doublet and charged lepton singlet of the SM, the singlet neutrinos N_R, S of the inverse seesaw model transform as triplet 3_1 of S_4. The SM Higgs doublet h transform as singlet under S_4. The different flavon fields Φ's are chosen in order to get the desired mass matrices and mixing. The Yukawa Lagrangian for the particle content shown in Table 3.1 reads

$$-\mathcal{L}^I = y\bar{L}HN_R + y_M N_R S \Phi_R + y'_M N_R S \Phi'_R + y_s S S \Phi_s \qquad (3.2.4)$$

The following flavon alignments are required to get a desired neutrino mass matrix and leptonic mixing.

$$\langle \Phi_R \rangle = v_R(1,0,0),\ \langle \Phi'_R \rangle = v'_R,\ \langle \Phi_s \rangle = v_s,\ \langle H^0 \rangle = v_h$$

In order to implement this flavon alignment in the inverse seesaw mechanism we note that m_D is connected to v_h and M is determined by the VEV v_R and v'_R.

	$\bar{L}$	N_R	l_R	H	S	Φ_R	Φ'_R	Φ_s	Φ_l	$\Phi_l{}'$	$\Phi_l{}''$
$SU(2)_L$	2	1	1	2	1	1	1	1	1	1	1
S_4	3_1	3_1	3_1	1_1	3_1	3_1	1_1	1_1	3_1	3_2	1_1
Z_2	+	+	+	+	-	-	-	+	+	+	+
Z_3	ω^2	ω	1	1	1	ω^2	ω^2	1	ω	ω	ω
$U(1)_L$	-1	1	1	0	-1	0	0	2	0	0	0

Table 3.1: Fields and their transformation properties under $SU(2)_L$ gauge symmetry as well as the $S_4 \times Z_2 \times Z_3 \times U(1)_L$ symmetry

In this way, the order of magnitude estimate of light neutrino mass from the Eq. (3.2.3) is $m_\nu \propto \frac{v_h^2}{(v_R+v'_R)^2}\mu$. Here v_h is of the order of electroweak symmetry breaking (EWSB) scale, v_R and v'_R can be taken of the order of TeV scale or more. Therefore, to get m_ν in sub-eV, μ which is coming from the VEV of Φ_S should be of the order of keV. Such a small vev can be naturally achieved from the soft $U(1)_L$ symmetry breaking terms in the scalar potential. For example, a term $\mu_1\Phi_s H^\dagger H$ will generate an induced VEV of Φ_s given by $v_s = \frac{\mu_1 v_h^2}{M^2_{\Phi_s}}$. This can be adjusted to be keV by choosing a small enough μ_1. By the same naturalness argument as before, such a small μ_1 is natural. Also, since the $U(1)_L$ symmetry is explicitly broken (softly) by the scalar potential, there is no danger of generating massless Goldstone boson that can result after spontaneous breaking of global $U(1)_L$ symmetry.

Decomposition of the various terms present in the Eq. (3.2.4) into singlets can be achieved using the S_4 tensor product rules given in the Section 1.8 of Chapter 1.

$$y\bar{L}_i N_{jR} H = y(L_1N_{1R} + L_2N_{2R} + L_3N_{3R})v_h \tag{3.2.5}$$

$$\begin{aligned} y_M N_{iR} S_j \Phi_R &= y_M[(N_{2R}S_3 + N_{3R}S_2)\Phi_{1R} + (N_{1R}S_3 + N_{3R}S_1)\Phi_{2R} + (N_{1R}S_2 + N_{2R}S_1)\Phi_{3R}] \\ &= y_M[(N_{2R}S_3 + N_{3R}S_2)]v_R \end{aligned} \tag{3.2.6}$$

$$y'_M N_{iR} S_j \Phi'_R = y'_M(S_1N_{1R} + S_2N_{2R} + S_3N_{3R})v'_R \tag{3.2.7}$$

$$y_s S S \Phi_s = y_s(S_1S_1 + S_2S_2 + S_3S_3)v_s \tag{3.2.8}$$

The chosen flavon alignments allow us to have different matrices involved in inverse seesaw formula as follows

$$m_D = y \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} v_h, \ \mu = y_s \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} v_s, \ M = \begin{pmatrix} y'_M v'_R & 0 & 0 \\ 0 & y'_M v'_R & y_M v_R \\ 0 & y_M v_R & y'_M v'_R \end{pmatrix} \tag{3.2.9}$$

The above three matrices lead to the following light neutrino mass matrix under ISS framework

$$m_\nu = U_\nu m_\nu^{o(diag)} U_\nu^T. \tag{3.2.10}$$

Using Eq. (3.2.9) in Eq. (3.2.3) the light neutrino mass matrix is found to be

$$m_\nu^o = \begin{pmatrix} \frac{1}{a^2} & 0 & 0 \\ 0 & \frac{a^2+b^2}{(b^2-a^2)^2} & -\frac{2ab}{(b^2-a^2)^2} \\ 0 & -\frac{2ab}{(b^2-a^2)^2} & \frac{a^2+b^2}{(b^2-a^2)^2} \end{pmatrix} \tag{3.2.11}$$

where, $a = y'_M v'_R/(\sqrt{y_s v_s} y v_h)$ and $b = y_M v_R/(\sqrt{y_s v_s} y v_h)$. The eigenvalues of this light neutrino mass matrix are

$$m_1 = \frac{1}{(a+b)^2}, \ m_2 = \frac{1}{(a-b)^2}, \ m_3 = \frac{1}{a^2}$$

which satisfy the neutrino mass sum rule

$$\frac{1}{\sqrt{m_1}} = \frac{2}{\sqrt{m_3}} - \frac{1}{\sqrt{m_2}} \tag{3.2.12}$$

Now the Lagrangian for the charged leptons can be written in terms of dimension five operators as [48]

$$-\mathcal{L}^l = \frac{y_l}{\Lambda} \bar{L} l_R H \Phi_l + \frac{y_l{}'}{\Lambda} \bar{L} l_R H \Phi_l{}' + \frac{y_l{}''}{\Lambda} \bar{L} l_R H \Phi_l{}'' \tag{3.2.13}$$

The authors of [48] considered additional messenger fields χ, χ^c such that this effective Lagrangian for charged leptons can be obtained after integrating out these heavy messenger fields. The following flavon alignments allow us to have the desired mass matrix corresponding to the charged lepton sector

$$\langle \Phi_l \rangle = v_l(1,1,1), \ \langle \Phi_l{}' \rangle = v_l{}'(1,1,1), \ \langle \Phi_l{}'' \rangle = v_l''$$

The charged lepton mass matrix is then given by

$$m_l^0 = \begin{pmatrix} y_l''v_l'' & y_lv_l - y_l'v_l' & y_lv_l + y_l'v_l' \\ y_lv_l + y_l'v_l' & y_l''v_l'' & y_lv_l - y_l'v_l' \\ y_lv_l - y_l'v_l' & y_lv_l + y_l'v_l' & y_l''v_l'' \end{pmatrix} \frac{v_h}{\Lambda} \tag{3.2.14}$$

As mentioned in [52] the charge lepton mass matrix m_l is diagonalised on the left by the magic matrix U_ω given by

$$U_\omega = 1/\sqrt{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \tag{3.2.15}$$

(with $\omega = \exp 2i\pi/3$). Now we know that the leptonic mixing matrix is given by

$$U = U_{\mathrm{TBM}} = U_l^\dagger U_\nu$$

where U_l corresponds to the identity matrix if the charged lepton mass matrix is diagonal. Since in our work, the charged lepton mass matrix is non-diagonal and is nothing but the magic matrix U_ω given by Eq. (3.2.15), the leptonic mixing matrix is

$$U_{\mathrm{TBM}} = U_\omega^\dagger U_\nu$$

The desired structures of the mass and mixing matrices written above have been made possible due to chosen flavour symmetries of the theory. For example, as required by the structure of the inverse seesaw mass matrix given in Eq. (3.2.2), there should not be any mass term involving ν_L and S. However, the coupling between ν_L and S is not forbidden by the SM gauge symmetry as well as S_4 flavour symmetry. In this regard, the additional $Z_2 \times Z_3$ symmetry and the chosen charges of ν_L, S under it keep the unwanted coupling of ν_L and S through the Higgs doublet H away. Similarly, the (22) term of the inverse seesaw mass matrix (3.2.2) or the mass term involving N_R, N_R should also be forbidden. However, the SM gauge symmetry as well as the S_4 flavour symmetry and $U(1)_L$ global symmetry can not prevent a term like $\Phi_s N_R N_R$ which will introduce a non-zero (22) entry into the inverse seesaw mass matrix. Therefore, the additional $Z_2 \times Z_3$ symmetry and non-trivial charges of N_R under this has to be chosen to keep such a term away from the Lagrangian. As mentioned above, the approximate $U(1)_L$ global symmetry helps in generating small (33) entry of the inverse seesaw mass matrix naturally, without any fine tuning of parameters. Thus, all the additional symmetries $Z_2 \times Z_3 \times U(1)_L$ play a crucial role in generating the desired structure of the inverse seesaw mass matrix along with the desired leptonic mixing.

3.3 Origin of non-zero θ_{13} and dark matter

Since $\theta_{13} = 0$ has already been ruled out by several neutrino experiments, one has to go beyond the TBM framework discussed in the previous work. This can simply be done in two different ways: giving corrections to the neutrino mass matrix or the charged lepton mass matrix. Both of these corrections will change the leptonic mixing matrix in a way to generate non-zero θ_{13}.

3.3.1 Correction to neutrino mass matrix

The model discussed above can be extended by the particle content shown in Table 3.2 charged under an additional Z_2^{Dark} symmetry guaranteeing the stability of the dark matter candidate. This additional field content will introduce a few

	$SU(2)_L$	S_4	Z_2	Z_3	$U(1)_L$	Z_2^{Dark}
η	2	1	1	1	0	-1
ψ_R	1	3	1	ω	1	-1
Φ'_s	1	1	1	ω	-2	1
Φ_ψ	1	3	1	ω	-2	1

Table 3.2: Fields responsible for generating non-zero θ_{13} as well as dark matter with their respective transformations under the symmetry group of the model.

more terms in the Yukawa Lagrangian given as

$$\mathcal{L}^I \supset h\bar{L}\psi_R\eta + y_\psi\psi_R\psi_R\Phi'_s + y'\psi_R\psi_R\Phi_\psi \tag{3.3.1}$$

The extra scalar doublet η odd under the Z_2^{Dark} symmetry introduces several other terms in the scalar potential. The most relevant terms are the interactions

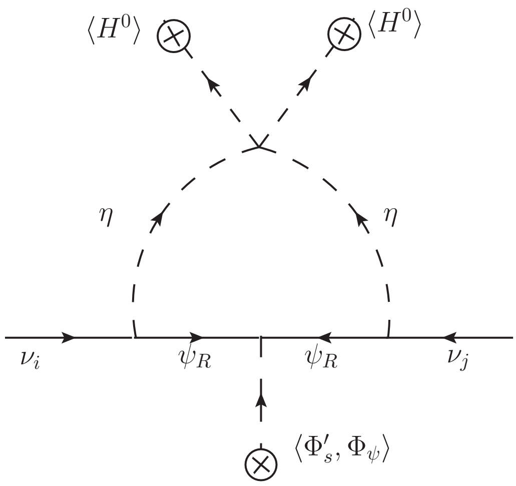

Figure 3.1: Radiative generation of non-zero θ_{13} from the light neutrino sector

with the standard model Higgs h which are relevant for neutrino mass and dark matter analysis. These relevant terms of the scalar potential can be written as

$$V(H,\eta) \supset \mu_1^2|H|^2 + \mu_2^2|\eta|^2 + \frac{\lambda_1}{2}|H|^4 + \frac{\lambda_2}{2}|\eta|^4 + \lambda_3|H|^2|\eta|^2 + \lambda_4|H^\dagger\eta|^2 + \{\frac{\lambda_5}{2}(H^\dagger\eta)^2 + \text{h.c.}\} \quad (3.3.2)$$

Using the expression from [35] of one-loop neutrino mass

$$(m_\nu)_{ij} = \frac{h_{ik}h_{jk}M_k}{16\pi^2}\left(\frac{m_R^2}{m_R^2 - M_k^2}\ln\frac{m_R^2}{M_k^2} - \frac{m_I^2}{m_I^2 - M_k^2}\ln\frac{m_I^2}{M_k^2}\right) \quad (3.3.3)$$

Here $m^2_{R,I}$ are the masses of scalar and pseudoscalar part of η^0 and M_k the mass of singlet fermion ψ_R in the internal line. The index $i,j = 1,2,3$ runs over the three fermion generations as well as three copies of ψ. For $m_R^2 + m_I^2 \approx M_k^2$, the above expression can be simply written as

$$(m_\nu)_{ij} \approx \frac{\lambda_5 v_h^2}{32\pi^2}\frac{h_{ik}h_{jk}}{M_k} = \frac{m_I^2 - m_R^2}{32\pi^2}\frac{h_{ik}h_{jk}}{M_k} \quad (3.3.4)$$

where $m_I^2 - m_R^2 = \lambda_5 v_h^2$ is assumed ignoring the quartic terms of η with other flavon fields. This formula for light neutrino mass is written in a basis where the mass matrix of the intermediate fermion ψ is diagonal which is true if only Φ'_s contributes to its mass $M_k = y_\psi\langle\Phi'_s\rangle$ due to the structure of S_4 tensor product $\psi_R\psi_R\Phi'_s = (\psi_{R1}\psi_{R1} + \psi_{R2}\psi_{R2} + \psi_{R3}\psi_{R3})\Phi'_s$. However, due to the S_4 triplet assignment to the other scalar Φ_ψ, the mass matrix of ψ_R becomes non-diagonal of the form

$$M_\psi = \begin{pmatrix} y_\psi v'_s & y'_\psi v_{\psi 3} & y'_\psi v_{\psi 2} \\ y'_\psi v_{\psi 3} & y_\psi v'_s & y'_\psi v_{\psi 1} \\ y'_\psi v_{\psi 2} & y'_\psi v_{\psi 1} & y_\psi v'_s \end{pmatrix}, \tag{3.3.5}$$

where $\langle \Phi_\psi \rangle = (v_{\psi 1}, v_{\psi 2}, v_{\psi 3})$ is the vacuum alignment of the flavon field Φ_ψ. Also the S_4 product rules dictate the Yukawa matrix h_{ij} to be diagonal in flavour space. Therefore, the new contribution to the light neutrino mass matrix will assume a structure similar to M_ψ. We can parameterise this correction, in general as

$$\delta m_\nu = \begin{pmatrix} x_\nu & y_\nu & z_\nu \\ y_\nu & x_\nu & w_\nu \\ z_\nu & w_\nu & x_\nu \end{pmatrix} \tag{3.3.6}$$

In this particular setup, the fermion ψ_R carries lepton number, and since lepton number is only softly broken within an inverse seesaw framework, one expects the VEV's of Φ'_s, Φ_ψ to be small say, of the order of keV in a TeV scale inverse seesaw model discussed above. Therefore, the dark matter in this model is a keV singlet fermion ψ_R. On the other hand, if ψ_R does not carry a lepton number, then the scalar doublet η carries a lepton number and the one-loop contribution can be generated with the particle content shown in Table 3.3. The Yukawa

	$SU(2)_L$	S_4	Z_2	Z_3	$U(1)_L$	Z_2^{Dark}
η	2	1	1	1	1	-1
ψ_R	1	3	1	ω	0	-1
Φ'_s	1	1	1	ω	0	1
Φ_ψ	1	3	1	ω	0	1
Δ_L	3	1	1	1	0	1

Table 3.3: Fields responsible for generating non-zero θ_{13} as well as dark matter with their respective transformations under the symmetry group of the model.

Lagrangian corresponding to this new field content is

$$\mathcal{L}^I \supset h\bar{L}\psi_R\eta + y_\psi\psi_R\psi_R\Phi'_s + y'\psi_R\psi_R\Phi_\psi \tag{3.3.7}$$

These relevant terms of the scalar potential can be written as

$$V(H,\eta,\Delta_L) \supset \mu_1^2|H|^2 + \mu_2^2|\eta|^2 + \frac{\lambda_1}{2}|H|^4 + \frac{\lambda_2}{2}|\eta|^4 + \lambda_3|H|^2|\eta|^2 + \lambda_4|H^\dagger\eta|^2 \tag{3.3.8}$$

$$+\{\frac{\lambda_5}{2}\eta^2\Delta_L\Phi_s + \text{h.c.}\}, \tag{3.3.9}$$

In this case, the fermion ψ_R can acquire a diagonal mass term due to the coupling with Φ_s' flavon and also acquire non diagonal mass terms from the flavon field Φ_ψ. The combined mass matrix for ψ_R therefore, has a similar structure to the one shown in Eq. (3.3.5). Since neither ψ_R nor Φ_ψ carries any lepton number, their mass and VEV respectively are not constrained to be small from naturalness argument. Also, the triplet scalar Δ_L does not couple to the leptons at tree level as it does not carry any lepton number. The corresponding neutrino mass diagram at one loop is shown in figure 3.2. This is equivalent to a radiative type II seesaw mechanism. In this case, the scalar doublet η can be naturally lighter than ψ_R and hence can be a dark matter candidate. We discuss this dark matter candidate in details later, specially with reference to its interactions with the light neutrinos, responsible for generating non-zero θ_{13}. In both these cases, the correction to the light neutrino mass matrix can be parameterised as Eq. (3.3.6). One can then write down the complete light neutrino mass matrix as

$$m_\nu = m_\nu^0 + \delta m_\nu = U_{\text{PMNS}} m_\nu^{\text{diag}} U_{\text{PMNS}}^T \tag{3.3.10}$$

where the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) leptonic mixing matrix can be parametrized as

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} U_{\text{Maj}} \tag{3.3.11}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ and δ is the leptonic Dirac CP phase. The diagonal matrix $U_{\text{Maj}} = \text{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)})$ contains the Majorana CP phases α, β which remained unknown. For NH, we can write $m_\nu^{\text{diag}} = \text{diag}(m_1, \sqrt{m_1^2 + \Delta m_{21}^2}, \sqrt{m_1^2 + \Delta m_{31}^2})$ and $m_\nu^{\text{diag}} = \text{diag}(\sqrt{m_3^2 + \Delta m_{23}^2 - \Delta m_{21}^2}, \sqrt{m_3^2 + \Delta m_{23}^2}, m_3)$ for

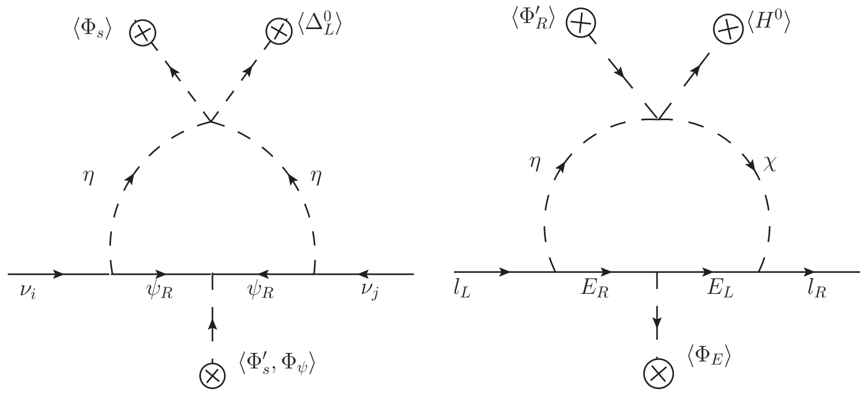

Figure 3.2: Radiative generation of non-zero θ_{13} from the light neutrino sector (left panel) and charged lepton sector (right panel)

IH. Using the 3σ values of neutrino parameters, we can find the model parameters in $m_\nu^0 + \delta m_\nu$ which can give rise to the correct neutrino phenomenology.

3.3.2 Correction to charged lepton mass matrix

Similar to the above, one can also give a radiative correction to the charged lepton mass matrix, by considering the presence of vector like charged fermions instead of neutral ones. The relevant particle content is shown in Table 3.4. The Yukawa Lagrangian corresponding to this new field content is

$$\mathcal{L}^I \supset h\bar{L}E_R\eta^\dagger + h'\bar{l}_R E_L\chi + M_E\bar{E}_L E_R + y_E\Phi_E\bar{E}_L E_R \tag{3.3.12}$$

These relevant terms of the scalar potential can be written as

$$\begin{aligned} V \supset\ & \mu_1^2|H|^2 + \mu_2^2|\eta|^2 + \frac{\lambda_1}{2}|H|^4 + \frac{\lambda_2}{2}|\eta|^4 + \lambda_3|H|^2|\eta|^2 + \lambda_4|H^\dagger\eta|^2 \\ & + \{\frac{\lambda_5}{2}(H^\dagger\eta)^2 + \text{h.c.}\} + \lambda_6 H^\dagger\eta\chi^\dagger\Phi_R' \end{aligned} \tag{3.3.13}$$

The corresponding Feynman diagram for one-loop charged lepton mass is shown in figure 3.2 (right panel). One can write down the one-loop expression similar to the one written for one-loop neutrino masses. Here also, the mass matrix of vector like charged leptons acquire a similar structure as shown for neutral fermion ψ_R in Eq. (3.3.5). Also the Yukawa matrix related to the coupling of $\bar{l}_L E_R\eta$ or $\bar{l}_R E_L\chi$ is restricted to be diagonal due to S_4 product rules. Therefore, one can parameterise the correction to the charged lepton mass matrix as

$$\delta m_l = \begin{pmatrix} a_l & b_l & c_l \\ b_l^s & a_l & d_l \\ c_l^s & d_l^s & a_l \end{pmatrix} \tag{3.3.14}$$

Adding this correction to the leading order charged lepton mass matrix given

	$SU(2)_L$	S_4	Z_2	Z_3	$U(1)_L$	Z_2^{Dark}
η	2	1	1	1	0	-1
χ	1	1	1	ω^2	0	-1
$E_{L,R}$	1	3	1	ω	1	-1
Φ_E	1	3	1	1	0	1

Table 3.4: Fields responsible for generating non-zero θ_{13} as well as dark matter with their respective transformations under the symmetry group of the model.

in Eq. (3.2.14) should give rise to a different diagonalising matrix U_l of charged leptons. The structure of this matrix will depend upon the parameters a_l, b_l, c_l, d_l which can be constrained from the requirement of producing the correct leptonic mixing matrix after multiplying with U_ν, the diagonalising matrix of light neutrino mass matrix. From the tree level model one can find $U_\nu = U_\omega U_{\text{TBM}}$. Now, the total charged lepton mass matrix is

$$m_l = m_l^0 + \delta m_l = U_L m_l^{\text{diag}} U_R^\dagger \tag{3.3.15}$$

where $U_{L,R}$ are unitary matrices that can diagonalise the complex charged lepton mass matrix. Here m_l^{diag} is the known diagonal charged lepton mass matrix. The unitary matrix U_L goes into the observed leptonic mixing matrix and hence can be calculated as $U_L = U_\nu U_{\text{PMNS}}^\dagger$ which can be written in terms of known U_ν from the leading order model and the known PMNS mixing matrix. We parameterise the another unitary matrix U_R in terms of three mixing angles and one phase and vary them randomly in $0 - \pi/4$ for angles and $0 - 2\pi$ for phase. Thus, we can calculate the charged lepton mass matrix in terms of known parameters as well as randomly generated values of U_R. For each possible such charged lepton mass matrix, we can then solve the above Eq. (3.3.15) and calculate the model parameters such that correct leptonic mixing can be achieved. In this model, the dark matter candidate can either be a scalar doublet η or a scalar singlet χ. We discuss their dark matter phenomenology below specially with reference to their interactions with the charged leptons.

3.4 Dark matter

In the very early epochs of the Universe, the abundance of a typical WIMP DM relic particle (η) is usually taken to be the equilibrium abundance. When the temperature of the radiation dominated Universe cools down below $T \sim m_\eta$, η becomes non-relativistic and quickly after that it also decouples from the thermal bath and its abundance freezes out. The final relic abundance of such a particle η which was in thermal equilibrium at earlier epochs can be calculated by solving the Boltzmann equation

$$\frac{dn_\eta}{dt} + 3Hn_\eta = -\langle\sigma v\rangle(n_\eta^2 - (n_\eta^{\rm eqb})^2) \tag{3.4.1}$$

where n_η is the number density of the DM particle η and $n_\eta^{\rm eqb}$ is the equilibrium number density. Also, H is the Hubble expansion rate of the Universe and $\langle\sigma v\rangle$ is the thermally averaged annihilation cross-section of the DM particle η. It is clear from this equation that when η was in thermal equilibrium, the right hand side of it vanishes and the number density of DM decreases with time only due to the expansion of the Universe, as expected. The approximate analytical solution of the above Boltzmann equation gives [53, 54]

$$\Omega_\chi h^2 \approx \frac{1.04 \times 10^9 x_F}{M_{Pl}\sqrt{g_*}(a + 3b/x_F)} \tag{3.4.2}$$

where $x_F = m_\chi/T_F$, T_F is the freeze-out temperature, g_* is the number of relativistic degrees of freedom at the time of freeze-out and $M_{Pl} \approx 10^{19}$ GeV is the Planck mass. Here, x_F can be calculated from the iterative relation

$$x_F = \ln \frac{0.038 g M_{\rm Pl} m_\chi < \sigma v >}{g_*^{1/2} x_F^{1/2}} \tag{3.4.3}$$

Typically, DM particles with electroweak scale mass and couplings freeze out at temperatures in the range $x_F \approx 20 - 30$. The expression for relic density also has a more simplified form given as [55]

$$\Omega_\chi h^2 \approx \frac{3 \times 10^{-27}\mathrm{cm}^3\mathrm{s}^{-1}}{\langle \sigma v \rangle} \tag{3.4.4}$$

In the model discussed in the previous section, there can be two different types of DM candidates, the lightest neutral particle under the Z_2^{Dark} symmetry. In the model with corrections to neutrino sector, either the neutral fermion ψ_R or the neutral component of the scalar doublet η can be DM depending on their masses whereas in the latter model with corrections to the charged lepton sector, only the scalar DM is possible. To keep the discussion same for both these models, we briefly discuss scalar DM phenomenology in this work. The scalar DM relic abundance calculation has already been done in several works [56–60]. Typically, correct relic abundance can be satisfied for two regions of DM mass in such a model: one below the W boson mass threshold and another around 550 GeV or more. Here we focus mainly on the low mass regime where the dominant annihilation channel of DM is the one through Higgs portal interactions. Also, depending on the mass difference between different components of the scalar doublet η, coannihilations can also play a non-trivial role. In the limit where Higgs portal and coannihilation effects are sub-dominant, the DM can annihilate through the lepton portal interactions which are also relevant for correct neutrino phenomenology discussed above. Here we briefly comment on the lepton portal interaction and its role in generating DM relic abundance using the approximate analytical formula mentioned above.

It is straightforward to see from the Lagrangian that the scalar DM can annihilate into leptons through a process mediated by heavy fermions ψ or $E_{L,R}$. The corresponding annihilation cross-section is given by [61]

$$\sigma v = \frac{v^2 h^4 m_\eta^2}{48\pi(m_\eta^2 + m_\psi^2)^2} \tag{3.4.5}$$

With $v \sim 0.3c$ is the typical relative velocity of the two DM particles at the freeze out temperature, η is the relic particle (DM), h is the Yukawa coupling, m_η the relic mass, m_ψ is the mass of the gauge singlet mediating the annihilation. We then vary the DM mass and the Yukawa coupling for different benchmark values of mediator masses and constrain the parameter space from the requirement of generating the correct DM relic abundance. It should be noted that, there are also constraints from DM direct detection experiments like LUX [62] which currently rules out DM-nucleon spin independent cross section above around 2.2×10^{-46} cm^2 for DM mass of around 50 GeV. However, the lepton portal interactions can not mediate DM-nucleon interactions and hence such bounds are weak in these cases. In fact, such null results at direct detection experiments will push lepton portal interactions of DM into a more favourable regime.

3.5 Neutrinoless double beta decay

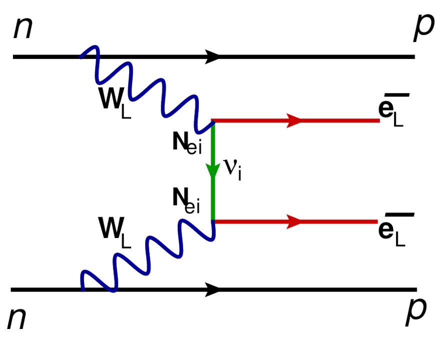

Figure 3.3: Feynman diagram contributing to neutrinoless double beta decay due to light Majorana neutrino exchanges [14].

The neutrinoless double beta decay (NDBD) is a lepton number violating process where a heavier nucleus decays into a lighter one and two electrons $(A, Z) \to (A, Z+2)+2e^-$ without any antineutrinos in the final state. If the light neutrinos of SM are Majorana fermions, then they can contribute to NDBD through the interactions shown in the Feynman diagram of figure 3.3. The amplitude of this light neutrino contribution is

$$A_{\nu LL} \propto G_F^2 \sum_i \frac{m_i U_{ei}^2}{p^2} \tag{3.5.1}$$

with p being the average momentum exchange for the process. In the above expression, m_i are the masses of light neutrinos for $i = 1, 2, 3$ and U is the PMNS leptonic mixing matrix mentioned earlier. The corresponding half-life of neutrinoless double beta decay can be written as

$$\frac{1}{T^{0\nu}_{1/2}} = G^{0\nu}_{01}\Big(|\mathcal{M}^{0\nu}_{\nu}(\eta^{L}_{\nu})|^2\Big) \tag{3.5.2}$$

where $\eta^L_\nu = \sum_i \frac{m_i U^2_{ei}}{m_e}$ with m_e being the mass of electron. Also, $\mathcal{M}^{0\nu}_\nu$ is the nuclear matrix element. The recent bound from the KamLAND-Zen experiment constrains $0\nu\beta\beta$ half-life [63]

$$T^{0\nu}_{1/2}(\text{Xe136}) > 1.1 \times 10^{26} \text{ yr}$$

which is equivalent to $|M^{ee}_\nu| < (0.06 - 0.16)$ eV at 90% C.L. where M^{ee}_ν is the effective neutrino mass given by

$$M^{ee}_\nu = U^2_{ei} m_i \tag{3.5.3}$$

Here U_{ei} are the elements of the first row of the PMNS mixing matrix. More explicitly, it is given by

$$M^{ee}_\nu = m_1 c^2_{12} c^2_{13} + m_2 s^2_{12} c^2_{13} e^{2i\alpha} + m_3 s^2_{13} e^{2i\beta} \tag{3.5.4}$$

Thus, the NDBD half-life is sensitive to the Majorana phases and the lightest neutrino mass as well, which remain undetermined at neutrino oscillation experiments. In the present model, the light neutrino contribution is the only dominant contribution. We check the predictions of our model for NDBD effective mass for both the cases and compare with the experimental bounds.

Parameters	Normal Hierarchy (NH)	Inverted Hierarchy (IH)
$\frac{\Delta m^2_{21}}{10^{-5}\text{eV}^2}$	$7.03 - 8.09$	$7.02 - 8.09$
$\frac{\lvert\Delta m^2_{3l}\rvert}{10^{-3}\text{eV}^2}$	$2.407 - 2.643$	$2.399 - 2.635$
$\sin^2\theta_{12}$	$0.271 - 0.345$	$0.271 - 0.345$
$\sin^2\theta_{23}$	$0.385 - 0.635$	$0.393 - 0.640$
$\sin^2\theta_{13}$	$0.01934 - 0.02392$	$0.01953 - 0.02408$
δ	$0^\circ - 360^\circ$	$145^\circ - 390^\circ$

Table 3.5: Global fit 3σ values of neutrino oscillation parameters [64]. Here $\Delta m^2_{3l} \equiv \Delta m^2_{31}$ for NH and $\Delta m^2_{3l} \equiv \Delta m^2_{32}$ for IH.

3.6 Results and discussions

We first parametrize the light neutrino mass matrix in terms of the 3σ global fit data available [64] which are summarised in Table 3.5. For the correction to the neutrino sector case, we then use Eq. (3.3.10) to relate the light neutrino mass matrix predicted by the model with the one parametrized by the global fit data. The leading order neutrino mass matrix given by Eq. (3.2.11) contains two complex parameters a, b whereas the correction to light neutrino mass is made up of four complex parameters x, y, z, w as seen from Eq. (3.3.6). The parametric form of light neutrino mass matrix is complex symmetric and hence contains six complex elements. Therefore, one can exactly solve the system of equations arising from Eq. (3.3.10) in order to evaluate the model parameters in terms of the known neutrino parameters. To be more precise, there are in fact five complex equations and one constraints arising from Eq. (3.3.10). This is due to the fact that in the total neutrino mass matrix predicted by the model, we have the 22 and 33 entries equal. This in fact restricts the light neutrino parameters, as it gives rise to two real equations involving the light neutrino parameters. We first solve these system of equations and generate the light neutrino parameters which satisfy them. For the resulting light neutrino parameters, we solve the other five complex equations to evaluate the model parameters. Since we have six model parameters and only five equations now, we vary the parameter x in the correction term (3.3.6) randomly in a range $10^{-6} - 10^{-1}$ eV. Since there are nine neutrino parameters namely, three masses, three angles and three phases, one can in general, show the variation of model parameters in terms of all of these nine parameters which are being varied randomly in their allowed ranges. Here we show only a few of them for illustrative purposes. For example, we show the variation of some of the model parameters in terms of the light neutrino parameters in figure 3.4, 3.5, 3.6, 3.7 and 3.8. This shows that the model parameters in the leading order and the correction mass matrices can not be arbitrary, but have to be within some specific ranges in order to be consistent with correct light neutrino data. From the figures 3.4 and 3.5 it is seen that the parameters of the leading order light neutrino mass matrix are in the range

$a, b \approx 1 - 10$ eV$^{-1/2}$. We recall the expressions for a, b in terms of the model parameters $a = y'_M v'_R/(\sqrt{y_s v_s} y v_h)$ and $b = y_M v_R/(\sqrt{y_s v_s} y v_h)$ mentioned earlier. Taking the lepton number violating term $\mu = y_s v_s \approx 1$ keV, the VEV of Higgs doublet at electroweak scale $v_h \approx 100$ GeV and the VEV of the other scalars Φ_R, Φ'_R around a TeV that is, $v_R, v'_R \approx 1$ TeV, our numerical results suggest that

$$\frac{y_M}{y} = \frac{y'_M}{y} \approx 10 - 1000 \tag{3.6.1}$$

in order to satisfy the correct neutrino data. This can be achieved by suitable tuning of the Dirac Yukawa y relative to $y_M = y'_M$. On the other hand, from the figures 3.6, 3.7 and 3.8, it can be seen that the correction terms to the light neutrino mass matrix lie in the sub-eV regime. The one loop correction term shown in Eq. (3.3.3) can be approximated for $m_R^2 + m_I^2 \approx M_k^2$, the above expression can be simply written as

$$(m_\nu)_{ij} \approx \frac{\lambda_5 v_h^2}{32\pi^2} \frac{h_{ik} h_{jk}}{M_k} = \frac{m_I^2 - m_R^2}{32\pi^2} \frac{h_{ik} h_{jk}}{M_k} \tag{3.6.2}$$

If the heavy neutrino mass M_k is around a TeV, then for $m_I^2 - m_R^2 \approx 1$ GeV, one can generate sub eV scale corrections ~ 0.01 eV if the corresponding Yukawa couplings are fine tuned to $h \approx 10^{-3}$. In the model with corrections to the leading order charged lepton mass matrix, we first find out the diagonalising matrix of light neutrino mass matrix as $U_\nu = U_\omega U_{\text{TBM}}$ using the leading order results mentioned before. Since the light neutrino mass matrix remains the same after the charged lepton correction, U_ν also remains same. However the addition of correction will change the left diagonalising matrix of charged lepton mass matrix from the magic matrix U_ω to something else, denoted by $U_L = U_\nu U^\dagger_{\text{PMNS}}$. Now, using Eq. (3.3.15), one can relate the complete charged lepton mass matrix predicted by the model, with the parametrized one given by the right hand side of Eq. (3.3.15). The total charged lepton mass matrix can be written as

$$m_l = m_l^0 + \delta m_l = \begin{pmatrix} x + a_l & y - z + b_l & y + z + c_l \\ y + z + b_l^s & x + a_l & y - z + d_l \\ y - z + c_l^s & y + z + d_l^s & x + a_l \end{pmatrix} \tag{3.6.3}$$

which contains ten complex parameters. Here x, y, z correspond to $y_l'' v_l'', y_l v_l, y_l' v_l'$ respectively in the leading order charged lepton mass matrix (3.2.14). Also there

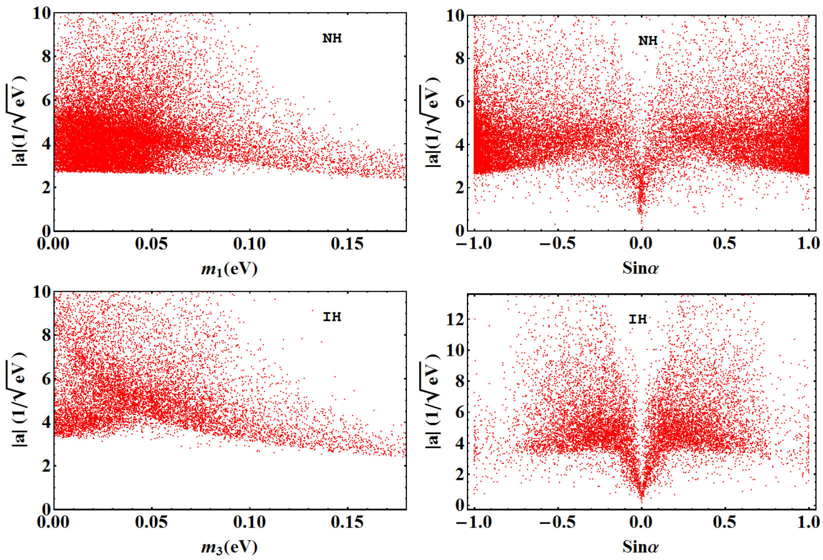

Figure 3.4: Model parameter as a function of the lightest neutrino mass and Majorana phase α.

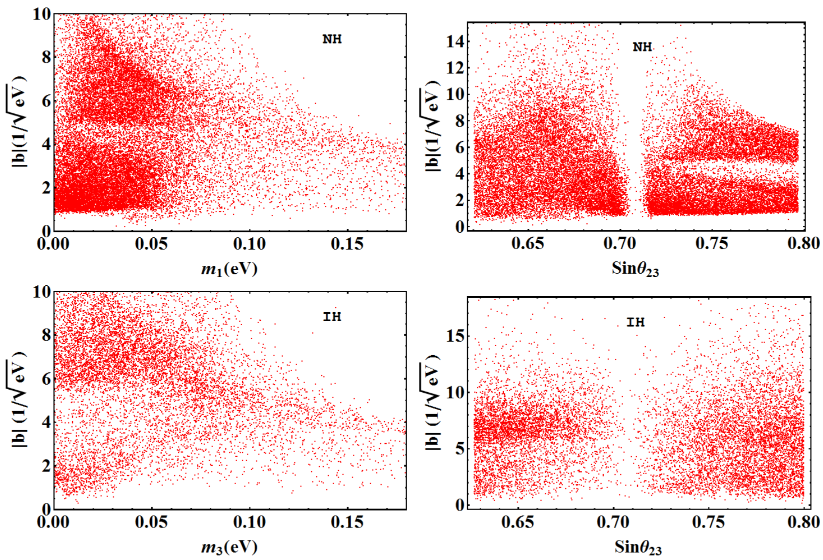

Figure 3.5: Model parameters as a function of the lightest neutrino mass and the atmospheric mixing angle θ_{23}.

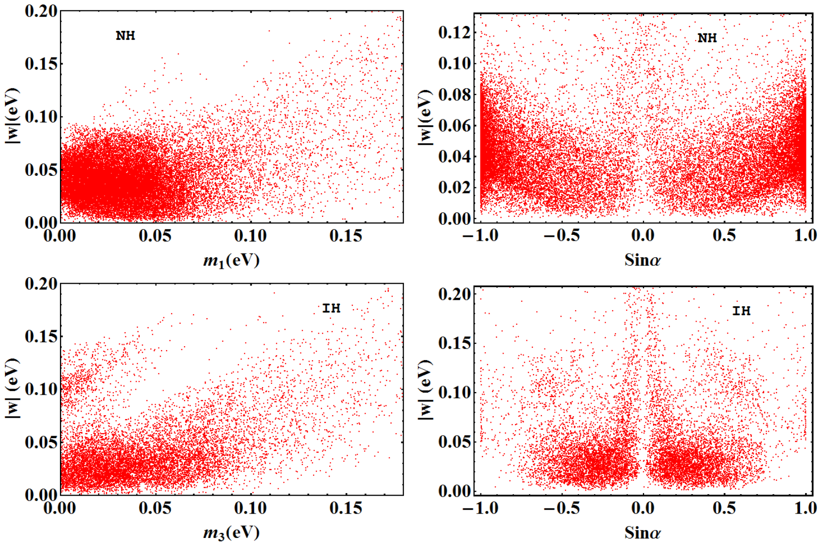

Figure 3.6: Corrections parameter(correction to neutrino mass matrix) as a function of lightest neutrino mass and Majorana phase α.

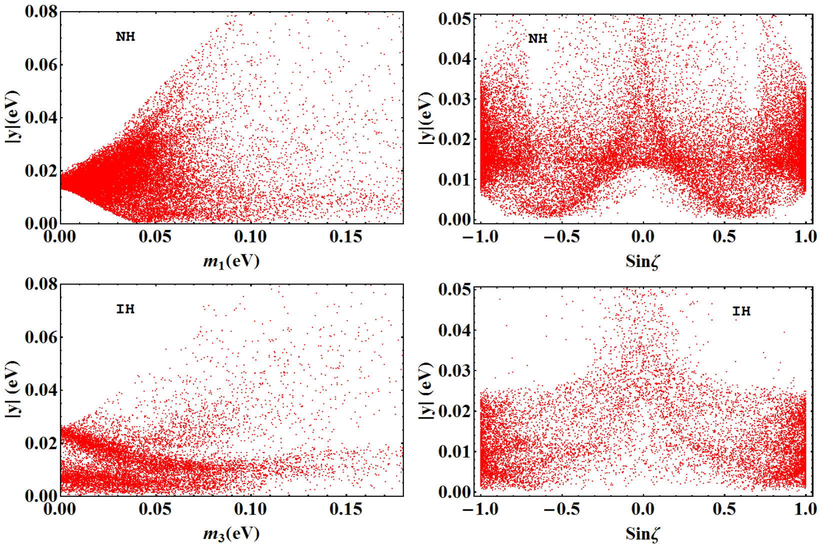

Figure 3.7: Corrections parameter(correction to neutrino mass matrix) as a function of lightest neutrino mass and Majorana phase ζ.

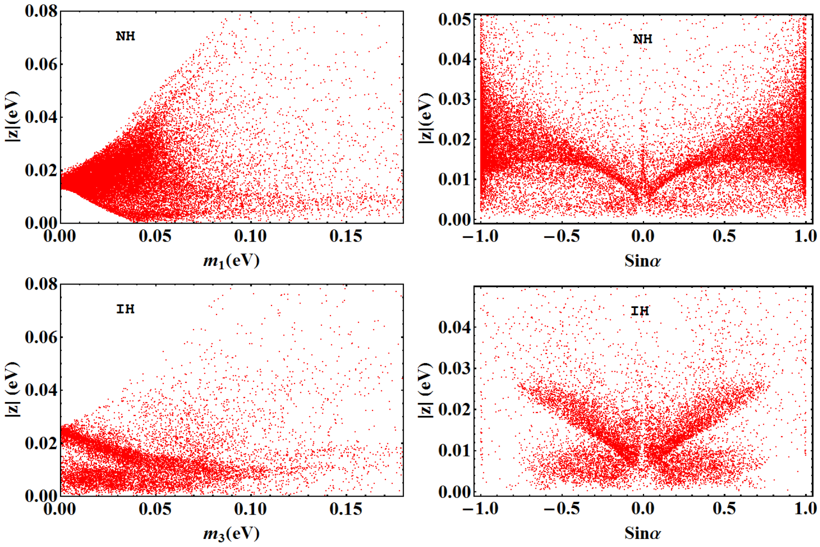

Figure 3.8: Corrections parameter(correction to neutrino mass matrix) as a function of lightest neutrino mass and Majorana phase α.

are two constraints in the parametrized charged lepton mass matrix due to fact that the 11, 22 and 33 elements are equal. This severely constraints the mixing angles and phases. Since the angles contained in U_L are related to the PMNS mixing angles, they can not be tuned arbitrarily. This forces some of the angles in U_R to take very small values in order to satisfy these two constraints. The tiny values are required in order to compensate for the large hierarchy in charged lepton masses which enters the 11, 22 and 33 elements of the mass matrix. We first solve these constraints numerically and then find the model parameters for those allowed values of mixing angles. We vary x, y, z randomly in $10^{-6} - 1.0$ GeV and evaluate other model parameters $a_l, b_l, c_l, d_l, b_l^s, c_l^s, d_l^s$ from the requirement of producing the correct leptonic mixing data. Unlike the earlier model with corrections to the neutrino mass matrix, here we get very few number of allowed points. For illustrative purposes we show the variation of a_l, b_l, c_l, d_l with some light neutrino parameters in figure 3.9 and 3.10. Since these one loop correction terms lie in the sub GeV regime, one can generate them without much fine tuning in the corresponding Yukawa couplings. For the same set of allowed parame-

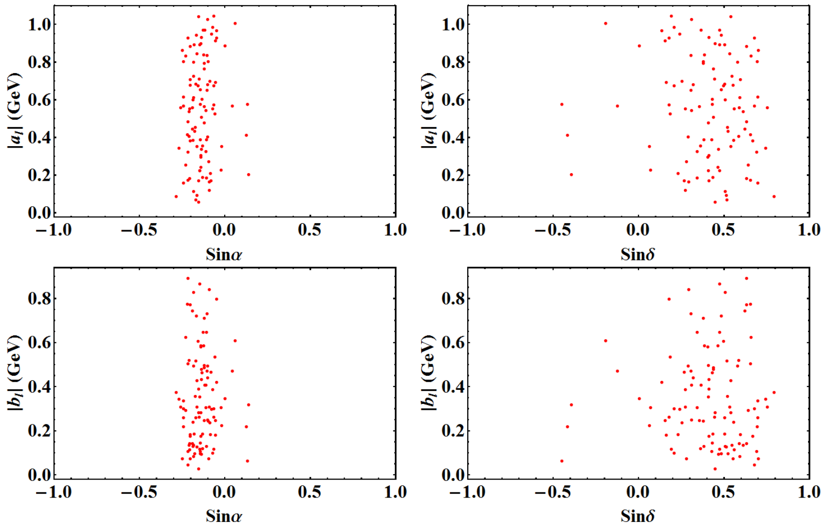

Figure 3.9: Correction parameters as a function of Majorana and Dirac phases while giving correction to the charged lepton mass matrix.

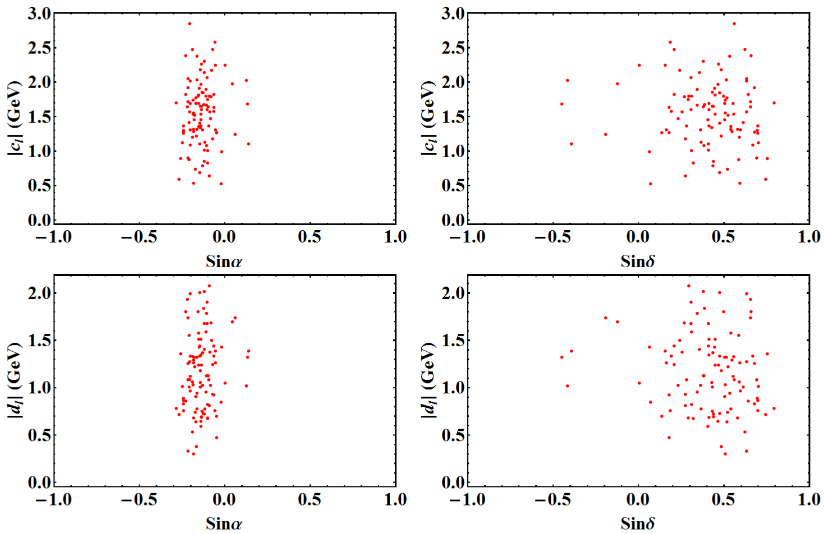

Figure 3.10: Correction parameters as a function of Majorana and Dirac phases while giving correction to the charged lepton mass matrix.

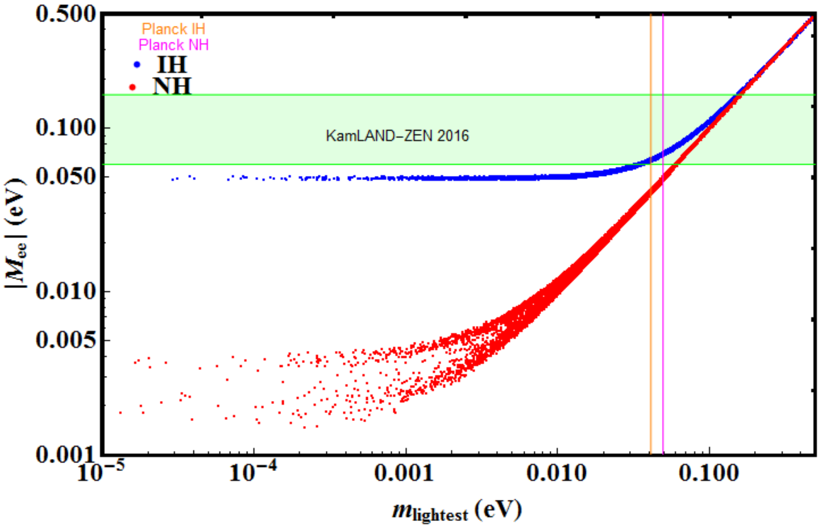

Figure 3.11: Variation of effective neutrino mass with the lightest neutrino mass in the model with neutrino mass correction. The purple line indicates the PLANCK bound on the sum of absolute neutrino masses. The green band shows the KamLAND-ZEN upper bound [63] on the effective neutrino mass.

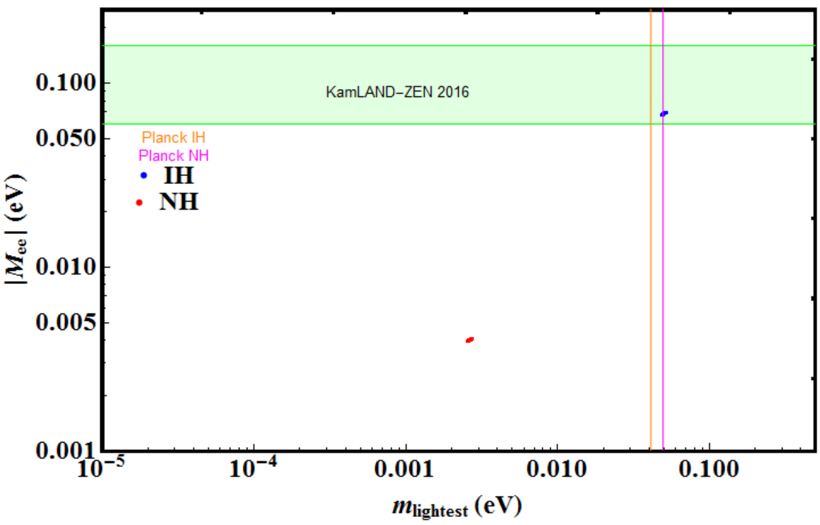

Figure 3.12: Variation of effective neutrino mass with the lightest neutrino mass in the model with charged lepton correction. The purple line indicates the PLANCK bound on the sum of absolute neutrino masses. The green band shows the KamLAND-ZEN upper bound [63] on the effective neutrino mass.

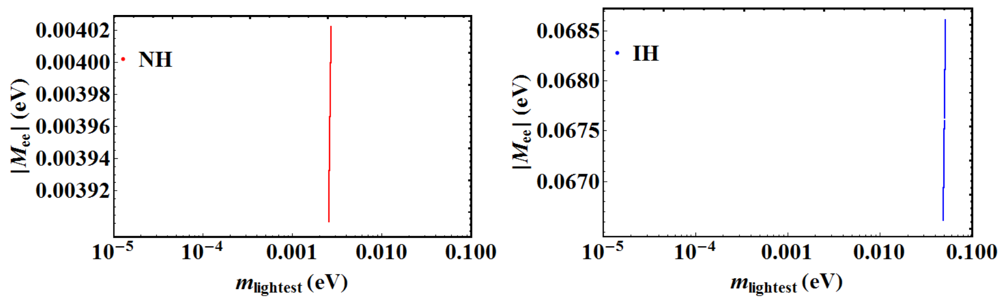

Figure 3.13: Variation of effective neutrino mass with the lightest neutrino mass in the model with charged lepton correction.

ters, numerically evaluated for both the models, we also calculate the respective predictions for neutrinoless double beta decay and plot it as a function of the lightest neutrino mass. Figure 3.11 shows the predictions for effective neutrino mass for both the hierarchies in the model where $\theta_{13} \neq 0$ is generated from neutrino sector itself. As expected, the inverted hierarchy predictions lie very close to the upper bound on M_{ee} from KamLAND-Zen experiment [63]. Similarly, fig 3.12 shows the predictions for effective neutrino mass M_{ee} for the second model where the charged lepton mass matrix is given a correction to generate non-zero θ_{13}. Due to very few number of allowed points in this case, the predicted values of M_{ee} are seen as a dot for both the hierarchies. This is also due to the fact the neutrino mass sum rule Eq. (3.2.12) is valid in this case which restricts the lightest neutrino mass to a small range of values. As can be seen from figure 3.12, the latest KamLAND-Zen data already disfavour this case for inverted hierarchy. If we zoom the points near the two dots in figure 3.12, they look like the points shown in figure 3.13. It is interesting to note that in both the models, the Planck bound on the sum of absolute neutrino mass $\sum_i |m_i| < 0.17$ eV [34] results in an upper bound on the lightest neutrino mass as $m_{\text{lightest}} \leq 0.04939$ eV for normal hierarchy, $m_{\text{lightest}} \leq 0.0414$ eV for inverted hierarchy, if we use the best fit values of mass squared differences. Interestingly this bound almost coincides with the bound from the KamLAND-Zen experiment as seen from figure 3.11. Finally we show the allowed range of dark matter mass and its couplings to leptons from the requirement of satisfying correct dark matter relic abundance criteria in figure 3.14. As expected, higher the values of mediator mass, the larger Yukawa cou-

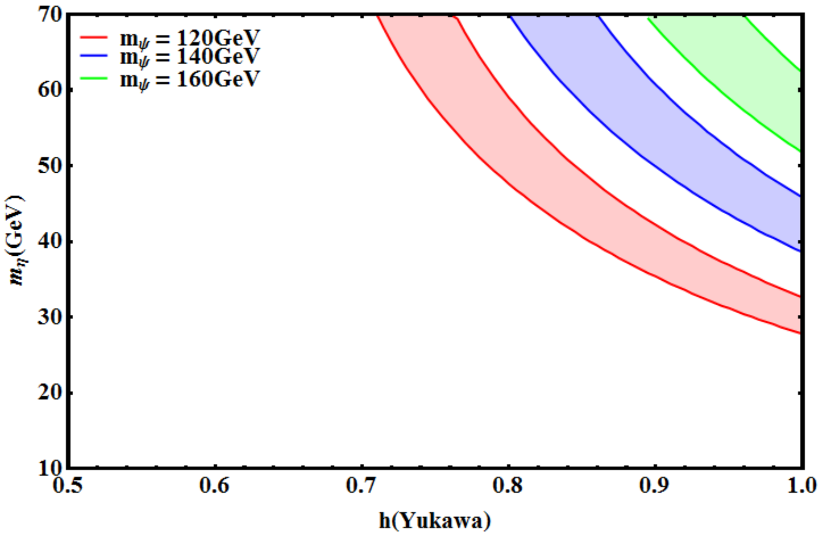

Figure 3.14: Dark matter mass as a function of Yukawa coupling keeping the mediator mass fixed for each plots, such that the constraints on the DM relic abundance is satisfied.

plings are needed to give rise to the correct relic abundance. Such large Yukawa couplings and smaller mediator masses favourable from lepton portal limit of DM will make the charged lepton correction case more favourable. This is because, one needs suppressed Yukawa couplings or large mediator mass in order to generate sub-eV corrections to light neutrino mass, than generating sub-GeV corrections to the charged lepton mass matrix.

3.7 Conclusion

We have studied a TeV scale inverse seesaw model based on S_4 flavour symmetry which can naturally generate correct light neutrino masses with Tri-Bimaximal type mixing at leading order. The model also predicts a neutrino mass sum rule that can further predict the value of the lightest neutrino mass, that can be tested at experiments like neutrinoless double beta decay. Since TBM mixing has already been ruled out by the latest neutrino oscillation data, we consider two possible ways of generating non-zero θ_{13} which automatically take dark matter into account. The idea is based on the scotogenic mechanism of neutrino mass generation, where neutrino mass arises at one loop level with DM particles going inside the loop. We first give such a one loop correction to the leading order light neutrino mass matrix and numerically evaluate the model parameters from the requirement of satisfying the correct neutrino data. This however, disturbs the mass sum rule prediction of the original model. The dark matter candidate in such a case could either be a singlet neutral fermion or the neutral component of a scalar doublet, depending whichever is lighter. We also study the possibility of generating $\theta_{13} \neq 0$ by giving a correction to the charged lepton sector. Such a case is found to be more constrained from the requirement of satisfying the correct neutrino data. We find much narrower ranges of points in terms of light neutrino parameters which can bring the model predictions closer to the observed data. Consistency with light neutrino data also requires the right diagonalising matrix of charged lepton to have very small mixing angles. The DM candidate in this case is the neutral component of a scalar doublet.

We also study the predictions for neutrinoless double beta decay and found that the charged lepton correction case with inverted hierarchy is disfavoured by the latest KamLAND-Zen data. The predictions for effective neutrino mass in this model is very specific and confined to a tiny region around a particular value of lightest neutrino mass. This is due to the neutrino mass sum rule which forces the lightest neutrino mass to remain within a very narrow range. We also find the allowed parameter space for scalar dark matter from the requirement of producing the correct neutrino data, ignoring the Higgs portal and gauge mediated annihilations. Such lepton portal annihilations are efficient for large Yukawa couplings or smaller mediator masses. Since the same Yukawa couplings and mediator mass go into the one loop correction for both neutrino and charged lepton mass matrix, the charged lepton correction is more favourable from lepton portal scalar DM point of view. As mentioned before, this is due to the fact that large Yukawa or small mediator mass will be able to generate sub-GeV corrections to charged lepton mass matrix more naturally than generating sub-eV corrections to light neutrino mass matrix. Also, the charged lepton correction case is much more predictive, as obvious from a much narrower region of allowed parameter space compared to the model with neutrino mass correction.

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"It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiments, it's wrong."

Richard Feynman

4

Non-zero θ_{13} and baryon asymmetry of the universe in a TeV scale seesaw model with A_4 flavour symmetry

In this chapter we study the possibility of generating non-zero reactor mixing angle θ_{13} and baryon asymmetry of the Universe within the framework of an A_4 flavour symmetric model. Using the conventional type I seesaw mechanism we construct the Dirac and Majorana mass matrices which give rise to the correct light neutrino mass matrix. Keeping the right handed neutrino mass matrix structure trivial so that it gives rise to a (quasi) degenerate spectrum of heavy neutrinos suitable for resonant leptogenesis at TeV scale, we generate the non-trivial structure of Dirac neutrino mass matrix that can lead to the light neutrino mixing through type I seesaw formula. Interestingly, such a setup naturally leads to non-zero θ_{13} due to the existence of anti-symmetric contraction of the product of two triplet representations of A_4. Such antisymmetric part of triplet products usually vanish for right handed neutrino Majorana mass terms, leading to $\mu-\tau$ symmetric scenarios in the most economical setups. We constrain the model parameters from the requirement of producing the correct neutrino data as well as baryon asymmetry of the Universe for right handed neutrino mass scale around TeV. The A_4 symmetry is augmented by additional $Z_3 \times Z_2$ symmetry to make sure that the splitting between right handed neutrinos required for resonant leptogenesis is generated only by next to leading order terms, making it naturally small. We find that the inverted hierarchical light neutrino masses give more allowed parameter space consistent with neutrino and baryon asymmetry data.

4.1 Introduction

Observations of tiny but non-zero neutrino mass and large leptonic mixing [1–6] have been one of the most compelling evidences suggesting the presence of beyond standard model (BSM) physics. The present status of different neutrino parameters can be found in the latest global fit analysis [7, 8], summarised in Table 4.1. It can be seen that out of the three leptonic mixing angles, the solar and atmospheric angles are reasonably large while the reactor mixing angle is relatively small. On the other hand, only two mass squared differences are measured experimentally, keeping the lightest neutrino mass still an unknown parameter. Also the mass ordering is not settled yet, allowing both normal hierarchy (NH) as well as inverted hierarchy (IH). Cosmology experiments can however, put an upper bound on the lightest neutrino mass from the measurement of the sum of absolute neutrino masses $\sum_i |m_i| < 0.17$ eV [9]. Although the solar and atmospheric mixing angles $(\theta_{12}, \theta_{23})$ were known to have large values, the discovery of non-zero θ_{13} is somewhat recent [3–6]. The leptonic Dirac CP phase δ is not yet measured experimentally, though a recent measurement hinted at $\delta \approx -\pi/2$ [10]. If neutrinos are Majorana fermions, then two other CP phases appear, which do not affect neutrino oscillation probabilities and hence remain undetermined in such experiments. They can however be probed at experiments looking for lepton number (L) violating processes like neutrinoless double beta decay.

Parameters	NH [7]	IH [7]	NH [8]	IH [8]
$\frac{\Delta m^2_{21}}{10^{-5}\text{eV}^2}$	$7.03 - 8.09$	$7.02 - 8.09$	$7.05 - 8.14$	$7.05 - 8.14$
$\frac{\|\Delta m^2_{31}\|}{10^{-3}\text{eV}^2}$	$2.407 - 2.643$	$2.399 - 2.635$	$2.43 - 2.67$	$2.37 - 2.61$
$\sin^2\theta_{12}$	$0.271 - 0.345$	$0.271 - 0.345$	$0.273 - 0.379$	$0.273 - 0.379$
$\sin^2\theta_{23}$	$0.385 - 0.635$	$0.393 - 0.640$	$0.384 - 0.635$	$0.388 - 0.638$
$\sin^2\theta_{13}$	$0.0193 - 0.0239$	$0.0195 - 0.0240$	$0.0189 - 0.0239$	$0.0189 - 0.0239$
δ	$0 - 360^\circ$	$145^\circ - 391^\circ$	$0 - 360^\circ$	$0^\circ - 31^\circ, 142^\circ - 360^\circ$

Table 4.1: Global fit 3σ values of neutrino oscillation parameters [7, 8].

The standard model (SM) of particle physics, in spite of its astonishing success as a low energy theory of fundamental particles and their interactions (except gravity), can not explain the origin of neutrino mass at renormalisable level. Due to the absence of right handed neutrinos, there is no coupling of the Higgs field responsible for the origin of mass, with neutrinos. Even if right handed neutrinos are introduced, one requires a Yukawa coupling with the Higgs of the order 10^{-12} in order to generate sub eV neutrino masses. It also introduces a new scale, equal to the bare mass term of the right handed neutrinos that can neither be explained nor prevented within the SM. In an effective field theory setup, one can generate light neutrino masses through the dimension five effective operator [11] so that neutrino masses are naturally light due to the suppression by a cut-off scale Λ. Such an operator can be realised within several BSM frameworks after integrating out the heavy fields. Such renormalisable BSM frameworks are popularly known as seesaw models [12–14]. Apart from the tiny mass of neutrinos, another puzzling observation is their large mixing angles, in sharp contrast with small mixing angles in the quark sector. This may also be a hint that the CP violation in the leptonic sector is large compared to quark sector. If this is true, then it can have non trivial implications for cosmology as the quark sector CP violation is found to be too small to generate the observed matter antimatter asymmetry of the Universe, to be discussed in details below. The observed large mixing in the leptonic sector has motivated the study of different flavour symmetry models that can predict such mixing patterns. One of the very popular flavour symmetric scenarios is the one that predicts a $\mu - \tau$ symmetric light neutrino mass matrix that predicts $\theta_{13} = 0, \theta_{23} = \frac{\pi}{4}$ whereas the value of θ_{12} depends upon the particular realisation of this symmetry [15]. Among different possible realisations, the Tri-Bimaximal (TBM) [16–18] mixing pattern which predicts $\theta_{12} = 35.3^o$ has probably been the most studied one. In fact, this mixing pattern was consistent with light neutrino data, prior to the discovery of non-zero θ_{13}. Such mixing patterns can naturally be realised within several non-abelian discrete flavour symmetry models [19, 20]. Among them, the discrete group A_4 which is the group of even permutations of four objects, can reproduce the TBM mixing in the most economical way [21–25]. Since the latest neutrino oscillation data is not consistent with $\theta_{13} = 0$ and hence TBM mixing, one has to go beyond the minimal $\mu - \tau$ symmetric framework. Since the measured value of θ_{13} is small compared to the other two, one can still consider the validity of $\mu - \tau$ symmetry at the leading order and generate non-zero θ_{13} by adding small $\mu - \tau$ symmetry breaking perturbations. Such corrections can originate from the charged lepton sector or the neutrino sector itself like for example, in the form of a new contribution to the neutrino mass matrix. This has led to several works including [26–36] within different BSM frameworks.

Apart from the issue of tiny neutrino mass and large leptonic mixing, another serious drawback of the SM is its inability to explain the observed baryon asymmetry of the Universe. The observed baryon asymmetry is often quoted as the baryon to photon ratio [9]

$$\eta_B = \frac{n_B - n_{\overline{B}}}{n_\gamma} = 6.04 \pm 0.08 \times 10^{-10} \tag{4.1.1}$$

If the Universe had started in a baryon symmetric manner then one has to satisfy the Sakharov's conditions [37]: baryon number (B) violation, C and CP violation, departure from thermal equilibrium. One popular BSM scenario that can generate a net baryon asymmetry is leptogenesis. For a review, one may refer to [38]. As outlined in the original proposal by Fukugita and Yanagida thirty years back [39], this mechanism can satisfy all the Sakharov's conditions [37] required to be fulfilled in order to produce a net baryon asymmetry. Here, a net leptonic asymmetry is generated first which gets converted into baryon asymmetry through $B + L$ violating electroweak sphaleron transitions [40]. The interesting feature of this scenario is that the required lepton asymmetry can be generated through out of equilibrium decay of the same heavy fields that take part in the seesaw mechanism. Although a the BSM framework explaining the baryon asymmetry could be completely decoupled from the one explaining leptonic mass and mixing, it is more economical and predictive if the same model can account for both the observed phenomena. In the conventional type I seesaw mechanism for example, the heavy right handed neutrino decay generate the required lepton asymmetry which not only depends upon the scale of right handed neutrino mass, but also on the leptonic CP violation, which can be probed at ongoing oscillation experiments. For a hierarchical spectrum of right handed neutrinos, there exists a lower bound on the right handed neutrino mass $M_R > 10^9$ GeV, popularly known as the Davidson-Ibarra bound [41], from the requirement of successful leptogenesis. One can however bring down the scale of right handed neutrino mass within the framework of resonant leptogenesis [42–45].

Motivated by this, we study an A_4 flavour symmetric model that can simultaneously explain the correct neutrino data as well as the baryon asymmetry through TeV scale resonant leptogenesis. Keeping the right handed neutrino mass matrix trivial, giving rise to a degenerate spectrum, we first try to obtain the non-trivial Dirac neutrino mass matrix responsible for non-trivial structure of the light neutrino mass matrix, to be obtained using the type I seesaw formula. We generate this non-trivial structure of Dirac neutrino mass matrix using a flavon field which, along with the lepton doublets and right handed neutrinos transform as A_4 triplets. We find that this choice automatically gives rise to non-zero θ_{13} as the resulting light neutrino mass matrix do not possess any $\mu - \tau$ symmetry. This is due to the antisymmetric term arising out of the products of two A_4 triplets. If we generate the non-trivial leptonic mixing from a non-trivial right handed neutrino mixing, like in the Altarelli-Feruglio type models [25], such anti-symmetric term vanishes due to Majorana nature of this mass term. This is however not true in case of Dirac mass term, resulting in a non-trivial

$\mu - \tau$ symmetry breaking structure in the most general case. We compare the light neutrino mass matrix derived from the model with the one from data and evaluate the model parameters for a particular choice of right handed neutrino mass scale. The minimal such scenario is found to be rather constrained with only a handful of allowed points that satisfy all the criteria from neutrino data point of view. We then feed these allowed points to the calculation of resonant leptogenesis and found agreement with the observed baryon asymmetry of the Universe. In the end we also briefly comment on the $\mu - \tau$ symmetric limit of these scenarios where the anti-symmetric coupling term is turned off by hand.

This paper is organised as follows. In section 4.2, we discuss our A_4 flavour symmetric model with the details of different mass matrices in the lepton sector. In section 4.3, we briefly outline the mechanism of resonant leptogenesis followed by the details of numerical analysis in section 4.4. We discuss our numerical results in section 4.5 and then briefly outline the $\mu - \tau$ symmetric limit of the model in section 4.6. We finally conclude in section 4.7.

4.2 The model

The discrete group A_4 is the group of even permutations of four objects or the symmetry group of a tetrahedron. It has twelve elements and four irreducible representations with dimensions n_i such that $\sum_i n_i^2 = 12$. These four representations are denoted by $\mathbf{1}, \mathbf{1}', \mathbf{1}''$ and $\mathbf{3}$ respectively. The product rules for these representations are given in Section 1.8 of Chapter 1. We consider a flavour symmetric model based on the discrete non-abelian group A_4 augmented by $Z_3 \times Z_2$ which predicts the specific structures of different 3×3 matrices involved in the type I seesaw in a natural and minimal way. The particle content of the model is shown in Table 4.2.

The Yukawa Lagrangian for the leptons can be written as

$$\mathcal{L}_Y \supset Y_e \bar{L} H \frac{\phi_E}{\Lambda} e_R + Y_\mu \bar{L} H \frac{\phi_E}{\Lambda} \mu_R + Y_\tau \bar{L} H_d \frac{\phi_E}{\Lambda} \tau_R + \frac{Y_s}{\Lambda} (\phi_\nu \bar{L})_{3_s} \tilde{H} N + \frac{Y_a}{\Lambda} (\phi_\nu \bar{L})_{3_a} \tilde{H} N \\ + Y_N (NN)_1 \xi + Y'_N (NN)_{1''} \xi \frac{\zeta\zeta}{\Lambda^2} + \text{h.c.} \tag{4.2.1}$$

	$\bar{L}$	e_R	μ_R	τ_R	N	H	ϕ_E	ϕ_ν	ξ	ζ
$SU(2)_L$	2	1	1	1	1	2	1	1	1	1
A_4	3	1	$1'$	$1''$	3	1	3	3	1	$1''$
Z_3	ω	ω^2	ω^2	ω^2	ω	1	1	ω	ω	1
Z_2	1	1	1	1	-1	1	1	-1	1	-1

Table 4.2: Fields and their transformation properties under $SU(2)_L$ gauge symmetry as well as the A_4 symmetry

Using the A_4 product rules given in Section 1.8 of Chapter 1, we can write down the relevant leptonic mass matrices corresponding to the above Lagrangian. We denote the vacuum expectation value (vev) of the Higgs to be v_H and choose a specific flavon vev alignment $\langle\phi_E\rangle = (v_E, 0, 0), \langle\phi_\nu\rangle = (v_\nu, v_\nu, v_\nu)$. The resulting charged lepton mass matrix is

$$M_l = \frac{v_H v_E}{\Lambda}\begin{pmatrix} Y_e & 0 & 0 \\ 0 & Y_\mu & 0 \\ 0 & 0 & Y_\tau \end{pmatrix} \tag{4.2.2}$$

The Dirac neutrino mass matrix is given by

$$M_D = \frac{v_H v_\nu}{\Lambda}\begin{pmatrix} \frac{2}{3}Y_s & -(\frac{Y_s}{3}+\frac{Y_a}{2}) & -(\frac{Y_s}{3}-\frac{Y_a}{2}) \\ -(\frac{Y_s}{3}-\frac{Y_a}{2}) & \frac{2}{3}Y_s & -(\frac{Y_s}{3}+\frac{Y_a}{2}) \\ -(\frac{Y_s}{3}+\frac{Y_a}{2}) & -(\frac{Y_s}{3}-\frac{Y_a}{2}) & \frac{2}{3}Y_s \end{pmatrix} \tag{4.2.3}$$

Considering only upto dimension five terms, the right handed neutrino mass matrix can be written as

$$M_R = 2Y_N v_\xi \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{4.2.4}$$

where v_ξ is the vev of the flavon ξ. The light neutrino mass matrix can be generated using type I seesaw

$$-M_\nu = M_D M_R^{-1} M_D^T = \frac{1}{c}\begin{pmatrix} -2(a^2-3b^2) & (a^2+6ab-3b^2) & (a^2-6ab-3b^2) \\ (a^2+6ab-3b^2) & (a^2-6ab-3b^2) & -2(a^2-3b^2) \\ (a^2-6ab-3b^2) & -2(a^2-3b^2) & (a^2+6ab-3b^2) \end{pmatrix} \tag{4.2.5}$$

where $a = \frac{1}{\Lambda} Y_a v_H v_\nu, b = \frac{2}{3\Lambda} Y_s v_H v_\nu, c = 2Y_N v_\xi$. Diagonalisation of this mass matrix gives the eigenvalues as

$$m_1 = 0, \quad m_2 = -\frac{3}{c}(a^2 + 3b^2), \quad m_3 = \frac{3}{c}(a^2 + 3b^2) \tag{4.2.6}$$

which clearly disagrees with the neutrino mass data that gives $\Delta m^2_{21} \neq 0$. Even if we lift the degeneracy of the right handed neutrino mass matrix as

$$M_R = \begin{pmatrix} c & 0 & 0 \\ 0 & 0 & c \\ 0 & c & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & d \\ 0 & d & 0 \\ d & 0 & 0 \end{pmatrix} \tag{4.2.7}$$

we still have degenerate light neutrino mass eigenvalues

$$m_1 = 0, \quad m_2 = -\frac{3(a^2 + 3b^2)}{\sqrt{c^2 - cd + d^2}}, \quad m_3 = \frac{3(a^2 + 3b^2)}{\sqrt{c^2 - cd + d^2}} \tag{4.2.8}$$

which is disallowed by neutrino data. Choosing a more general vacuum alignment $\langle \phi_\nu \rangle = (v_{\nu 1}, v_{\nu 2}, v_{\nu 3})$, the Dirac neutrino mass matrix can be written as

$$M_D = \frac{v_H}{\Lambda} \begin{pmatrix} \frac{2}{3} Y_s v_{\nu 1} & -(\frac{Y_s}{3} + \frac{Y_a}{2}) v_{\nu 3} & -(\frac{Y_s}{3} - \frac{Y_a}{2}) v_{\nu 2} \\ -(\frac{Y_s}{3} - \frac{Y_a}{2}) v_{\nu 3} & \frac{2}{3} Y_s v_{\nu 2} & -(\frac{Y_s}{3} + \frac{Y_a}{2}) v_{\nu 1} \\ -(\frac{Y_s}{3} + \frac{Y_a}{2}) v_{\nu 2} & -(\frac{Y_s}{3} - \frac{Y_a}{2}) v_{\nu 1} & \frac{2}{3} Y_s v_{\nu 3} \end{pmatrix} \tag{4.2.9}$$

Denoting $a = \frac{v_H}{\Lambda} \frac{1}{3} Y_s v_{\nu 1}, b = \frac{v_H}{\Lambda} \frac{1}{3} Y_a v_{\nu 1}, c = \frac{v_H}{\Lambda} \frac{1}{3} Y_s v_{\nu 2}, d = \frac{v_H}{\Lambda} \frac{1}{3} Y_s v_{\nu 3}$ we can write the Dirac neutrino mass matrix as

$$M_D = \begin{pmatrix} 2a & -d - \frac{bd}{a} & -c + \frac{bc}{a} \\ -d + \frac{bd}{a} & 2c & -a - b \\ -c - \frac{bc}{a} & -a + b & 2d \end{pmatrix} \tag{4.2.10}$$

In this notation, the light neutrino mass matrix elements are given by

$$\begin{aligned}
(-M_\nu)_{11} &= \frac{4a^4 + 2a^2 cd - 2b^2 cd}{a^2 f} \\
(-M_\nu)_{12} &= \frac{a^2(-d) + 4abd - 2ac^2 + b^2 d + 2bc^2}{af} \\
(-M_\nu)_{13} &= -\frac{a^2 c + 4abc + 2ad^2 - b^2 c + 2bd^2}{af} \\
(-M_\nu)_{22} &= \frac{\left(d - \frac{bd}{a}\right)^2 - 4c(a+b)}{f} \\
(-M_\nu)_{23} &= \frac{a^4 - a^2\left(b^2 - 5cd\right) - b^2 cd}{a^2 f} \\
(-M_\nu)_{33} &= \frac{\frac{c^2(a+b)^2}{a^2} + 4d(b-a)}{f}
\end{aligned} \tag{4.2.11}$$

where $f = 2Y_N v_\xi$ is the non-zero entry in the right handed neutrino mass matrix given by Eq. (4.2.4). In this case, the resulting light neutrino mass matrix can give rise to the correct mass squared differences as well as mixing angles including non-zero θ_{13}. At the dimension five level however, the right handed neutrinos remain degenerate. As we discuss below, for successful resonant leptogenesis, the right handed neutrinos must have tiny splittings which can be generated at dimension six level in the model. This higher order contribution to the right handed neutrino mass matrix can be written as

$$\delta M = \begin{pmatrix} 0 & 0 & r_1 \\ 0 & r_1 & 0 \\ r_1 & 0 & 0 \end{pmatrix} \tag{4.2.12}$$

where $r_1 = Y'_N v_\xi \frac{v_\zeta^2}{\Lambda^2}$ with v_ζ being the vev of the flavon ζ. Such a small higher order term does not affect light neutrino masses and mixings considerably.
It should be noted that, we have used the A_4 product rules in T diagonal basis, as given in Section 1.8 of Chapter 1. This is justified in the diagonal charged lepton and Majorana light neutrino mass limit. In the S diagonal basis, the charged lepton mass matrix is non-diagonal and the light neutrino mass matrix will also have a different structure due to the difference in the triple product rules.

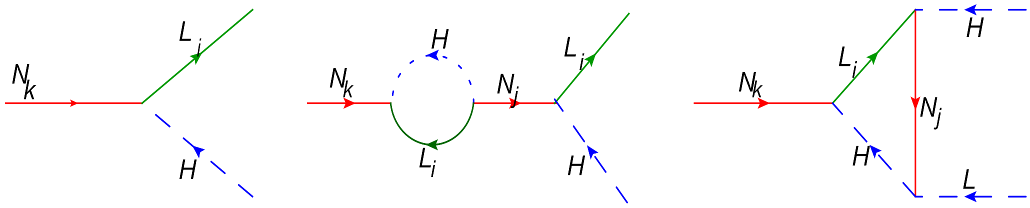

Figure 4.1: Decay modes of right handed neutrino in type I seesaw

4.3 Resonant leptogenesis

As referred by Fukugita and Yanagida [39], the out of equilibrium and CP violating decay of heavy Majorana neutrinos affords a natural way to produce the needed lepton asymmetry, as evinced in figure 4.1. The asymmetry generated by the decay of the lightest right handed neutrino into lepton and Higgs is given by,

$$\epsilon_{N_k} = -\sum \frac{\Gamma(N_k \to L_i + H^*) - \Gamma(N_k \to L_i + H)}{\Gamma(N_k \to L_i + H^*) + \Gamma(N_k \to L_i + H)} \tag{4.3.1}$$

This lepton asymmetry is converted to the baryon asymmetry through electroweak sphaleron transitions allowing us to reproduce the observed baryon asymmetry of the Universe. As mentioned before, resonant leptogenesis is a viable alternative to high scale or vanilla leptogenesis scenarios [42–45] within the context of a TeV scale minimal seesaw scenarios. Since a hierarchical spectrum of right handed neutrinos can not give rise to the required asymmetry at TeV scale, this mechanism gives a resonance enhancement to the lepton asymmetry by considering a very small mass splitting between the two heavy neutrinos, of the order of their average decay width.

The lepton asymmetry can be found from the following formula [46, 47],

$$\epsilon_{il} = \sum_{j\neq i} \frac{\mathrm{Im}[Y_{\nu_{il}} Y^*_{\nu_{jl}} (Y_\nu Y_\nu^\dagger)_{ij}] + \frac{M_i}{M_j}\mathrm{Im}[Y_{\nu_{il}} Y^*_{\nu_{jl}} (Y_\nu Y_\nu^\dagger)_{ji}]}{(Y_\nu Y_\nu^\dagger)_{ii} (Y_\nu Y_\nu^\dagger)_{jj}} f_{ij} \tag{4.3.2}$$

with the regulator f_{ij} being given as

$$f_{ij} = \frac{(M_i^2 - M_j^2) M_i \Gamma_j}{(M_i^2 - M_j^2)^2 + M_i^2 \Gamma_j^2}.$$

Here, $\Gamma_i = \frac{M_i}{8\pi}(Y_\nu Y_\nu^\dagger)_{ii}$ as the tree level heavy-neutrino decay width and Y_ν is the effective coupling between heavy and light neutrinos. Now, there is a similar contribution ϵ'_{il} to the CP asymmetry from RH neutrino oscillation [47–49]. Its form is given by Eq. (4.3.2) with the replacement f_{ij} by f'_{ij}, where

$$f'_{ij} = \frac{(M_i^2 - M_j^2) M_i \Gamma_j}{(M_i^2 - M_j^2)^2 + (M_i \Gamma_i + M_j \Gamma_j)^2 \frac{\det[\mathrm{Re}(Y_\nu Y_\nu^\dagger)]}{(Y_\nu Y_\nu^\dagger)_{ii} (Y_\nu Y_\nu^\dagger)_{ii}}}.$$

The total CP asymmetry is therefore is the summation of these two $\epsilon^T_{il} = \epsilon_{il} + \epsilon'_{il}$. Taking into account of the appropriate efficiency and dilution factors [50], one can write the final baryon asymmetry as

$$\eta_B = \frac{n_B - n_{\overline{B}}}{n_\gamma} \simeq -\frac{28}{51}\frac{1}{27}\frac{3}{2}\sum_{l,i} \frac{\epsilon_{il}}{K_l^{\mathrm{eff}} \mathrm{min}(z_c, z_l)} \tag{4.3.3}$$

where, $z_c = \frac{M_N}{T_c}$, $T_c \sim 149$ GeV being the critical temperature, $z_l \simeq 1.25\log(25K_l^{\rm eff})$ [50] and $K_l^{\rm eff} = \kappa_l \sum_i K_i B_{il}$, with $K_i = \Gamma_i/H_N$ being the wash out factor. The Hubble parameter for radiation dominated Universe is $H_N = 1.66\sqrt{g_*}M_N^2/M_{\rm Pl}$ at $T = M_N$ and $g^* \simeq 106.75$ is the relativistic degrees of freedom at high temperatures. B_{il}'s are the branching ratios of the N_i decay to leptons of lth flavor: $B_{il} = \frac{|Y_{\nu il}|^2}{(Y_\nu Y_\nu^\dagger)_{ii}}$. The factor κ is given by

$$\kappa_l = 2\sum_{i,jj\neq i} \frac{{\rm Re}[Y_{\nu il}Y^*_{\nu jl}(YY^\dagger)_{ij}] + {\rm Im}[(Y_{\nu il}Y^*_{\nu jl})^2]}{{\rm Re}[(Y^\dagger Y)_{ll}\{(YY^\dagger)_{ii} + (YY^\dagger)_{jj}\}]}\left(1 - 2i\frac{M_i - M_j}{\Gamma_i + \Gamma_j}\right)^{-1} \tag{4.3.4}$$

As seen from the expression (4.3.2), the lepton asymmetry is dependent on the elements of the Dirac Yukawa coupling matrix. Therefore it can be said that, the same sets of model parameters which are supposed to yield correct neutrino phenomenology are also responsible to yield an enhanced lepton asymmetry, later on generating the observed BAU.

4.4 Numerical analysis

As discussed before, the most general form of Dirac neutrino mass matrix (assuming a degenerate right handed neutrino mass spectrum) can give rise to a light neutrino mass matrix from type I seesaw formula, which is consistent with $\theta_{13} \neq 0$. This is due to the presence of anti-symmetric part of A_4 triple product that explicitly breaks $\mu - \tau$ symmetry leading to the generation of $\theta_{13} \neq 0$. Within the minimal setup, the light neutrino mass matrix is given by Eq. (4.6.2), which contains five parameters a, b, c, d, f that can in general be complex. Since this corresponds to degenerate heavy neutrino masses which can not give rise to successful leptogenesis, we can break the degeneracy by including higher order contribution to the right handed neutrino mass matrix as discussed above. Taking this correction into account, we can write the right handed neutrino mass matrix as

$$M = M_R^0 + \delta M_R = \begin{pmatrix} f & 0 & g \\ 0 & g & f \\ g & f & 0 \end{pmatrix} \tag{4.4.1}$$

This has eigenvalues $f+g, -\sqrt{f^2-fg+g^2}, \sqrt{f^2-fg+g^2}$ where, f is the leading order right handed neutrino mass and g is the parameter creating tiny mass splitting. As mentioned earlier, these parameters are related to the Lagrangian parameters as

$$a = \frac{v_H}{\Lambda}\frac{1}{3}Y_s v_{\nu 1}, b = \frac{v_H}{\Lambda}\frac{1}{3}Y_a v_{\nu 1}, c = \frac{v_H}{\Lambda}\frac{1}{3}Y_s v_{\nu 2}, d = \frac{v_H}{\Lambda}\frac{1}{3}Y_s v_{\nu 3}, f = 2Y_N v_\xi, g = Y'_N v_\xi \frac{v_\zeta^2}{\Lambda^2}$$

For numerical analysis part we first fix the scale of leptogenesis by fixing the leading right handed neutrino mass or the parameter f to be 5 TeV, say. The range of g has been chosen in such a way that we can have a tiny Majorana mass splitting required for successful leptogenesis without affecting the neutrino parameters being from their correct 3σ bound. For satisfying neutrino phenomenology and explaining leptogenesis, g has been varied randomly from 10^{-6} to 10^{-5} GeV which gives lepton asymmetry of an order around 10^{-7} or more. Since g is very small compared to f, its effects on light neutrino masses and mixing is not substantial. Yet, we include it while discussing the compatibility of the model with neutrino data. Thus, after making the choice of f and the range of g, we are left with four model parameters a, b, c, d that can be calculated by comparing the mass matrix predicted by the model with the one we can construct in terms of light neutrino parameters.

The leptonic mixing matrix can be written in terms of the charged lepton diagonalising matrix (U_l) and light neutrino diagonalising matrix U_ν as

$$U_{\text{PMNS}} = U_l^\dagger U_\nu \tag{4.4.2}$$

In the simple case where the charged lepton mass matrix is diagonal which is true in our model, we can have $U_l = \mathbb{1}$. Therefore we can write $U_{\text{PMNS}} = U_\nu$. Now we can write the complete light neutrino mass matrix as

$$m_\nu = U_{\text{PMNS}} m_\nu^{\text{diag}} U_{\text{PMNS}}^T \tag{4.4.3}$$

where the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) leptonic mixing matrix can be parameterized as

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} U_{\mathrm{Maj}} \tag{4.4.4}$$

where $c_{ij} = \cos\theta_{ij},\ s_{ij} = \sin\theta_{ij}$ and δ is the leptonic Dirac CP phase. The diagonal matrix $U_{\mathrm{Maj}} = \mathrm{diag}(1, e^{i\alpha}, e^{i(\zeta+\delta)})$ contains the undetermined Majorana CP phases α, ζ. We have $m_\nu^{\mathrm{diag}} = \mathrm{diag}(m_1, \sqrt{m_1^2 + \Delta m_{21}^2}, \sqrt{m_1^2 + \Delta m_{31}^2})$ for normal hierarchy (NH) and $m_\nu^{\mathrm{diag}} = \mathrm{diag}(\sqrt{m_3^2 + \Delta m_{23}^2 - \Delta m_{21}^2}, \sqrt{m_3^2 + \Delta m_{23}^2}, m_3)$ for inverted hierarchy (IH).

For a fixed value of right handed neutrino mass, we can now compare the light neutrino mass matrix predicted by the model and the one calculated from the light neutrino parameters. Since there are four undetermined complex parameters of the model, we need to compare four elements. Without any loss of generality, we equate $(12), (13), (22), (33)$ elements of both the mass matrices. We vary the light neutrino parameters in their allowed 3σ ranges and vary the lightest neutrino mass $m_{\mathrm{lightest}} \in (10^{-6}, 0.1)$ eV and calculate the model parameters a, b, c, d for each set of values of neutrino parameters. However, the light neutrino mass matrix has two more independent elements as any general 3×3 complex symmetric mass matrix has six independent complex elements. On the other hand, once a, b, c, d are calculated from the equations $(M_\nu)_{12} = (m_\nu)_{12}, (M_\nu)_{13} = (m_\nu)_{13}, (M_\nu)_{22} = (m_\nu)_{22}, (M_\nu)_{33} = (m_\nu)_{33}$, the other two elements $(M_\nu)_{11}, (M_\nu)_{23}$ are automatically determined. Since every set of values of a, b, c, d corresponds to a particular set of light neutrino parameters, we can calculate the other two light neutrino mass matrix elements $(m_\nu)_{11}, (m_\nu)_{23}$ for the same set of neutrino parameters. For consistency, one needs to make sure that these two elements calculated for the neutrino mass matrix predicted by the model M_ν and the ones from light neutrino parameters m_ν are equal to each other. It turns out that these two constraints tightly restrict the light neutrino parameters to a set of very specific values, resulting in a very predictive scenario. We randomly generate ten million light neutrino parameters to calculate the four model parameters a, b, c, d and restrict the parameters to only those ones which satisfy $|(m_\nu)_{11} - (M_\nu)_{11}| < 10^{-5}, |(m_\nu)_{23} - (M_\nu)_{23}| < 10^{-5}$. Here a tolerance of 10^{-5} is chosen to decide the equality between the two elements.

After finding the model parameters a, b, c, d as well as the light neutrino parameters satisfying the constraints relating the two elements of the mass matrices constructed from the model and neutrino data respectively, we calculate the lepton asymmetry for the same set of allowed parameters. The effective Dirac Yukawa coupling matrix (Y_ν) relating heavy neutrinos to the light ones appearing in the lepton asymmetry formula is considered to have the same structure as the Dirac neutrino mass matrix given in Eq. (4.2.9). Since the corrected form of the heavy neutrino mass matrix is non-diagonal (given by Eq. (4.4.1)), we first diagonalise it and find the corresponding diagonalising matrix U_R. To keep the analysis in this basis we transform the Dirac Yukawa coupling matrices as $Y_\nu \to Y_\nu U_R$ with $U_R^* M_R U_R^\dagger = \text{diag}(M_1, M_2, M_3)$. We then calculate the baryon asymmetry for the light neutrino parameters that are consistent with neutrino data as well as the model restrictions discussed above.

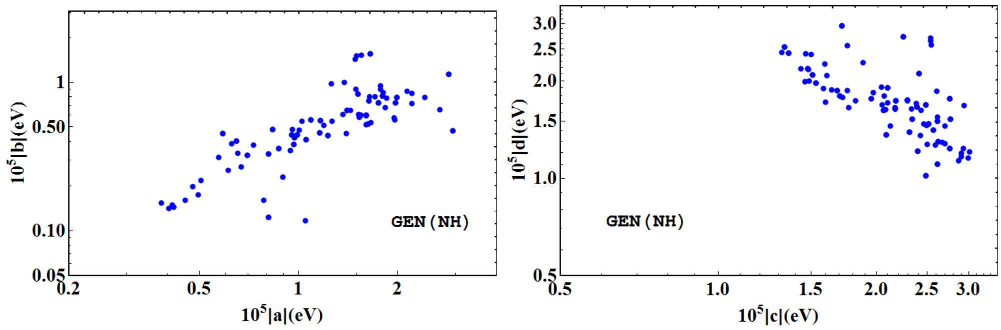

Figure 4.2: Correlation between different model parameters for NH. The label Gen refers to the most general structure of the mass matrix discussed.

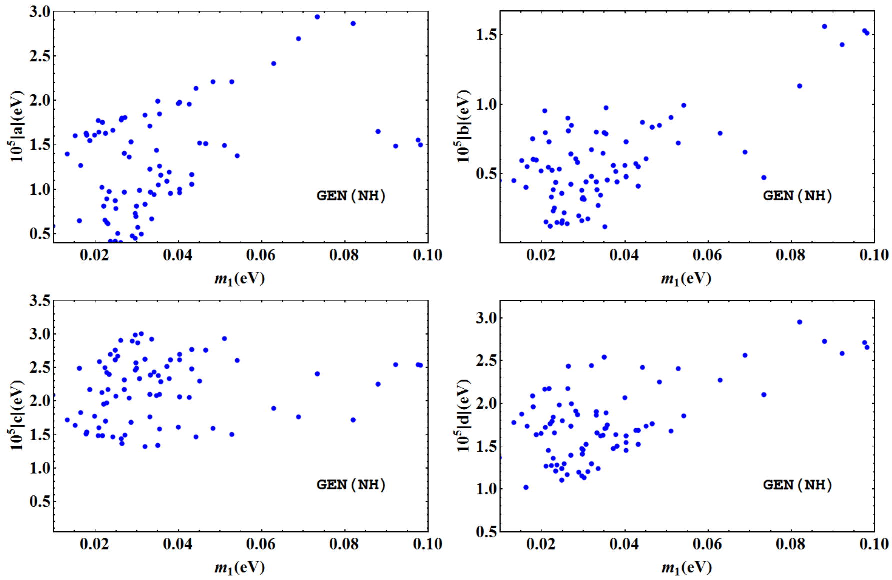

Figure 4.3: Model parameters as a function of the lightest neutrino mass for NH. The label Gen refers to the most general structure of the mass matrix discussed.

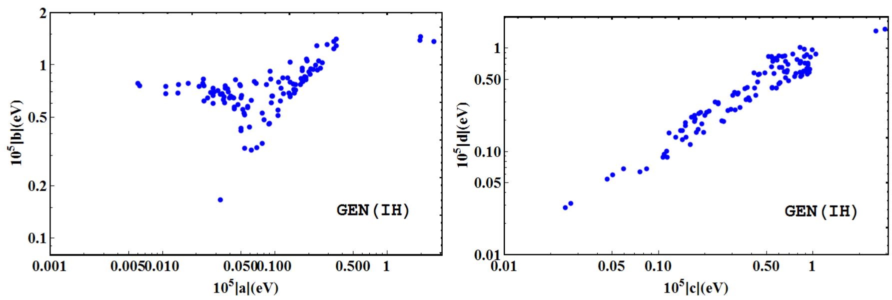

Figure 4.4: Correlation between different model parameters for IH. The label Gen refers to the most general structure of the mass matrix discussed.

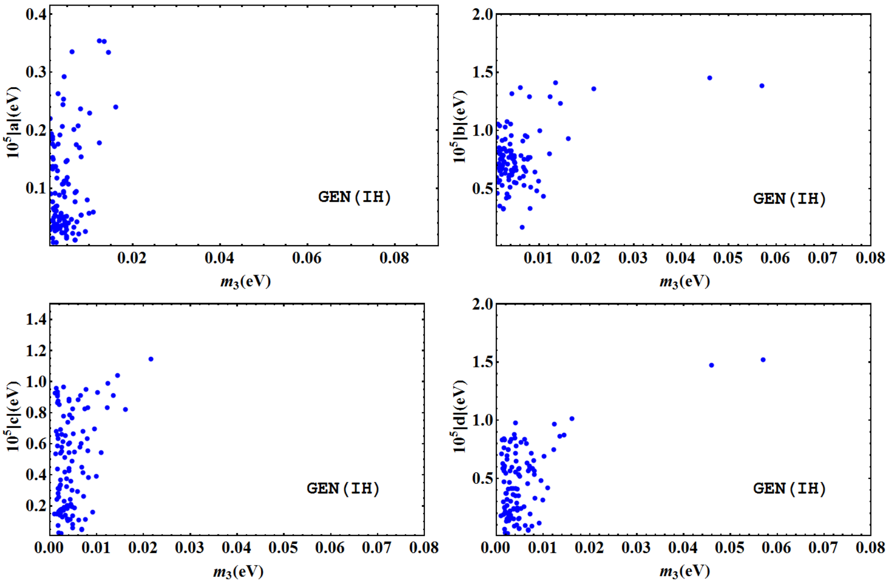

Figure 4.5: Model parameters as a function of the lightest neutrino mass for IH. The label Gen refers to the most general structure of the mass matrix discussed.

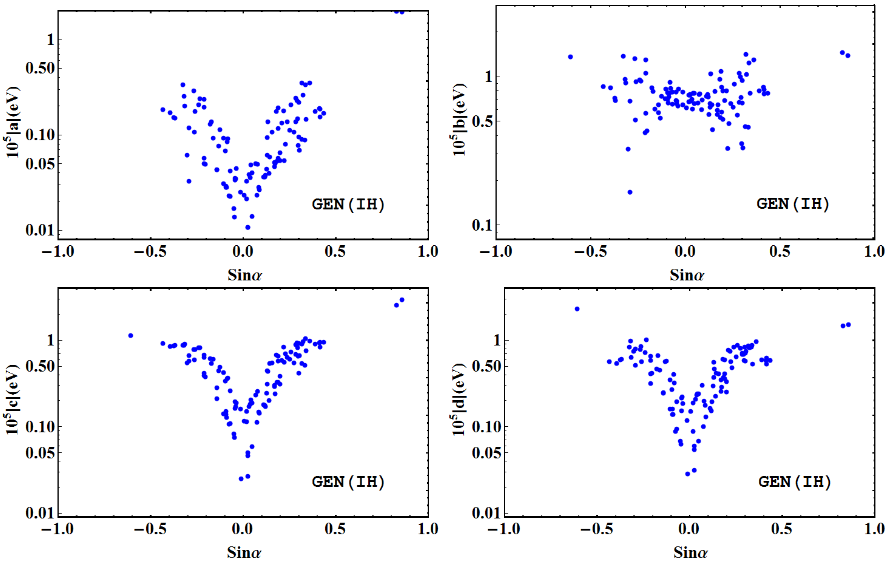

Figure 4.6: Model parameters as a function of one of the Majorana phases α for IH. The label Gen refers to the most general structure of the mass matrix discussed.

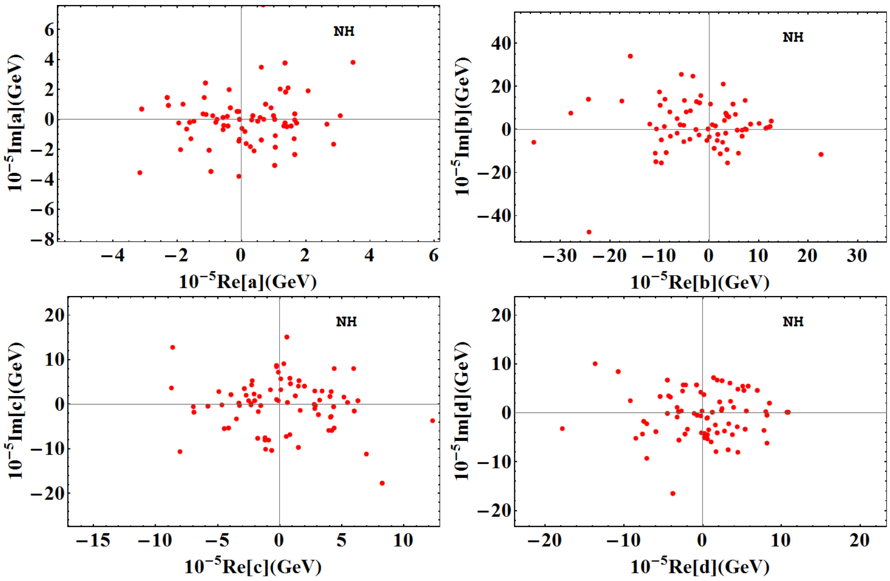

Figure 4.7: Real and imaginary parts of the model parameters for NH with the most general structure of the mass matrix discussed in the text.

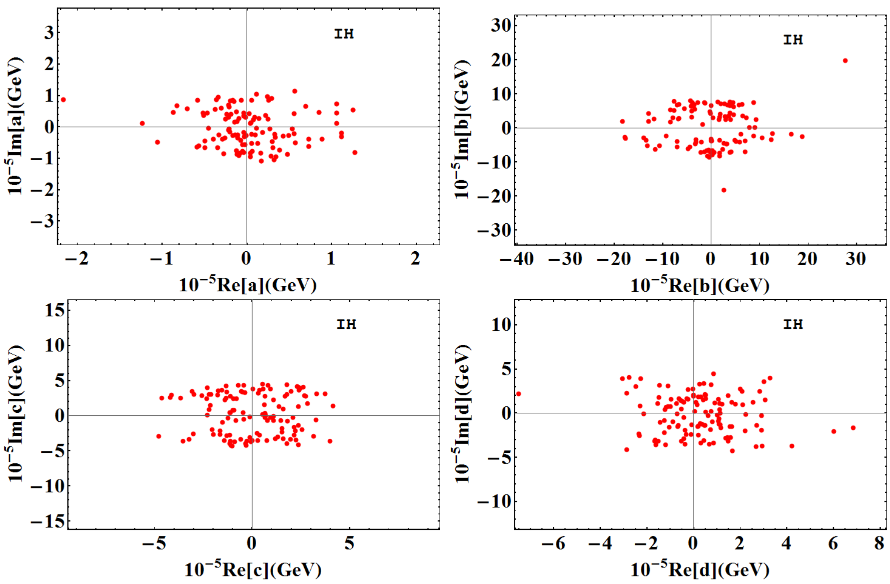

Figure 4.8: Real and imaginary parts of the model parameters for IH with the most general structure of the mass matrix discussed in the text.

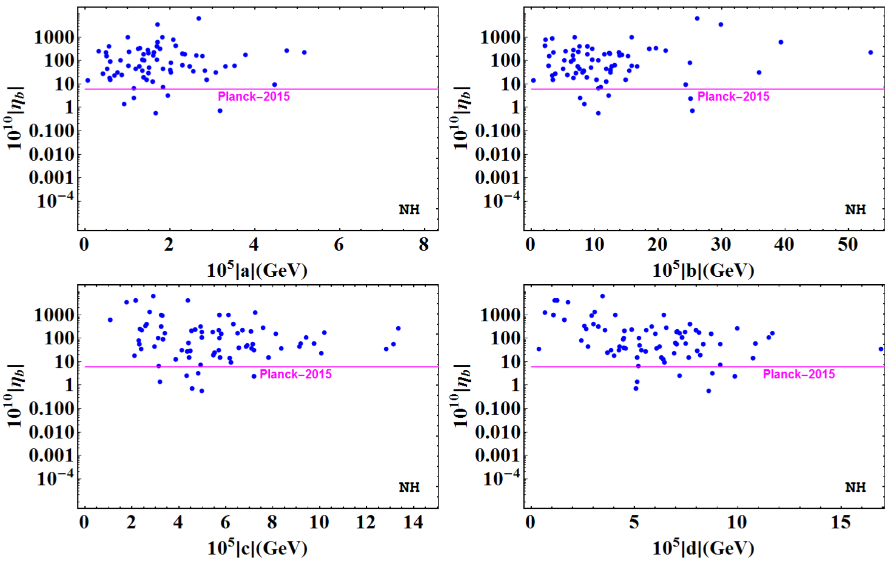

Figure 4.9: Baryon asymmetry as a function of model parameters for NH with a horizontal pink line corresponding to the Planck bound $\eta_B = 6.04 \pm 0.08 \times 10^{-10}$ [9].

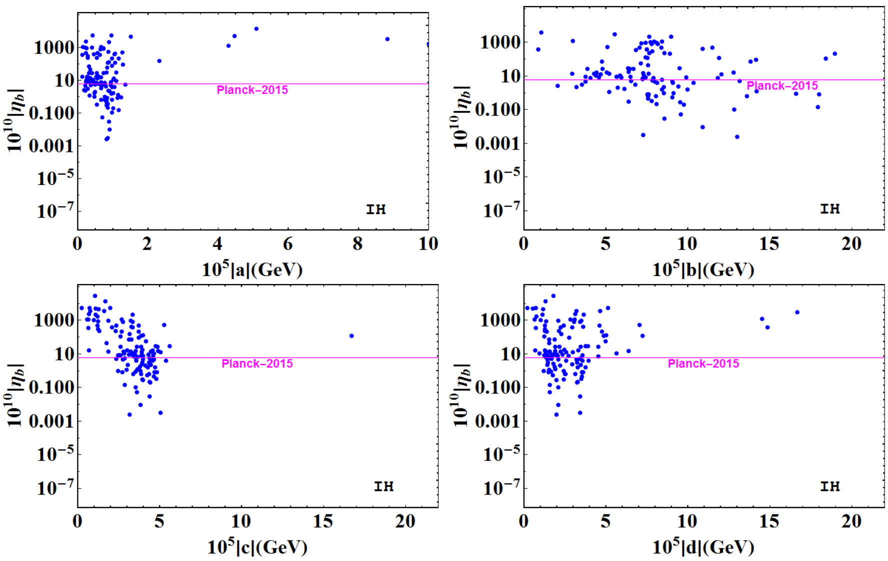

Figure 4.10: Baryon asymmetry as a function of model parameters for IH with a horizontal pink line corresponding to the Planck bound $\eta_B = 6.04 \pm 0.08 \times 10^{-10}$ [9].

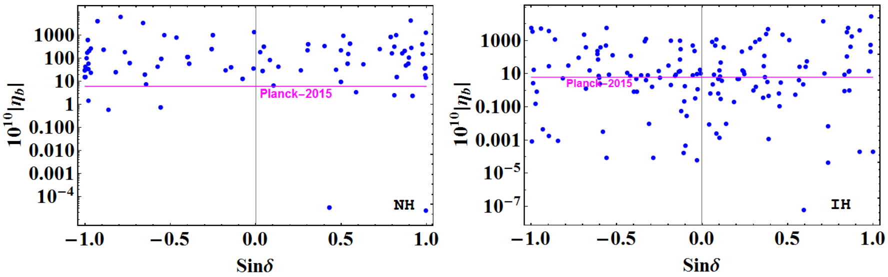

Figure 4.11: Baryon asymmetry as a function of Dirac CP phase for NH and IH with a horizontal pink line corresponding to the Planck bound $\eta_B = 6.04 \pm 0.08 \times 10^{-10}$ [9].

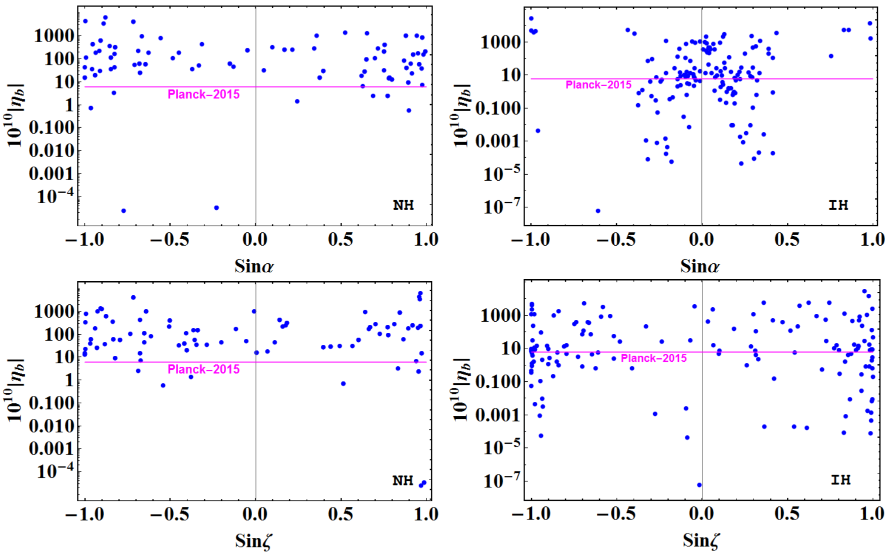

Figure 4.12: Baryon asymmetry as a function of Majorana CP phases for NH and IH with a horizontal pink line corresponding to the Planck bound $\eta_B = 6.04 \pm 0.08 \times 10^{-10}$ [9].

4.5 Results and discussion

Following the procedures outlined in the previous section, we first randomly generate the light neutrino parameters in their 3σ range [7] and for each set of values, we calculate the model parameters a, b, c, d using four equations. We then apply the constraints relating other two elements of the neutrino mass matrix and find the constrained parameter space obeying them. For normal hierarchy, we show the correlation between these model parameters in figure 4.2. Since a, b, c, d denote the strength of the Dirac neutrino mass, we can see that they lie near or below the MeV scale so that the correct light neutrino mass is generated from type I seesaw formula where the right handed neutrino scale is fixed at 5 TeV. We also show the variation of the same model parameters with the lightest neutrino mass m_1 for normal hierarchy in figure 4.3. It can be seen that the allowed lightest neutrino mass can have values in the range $0.01 - 0.1$ eV, that can be sensitive to $0\nu\beta\beta$ experiments. In fact, the region of parameter space near $m_1 \sim 0.1$ eV will be ruled out by latest bounds from $0\nu\beta\beta$ experiments as well as the cosmology upper bound on the sum of absolute neutrino masses [9]. We show similar correlations for inverted hierarchy in figure 4.4 and 4.5. The overall features of these correlation plots are similar the ones for normal hierarchy, shown in figure 4.2 and 4.3. However, for inverted hierarchy, we see a preference for smaller values of lightest neutrino mass, close to 0.01 eV, away from the upper bounds set by $0\nu\beta\beta$ and cosmology data. We then show some interesting correlations between the model parameters for inverted hierarchy with one of the Majorana CP phases in figure 4.6. This figure also shows that the requirement of satisfying correct neutrino data constrains this CP phase to a range $|\sin\alpha| < 0.5$.

We also check if there are any correlations among the known neutrino parameters in this analysis. This could arise due to the fact that there are only four parameters a, b, c, d that we are solving for by using more number of input parameters, leading to an over-constrained system. However, we did not find any such correlations between the known neutrino parameters. This is primarily due to the fact that the model parameters a, b, c, d are in general complex and hence they represent a set of eight real parameters. We show their real and imaginary parts separately in figure 4.7 and 4.8 for normal and inverted hierarchies respectively. The imaginary parts of the model parameters are the source of CP phases in this model and hence play a crucial role in generating the leptonic asymmetries.

After finding the allowed neutrino as well as model parameters from the requirement of satisfying the latest neutrino oscillation data, we feed them to the calculation of the baryon asymmetry through resonant leptogenesis. The resulting values of η_B are shown for normal hierarchy as a function of the model parameters in figure 4.9. We can see that there are several points which satisfy the Planck 2015 bound on baryon asymmetry $\eta_B = 6.04 \pm 0.08 \times 10^{-10}$ [9]. We find more allowed parameters that satisfy the Planck bound for inverted hierarchy, as can be seen from the plots shown in figure 4.10. We also show the baryon asymmetry versus Dirac CP phase δ in figure 4.11. It can be seen from this plot that, we do not see preference for any particular value of Dirac CP phase. To show the variation of η_B with Majorana CP phases, we show the plots in figure 4.12 for both normal and inverted hierarchy.

Here we note that there is a difference of around nine order of magnitudes between the mass splitting between the right handed neutrinos (of keV order) and their masses (of TeV order). Although in this model we have generated such tiny mass splitting naturally, by forbidding it at leading order and generating it only at higher orders (mass splitting term is suppressed by Λ^2 compared to the dimension four mass term without any suppression, as discussed above), we still need to make sure that these splittings are stable under quantum corrections. That is, if we generate this tiny splitting naturally at the scale of the flavour symmetry breaking $\sim \Lambda$, such splittings should not be disturbed significantly while running them down to the scale at which the lepton asymmetry is being generated $T \sim M_R \sim \mathcal{O}(\text{TeV})$. Several earlier works discussed such radiative origin of mass splittings [51] by considering a degenerate spectrum at high energy scale [47, 52]. Such splittings at the scale of leptogenesis ($T \sim M_R$) originating from renormalisation group (RG) effects from a scale Λ to M_R can be estimated as

$$\Delta M_R^{\text{RG}} \approx -\frac{M_R}{8\pi^2} \ln\left(\frac{\Lambda}{M_R}\right) \text{Re}[Y_\nu^\dagger(\Lambda) Y_\nu(\Lambda)] \tag{4.5.1}$$

The effective Yukawa couplings Y_ν here can be derived from the model parameters a, b, c, d by taking their ratio with the Higgs vev $v_H \sim 100$ GeV. As seen from the figures 4.7, 4.8, the parameters a, b, c, d can be as large as of order 10^{-4} GeV and hence the effective Yukawa couplings Y_ν will be of the order of 10^{-6}. Thus, the mass splitting from RG effects can be estimated to be approximately

$$\Delta M_R^{\mathrm{RG}} \approx (x-3)\times 3\times 10^{-11}\ \mathrm{GeV}$$

where $\Lambda = 10^x$ GeV, $M_R \sim \mathcal{O}(\mathrm{TeV})$ is used. Therefore, the splitting from RG effects is usually small for TeV scale M_R and the values of Yukawa couplings we have in our model. In fact, as pointed out by [48], pure radiative splitting scenario gives rise to vanishing lepton asymmetry at order $\mathcal{O}(Y_\nu^4)$, showing more preference to non-minimal scenario where splitting is generated by extra term in the Lagrangian, like the one we have in our model.

4.6 $\mu-\tau$ Symmetric limit of the model

In the most general case discussed above, the light neutrino mass matrix derived from the type I seesaw formula turns out to break $\mu-\tau$ symmetry resulting in non-zero θ_{13}. The anti-symmetric part of the triplet multiplications $\frac{Y_a}{\Lambda}(\phi_\nu \bar{L})_{3_a}\tilde{H}N$ in the Dirac mass term is responsible for breaking the $\mu-\tau$ symmetry and in the limit of $Y_a \to 0$, the $\mu-\tau$ symmetry in the light neutrino mass matrix can be recovered. In this limit, for the simple flavon vev alignment $\langle\phi_E\rangle = (v_E, 0, 0)$, $\langle\phi_\nu\rangle = (v_\nu, v_\nu, v_\nu)$, the charged lepton mass matrix is diagonal as before whereas the Dirac neutrino mass matrix takes a simpler form given by

$$M_D = \begin{pmatrix} 2a & -a & -a \\ -a & 2a & -a \\ -a & -a & 2a \end{pmatrix} \tag{4.6.1}$$

where $a = \frac{v_H}{\Lambda}\frac{1}{3}Y_s v_\nu$. Using the right handed neutrino mass matrix given by Eq. (4.2.4), the light neutrino mass matrix from type I seesaw formula can be written as

$$-M_\nu = M_D M_R^{-1} M_D^T = \frac{3a^2}{b}\begin{pmatrix} 2 & -1 & -1 \\ -1 & -1 & 2 \\ -1 & 2 & -1 \end{pmatrix} \tag{4.6.2}$$

where $b = 2Y_N v_\xi$. This light neutrino mass matrix is clearly $\mu-\tau$ symmetric but it predicts two degenerate massive neutrinos and one massless neutrino, inconsistent with the observed mass squared differences.

We suitably modify the field content to arrive at a more realistic $\mu-\tau$ symmetric light neutrino mass matrix, as shown in Table 4.3. In the limit of vanishing

	$\bar{L}$	e_R	μ_R	τ_R	N	H	ϕ_E	ϕ_ν	ξ	ζ	η
$SU(2)_L$	2	1	1	1	1	2	1	1	1	1	1
A_4	3	1	$1'$	$1''$	3	1	3	3	1	$1''$	1
Z_3	ω	ω^2	ω^2	ω^2	ω	1	1	ω	ω	1	ω
Z_2	1	1	1	1	-1	1	1	-1	1	-1	-1

Table 4.3: Fields and their transformation properties under $SU(2)_L$ gauge symmetry as well as the A_4 symmetry in the $\mu-\tau$ symmetric limit.

antisymmetric part of the A_4 triplet products, the Yukawa Lagrangian for the Dirac neutrino mass terms can be written as

$$\mathcal{L}_Y \supset \frac{Y_s}{\Lambda}(\phi_\nu \bar{L})_{3_s}\tilde{H}N + \frac{Y'}{\Lambda}(\bar{L}N)_1\tilde{H}\eta + \text{h.c.} \tag{4.6.3}$$

In this case, the Dirac neutrino mass matrix can be written as

$$M_D = \begin{pmatrix} a+2b & -b & -b \\ -b & 2b & a-b \\ -b & a-b & 2b \end{pmatrix}. \tag{4.6.4}$$

where $b = \frac{v_H}{\Lambda}\frac{1}{3}Y_s v_\nu, a = \frac{v_H}{\Lambda}\frac{1}{3}Y' v_\eta$, with v_η being the vev of the flavon field η. Using the same leading order right handed neutrino mass matrix given by Eq. (4.2.4), we can derive a $\mu-\tau$ symmetric light neutrino mass matrix using the type I seesaw formula. In fact, this gives rise to TBM type mixing, one of the widely studied neutrino mixing framework which was consistent with neutrino data prior to the discovery of non-zero θ_{13}. Since the TBM can still be considered as a leading order approximation due to the smallness of θ_{13} compared to other mixing angles, such a scenario can be realistic provided a small deviation to it can be realised in order to generate non-zero θ_{13}. This can be done simply by incorporating another flavon field ψ that has the following transformation

$$\psi(SU(2)_L : 1,\ A_4 : 1',\ Z_3 : \omega,\ Z_2 : -1)$$

This allows one more contribution to Dirac neutrino mass term in the form of

$$\mathcal{L}_Y \supset \frac{Y''}{\Lambda}(\bar{L}N)_{1''}\tilde{H}\psi + \text{h.c.} \tag{4.6.5}$$

After the flavon field ψ gets a vev v_ψ, this introduces a $\mu-\tau$ symmetry breaking correction to the Dirac mass term given by

$$\delta M_D = \begin{pmatrix} 0 & 0 & f \\ 0 & f & 0 \\ f & 0 & 0 \end{pmatrix} \tag{4.6.6}$$

where $f = \frac{v_H}{\Lambda}\frac{1}{3}Y''v_\psi$. Since this is a limiting case of the most general case based on an assumption of vanishing antisymmetric terms, we do not perform any numerical calculations for this scenario. The calculations will be similar to generic A_4 models where non-zero θ_{13} is generated by considering corrections to a leading order $\mu-\tau$ symmetric light neutrino mass matrix. For example, the work [34] considered such a scenario.

4.7 Conclusion

We have studied an extension of the standard model by discrete flavour symmetry $A_4 \times Z_3 \times Z_2$ that can simultaneously explain the correct neutrino oscillation data and the observed baryon asymmetry of the Universe. Considering a TeV scale type I seesaw we adopt the mechanism of resonant leptogenesis to generate a lepton asymmetry through out of equilibrium CP violating decay of right handed neutrinos which later gets converted into the required baryon asymmetry through electroweak sphalerons. The field content and its transformation under the flavour symmetry are chosen in such a way that the leading order right handed neutrino mass matrix has a trivial structure giving a degenerate spectrum. The tiny splitting between the right handed neutrino masses (required for resonant leptogenesis) arises through higher dimension mass terms, naturally suppressing the splitting. Due to the trivial structure of the right handed neutrino mass matrix, the leptonic mixing arises through the non-trivial structure of the Dirac neutrino mass matrix within a type I seesaw framework. This automatically leads to a $\mu - \tau$ symmetry breaking light neutrino mass matrix due to the existence of anti-symmetric terms arising from product of two triplet representations of A_4. Although such terms vanish for right handed neutrino mass matrix due to the Majorana nature, they do not vanish in general for Dirac neutrino mass matrix. Within a minimal setup, we then compare the $\mu - \tau$ symmetry breaking light neutrino mass matrix with the one constructed from light neutrino parameters and find the model parameters, while fixing the right handed neutrino mass at 5 TeV. Since there are only four independent complex parameters of the model that can be evaluated comparing four mass matrix elements, it gives rise to two constraints due to the existence of six independent complex elements of a light neutrino mass matrix which is complex symmetric if the light neutrinos are of Majorana type. These two constraints severely restrict the allowed parameter space to a narrow range, which we evaluate numerically by doing a random scan of ten million neutrino data points in the allowed 3σ range, for both normal and inverted hierarchical patterns of light neutrino masses. Among the unknown light neutrino parameters namely, the lightest neutrino mass, one Dirac and two Majorana CP phases, we get some interesting restrictions on some of these parameters from the requirement of satisfying the correct neutrino data within the model framework.

After finding the model and neutrino parameters consistent with the basic setup, we then feed the allowed parameters to the resonant leptogenesis formulas and calculate the baryon asymmetry of the Universe. We find that both the normal and inverted hierarchical scenarios can satisfy the Planck 2015 bound on baryon asymmetry $\eta_B = 6.04 \pm 0.08 \times 10^{-10}$ [9]. We however get more allowed points for the inverted hierarchical scenario compared to the normal one. Finally, we also briefly outline the $\mu - \tau$ symmetric limit of the model taking the approximation of vanishing anti-symmetric triplet product term and a possible way to generate non zero θ_{13} in that scenario. We however, do not perform any separate numerical calculation in this limiting scenario. We find it interesting that, just by trying to generate leptonic mixing through a non-trivial Dirac neutrino mass term automatically leads to broken $\mu - \tau$ symmetry, automatically generating non-zero θ_{13}. This is in fact a more economical way to generate the correct neutrino oscillation data than taking the usual route of generating $\mu - \tau$ symmetric mass matrix at leading order followed by some next to leading order corrections responsible for generating $\theta_{13} \neq 0$ which was the usual procedure adopted after the discovery of non-zero θ_{13} in 2012. It is also interesting that the model can naturally generate the tiny mass splitting between right handed neutrinos and generate the required baryon asymmetry through the mechanism of resonant leptogenesis. Such TeV scale seesaw scenario can also have some other interesting implications in collider as well as rare decay experiments like lepton flavour violation, details of which can be found elsewhere. Also, such a TeV scale seesaw scenario can play a non-trivial role in restoring the electroweak vacuum stability as discussed recently by the authors of [46].

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"Real museums are places where Time is transformed into spaces"

Orhan Pamuk in "The Museum of Innocence"

5

Neutrino phenomenology with S_4 flavor symmetry in inverse and type II seesaw

In this chapter we have exercised an inverse seesaw model based on the S_4 flavor symmetry with an adaptation of type II seesaw mechanism. The leading order neutrino mass is explained under the scheme of ISS, which is later on accompanied by the type II seesaw mechanism in order to reproduce non-zero reactor mixing angle. The type II seesaw perturbation at the same time yields the other oscillation parameters undeviated from their correct 3σ range. A detailed analysis has been performed by varying the Dirac Yukawa coupling and type II seesaw strength which together play a crucial role in obtaining the oscillation parameters in agreement with the recent experiments. We calculate the contribution to the effective mass governing $0\nu\beta\beta$ decay assuming it to take place through the exchange of light neutrinos.

5.1 Introduction

On 4th of July 2012, there was a milestone discovery by the ATLAS and CMS collaborations at the CERN Large Hadron Collider (LHC), of the last missing entity of the standard model of particle physics i.e., the Higgs Boson which is electrically neutral and said to have a mass around 125 GeV. Till date this discovery seems to complete the menagerie of the particles of the SM and can address many observable phenomena of the SM. In addition this discovery also gave a concrete explanation of the elementary particles getting masses by interacting with the Higgs Boson. However the SM in spite of this new discovery is unable to address many issues. Existence of tiny neutrino mass and its large mixing angles is one such observation which the SM is unable to account for.

There has been several theoretical models proposed so far, in order to investigate the neutrino oscillation parameters in detail. After the announcement made by T2K [1] and RENO [2] and Daya Bay [3] the reactor mixing angle also gained very much interest in recent years. In this context a class of particle physics models have been suggested explaining the origin of non-zero reactor angle at the same time keeping other neutrino oscillation parameters consistent with experiments. In the neutrino sector also there are several issues which are yet to be addressed like which hierarchy pattern the neutrino mass follows, then the octant of the atmospheric mixing angle and finally, whether neutrino is a Dirac or Majorana particle. Neutrino less double beta decay is considered as the most profound evidence in support of the Majorana nature of the neutrino. All of these puzzles motivate us to extend the SM particle sector which we generally call as a SM extension.

As reported by many observations and experiments which are dedicated to neutrino oscillation phenomena, it is a well established fact that neutrinos are massive, however small there mass is. This fact demands a justification from the point of theoretical model building. We know that for the explanation of non zero neutrino mass one needs to go BSM. This journey starts with inclusion of SM scalar and fermion sector. The fermion sector is extended with a number of right handed gauge singlets which after coupling with left handed lepton doublets and SM Higgs, seem to generate neutrino mass. This whole scenario is mathematically realized via the implementation of Weinberg's Dimension 5 operator [4, 5], which beautifully explains the nonzero neutrino mass without a fine tuning of the Yukawa coupling. Inclusion of RH gauge singlets or RH neutrinos (RHN) in the standard model helps to explain the tiny neutrino mass with the help of Weinberg's operator in the frame work of type I seesaw mechanism. Generally type I seesaw indicates the RHN mass scale to be around 10^{15} to 10^{16} GeV, which is quite high in comparison to the present accessible energy status of the LHC and other particle accelerators. In this situation inverse seesaw (ISS) mechanism gives a road map connecting light neutrino mass to such a RHN mass scale which may be accessible at the colliders in near future. In this work we have studied an ISS scheme described by the authors in [6]. The ISS presents the explanation for sub-eV neutrino mass by means of keeping the RHN mass around few TeV. We also adopt type II seesaw mechanism in order to reproduce non-vanishing reactor angle. With this motivation of studying the oscillation parameters we proceed towards building an ISS and type II based model adopting the S_4 flavor symmetry group.

Having set the stage with so many RHNs it may be a ground for the search of Neutrinoless double decay (NDBD) which crucially involves RHNs. NDBD process is a distinctive probe for the determination of Majorana nature of neutrinos. This is a process which has non-trivial implications for particle physics and cosmology, although its observation still remained elusive. The search for this process constitutes another new province of neutrino physics permitting us to look for possible CP violation effects in lepton number violating processes. With this motivation in the light of the presented model we have also studied the effective mass prediction to neutrinoless double beta decay assuming it to take place via to light neutrino exchanges.

This chapter is organized as follows. In Section. 5.2 we present the ISS model with type II seesaw. Section. 5.3 has been dedicated for neutrinoless double beta decay. In Section. 5.4 is kept for numerical analysis. We discuss the results in Section. 5.5. Finally, in Section. 5.6 we end up with our conclusion.

5.2 Structure of the model

As already mentioned in the earlier chapters the ISS scheme manifests in addressing a sub-eV neutrino mass scale at the cost of proposing the RHN mass at a scale much lower than that associated with the Type I seesaw [7–12]. If we recall the ISS Lagrangian it goes like as follows.

$$\mathcal{L} = -\bar{\nu}_L m_D \nu_R - \bar{S}_L M \nu_R - \frac{1}{2}\bar{S}_L \mu S_L^C + \mathrm{H.}C.. \tag{5.2.1}$$

The ISS model contains a pair of two gauge singlet leptons, ν_R and S respectively. μ is the lepton number violating scale here, which plays a non-trivial role in this seesaw scheme. The fact that if μ takes a zero value, then the lepton number symmetry is preserved , leading to a vanishing light neutrino mass. This fact demands a nonzero μ scale to bring a non-zero light neutrino mass. The scale of μ adds a new dimension in the mass generation mechanism of light neutrinos via accommodating a TeV scale RHN. This feature makes the ISS unique among all the seesaw schemes. Essentially the scale of the lepton number violating parameter μ is of order keV for an electro-weak scale Dirac neutrino mass m_D. In this scheme the effective light neutrino mass is obtained from the following equation.

$$m_\nu^I = m_D^T (M^T)^{-1} \mu M^{-1} m_D, \tag{5.2.2}$$

In this work Eq. (5.2.2) decides the structure of the leading order light neutrino mass. To implement the ISS scheme and to get a desired light neutrino mass structure we extend the SM scalar structure with the inclusion of five flavons which transform as singlets and one additional Higgs η transforming as doublet under $SU(2)$ symmetry group. Three right handed gauge singlets ν_R are introduced, which are supposed to transform as a triplet 3_1 of S_4. We assign the SM type Higgs η to the triplet 3_1 of S_4. The additional three SM fermion singlets 'S_i' are assumed to transform as an S_4 triplet 3_1. The charge assignments of the particle content of the model is presented in Table 5.1.

The Yukawa Lagrangian relevant for the above particle content is given by,

$$\mathcal{L}^I = y_D \bar{L} \nu_R \eta + {y_D}' \bar{L} \nu_R h + y_M \nu_R S \Phi_R + y_s S S \Phi_s, \tag{5.2.3}$$

	$\bar{L}$	ν_R	l_R	h	η	S	Φ_R	Φ_s	Φ_l	$\Phi_l{}'$	$\Phi_l{}''$
$SU(2)_L$	2	1	1	2	2	1	1	1	1	1	1
S_4	3_1	3_1	3_1	1_1	3_1	3_1	1_1	1_1	3_1	3_2	1_1
Z_2	+	-	+	-	-	-	+	+	+	+	+

Table 5.1: Fields and their transformation properties under $SU(2)_L$, the S_4 flavor symmetry and Z_2 flavor symmetry

The following flavon alignments help us to get a desired neutrino mass matrix.

$$\langle\Phi_R\rangle = v_R,\ \langle\Phi_s\rangle = v_s,\ \langle h\rangle = v_h,\ \langle\eta\rangle = v_\eta(1,0,0).$$

Decomposition of the various terms present in the Eq. (5.2.3) into singlets following the S_4 rules mentioned in Section 1.8 of Chapter 1 (for detail please see [13]), has been shown as follows

$$\begin{aligned}
y_D\bar{L}_i\nu_{jR}\eta &= y_D[(L_2\nu_{3R}+L_3\nu_{2R})\eta_1+(L_1\nu_{3R}+L_3\nu_{1R})\eta_2+(L_2\nu_{1R}+L_1\nu_{2R})\eta_3]\\
&= y_D(L_2\nu_{3R}+L_3\nu_{2R})v_\eta,\\
y_D{}'\bar{L}_i\nu_{jR}h &= y_D{}'(L_1\nu_{1R}+L_2\nu_{2R}+L_3\nu_{3R})v_h,
\end{aligned}$$

$$\begin{aligned}
y_M\nu_{iR}S_j\Phi_R &= y_M(S_1\nu_{1R}+S_2\nu_{2R}+S_3\nu_{3R})v_R,\\
y_sSS\Phi_s &= y_s(S_1S_1+S_2S_2+S_3S_3)v_s.
\end{aligned}$$

With the help of the S_4 product rules and using the chosen flavon alignments mentioned above we can design the following mass matrices.

$$m_D = y\begin{pmatrix} v_h & 0 & 0\\ 0 & v_h & v_\eta\\ 0 & v_\eta & v_h\end{pmatrix},\mu = y_s\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}v_s, M = y_R\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}v_R. \tag{5.2.4}$$

We notice that, m_D is connected to v_η and v_h, and M is determined by the VEV v_R. In this way, the order of magnitude involved in the Eq. 5.2.2 is such that, $m_\nu \propto \frac{(v_\eta+v_h)^2}{v_R^2}\mu$. Here v_η and v_h are of the order of electroweak breaking, v_R is of the order of TeV scale. Therefore, to get m_ν in sub-eV, μ which is coming from the VEV of Φ_S should be of the order of keV.

Type II seesaw with triplet Higgs

We have chosen type II seesaw for reproducing non-zero θ_{13}. For the implementation of type II seesaw mechanism the SM is extended by the inclusion of an additional $SU(2)_L$ triplet scalar field Δ. Apart having bilinear and quartic terms the triplet also has a trilinear mass term $\mu_{\phi\Delta}$ which generates an induced VEV for the neutral component of the Higgs triplet as $\Delta^0 = v_\Delta\sqrt{2}$ where, $v_\Delta \simeq \mu_{\phi\Delta} v^2/\sqrt{2}M_\Delta^2$ [14–18]. The type II seesaw contribution to light neutrino mass is given by

$$m_{LL}^{II} = f_\nu v_\Delta, \tag{5.2.5}$$

In the low scale type II seesaw mechanism operative at the TeV scale, barring the naturalness issue, one can consider a very small value of the trilinear mass parameter to be

$$\mu_{\phi\Delta} \simeq 10^{-8} GeV.$$

The sub-eV scale light neutrino mass with type II seesaw mechanism constrains the corresponding Majorana Yukawa coupling as

$$f_\nu^2 < 1.4 \times 10^{-5} (\frac{M_\Delta}{1TeV}).$$

Within the reasonable value of $f_\nu \simeq 10^{-2}$, the triplet Higgs scalar VEV is $v_\Delta \simeq 10^{-7} GeV$ which is in agreement with oscillation data. The Yukawa Lagrangian for the type II seesaw part is given by,

$$\mathcal{L}^{II} = f_\nu \frac{LL\zeta\triangle}{\Lambda} + f_\nu \frac{LL\xi\triangle}{\Lambda} \tag{5.2.6}$$

Where, Λ is the cutoff scale. With the type II perturbation the Lagrangian takes the following form

$$\mathcal{L} = y_D \bar{L}\nu_R\eta + y_D{}'\bar{L}\nu_R h + y_M \nu_R S\phi_R + y_s SS\phi_s + f_\nu \frac{LL\zeta\triangle}{\Lambda} + f_\nu \frac{LL\xi\triangle}{\Lambda}. \tag{5.2.7}$$

The first four terms of the above equation are considered to be the leading order contribution, and the last two terms are for the perturbation to generate non-zero θ_{13}. As summarized in the Table 5.1 the $SU(2)_L$ lepton doublets are supposed to transform as S_4 triplets. The $SU(2)_L$ triplet Higgs field Δ_L is supposed to

transform as a S_4 singlet. We have introduced two more flavon fields ξ and ζ which are assumed to transform as 2 and 3_1 of S_4 respectively.
The decomposition of the $\frac{LL\zeta\Delta}{\Lambda}$ term into S_4 singlet with the multiplication rules can be shown as follows

$$LL\zeta\Delta = (L_2L'_2 - L_3L'^{)}_3\zeta_1\Delta$$
$$3_1 \times 3_1 \sim 2$$
$$2 \times 2 \sim 1$$
$$1 \times 1 \sim 1$$

The decomposition of the $\frac{LL\xi\Delta}{\Lambda}$ term into S_4 singlet with the multiplication rules is given by

$$LL\xi\Delta = (-L_2L_1 - L_1L_2 + L_1L_3 + L_3L_1)v_\xi\Delta$$
$$3_1 \times 3_1 \sim 3_1$$
$$3_1 \times 3_1 \sim 1$$

The flavon alignments which help in constructing the m_{LL}^{II} matrix are as follows

$$\Delta \sim v_\Delta, \; <\zeta>\sim \sqrt{2}v_\zeta(1,0), \; <\xi>\sim v_\xi(0,1,-1).$$

ζ and ξ are assumed to take the VEV in the same scale $v_\zeta = v_\xi = \Lambda$. With these flavon alignments the following structure for the type II seesaw mass matrix m_{LL}^{II} is constructed.

$$m_{LL}^{II} = \begin{pmatrix} 0 & -w & w \\ -w & w & 0 \\ w & 0 & -w \end{pmatrix}. \tag{5.2.8}$$

The three matrices (5.2.4) lead to the following leading order light neutrino mass matrix under the ISS framework.

$$m_\nu = y^2 U_\nu m_0^{\text{diag}} U_\nu^T, \tag{5.2.9}$$

where, m_0 is a non-diagonal matrix given by Eq. (5.4.1). The two Yukawa couplings are supposed to have the same numerical value, $y_D = {y_D}' = y$, which governs the interactions shown by the first two terms in the Eq. (5.2.3). Now the Lagrangian for the charged lepton mass is ,

$$\mathcal{L}^l = y_l \bar{L} l_R \Phi_l + y_l' \bar{L} l_R \Phi_l' + y_l'' \bar{L} l_R \Phi_l'', \tag{5.2.10}$$

The following flavon alignments allow us to have the mass matrix corresponding to the charged lepton sector as given by the Eq. (5.2.11).

$$< \Phi_l >= v_l(1,1,1), \ < \Phi_l' >= v_l'(1,1,1), \ < \Phi_l'' >= v_l''$$

S_4 product rules and the chosen vev alignments yield the charged lepton mass matrix as follows,

$$m_l = \begin{pmatrix} y_l'' v_l'' & y_l v_l - y_l' v_l' & y_l v_l + y_l' v_l' \\ y_l v_l + y_l' v_l' & y_l'' v_l'' & y_l v_l - y_l' v_l' \\ y_l v_l - y_l' v_l' & y_l v_l + y_l' v_l' & y_l'' v_l'' \end{pmatrix}, \tag{5.2.11}$$

As the charge lepton matrix in non-diagonal, the charge lepton mass matrix m_l is diagonalized by the magic matrix U_ω exhibited in the following equation.

$$U_\omega = 1/\sqrt{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \tag{5.2.12}$$

(with $\omega = \exp 2i\pi/3$). Considering up to leading order ISS, one can write,

$$U_{TBM} = U_l^\dagger U_\nu.$$

where, U_l corresponds to the identity matrix if the charged lepton mass matrix is diagonal. Since in our work, the charged lepton mass matrix is non-diagonal, U_l is nothing but the magic matrix U_ω given by Eq. (5.2.12). Now in the basis, where charged lepton is diagonal

$$U_\nu \to U_{\mathrm{TBM}} = U_\omega^\dagger U_\nu,$$

The Eq. (5.2.9) implies that,

$$U_\omega m_\nu U_\omega^{-1} = y^2 U_\omega U_\nu m_o^{\mathrm{diag}} U_\nu^T U_\omega^{-1} \implies m_\nu^{\mathrm{TBM}} = y^2 U_{\mathrm{TBM}} m_o^{diag} U_{\mathrm{TBM}}^T. \tag{5.2.13}$$

5.3 Neutrinoless double beta decay

Neutrinoless double beta decay (NDBD) is a process where two protons are converted into two electrons with no neutrinos in the final state, leading to the violation of lepton number by two units.

$$N(A,Z) \to N(A,Z+2) + e^- + e^-.$$

This violation of lepton number makes this process a strong probe of Majorana neutrinos [19–22]. The time period of NDBD is dependent on the effective mass m_ν^{ee} as we have already mentioned in Chapter 2. If we recall the effective mass formula it is given by,

$$|m_\nu^{ee}| = |U_{ei}^2 m_i|, \tag{5.3.1}$$

There may have several contributions [23, 24] that influence the effective mass prediction. In this work since we have considered the ISS mechanism to explain tiny neutrino mass, only relevant contribution will come from the process occurred due to SM light neutrino exchanges. On the other hand the contribution from the triplet Higgs is of the order of $10^{-13} m_i$ which is relatively concealed as compared to the dominant contribution [25]. One can determine the effective neutrino mass from the following expression given by,

$$m_{\nu,LL}^{ee} \simeq U_{e1}^2 m_1 + U_{e2}^2 e^{2i\alpha} m_2 + U_{e3}^2 e^{2i\beta} m_3. \tag{5.3.2}$$

From the effective mass formula it is clear that $m_{\nu,LL}^{ee}$ solely depends on the matrix elements of the first row of the $U_{\rm PMNS}$ mixing matrix and the light neutrino mass eigenvalues. The matrix elements are functions of the neutrino mixing angles. Therefore, $m_{\nu,LL}^{ee}$ for the present model under discussion is dependent on the predictions regarding the oscillation parameters, made by the model. At the same time the light neutrino mass eigenvalues are different for different mass hierarchy patterns, normal and/or inverted. This fact clearly indicates that, the effective mass will be different for different mass hierarchy patterns. We evince the plots for effective mass prediction due to SM light neutrino exchanges in figure 5.9.

5.4 Numerical analysis

The global fit neutrino oscillation data used for this work is taken from [26], which is exhibited in Table 5.2.

Oscillation parameters	bfp(NO)	3σ Cl(NO)	bfp(IO)	3σ Cl(IO)
$\Delta m^2_{21}[10^{-5}eV^2]$	7.5	(7.02 , 8.07)	7.5	(7.02 , 8.07)
$\Delta m^2_{31}/\Delta m^2_{23}(NO/IO)[10^{-3}eV^2]$	2.457	(2.317 , 2.607)	-2.449	(-2.590 , -2.307)
$\sin^2\theta_{12}$	0.304	(0.270 , 0.344)	0.304	(0.270 , 0.34)
$\sin^2\theta_{13}$	0.0218	(0.0186 , 0.0250)	0.0219	(0.0188 , 0.0251)
$\sin^2\theta_{23}$	–	0.381-0.643	–	(0.388 , 0.644)

Table 5.2: Gobal fit oscillation data from reference [26]

Each value of y (which is present in the m_D) gives rise to various sets of the neutrino mass matrix parameters a, b.

As shown in [6] that the ISS mechanism also yields some potential way to obtain TBM mixing pattern. In the present analysis, we consider $M \propto I$, $\mu \propto I$ and $m_D \propto M_0$. These three matrices give rise to neutrino mass matrix which is of TBM pattern, that naturally accounts for vanishing θ_{13}. We parameterize the light neutrino mass matrix obtained from the ISS realization with the help of recent neutrino oscillation data given in Table 5.2. Using Eq. (5.2.4) the light neutrino mass matrix is found to be

$$M_\nu^0 = y^2 \frac{y_s v_v}{y_R^2 v_R^2} \begin{pmatrix} v_h^2 & 0 & 0 \\ 0 & v_h^2 + v_\eta^2 & 2v_h v_\eta \\ 0 & 2v_h v_\eta & v_h^2 + v_\eta^2 \end{pmatrix}$$

Now, if we define some parameters $a = \frac{\sqrt{y_s v_s}}{y_R v_R} v_h$ and $b = \frac{\sqrt{y_s v_s}}{y_R v_R} v_\eta$ the light neutrino mass matrix can take the following form given by Eq. (5.2.11).

$$M_\nu^0 = y^2 \begin{pmatrix} a^2 & 0 & 0 \\ 0 & a^2 + b^2 & 2ab \\ 0 & 2ab & a^2 + b^2 \end{pmatrix} = y^2 m_o, \tag{5.4.1}$$

then we solve for a and b with the help of two mass squared splittings taken from the global fit oscillation data. While finding the solutions for a and b each time we vary the Yukawa coupling y present in Eq. (5.2.9). For each Yukawa value we get different sets of solutions for a and b. Choosing each set of a, b values and that particular Yukawa coupling chosen for a particular set we get sets of light neutrino mass matrices. Once we get these light neutrino mass matrix, to this we add the type II perturbation matrix to reproduce nonzero θ_{13}. After adding the perturbation we get the neutrino mass matrix as follows,

$$M_\nu = M_\nu^0 + m_\nu^{II} = y^2 U_{TBM} m_\nu^{diag} U_{TBM}^T + m_\nu^{II}.$$

The numerical value of the perturbation term $w = f_\nu v_\Delta$ critically depends upon the Majorana coupling f_ν, trilinear mass parameter $\mu\phi\Delta$ and M. Accordingly, we vary the type II seesaw strength from 10^{-6} to 0.01 eV to produce non-zero θ_{13}. It is observed from the figures 5.1 and 5.2 that the type II seesaw strength of 10^{-3} eV is generating the non-zero θ_{13} in the 3σ range of all the cases. After getting the complete mass matrix we diagonalize it. After diagonalization the mass eigenvalues are found to be $m_1 = y^2 a^2$, $m_2 = y^2(a+b)^2$, $m_3 = y^2(a-b)^2$. Now the elements of these diagonalized matrices are associated with the parameters of the model and the type II perturbation term. The set of a, b values obtained for each y value and chosen for analysis are listed in Table 5.3, Table 5.4, Table 5.5 for NH case and Table 5.6, Table 5.7, Table 5.8 for IH case.

Parameters	$y = 0.99$	$y = 0.992$	$y = 0.994$	$y = 0.996$	$y = 0.998$	$y = 1$
a	0.0633626	0.0632349	0.0631076	0.0624	0.0622749	0.062729
b	0.161879	0.161552	0.161227	0.16017	0.159849	0.16026

Table 5.3: Values of a, b obtained by solving for NH case with best fit central value of 3σ deviations

Parameters	$y = 0.99$	$y = 0.992$	$y = 0.994$	$y = 0.996$	$y = 0.998$	$y = 1$
a	0.0641786	0.0640492	0.0639203	0.063792	0.0636641	0.0635368
b	0.164422	0.16409	0.16376	0.163431	0.163104	0.162777

Table 5.4: Values of a, b obtained by solving for NH case with a upper bound of 3σ deviations

Parameters	$y = 0.99$	$y = 0.992$	$y = 0.994$	$y = 0.996$	$y = 0.998$	$y = 1$
a	0.0625069	0.0623809	0.0622554	0.0621304	0.0620059	0.0618818
b	0.159456	0.159135	0.158815	0.158496	0.158178	0.157862

Table 5.5: Values of a, b obtained by solving for NH case with an lower bound of 3σ deviations

Parameters	$y = 0.99$	$y = 0.992$	$y = 0.994$	$y = 0.996$	$y = 0.998$	$y = 1$
a	0.0640348	0.0639057	0.0637771	0.063649	0.0635215	0.0633944
b	0.162732	0.162404	0.162077	0.161752	0.161428	0.161105

Table 5.6: Values of a, b obtained by solving for IH case with best fit central value of 3σ deviations

Parameters	$y = 0.99$	$y = 0.992$	$y = 0.994$	$y = 0.996$	$y = 0.998$	$y = 1$
a	0.0647916	0.064661	0.0645309	0.0644013	0.0642722	0.0641437
b	0.165197	0.164864	0.164533	0.164202	0.163873	0.163545

Table 5.7: Values of a, b obtained by solving for IH case with a upper bound of 3σ deviations

Parameters	$y = 0.99$	$y = 0.992$	$y = 0.994$	$y = 0.996$	$y = 0.998$	$y = 1$
a	0.0631378	0.0630105	0.0628838	0.0627575	0.0626317	0.0625065
b	0.160259	0.159935	0.159614	0.159293	0.158975	0.158656

Table 5.8: Values of a, b obtained by solving for IH case with an lower bound of 3σ deviations

5.5 Results and discussions

A thorough analysis has been carried out to check whether the oscillation parameters are near to reach or not by taking the upper and lower bound of 3σ deviation as well. We fit the values of oscillation parameters using recent cosmological constraints for both normal and inverted mass ordering. We compute all the oscillation parameters also by varying the type II seesaw strength. Variation of type II seesaw strength with the non-vanishing θ_{13}, for both hierarchy patterns have been shown in figure 5.1 using the best fit values and figure 5.2 for the extremum bounds of 3σ deviations. The production of other oscillation parameters, e.g. the two mixing angles and two mass squared splitting as a function of nonzero θ_{13} has been shown in the figure 5.3, figure 5.4 and figure 5.5 for NH case, figure 5.6, figure 5.7 and figure 5.8 for IH case for different values of Yukawa coupling. The sum of absolute masses has also been calculated to see whether it satisfies the Planck upper bound or not. Seeing that, the sum of absolute neutrino masses can give some clue on neutrinoless double beta decay, a little study has been performed to check whether the presented model is able to contribute to the $0\nu\beta\beta$ physics. In figure 5.9 we plot for the contribution of the effective mass to $0\nu\beta\beta$ decay due to light neutrino exchanges for standard contribution. Also the effective mass prediction has been studied varying the type II strength. A comparison among the various sets of results has been shown in the Table 5.9. From the results obtained as clear from the plots 5.1-5.8, we can get the following observations.

- The Yukawa coupling is varied from 0.992 to 1 which is demanded by the neutrino parameters to being in their allowed 3σ range.
- The non-zero value of θ_{13} has been found to be consistent with the variation of type II seesaw strength for both kinds of hierarchy patterns.
- The proposed model is able to evince a good neutrino phenomenology within the NH framework while taking into consideration the lower bound of 3σ deviation. All the oscillation parameters have been obtained within the correct 3σ range for any value of Yukawa coupling ranging from 0.992

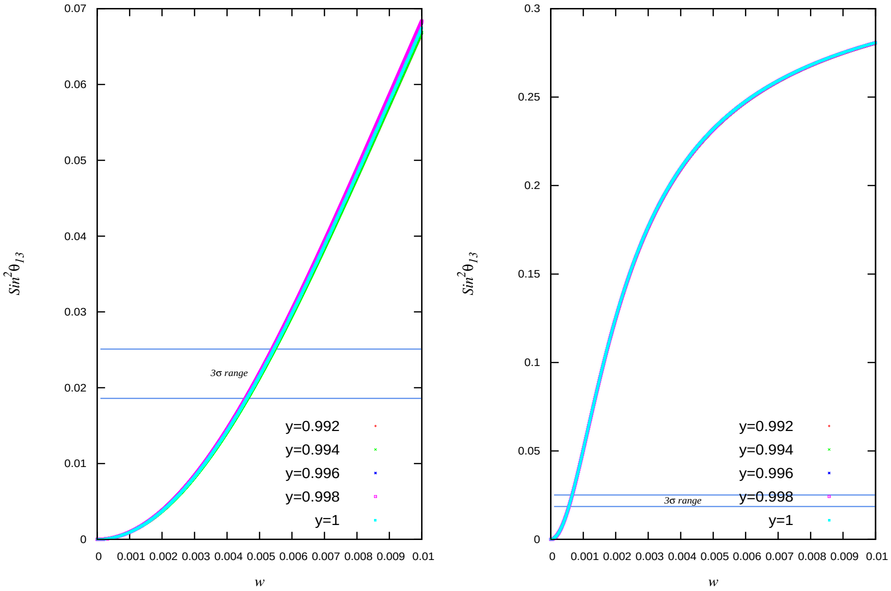

Figure 5.1: Generation of non-zero $sin^2\theta_{13}$, varying the type II strength for best fit mass squared splittings for NH (left panel) and IH (right panel) case.

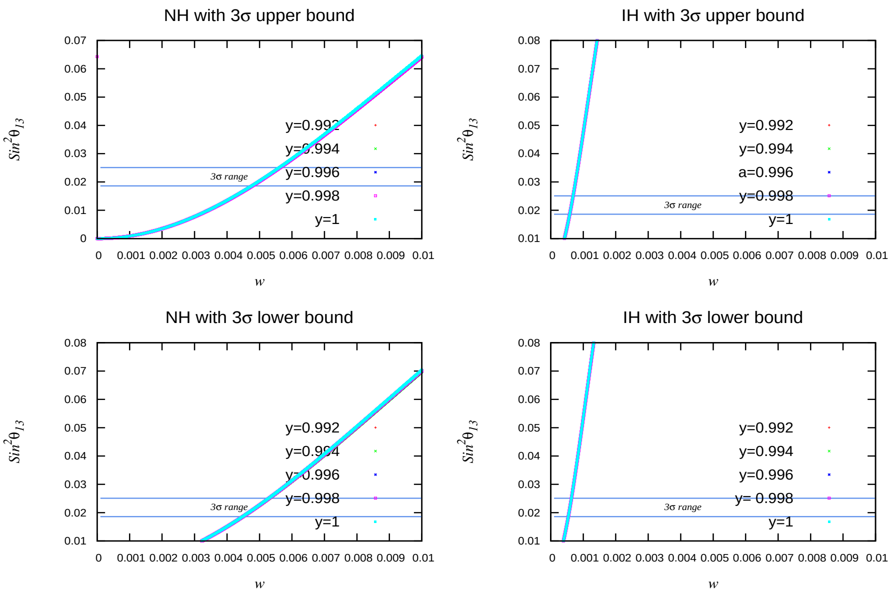

Figure 5.2: Generation of non-zero $sin^2\theta_{13}$, varying the type II strength using lower and upper bound of 3σ deviations.

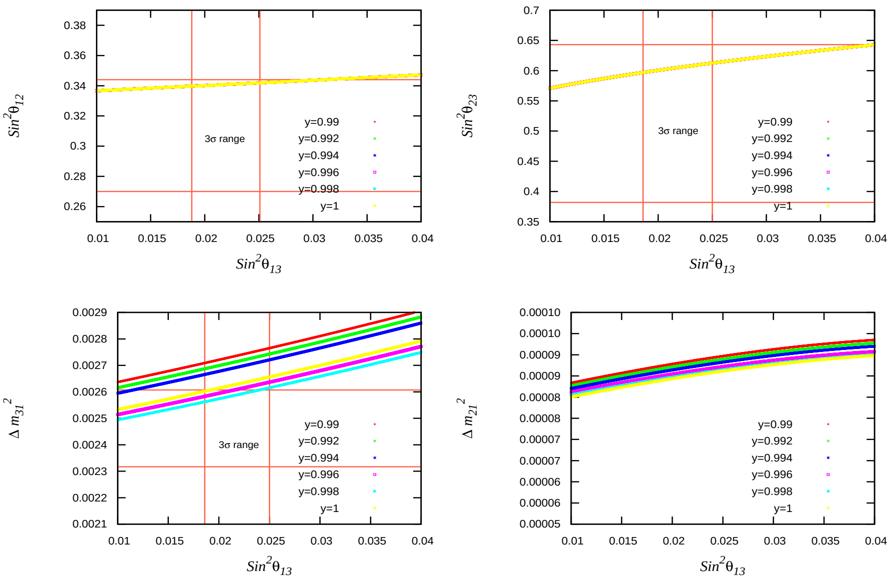

Figure 5.3: Variation of $sin^2\theta_{12}$, $sin^2\theta_{23}$,Δm_{31}^2 and Δm_{21}^2 with $sin^2\theta_{13}$ for NH case with best fit values

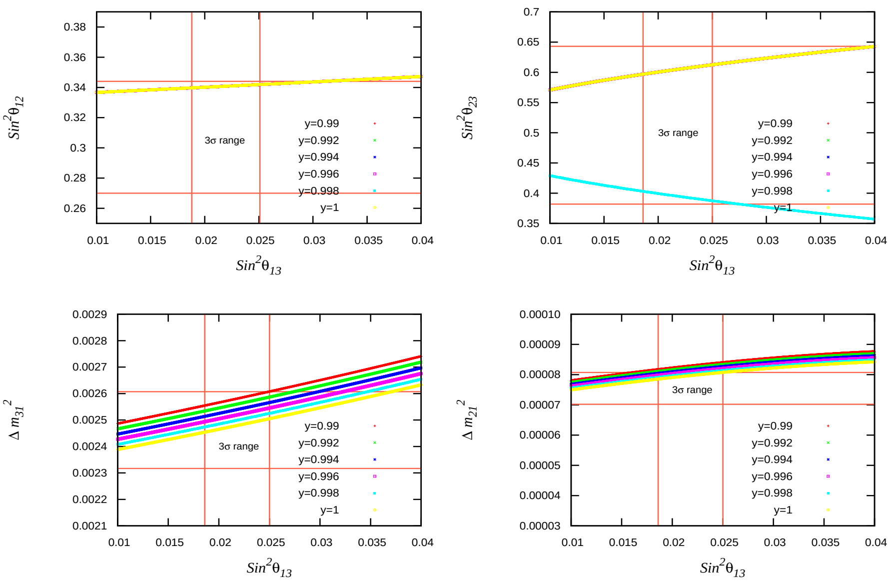

Figure 5.4: Variation of $sin^2\theta_{12}$, $sin^2\theta_{23}$,Δm_{31}^2 and Δm_{21}^2 with $sin^2\theta_{13}$ for NH case with lower bound of 3σ deviation

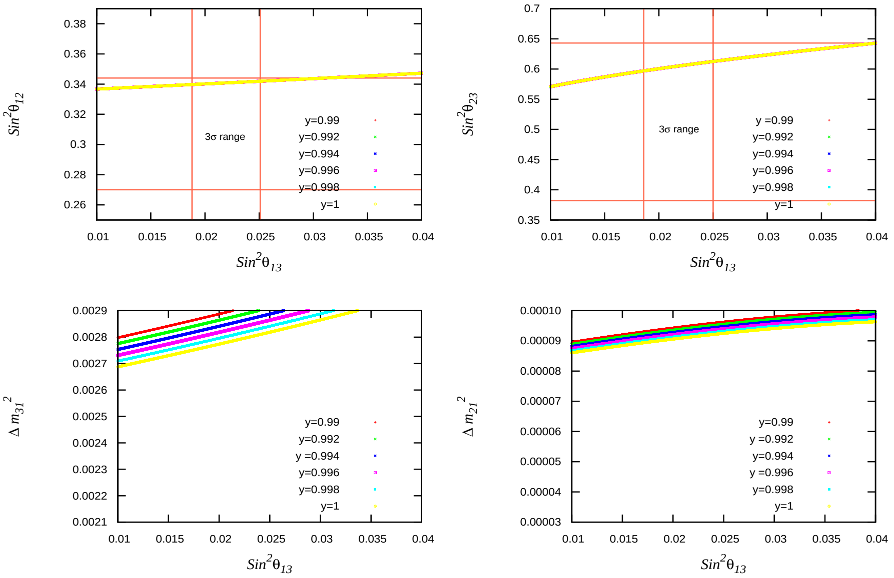

Figure 5.5: Variation of $sin^2\theta_{12}$, $sin^2\theta_{23}$,Δm^2_{31} and Δm^2_{21} with $sin^2\theta_{13}$ for NH case with with upper bound of 3σ deviation

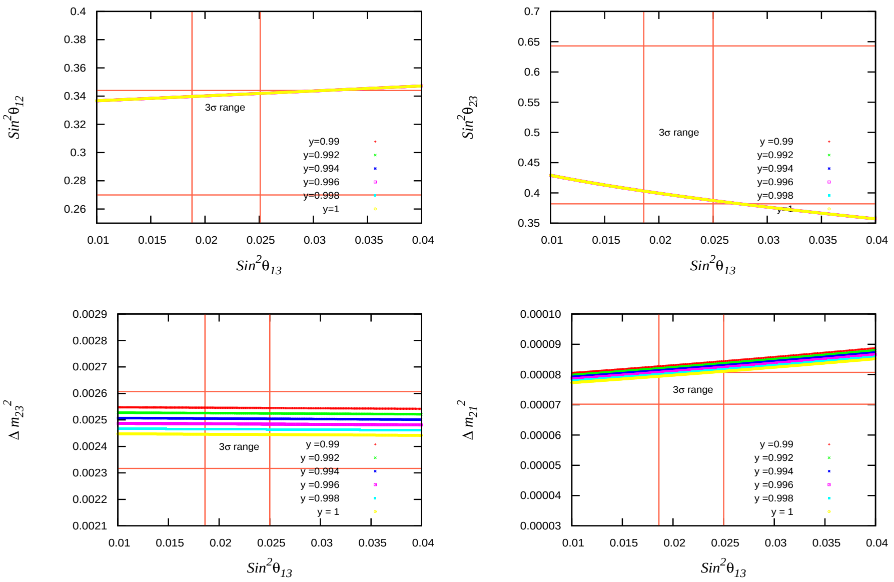

Figure 5.6: Variation of $sin^2\theta_{12}$, $sin^2\theta_{23}$,Δm^2_{23} and Δm^2_{21} with $sin^2\theta_{13}$ for IH case with best fit value

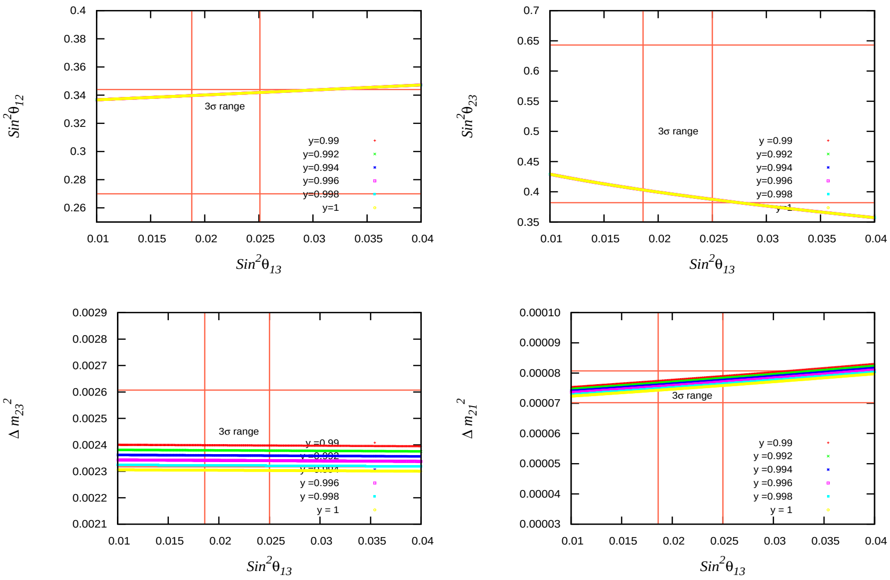

Figure 5.7: Variation of $sin^2\theta_{12}$, $sin^2\theta_{23}$,Δm^2_{23} and Δm^2_{21} with $sin^2\theta_{13}$ for IH case with lower bound of 3σ deviation

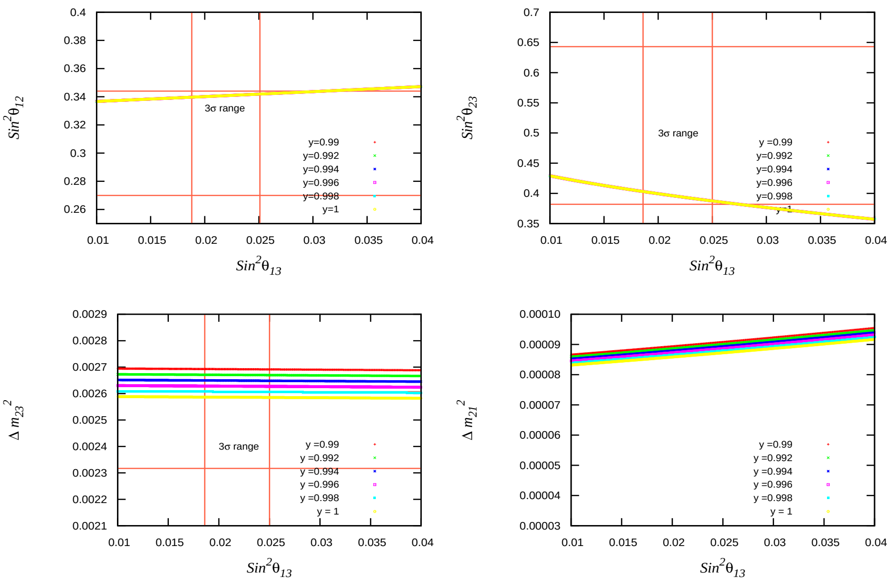

Figure 5.8: Variation of $sin^2\theta_{12}$, $sin^2\theta_{23}$,Δm^2_{23} and Δm^2_{21} with $sin^2\theta_{13}$ for IH case with upper bound of 3σ deviation

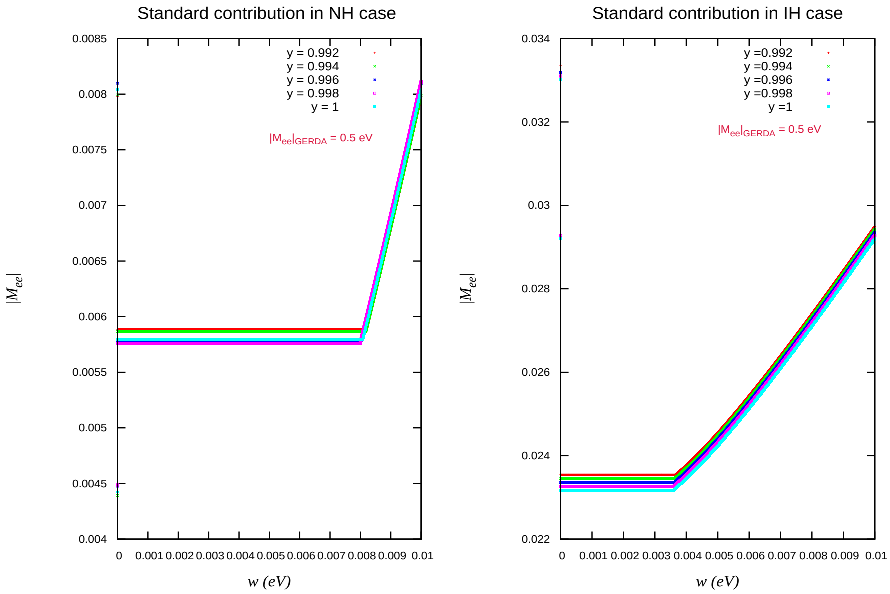

Figure 5.9: Variation of effective mass $|M_{ee}|$ (in eV)for the standard and non-standard contribution to $0\nu\beta\beta$ decay due to light neutrino exchanges [27].

to 1 for NH case. Taking the best fit value and the upper bound of 3σ deviation for NH case, the model is found to be unable to produce Δm^2_{21} and Δm^2_{31} within the correct 3σ range.

- It has been noticed that the proposed model also shows evidences for correct neutrino phenomenology using the best fit and lower 3σ bound for mass squared splittings in case of inverted hierarchy mass pattern.

- The effective mass predictions for $0\nu\beta\beta$ decay for both NH and IH are obtained in the vicinity of experimental results as shown in figure 5.9.

5.6 Conclusion

We have studied an S_4 based ISS model which is accompanied by the type II seesaw as a perturbation to the leading order ISS mass, from the need of bringing non vanishing reactor angle into account. We have chosen one ISS scheme among the seven schemes as listed by the authors in [6] and extended the study to a search for $\theta_{13} \neq 0$. The entire study has been performed from a different aspect;

Model	θ_{13}	θ_{12}	θ_{23}	Δm^2_{21}	$\Delta m^2_{31}, \Delta m^2_{23}$	$\sum_{i=1,2,3}$ mod m_i
NH (bfp)	✓	✓	✓	×	✓	✓
NH (lower 3σ bound)	✓	✓	✓	✓	✓	✓
NH (upper 3σ bound)	✓	✓	✓	×	×	✓
IH (bfp)	✓	✓	✓	✓	✓	✓
IH (lower 3σ bound)	✓	✓	✓	✓	✓	✓
IH (upper 3σ bound)	✓	✓	✓	×	✓	✓

Table 5.9: Summary of results obtained from various allowed mass schemes.

by extracting the Yukawa coupling (y) from the light neutrino mass matrix and varying it from and to a certain range, in order to check the parameter space of the Yukawa coupling strength and the global fit neutrino parameters. For NH mass pattern it is seen that only the lower bound of the mass squared splittings can give a solution for the model parameters a, b who further give rise to the other oscillation parameters in correct 3σ range for the chosen range of Yukawa coupling (y). For the same Yukawa coupling range the model prediction is more sensitive to IH mass pattern, as we obtain the oscillation parameters in agreement with experiments, while scanning the mass squared splittings from the lower bound of the 3σ bound to the best fit central value. Thus, we can conclude that a broader region of parameter space exists in case of IH. From the type II seesaw term we have the type II strength w which reproduces non-zero θ_{13} in the correct 3σ range. We have also studied the effective mass prediction for the contribution of NDBD. However, for both hierarchy pattern we get the effective mass within GERDA limit [27]. The variation of Yukawa coupling makes a better plot for a detailed study of the neutrino parameters. As we also have some complex solution for the model parameters a, b, considering them we can further go for lepton asymmetry study (by considering a non-degenerate M structure), as the complex nature of a, b may be a source of CP violation. The complex solutions of a, b almost yield the same neutrino phenomenology. Moreover, this study of variation of Yukawa coupling may have some effect on the order of lepton asymmetry that can account for the observed matter-antimatter asymmetry.

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"Begin at the beginning and go on till you come to the end: then stop"

Lewis Carroll in "Alice in Wonderland"

6

Conclusion

In this chapter we have discussed about the over all conclusion drawn from the aforementioned works exhibited in the earlier chapters which primarily deal with the study of connecting neutrino masses with modern cosmology such as origin of dark matter, and the baryon asymmetry of the universe (BAU). To explain the neutrino mass, class of seesaw mechanisms have been implemented, specially the inverse seesaw, type II seesaw and TeV scale type I seesaw. We have studied the possibility of generating non-zero reactor angle and origin of dark matter within a single frame work. We also have explored a way to reproduce origin of non-zero reactor angle and BAU inside the same setting. Effective mass prediction for NDBD has also been addressed very briefly.

We summarize the notable conclusions drawn from the present study, chapter wise in Sections **6.1 6.2, 6.3, 6.4** respectively.

6.1 Chapter 2

In **chapter 2** an A_4 based IH neutrino mass model originating from both Inverse and type II seesaw have been studied. Here ISS is implemented as a leading order contribution to the light neutrino mass matrix yielding zero reactor mixing and $m_3 = 0$. Then the type II seesaw has been used in order to produce non-Zero reactor mixing angle, which later on produces $m_3 \neq 0$ keeping the hierarchy as inverted only. We have studied the possibility of having a common parameter space where both the Neutrino oscillation parameters in the 3σ range and DM relic abundance has a better reach. With a proper choice of Yukawa coupling(y), right handed neutrino (mediator particle) mass (m_ψ) , and complex scalar (potential DM candidate) mass (m_χ) the variation in relic abundance as a function of Yukawa coupling has been shown. For a choice of Yukawa coupling between 0.994 to 0.9964, m_{DM} around 50 GeV, the mediator mass needs to fall around 153 GeV to match the correct relic abundance. The same Yukawa coupling has got a key role in generating the Neutrino oscillation parameters as well. We have studied the prospect of producing non-zero θ_{13} by introducing a perturbation to the light neutrino mass matrix using type II seesaw within the A_4 model. We have also determined the strength of the type II seesaw term which is responsible for the generation of non-zero θ_{13} in the correct 3σ range. We have also checked whether the proposed model can project about neutrinoless double beta decay or not. In context to the presented model we have found a wide range of parameter space where one may have a better reach for both neutrino and dark matter sector as well.

6.2 Chapter 3

In **chapter 3** we have studied a TeV scale inverse seesaw model based on S_4 flavor symmetry which can naturally generate correct light neutrino masses with Tri-Bi-maximal type mixing at leading order. The model also predicts a neutrino mass sum rule that can further predict the value of the lightest neutrino mass, that can be tested at experiments like neutrinoless double beta decay. Since

TBM mixing has already been ruled out by the latest neutrino oscillation data, we consider two possible ways of generating non-zero θ_{13} which automatically take dark matter into account. The idea is based on the scotogenic mechanism of neutrino mass generation, where neutrino mass arises at one loop level with DM particles going inside the loop. We first give such a one loop correction to the leading order light neutrino mass matrix and numerically evaluate the model parameters from the requirement of satisfying the correct neutrino data. This however, disturbs the mass sum rule prediction of the original model. The dark matter candidate in such a case could either be a singlet neutral fermion or the neutral component of a scalar doublet, depending whichever is lighter. We also study the possibility of generating $\theta_{13} \neq 0$ by giving a correction to the charged lepton sector. Such a case is found to be more constrained from the requirement of satisfying the correct neutrino data. We find much narrower ranges of points in terms of light neutrino parameters which can bring the model predictions closer to the observed data. Consistency with light neutrino data also requires the right diagonalising matrix of charged lepton to have very small mixing angles. The DM candidate in this case is the neutral component of a scalar doublet.

We also study the predictions for neutrinoless double beta decay and found that the charged lepton correction case with inverted hierarchy is disfavored by the latest KamLAND-Zen data. The predictions for effective neutrino mass in this model is very specific and confined to a tiny region around a particular value of lightest neutrino mass. This is due to the neutrino mass sum rule which forces the lightest neutrino mass to remain within a very narrow range. We also find the allowed parameter space for scalar dark matter from the requirement of producing the correct neutrino data, ignoring the Higgs portal and gauge mediated annihilations. Such lepton portal annihilations are efficient for large Yukawa couplings or smaller mediator masses. Since the same Yukawa couplings and mediator mass go into the one loop correction for both neutrino and charged lepton mass matrix, the charged lepton correction is more favourable from lepton portal scalar DM point of view. As mentioned before, this is due to the fact that large Yukawa or small mediator mass will be able to generate sub-GeV corrections to charged lepton mass matrix more naturally than generating sub-eV corrections to light neutrino mass matrix. Also, the charged lepton correction case is much more predictive, as obvious from a much narrower region of allowed parameter space compared to the model with neutrino mass correction.

6.3 Chapter 4

In **chapter 4** we have studied an extension of the standard model by discrete flavour symmetry $A_4 \times Z_3 \times Z_2$ that can simultaneously explain the correct neutrino oscillation data and the observed baryon asymmetry of the Universe. Considering a TeV scale type I seesaw we adopt the mechanism of resonant leptogenesis to generate a lepton asymmetry through out of equilibrium CP violating decay of right handed neutrinos which later gets converted into the required baryon asymmetry through electroweak sphalerons. The field content and its transformation under the flavour symmetry are chosen in such a way that the leading order right handed neutrino mass matrix has a trivial structure giving a degenerate spectrum. The tiny splitting between the right handed neutrino masses (required for resonant leptogenesis) arises through higher dimension mass terms, naturally suppressing the splitting. Due to the trivial structure of the right handed neutrino mass matrix, the leptonic mixing arises through the non-trivial structure of the Dirac neutrino mass matrix within a type I seesaw framework. This automatically leads to a $\mu - \tau$ symmetry breaking light neutrino mass matrix due to the existence of anti-symmetric terms arising from product of two triplet representations of A_4. Although such terms vanish for right handed neutrino mass matrix due to the Majorana nature, they do not vanish in general for Dirac neutrino mass matrix. Within a minimal setup, we then compare the $\mu - \tau$ symmetry breaking light neutrino mass matrix with the one constructed from light neutrino parameters and find the model parameters, while fixing the right handed neutrino mass at 5 TeV. Since there are only four independent complex parameters of the model that can be evaluated comparing four mass matrix elements, it gives rise to two constraints due to the existence of six independent complex elements of a light neutrino mass matrix which is complex symmetric if the light neutrinos are of Majorana type. These two constraints severely restrict the allowed parameter space to a narrow range, which we evaluate numerically by doing a random scan of ten million neutrino data points in the allowed 3σ range, for both normal and inverted hierarchical patterns of light neutrino masses. Among the unknown light neutrino parameters namely, the lightest neutrino mass, one Dirac and two Majorana CP phases, we get some interesting restrictions on some of these parameters from the requirement of satisfying the correct neutrino data within the model framework. After finding the model and neutrino parameters consistent with the basic setup, we then feed the allowed parameters to the resonant leptogenesis formulas and calculate the baryon asymmetry of the Universe. We find that both the normal and inverted hierarchical scenarios can satisfy the Planck 2015 bound on baryon asymmetry $\eta_B = 6.04 \pm 0.08 \times 10^{-10}$. We however get more allowed points for the inverted hierarchical scenario compared to the normal one. Finally, we also briefly outline the $\mu - \tau$ symmetric limit of the model taking the approximation of vanishing anti-symmetric triplet product term and a possible way to generate non zero θ_{13} in that scenario. We however, do not perform any separate numerical calculation in this limiting scenario. We find it interesting that, just by trying to generate leptonic mixing through a non-trivial Dirac neutrino mass term automatically leads to broken $\mu - \tau$ symmetry, automatically generating non-zero θ_{13}. This is in fact a more economical way to generate the correct neutrino oscillation data than taking the usual route of generating $\mu - \tau$ symmetric mass matrix at leading order followed by some next to leading order corrections responsible for generating $\theta_{13} \neq 0$ which was the usual procedure adopted after the discovery of non-zero θ_{13} in 2012. It is also interesting that the model can naturally generate the tiny mass splitting between right handed neutrinos and generate the required baryon asymmetry through the mechanism of resonant leptogenesis.

6.4 Chapter 5

In **chapter 5** we have studied an S_4 based ISS model which is accompanied by the type II seesaw as a perturbation to the leading order ISS mass, from the need of bringing non vanishing reactor angle into account. The entire study has been performed from a different aspect; by extracting the Yukawa coupling (y) from the light neutrino mass matrix and varying it from and to a certain range, in order to check the parameter space of the Yukawa coupling strength and the global fit neutrino parameters. For NH mass pattern it is seen that only the lower bound of the mass squared splittings can give a solution for the model parameters a, b who further give rise to the other oscillation parameters in correct 3σ range for the chosen range of Yukawa coupling (y). For the same Yukawa coupling range the model prediction is more sensitive to IH mass pattern, as we obtain the oscillation parameters in agreement with experiments, while scanning the mass squared splittings from the lower bound of the 3σ bound to the best fit central value. Thus, we can conclude that a broader region of parameter space exists in case of IH. From the type II seesaw term we have the type II strength w which is responsible for the generation of non-zero θ_{13} in the correct 3σ range. We have also studied the effective mass prediction for the contribution of NDBD. However, for both hierarchy pattern we get the effective mass within GERDA limit. The variation of Yukawa coupling makes a better plot for a detailed study of the neutrino parameters.

6.5 Future Prospects

The class of models discussed in the present study apart the exhibited result do also have strong predictions. The models under discussion can be further studied to explore some untouched portion of neutrino mixing theory such as possible mass sum rules in each model. Below we present some of the future plans.

- We can explore viable DM phenomenology, by considering several decay channels which we have not considered here. We can further explore the models for a detailed study of Baryogenesis via leptogenesis.

- The above discussed TeV scale seesaw scenario can also have some other interesting implications in collider as well as rare decay experiments like lepton flavour violation. Also, such a TeV scale seesaw scenario can play a non-trivial role in restoring the electroweak vacuum stability.

- We can explore the possibility of leptogenesis in the TeV scale ISS models presented here, by considering a non-degenerate M structure and utilizing the complex solutions for the model parameters, considering them as sources of CP violation.

List of Publications

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1. Mukherjee, A and Das, M. K. Neutrino phenomenology and scalar Dark Matter with A_4 flavor symmetry in Inverse and type II seesaw. *Nuclear Physics B*, 913:643-663, 2016.

2. Mukherjee, A, Borah, D. and Das, M. K. Common Origin of Non-zero θ_{13} and Dark Matter in an S_4 Flavour Symmetric Model with Inverse Seesaw. *Physical Review D*, 96(1):015014, 2017.

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5. Nath, N. M., Mukharjee, A., Das, M. K., and Sarma, J. K. $xF_3(x, Q^2)$ Structure Function and Gross-Llewellyn Smith Sum Rule with Nuclear Effect and Higher Twist Correction. *Communication in Theoretical Physics*, 66:663-670, 2016.

6. Mukherjee, A, Das, M. K., and Sarma, J. K. Normal hierarchy neutrino mass model revisited with leptogenesis. **arXiv:1803.08239** (Under communication with *Nuclear Physics B*)

7. Das, P., Mukherjee, A., and Das, M. K. Active and sterile neutrino phenomenology with A_4 based minimal extended seesaw, **arXiv:1805.09231** (Under communication with *Physical Review D*)

Papers Presented in Conference/ Workshop:

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1 A. Mukherjee and M. K. Das, X^{th} **BIENNIAL CONFERENCE OF PHYSICS ACADEMY OF NORTH EAST(PANE)**, St. Anthony's College, Shillong, Meghalaya. 10-12 Nov. 2016.

2. A. Mukherjee and M. K. Das, **XXII DAE-BRNS High Energy Physics Symposium 2016**, Dept. of Physics and Astrophysics, University of Delhi, 12-16 Dec. 2016.

3. A. Mukherjee, D. Borah, and M. K. Das, **National Symposium on Particles, Detectors and Instrumentation**, Tata Institute of Fundamental Research, 4-7 October, 2017.

4 A. Mukherjee and M. K. Das, **NAHEP - National Seminar in Nuclear and High Energy Physics**, Dept. of Physics, Gauhati University, Guwahati, 30-31 march, 2018.