50 on 50-GeV Muon Collider Storage Ring

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Abstract. Two modes are being considered for a 50 on 50-GeV muon collider: one being a high-luminosity ring with broad momentum acceptance (dp/p of ∼ 0.12%, rms) and the other lower luminosity with narrow momentum acceptance (dp/p of ∼ 0.003%, rms), or Higgs Factory. To reach the design luminosities, the value of beta at collision in the two rings must be 4 cm and 14 cm, respectively. In addition, the bunch length must be held comparable to the value of the collision beta to avoid luminosity dilution due to the hour-glass effect. To assist the rf system in preventing the bunch from spreading in time, the constraint of isochronicity is also imposed on the lattice. Finally, the circumference must be kept as small as possible to minimize luminosity degradation due to muon decay. Two lattice designs will be presented which meet all of these conditions. Furthermore, the high-luminosity and Higgs Factory lattice designs have been successfully merged into one physical ring with mutual components; the only difference being a short chicane required to match dispersion and floor coordinates from one lattice into the other.

INTRODUCTION

After one μ^+ bunch and one μ^- bunch have been accelerated to collision energy, the two bunches are injected into the collider ring, which is a fixed-field storage ring. Two cases are being considered for a 50 on 50-GeV collider: a ring with broad momentum acceptance (dp/p_{rms} of $\pm.12\%$) and high luminosity, and one with a much narrower momentum acceptance (dp/p_{rms} of $\pm.003\%$) and lower luminosity. The narrow-band machine is intended to resolve the width of the Higgs mass to high precision.

The two operational modes for the 100-GeV collider require different machine optics. The following sections discuss collider lattices for both the broad momentum application and the monochromatic mode.

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DESIGN CRITERIA

Stringent criteria have been imposed on the collider lattice designs in order to attain the specified luminosities. The first and most difficult criterion to satisfy is provision of an Interaction Region (IR) with extremely low β^* values at the collision point consistant with acceptable dynamic aperture. The required β^* values for the 100-GeV collider are 4 cm for the broad momentum-width case and 14 cm for the narrow-width case. These β^* values were tailored to match the longitudinal bunch lengths in order to avoid luminosity dilution from the hour-glass effect. Final-focus designs must also provide collimators and background sweep dipoles, and other provisions for protecting the magnets and detectors from muon-decay electrons. Effective schemes have been incorporated into the current lattices.

Another difficult constraint imposed on the lattice is that of isochronicity. A high degree of isochronicity is required in order to maintain the short bunch structure without excessive rf voltage. A final criterion especially important in the lower-energy colliders, is that the ring circumference be as small as feasible in order to minimize luminosity degradation through decay of the muons. To achieve small circumference requires high fields in the bending magnets as well as a compact, high dipole packing-fraction design. (To meet the small circumference demand, 8 T poletip fields have been assumed for all superconducting magnets.)

Some of these criteria conflict with one another. For example, the small value of β^* leads to large peak beta values in the final-focus quadrupoles and correspondingly large linear chromaticities in the IR. For the high-luminosity machine, local correction of the linear part of the IR chromaticity is required to achieve adequate momentum acceptance. Efficient chromatic correction in turn requires large positive values for dispersion in the correction sextupoles. Because of the short circumference condition, high dipole packing fractions must be maintained not only in the arcs, but in the local Chromatic Correction Section (CC) as well. One consequence of the high dipole concentration in the CC is that a small momentum compaction becomes difficult to maintain because of the large number of dipoles in regions of high positive dispersion, in conflict with the need for isochronicity. Control over the momentum compaction is achieved through appropriate design of the arcs. The following sections discuss a base ring design which approaches the limit of compactness for a 50-GeV collider lattice under isochronous conditions and with strong local chromatic correction.

OVERVIEW

For the 100 GeV CoM collider, two operating modes are contemplated: a high-luminosity case with broad momentum acceptance to accommodate a beam with a $\delta p/p$ of $\pm 0.12\%$ (rms), and one with a much narrower momentum acceptance and lower luminosity for a beam with $\delta p/p$ of $\pm 0.003\%$ (rms). For the broad momentum acceptance case, β^* must be 4 cm and for the narrow momentum acceptance case, 14 cm. In either case, the bunch length must be held comparable to the value of β^*.

The 100-GeV CoM ring has a roughly racetrack design with two circular arcs separated by an experimental insertion on one side, and a utility insertion for injection, extraction, and beam scraping on the other. The experimental insertion includes the interaction region (IR) followed by a local chromatic correction section and a matching section. The chromatic correction section is optimized to correct the ring's linear chromaticity, which is almost completely generated by the low beta quadurupoles in the IR. In designs of e^+e^- colliders, it has been found that local chromatic correction of the final focus is essential [1], as was found to be the case here.

Two 100 GeV lattice designs have been made; these are described below. The design has two optics modes: one mode has a β^* value of 4 cm with small transverse and large momentum acceptance; a second mode has a β^* value of 14 cm with large transverse and small, approximately monochromatic, momentum acceptance. Both lattices were merged into one physical, highly compact ring design with a total circumference of only about 345 m. The arc modules account for only about a quarter of the ring circumference.

The Interaction Region

Because of the dynamics of the cooling process, μ^+ and μ^- emerge from the cooling stage with roughly equal emittances. Initially unequal β^*s, or elliptical beams, were explored at the collision point. From an optics standpoint, elliptical beams are more manageable and less nonlinear than round beams in the design of Interaction Regions. Using a β^* ratio of 1:4 for the horizontal to vertical (factor of 2 in the relative beam sizes), however, causes a decrease in the luminosity of a factor of 2 and this was felt to be unacceptable. Therefore, the condition of round beams at the Interaction Point (IP) has been imposed in all current collider designs.

The need for different collision modes in the 100-GeV machine led to an Interaction Region design with two optics modes: one with broad momentum acceptance (dp/p of 0.12%, rms) and a collision β of 4 cm (Fig. 1), and the other basically monochromatic (dp/p of 0.003%, rms) and a larger collision β of 14 cm (Fig. 2). The low beta function values at the IP are mainly produced by three strong superconducting quadrupoles in the Final Focus Telescope (FFT) with pole-tip fields of 8 T. Because of significant, large-angle backgrounds from muon decay, a background-sweep dipole is included in the final-focus telescope and placed near the IP to protect the detector and the low-β quadrupoles [2]. It was found that this sweep dipole, 2.5 m long with an 8 T field, provides sufficient background suppression. The first quadrupole is located 5 m away from the interaction point, and the beta functions reach a maximum value of 1.5 km in the final focus telescope, when the maxima of the beta functions in both planes are equalized. For this maximum beta value, the quadrupole apertures must be at least 11 cm in radius to accommodate

$5\,\sigma$ of a 90 π mm mrad, 50-GeV muon beam (normalized rms emittance) plus a 2 to 3 cm thick tungsten liner [3]. The natural chromaticity of this interaction region is about -60.

The proximity of the final-focus quadrupoles to the IP determines the maximum beta and this value combined with the quadrupole strengths and lengths determine the natural chromaticity and, ultimately, the nonlinear behavior of the lattice. With poletip fields reaching 8T, the final-focus triplet in the 100-GeV collider remains short: quadrupole lengths range from .6 to 1.5 m. With such short quadrupoles, the peak beam size in the 100-GeV machine and, therefore, the natural chromaticity of its interaction region is almost completely a property of the IP to quadrupole spacing.

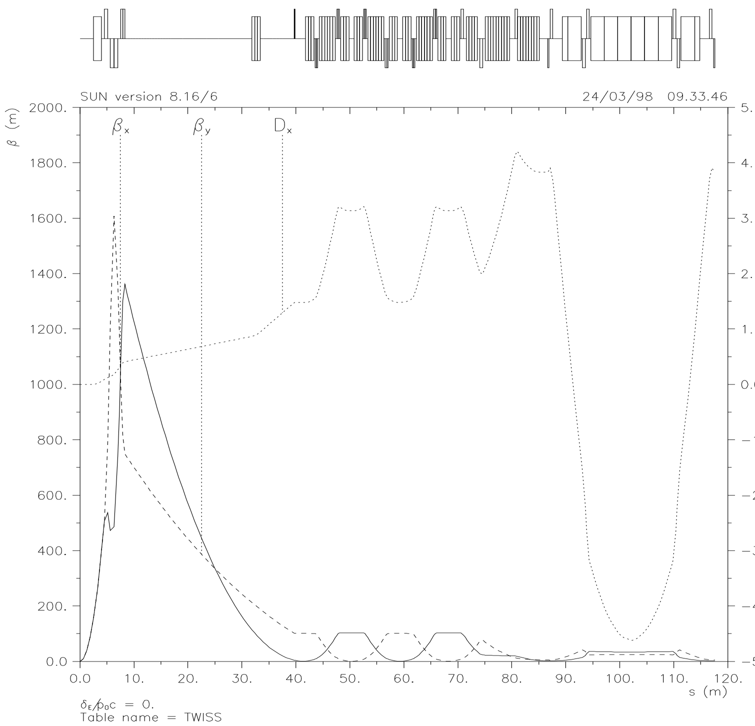

FIGURE 1. 4 cm β^* Mode showing half of the IR, local, chromatic correction, and one of three arc modules.

The optimum design of a very low-beta IR is to make the imaging as point to parallel as is practical to soften chromatic aberrations. The less the applied chromatic correction, the larger, in general, is the dynamic aperture. In the 100-GeV machine, circumference constraints require the IP to be imaged in a short distance; implying stronger than optimal focussing from the high-beta triplet. The IP image distance can be reduced by as much as 35 meters on either side of the IP; or about a 30% decrease in the ring circumference. The stronger quadrupole strengths do increase the linear chromaticity of the IR from about 60 to 85 in the vertical with little effect on the horizontal (assuming the triplet powering is

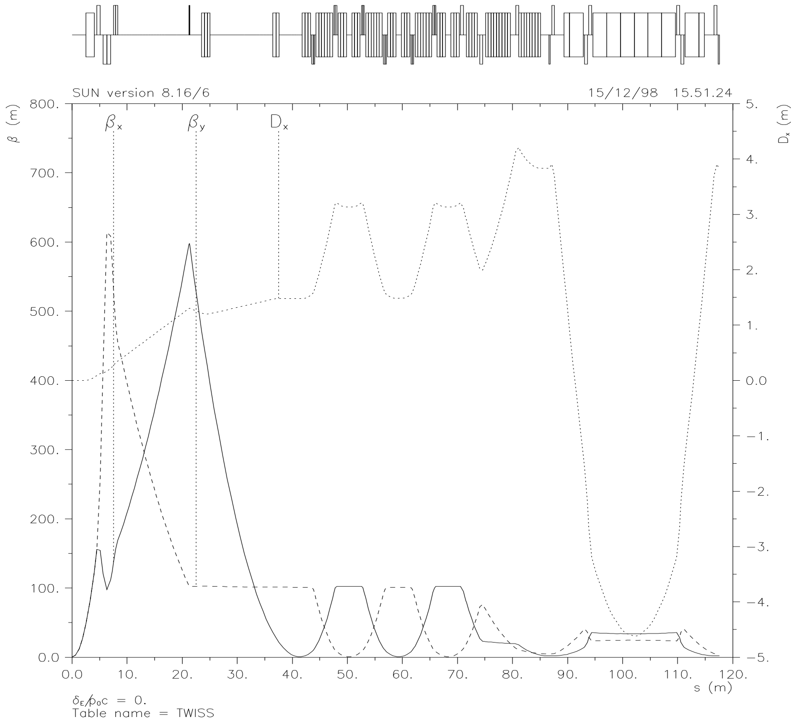

FIGURE 2. 14 cm β^* Mode showing half of the IR, local, chromatic correction, and one of three arc modules.

FDF). In practive, the demagnification is about halfway between a compact and an optimal, or soft-focussing IR. Some deterioration in dynamic aperture is evident with stronger focussing, although studies of high-order and phase dependencies are underway and careful tuning appears to meliorate these effects.

Initially, the powering of the triplet was chosen such that the vertical apertures in the near dipoles were minimized. This requires placing the vertical high-beta peak at the center of the triplet, so that the triplet sequence is FDF. This has the disadvantage in that the local chromatic correction is not as efficient (the higher the dispersion, the more efficient the correction). Higher values of dispersion are usually obtained at peaks in the horizontal beta function than in vertical beta peaks. The plane corrected first should be the one with the highest chromaticity; in this case the vertical. If the dispersion is lower, then the chromatic correction, even with π pairs of sextupoles, is not as efficient and generates stronger nonlinearities. These nonlinearities propagate and appear to be enhanced by the sextupoles of the opposite plane and can be correlated to an observed decrease in dynamic aperture in this plane.

In a test lattice, the triplet was powered in a DFD configuration out of concern for the dynamic aperture. The plane with the highest chromaticity and the highest achievable dispersion at the sextupoles was corrected closest to the source, effecting a more efficient chromatic correction. Nonlinear terms were amplified less by sextupoles in the opposite plane as was evidenced by a slight improvement in dynamic aperture. A questionable consequence of installing the horizontal high-dispersion peak nearest the IP was the unavoidable application of reverse bends to create a dispersion plateau (D'=0) after a defocussing quadrupole. (These reverse bends are not needed if vertical chromaticity correction is performed first because a dispersion plateau can follow a focussing quadrupole.) The net increase in circumference due to reverse bends and less efficient dipole packing in general brought the circumference up by at least 50 m; making the circumference more than 400 m when injection and scraping are included. The loss in muon lifetime was felt to outweight the small advantage to the optics of the ring. The final triplet powering remains as FDF with the vertical chromaticity beign corrected closest to the IP.

Chromatic Correction

Local chromatic correction of the muon collider interaction region is required to achieve broad momentum acceptance. With such a large aperture in the final-focus quadrupoles, adding dispersion to the final focus is not reasonable and therefore chromatic correction must take place in a specialized section. The basic approach developed by Brown [1] and others is implemented in the Chromatic Correction Region (CC) used here. The CC contains two pairs of sextupoles, one pair for each transverse plane, all located at locations with high dispersion. The sextupoles of each pair are located at positions of equal, high beta value in the plane (horizontal or vertical) whose chromaticity is to be corrected, and low beta value in the other plane. Moreover, the two sextupoles of each pair are separated by a betatron phase advance of π, and each sextupole has a phase separation of $(2n+1)\frac{\pi}{2}$ from the IP, where n is an integer. The result of this arrangement is that the geometric aberrations of each sextupole is canceled by its companion while the chromaticity corrections add.

An innovative module was developed specifically for chromatic correction (Fig. 3) and implemented first in the 4-TeV muon collider [4]. Its characteristics include a high-dispersion and high-beta plateau in one plane coincident with a deep minimum in beta in the opposite plane. The high-beta plateaus alternate between planes, with the single intervening deep minimum establishing a π phase advance between plateaus in the same plane. The sextupoles of each pair are centered about a minimum in the opposite plane ($\beta_{min} < 1$), which provides chromatic correction with minimal cross correlation between the planes. A further advantage to locating the opposite plane's minimum at the center of the sextupole, is that this point is $\frac{\pi}{2}$ away from, or "out of phase" with, the source of chromatic effects in the final focus quadrupoles; i.e. the plane not being chromatically corrected is treated like the IP in terms of phase to eliminate a second order chromatic aberration generated by an "opposite-plane" sextupole.

In this lattice example, the CC was optimized to be as short as possible with a smooth transition designed to place the first chromatic correction sextupole at

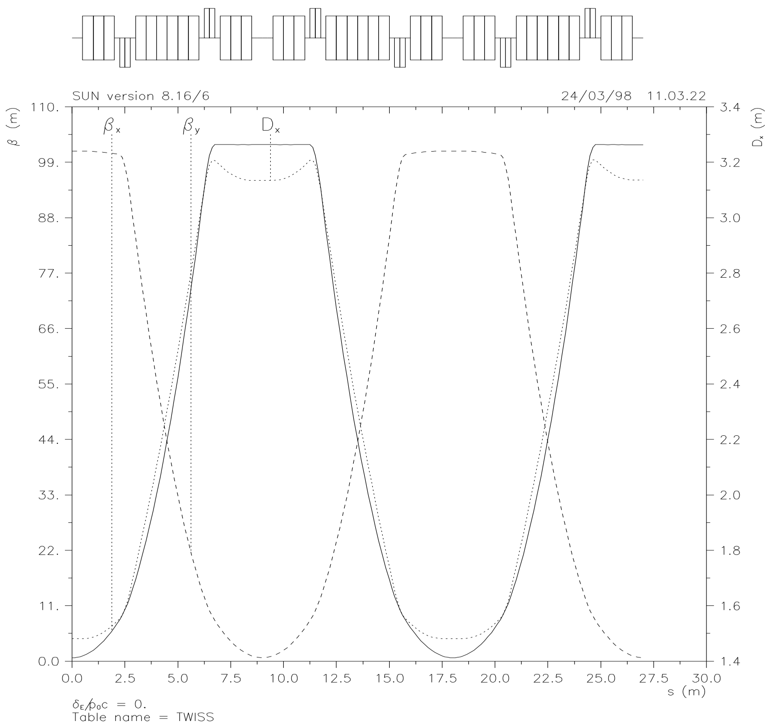

FIGURE 3. The Chromatic Correction Module.

the same phase as the high-beta point [4]. The β_{max} is only 100 m and the $\beta_{\mathrm{min}} = 0.7$ m, giving a β_{ratio} between planes of about 150, so the dynamic aperture is not compromised by a large amplitude-dependent tuneshift.

This large beta ratio, combined with the opposite-plane phasing, allows the sextupoles for the opposite planes to be interleaved, without significantly increasing the nonlinearity of the lattice. In fact, interleaving improved lattice performance compared to that of a non-interleaved correction scheme, due to a shortening of the chromatic correction section, which lowers its chromaticity contribution. The use of somewhat shallower beta-minima with less variation in beta through the sextupoles were also applied to soften the chromatic aberrations, although this caused a slight violation of the exact π phase advance separation between sextupole partners. The retention of an exact π phase advance difference between sextupoles was found to be less important to the dynamic aperture than elimination of minima with $\beta_{\mathrm{min}} < 0.5$ m.

This module, specifically optimized to perform chromatic correction, is particularly powerful in that it can accomodate long sextupoles without beta and phase changes taking place in the plane being corrected. However, because of finite element lengths and changes in the phase advance between sextuple as a function of energy, a tuneshift with amplitude is unavoidable, and depends most sensitively on the beta amplitude in the sextupole, but also on the length of the sextupole and the tune of the ring. Ultimately, a tuneshift with amplitude constricts the dynamic aperture and a tradeoff exists between momentum acceptance and transverse dynamic aperture. Lattice parameters, especially the beta values at the sextupoles and the phase advance around the ring, must be carefully tuned to optimize both acceptances simultaneously.

For the narrow band acceptance, local chromatic correction; i.e. the sextupoles are turned off. The momentum acceptance narrows to about a $\delta p/p$ of about $\pm 0.2\%$, while the transverse dynamic aperture increases rapidly to over 10σ at the central momentum.

The Arc

The arc module is shown in Fig. 4. It has the small beta functions characteristic of FODO cells, yet a large, almost separate, variability in the momentum compaction of the module which is a characteristic associated with the flexible momentum compaction module [5]. The small beta functions are achieved through the use of a doublet focusing structure which produces a low beta simultaneously in both planes. At the dual minima, a strong focusing quadrupole is placed to control the derivative of dispersion with little impact on the beta functions. (The center defocusing quadrupole is used only to clip the point of highest dispersion.) Ultimately a dispersion derivative can be generated which is negative enough to drive the dispersion negative through the doublet and the intervening waist. Negative values of momentum compaction as low as $\alpha = -0.13$ have been achieved, and $\gamma_t = 2\ i$, has been achieved with modest values of the beta function.

The entire ring was designed to control momentum compaction, even in the match section with connects the CC to the arc. This careful attention to momentum compaction for the isochronicity condition resulted in a circumference which is just under 350 m, as opposed to rings which were greater than the 400 m characteristic of earlier designs. The total momentum compaction contributions of the IR, CC, and matching sections is about 0.04. The total length of these parts is 173 m, while that of the the momentum-compaction-correcting arc is 93 m. From these numbers, it follows that this arc must and does have a negative momentum compaction of about -0.09 in order to offset the positive contributions from the rest of the ring.

RF System

The rf requirements depend on the momentum compaction of the lattice and on the parameters of the muon bunch. For the case of very low momentum spread, synchrotron motion is negligibe and the rf system is used solely to correct an energy spread generated through the impedance of the machine. For the cases of higher momentum spreads, there are two approaches. One is to make the momentum compaction zero to high order through lattice design. Then the synchrotron motion can be eliminated, and the rf is again only needed to compensate the induced energy

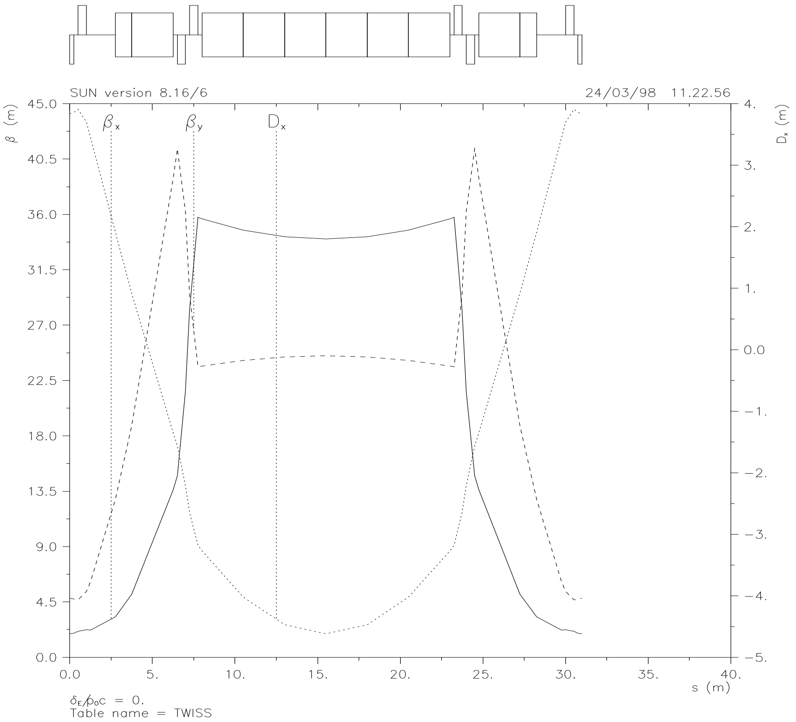

FIGURE 4. The 100-GeV CoM collider arc: a new flexible momentum compaction module.

spread correction. Alternatively, if some momentum compaction is retained, then a more powerful rf system is needed to maintain the specified short bunches. In either case, rf quadrupoles will be required to generate BNS damping of the transverse head-tail instability.

PERFORMANCE

A very preliminary calculation of the dynamic aperture without optimization of the lattice nor inclusion of errors and end effects is given in Fig. 4. One would expect that simply turning off the chromatic correction sextupoles in the 4 cm β^* mode would result in a linear lattice with a large transverse aperture. With only linear elements, the 4 cm β^* optics showed to be strongly nonlinear with limited on-momentum dynamic acceptance.

A normal form analysis using COSY INFINITY showed that the tune-shift-with-amplitude was large, which was the source of the strong nonlinearity in the seemingly linear lattice. To locate the source of this nonlinearity, a lattice consisting of the original IR and arcs only (no CC), was studied. Numerical studies showed that a similar dynamic aperture and tune-shift-with-amplitude terms. This ruled out the possibility that the dynamic aperture was limited by the low beta points in the local chromatic correction section and points to the IR as the source of the nonlinearity. (The findings were verified by S. Ohnuma who used a Runge-Kutta integrator to track through the IR and a linear matrix for the rest of the lattice.) Further analytical study using perturbation theory showed that the first-order contribution to the tune shift with amplitude is proportional to $\gamma_{x,y}^2$ and $\gamma_x\gamma_y$, which are large in this IR. These terms come from the nonlinear terms of p_x/p_0 and p_y/p_0, which, to the first order, equal the angular divergence of a particle. As a demonstration, a comparison to the LHC low-beta IR was done. Taking into account only the drift from the IP to the first quadrupole, the horizontal detuning at 10σ of the present IR ($\beta^* = 4$ cm) is 0.01, whereas the detuning of the entire LHC lattice is below 1e-4. This also explains the fact that the on-momentum aperture of the wide momentum spread mode remains roughly constant among different versions despite various correction attempts.

It was therefore concluded and later shown that the dynamic aperture of the more relaxed β^* of 14 cm would not have the same strong nonlinearities due to the reduced angular terms. In fact, the tune shift with amplitude was less by an order of magnitude; hence the large transverse acceptance shown in Fig. 5 (dashed line).

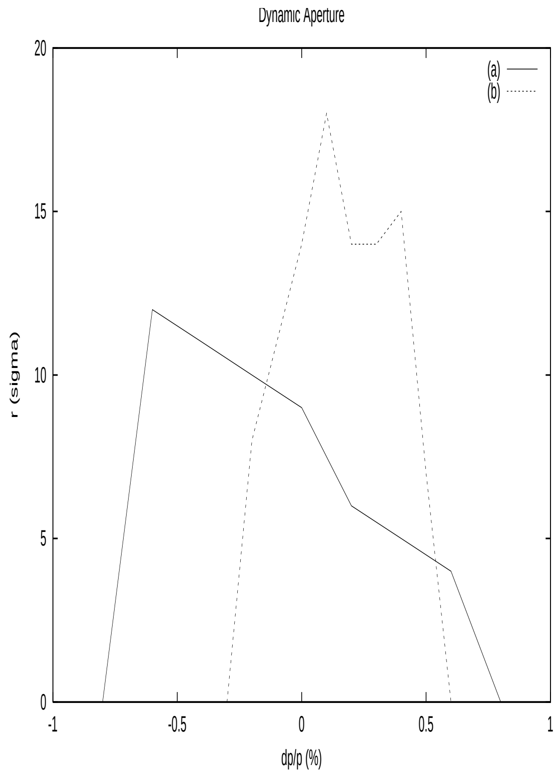

FIGURE 5. A preliminary dynamic aperture for the 4 cm β^* mode where σ (rms) $= 82\mu m$ (solid line) and the 14 cm β^* mode where σ (rms) $= 281\mu m$ (dashed line).

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