

PAPER • OPEN ACCESS

BABAR results on T and CPT symmetry in B^0 - \bar{B}^0 mixing and in $B^0 \rightarrow c\bar{c}K^0$ decays

To cite this article: Klaus R. Schubert 2017 *J. Phys.: Conf. Ser.* **873** 012023

View the [article online](#) for updates and enhancements.

Related content

- [Nuclear and Particle Physics: The \$B_0\$ - \$\bar{B}_0\$ system](#)
C Amsler
- [Recent BABAR Results](#)
Gerald Eigen and BABAR Collaboration
- [Discrete Symmetries CP, T, CPT](#)
J Bernabeu

BABAR results on T and CPT symmetry in B^0 - \bar{B}^0 mixing and in $B^0 \rightarrow c\bar{c}K^0$ decays

Klaus R. Schubert

Institut für Kernphysik, Johannes Gutenberg-Universität, Mainz, Germany

BABAR Collaboration

E-mail: klaus.schubert@tu-dresden.de

Abstract. The presentation reviews the results which have been obtained with the *BABAR* detector at the e^+e^- storage ring PEP-II at SLAC between 2004 and 2016 on T and CPT symmetry in B^0 - \bar{B}^0 mixing and in the decay amplitudes of $B^0 \rightarrow c\bar{c}K^0$ decays. All results are based on the measured time dependence of two-decay events from entangled B^0 - \bar{B}^0 pairs produced in decays of $\Upsilon(4S)$ mesons. Neither T nor CPT violation have been observed in B^0 - \bar{B}^0 mixing, also no CPT violation in the $B^0 \rightarrow c\bar{c}K^0$ decay amplitudes.

1. Introduction

Solving the puzzle of two different K^+ mesons decaying into final states with opposite parity [1], Lee and Yang [2] proposed in 1956 that P symmetry is broken in weak interactions since there were no convincing tests of P conservation in weak processes. Soon later, two experimental groups concurrently proved that P is not only violated in K^+ decays, but also in the decay chain $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ [3] and in β decays of ^{60}Co [4]. CP violation was discovered in 1964 in the decays $K^0 \rightarrow \pi^+\pi^-$ at late decay times [5]. In the following years, many authors asked if CPT could also be violated in Nature despite its validity in Lorentz-invariant QFT. CP violation implies that T or CPT or both are also violated. Bell and Steinberger [6] proposed in 1965 separate tests of T and CPT using a unitarity relation with the sum of CP violations in all K^0 decay modes. This relation could not yield numerical results before all essential inputs had been measured. The last essential input, the phase ϕ_{00} between the amplitudes for the decays of K_S^0 and K_L^0 into $\pi^0\pi^0$, was first determined in 1970 [7]. With this entry, the Bell-Steinberger unitarity analysis resulted in $\text{Re}(\epsilon) = (1.7 \pm 0.3) \times 10^{-3}$ and $\text{Im}(\delta) = (-0.3 \pm 0.4) \times 10^{-3}$ [8] immediately after the ϕ_{00} measurement. $\text{Re}(\epsilon)$ describes T violation in K^0 - \bar{K}^0 mixing, here established with $\sim 5\sigma$, and δ describes CPT violation therein, here compatible with zero. The most recent update of the analysis [9, 10] gives $\text{Re}(\epsilon) = (161.1 \pm 0.5) \times 10^{-5}$ and $\text{Im}(\delta) = (-0.7 \pm 1.4) \times 10^{-5}$.

In the 10 times heavier B^0 - \bar{B}^0 system, large CP and T violation was observed in 2001 in $B^0 \rightarrow c\bar{c}K^0$ decays [11, 12, 13], up to very recently without a determination of CPT -violating parameters. No CP violation has been observed in B^0 - \bar{B}^0 mixing so far. The present presentation discusses the contributions of the *BABAR* experiment to the determination of T - and CPT -violating parameters in mixing and decays of the $B^0(b\bar{d})$ - $\bar{B}^0(\bar{b}d)$ system.



2. CPT and T symmetries in B^0 - \bar{B}^0 mixing

Since mixing in the two-state system $\Psi = \psi_1 B^0 + \psi_2 \bar{B}^0$ proceeds on a similar time scale as weak decays, it is well described by the linear evolution equation

$$i \frac{\partial}{\partial t} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \left[\begin{pmatrix} m_{11} & m_{12} \\ m_{12}^* & m_{22} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix} \right] \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad (1)$$

with 7 real parameters m_{11} , m_{22} , Γ_{11} , Γ_{22} , $|m_{12}|$, $|\Gamma_{12}|$, and $\phi(\Gamma_{12}/m_{12})$. As long as mixing is “slow”, this description is independent of the dynamics which may include New Physics in addition to the Standard weak interaction; it is just the two-dimensional generalization of

$$i \frac{d\psi}{dt} = (m - i\Gamma/2)\psi \quad (2)$$

for a slowly decaying particle with wave-function $\psi(t)$. Two solutions of Eq. 1, B_H^0 (heavy) and B_L^0 (light), have an exponential decay law. In lowest order of z and $1 - |q/p|$, as used throughout this presentation, they are given by

$$\begin{aligned} B_H^0(t) &= e^{-\Gamma_H t/2 - im_H t} \left[p(1 + z/2) B^0 - q(1 - z/2) \bar{B}^0 \right] / \sqrt{2}, \\ B_L^0(t) &= e^{-\Gamma_L t/2 - im_L t} \left[p(1 - z/2) B^0 + q(1 + z/2) \bar{B}^0 \right] / \sqrt{2}, \end{aligned} \quad (3)$$

with 7 real observables m_H , $\Delta m = m_H - m_L$, Γ_H , $\Delta\Gamma = \Gamma_H - \Gamma_L$, $|q/p|$, $\text{Re}(z)$, and $\text{Im}(z)$. The 7 observables follow from the 7 parameters, e.g.,

$$\left| \frac{q}{p} \right| = 1 - \frac{2 \text{Im}(\Gamma_{12}/m_{12})}{4 + |\Gamma_{12}/m_{12}|^2}, \quad z = \frac{(m_{11} - m_{22}) - i(\Gamma_{11} - \Gamma_{22})/2}{\Delta m - i\Delta\Gamma/2}. \quad (4)$$

In the $K^0 - \bar{K}^0$ system with the sign conventions $\Delta m = m(K_L) - m(K_S)$ and $\Delta\Gamma = \Gamma(K_L) - \Gamma(K_S) < 0$, the traditionally used observables are $\text{Re}(\epsilon) = (1 - |q/p|)/2$ and $\delta = -z/2$.

From Eqs. 3, we obtain the transition rates as function of the evolution time t . With $\Gamma = (\Gamma_H + \Gamma_L)/2$ and $|\Delta\Gamma| \ll \Gamma$, they are

$$\begin{aligned} R(B^0 \rightarrow B^0) &= e^{-\Gamma t} [1 + \cos(\Delta m t) - \text{Re}(z)\Delta\Gamma t + 2 \text{Im}(z) \sin(\Delta m t)]/2, \\ R(\bar{B}^0 \rightarrow \bar{B}^0) &= e^{-\Gamma t} [1 + \cos(\Delta m t) + \text{Re}(z)\Delta\Gamma t - 2 \text{Im}(z) \sin(\Delta m t)]/2, \end{aligned} \quad (5)$$

$$\begin{aligned} R(B^0 \rightarrow \bar{B}^0) &= e^{-\Gamma t} [1 - \cos(\Delta m t)] \cdot |q/p|^2/2, \\ R(\bar{B}^0 \rightarrow B^0) &= e^{-\Gamma t} [1 - \cos(\Delta m t)] \cdot |p/q|^2/2. \end{aligned} \quad (6)$$

The rates in Eq. 5 depend only on z , they are not sensitive to T symmetry, and CPT symmetry requires equality of the two rates, i.e. $z = 0$. The rates in Eq. 6 depend only on $|q/p|$, they are not sensitive to CPT symmetry, and T symmetry requires equal rates, i.e. $|q/p| = 1$. Since $\Delta\Gamma$ is unknown, the first rates determine up to now only $\text{Im}(z)$, not $\text{Re}(z)$. Transitions into states decaying into CP eigenstates like $c\bar{c}K_S$, $c\bar{c}K_L$ are also sensitive to $\text{Re}(z)$ as shown later.

3. CPT and T symmetries in B^0 decays

If the decay amplitudes

$$A = \langle f | \mathcal{H} | B^0 \rangle \text{ and } \bar{A} = \langle \bar{f} | \mathcal{H} | \bar{B}^0 \rangle \quad (7)$$

have a single weak phase, CPT symmetry requires [14]

$$|\bar{A}/A| = 1. \quad (8)$$

If the amplitudes are sums of amplitudes with different weak phases, we can have $|\bar{A}/A| \neq 1$ inspite of CPT symmetry (“direct CP violation” and “direct T violation”) or $|\bar{A}/A| = 1$ inspite of CPT violation.

Tests of T symmetry require the interference of at least two contributing amplitudes to the same final state. Two early test examples are radiative decays of excited atoms and neutron decays. “Forbidden” atomic transitions proceed in E2 and M1 modes when selection rules exclude E1. In 1951, Lloyd [15] has shown that T symmetry requires $\text{Im}(A_{M1}/A_{E2}) = 0$ for the observable ratio of the two transition amplitudes. In 1957, Jackson *et al* [16] have shown that the beta decay of polarized neutrons allows to measure the phase between the axial-vector and vector coupling constants G_A and G_V . T symmetry requires this phase to be 0 or 180° , i. e. $\text{Im}(G_A/G_V) = 0$.

The same symmetry principle applies to B^0 decays into CP eigenstates like $c\bar{c}K_S^0$ with eigenvalues $CP = -1$ and $c\bar{c}K_L^0$ with $CP = +1$, where $c\bar{c} = J/\psi, \psi'$ and other charmonium states. With

$$A = \langle c\bar{c}K^0 | \mathcal{H} | B^0 \rangle \text{ and } \bar{A} = \langle c\bar{c}\bar{K}^0 | \mathcal{H} | \bar{B}^0 \rangle, \quad (9)$$

the well-obeyed $\Delta S = \Delta b$ rule

$$\langle c\bar{c}\bar{K}^0 | \mathcal{H} | B^0 \rangle = \langle c\bar{c}K^0 | \mathcal{H} | \bar{B}^0 \rangle = 0, \quad (10)$$

and negligible CP violation in K^0 - \bar{K}^0 mixing, we have

$$\begin{aligned} A_S &= \langle c\bar{c}K_S^0 | \mathcal{H} | B^0 \rangle = A/\sqrt{2}, \\ A_L &= \langle c\bar{c}K_L^0 | \mathcal{H} | B^0 \rangle = A/\sqrt{2}, \\ \bar{A}_S &= \langle c\bar{c}K_S^0 | \mathcal{H} | \bar{B}^0 \rangle = \bar{A}/\sqrt{2}, \\ \bar{A}_L &= \langle c\bar{c}K_L^0 | \mathcal{H} | \bar{B}^0 \rangle = -\bar{A}/\sqrt{2}. \end{aligned} \quad (11)$$

Because of mixing, an initial B^0 or \bar{B}^0 state decays into final states $c\bar{c}K_S^0$ or $c\bar{c}K_L^0$ with interfering amplitudes from both B^0 and \bar{B}^0 at the time of decay. T symmetry, therefore, requires a phase relation between A and \bar{A} . Since both states B^0 and \bar{B}^0 can be arbitrarily rephased, the phase of \bar{A}/A is unobservable. However, the phase of the parameter

$$\lambda = \frac{q\bar{A}}{pA} \quad (12)$$

with q and p from Eqs. 3 is an observable, and T symmetry requires [17, 13, 18]

$$\text{Im}(\lambda) = 0. \quad (13)$$

The observed CP and T violation in time-dependent B^0 decays with $\text{Im}(\lambda) \neq 0$ is often called CP violation in the interplay of decay and mixing since it is not a property of the decay dynamics alone. It requires the observable phase of $(q/p) \times \bar{A}/A$ to be different from 0 and 180° .

4. Earlier BABAR results on T and CPT symmetry tests in B^0 - \bar{B}^0 mixing

Neutral B mesons in the BABAR experiment are produced in the entangled two-particle state $(B^0\bar{B}^0 - \bar{B}^0B^0)/\sqrt{2}$ from $\Upsilon(4S)$ decays. With a flavor-specific first decay into $\ell^- X$ ($\ell^+ X$) at time t_1 , the remaining single-particle state is a B^0 (\bar{B}^0) at this time. The rates for flavor-specific

decays into $\ell^- X$ ($\ell^+ X$) at time $t_2 = t_1 + t$ are given by Eqs. 6. The t -dependent difference of $\ell^- \ell^-$ and $\ell^+ \ell^+$ pair rates determines, therefore, the T -symmetry parameter $|q/p|$. In 2006, *BABAR* analysed like-sign dilepton pairs from about 230×10^6 $\Upsilon(4S)$ decays [19], corresponding to about half of its finally collected data, resulting in

$$|q/p| (BABAR, \text{ dileptons}) = 1 - (0.8 \pm 1.7 \pm 1.9) \times 10^{-3}. \quad (14)$$

In 2013, with about 425×10^6 $\Upsilon(4S)$ decays [20], *BABAR* found

$$|q/p| (BABAR, D^{*\pm} X^0 \ell^\pm \text{ and } K^\pm X) = 1 - (0.3 \pm 0.8 \pm 1.8) \times 10^{-3}, \quad (15)$$

where the flavor of one B -meson is identified by a charged lepton and the slow charged pion from a $D^{*\pm}$ decay, and the flavor of the other B by using the angular distribution of a charged kaon from sequential $B \rightarrow DX \rightarrow K^\pm X'$ decays.

These two results, together with results from Belle, CLEO, LHCb and D0, lead to the present world average [21] of

$$|q/p| (\text{HFAG 2016}) = 1 + (1, 0 \pm 0.8) \times 10^{-3}, \quad (16)$$

with no observed T violation in $B^0\text{-}\bar{B}^0$ mixing.

In the same 2006 analysis determining $|q/p|$, *BABAR* [19] uses opposite-sign dilepton events for determining $\text{Im}(z)$. These events have the rate $R_{-+} = R(B^0 \rightarrow B^0)$ in Eq. 5 when the ℓ^- decay occurs first, and $R_{+-} = R(\bar{B}^0 \rightarrow \bar{B}^0)$ when the ℓ^+ is first. The two rates $R_{-+}(t)$ with $t = t_+ - t_-$ and $R_{+-}(t)$ with $t = t_- - t_+$ have the property $R_{+-}(t) = R_{-+}(-t)$, required by the two-decay-time formula [22] as a consequence of entanglement independent of CPT symmetry. *BABAR*'s 2006 result [19] from opposite-sign dilepton events is

$$\text{Im}(z) = (-14 \pm 7 \pm 3) \times 10^{-3}. \quad (17)$$

As long as $\Delta\Gamma$ in $B^0\text{-}\bar{B}^0$ mixing remains unknown, opposite-sign dilepton events do not determine $\text{Re}(z)$, see Eqs. 5. For completeness, I include in this Section a 2004 *BABAR* measurement of $\text{Re}(z)$ [23] using decay pairs $\ell^\pm X$ from one neutral B and $c\bar{c}K$ from the other B with a data set of about 90×10^6 $\Upsilon(4S)$ decays. The time-dependent rates for these decay pairs are derived in Section 6 where the same analysis is discussed with the final *BABAR* data set of about 470×10^6 $\Upsilon(4S)$ decays [24]. The 2004 result was

$$\text{Re}(z) = (-19 \pm 48 \pm 47) \times 10^{-3}. \quad (18)$$

5. The *BABAR* 2012 result on T violation in time-dependent $B^0 \rightarrow c\bar{c}K^0$ decays

The final result on CP violation in $B^0 \rightarrow c\bar{c}K^0$ decays has been published in 2009 [25] using about 470×10^6 $\Upsilon(4S)$ decays. Assuming CPT symmetry, the CP - and T -violating parameter $\text{Im}(\lambda)$ in the definition of Eqs. 9 and 12 has been determined from a fit to the four time-dependent rates of the decay pairs $\ell^+ X$ or $\ell^- X$, $c\bar{c}K_S^0$ or $c\bar{c}K_L^0$, with the result

$$\text{Im}(\lambda) = 0.687 \pm 0.028 \pm 0.012. \quad (19)$$

The flavor-specific decays $\ell^+ X$ and $\ell^- X$ include decays with high-momentum charged particles, charged kaons and charged pions from D^* decays in addition to high-momentum electrons and muons. The charmonium states $c\bar{c}$ include J/ψ , $\psi(2S)$, η_c and χ_{c1} in combination with K_S^0 and only J/ψ in combination with K_L^0 and $K^*(892)^0$. The K^* events with decays into $K_S^0 \pi^0$ contribute to both the K_S^0 and K_L^0 samples with the eigenvalues $CP = -1$ and $CP = +1$ respectively; decay angular distributions are used for the separation of the two contributions.

Using essentially the same event selection, omitting $J/\psi K^*$ and $\eta_c K_S^0$ decays, and following a suggestion of J. Bernab   [26, 27], *BABAR* [28] used the four time-dependent rates of the decay pairs $(\ell^- X, c\bar{c}K_L^0)$, $(\ell^+ X, c\bar{c}K_L^0)$, $(\ell^- X, c\bar{c}K_S^0)$ and $(\ell^+ X, c\bar{c}K_S^0)$ for a demonstration of “genuine” T violation in 2012. In contrast to testing a consequence of T -symmetry in the dynamics of a process, a genuine T -symmetry test [27] measures the detailed balance of two processes that are related by the exchange of initial and final state, i.e. the equality of the transition rates for $i \rightarrow f$ and $f \rightarrow i$. An example for a genuine T test in the strong interaction is the comparison of the cross sections for the nuclear reactions $^{24}\text{Mg} + \alpha \rightarrow ^{27}\text{Al} + p$ and $^{27}\text{Al} + p \rightarrow ^{24}\text{Mg} + \alpha$ by von Witsch *et al* in 1968 [29].

For the four decay pairs $\ell^\pm X$, $c\bar{c}K_{S,L}^0$, there are four independent rates, but for the clarity of demonstration, *BABAR* determined eight rates separating into $t(c\bar{c}K) - t(\ell X) > 0$ $t(c\bar{c}K) - t(\ell X) < 0$. The separated rates are related by the two-decay-time formula [22] following from entanglement of the B^0 - \bar{B}^0 pair independent of any discrete symmetry, as already discussed for dilepton events in the text above Eq. 17. Table 1 shows the numbering of the eight chosen rates for the following discussion.

Table 1. Decay pairs in Ref. [28] for the measurement of the decay-time dependences with the coefficients $C_1 \cdots C_8$ and $S_1 \cdots S_8$. Rows 3 gives the initial state of the second B as prepared by the first decay, row 4 the final state as defined by the second decay of the decay pair.

| i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 1st decay | $\ell^- X$ | $\ell^+ X$ | $\ell^- X$ | $\ell^+ X$ | $c\bar{c}K_L$ | $c\bar{c}K_L$ | $c\bar{c}K_S$ | $c\bar{c}K_S$ |
| 2nd decay | $c\bar{c}K_L$ | $c\bar{c}K_L$ | $c\bar{c}K_S$ | $c\bar{c}K_S$ | $\ell^- X$ | $\ell^+ X$ | $\ell^- X$ | $\ell^+ X$ |
| initial state | B^0 | \bar{B}^0 | B^0 | \bar{B}^0 | B_- | B_- | B_+ | B_+ |
| final state | B_+ | B_+ | B_- | B_- | \bar{B}^0 | B^0 | \bar{B}^0 | B^0 |

Without assuming CPT symmetry in mixing, but assuming CPT symmetry in the amplitudes A for the decay $B^0 \rightarrow c\bar{c}K^0$ and \bar{A} for $\bar{B}^0 \rightarrow \bar{K}^0$, i.e. $|\bar{A}/A| = 1$, the first decay of a B^0 - \bar{B}^0 pair into $c\bar{c}K_S^0$ prepares a remaining single- B -meson state with $c\bar{c}K$ decays into only $c\bar{c}K_L^0$ at $t = 0$, called B_+ . If the first decay is $c\bar{c}K_L^0$, the prepared state B_- decays into $c\bar{c}K$ states with only $c\bar{c}K_S^0$ at $t = 0$. In the same way, a first decay into $\ell^+ X$ ($\ell^- X$) prepares a remaining B in the state \bar{B}^0 (B^0). With $t = t(c\bar{c}K) - t(\ell X)$ for the first four and $t = t(\ell X) - t(c\bar{c}K)$ for the second four rates in Table 1, all eight measured rates are parametrized by

$$R_i(t) = N_i e^{-\Gamma t} (1 + C_i \cos \Delta m t + S_i \sin \Delta m t), \quad (20)$$

and the 16 coefficients $C_1 \cdots S_8$ are determined by eight fits including the resolution between the measured times \hat{t} and the true times t . The 3rd and 4th row in Table 1 give the initial state of the second B as prepared by the first decay and the final state as defined by the second decay of the decay pair. Detailed balance between the time-dependent transitions $B^0 \rightarrow B_+$ and $B_+ \rightarrow B^0$ requires $R_8(t) = R_1(t)$, i.e. $C_8 = C_1$ and $S_8 = S_1$, and the same for the other three exchanges of initial and final state. *BABAR* [28] finds that all C_i are compatible with zero, but the differences $S_8 - S_1$, $S_7 - S_2$, $S_6 - S_3$ and $S_5 - S_4$ are all different from zero with a combined significance of 14 standard deviations, strongly demonstrating T -symmetry violation in B^0 -meson decays.

In the same publication [28], *BABAR* also demonstrates CPT symmetry by measuring the difference of the time-dependent transition rates for $B^0 \rightarrow B_+$ and $B_+ \rightarrow \bar{B}^0$ and the three other differences between CPT -related rates. All four differences are compatible with zero.

Since CPT asymmetry in the decay amplitudes has been assumed, this is a CPT test in $B^0\text{-}\bar{B}^0$ mixing. It is a qualitative test only; no values for $\text{Re}(z)$ and $\text{Im}(z)$ have been determined. This determination was performed by $BABAR$ in 2016 [24] and is discussed in the next Section.

6. The $BABAR$ 2016 results on CPT in $B^0\text{-}\bar{B}^0$ mixing and $B^0 \rightarrow c\bar{c}K^0$ decays

The eight time-dependent rates for the decays of entangled $B^0\text{-}\bar{B}^0$ pairs in Table 1 are sensitive to $|\lambda|$, $\text{Im}(\lambda)$, $\text{Re}(z)$ and $\text{Im}(z)$, where z is defined in Eqs. 4 and λ in Eq. 12 with q/p from Eqs. 4 and A and \bar{A} from Eqs. 9. Assuming

- (i) $\Delta\Gamma = \Gamma_H - \Gamma_L = 0$,
- (ii) absence of decays $B^0 \rightarrow c\bar{c}\bar{K}^0$ and $\bar{B}^0 \rightarrow c\bar{c}K^0$, and
- (iii) negligible CP violation in $K^0\text{-}\bar{K}^0$ mixing,

the eight rates are of the form in Eq. 20 with $t = t(c\bar{c}K) - t(\ell X)$ for the first four and $t = t(\ell X) - t(c\bar{c}K)$ for the second four. In lowest order of the small quantities z , $|q/p| - 1$, and $|\lambda| - 1$, the coefficients $C_1 \cdots C_4$ and $S_1 \cdots S_4$ are given by

$$\begin{aligned}
 C_1(B^0 \rightarrow c\bar{c}K_L) &= +(1 - |\lambda|) - \text{Re}(\lambda) \text{Re}(z) - \text{Im}(\lambda) \text{Im}(z), \\
 C_2(\bar{B}^0 \rightarrow c\bar{c}K_L) &= -(1 - |\lambda|) + \text{Re}(\lambda) \text{Re}(z) - \text{Im}(\lambda) \text{Im}(z), \\
 C_3(B^0 \rightarrow c\bar{c}K_S) &= +(1 - |\lambda|) + \text{Re}(\lambda) \text{Re}(z) + \text{Im}(\lambda) \text{Im}(z), \\
 C_4(\bar{B}^0 \rightarrow c\bar{c}K_S) &= -(1 - |\lambda|) - \text{Re}(\lambda) \text{Re}(z) + \text{Im}(\lambda) \text{Im}(z), \\
 S_1 &= +\text{Im}(\lambda)/|\lambda| - \text{Re}(z)\text{Re}(\lambda)\text{Im}(\lambda) + \text{Im}(z)[\text{Re}(\lambda)]^2, \\
 S_2 &= -\text{Im}(\lambda)/|\lambda| - \text{Re}(z)\text{Re}(\lambda)\text{Im}(\lambda) - \text{Im}(z)[\text{Re}(\lambda)]^2, \\
 S_3 &= -\text{Im}(\lambda)/|\lambda| - \text{Re}(z)\text{Re}(\lambda)\text{Im}(\lambda) + \text{Im}(z)[\text{Re}(\lambda)]^2, \\
 S_4 &= +\text{Im}(\lambda)/|\lambda| - \text{Re}(z)\text{Re}(\lambda)\text{Im}(\lambda) - \text{Im}(z)[\text{Re}(\lambda)]^2.
 \end{aligned} \tag{21}$$

Because of entanglement of the $B^0\text{-}\bar{B}^0$ pair and the two-decay-time formula, the coefficients for the decay pairs with $i = 5 \cdots 8$ follow Eq. 21 with $C_i = C_{i-4}$ and $S_i = -S_{i-4}$.

The 16 coefficients $C_1 \cdots S_8$ have been determined in 2012 in the T -violation paper of $BABAR$ [28] together with their statistical and systematic covariance matrices. In the 2016 paper [24], these 16 results based on $BABAR$'s final data set with about 470×10^6 $\Upsilon(4S)$ decays are used to determine the four parameters $|\lambda|$, $\text{Im}(\lambda)$, $\text{Re}(z)$ and $\text{Im}(z)$ without repeating reconstruction and selection of the $\ell^\pm X$, $c\bar{c}K_{S,L}$ decay pairs. The four parameters are obtained in an iterative linearized χ^2 fit of the parameters to the expressions in Eqs. 21. Since the expressions contain $\text{Re}(z)$ only in products with $\text{Re}(\lambda)$ and the fit parameters $\text{Im}(\lambda)$ and $|\lambda|$ do not fix the sign of $\text{Re}(\lambda)$, additional information is used [30] for taking $\text{Re}(\lambda)$ negative. The χ^2 fit converges already in the second step and gives the results

$$\begin{aligned}
 \text{Im}(\lambda) &= 0.689 \pm 0.034 \pm 0.019, \quad |\lambda| = 0.999 \pm 0.023 \pm 0.017 \\
 \text{Im}(z) &= 0.010 \pm 0.030 \pm 0.013, \quad \text{Re}(z) = -0.065 \pm 0.028 \pm 0.014,
 \end{aligned} \tag{22}$$

where the first uncertainties are statistical and the second uncertainties are systematic, as in all results throughout this report. The final results are independent of the assumption $\Delta\Gamma = 0$ as controlled by a ‘‘toy Monte Carlo’’ simulation varying $\Delta\Gamma$ within one standard deviation of the present world average [21]. Inserting the world average for $|q/p|$ in Eq. 16 into the definition of λ , we obtain

$$|\bar{A}/A| = 0.999 \pm 0.023 \pm 0.017. \tag{23}$$

7. Final remarks

The uncertainty on the T -violation result $\text{Im}(\lambda)$ in Eqs. 22 is slightly larger than for *BABAR*'s final CP -violation result in Eq. 19, since that one is obtained from a slightly wider event selection and from assuming CPT symmetry. The three CPT -sensitive results $\text{Im}(z)$, $\text{Re}(z)$ and $|\bar{A}/A|$ are all in agreement with no CPT violation, assuming a single weak phase of the amplitude A . The $\text{Im}(z)$ result has a larger uncertainty than *BABAR*'s dilepton result in Eq. 17 since that one is based on a larger number of decay pairs. But with the small and unknown value for $\Delta\Gamma$, dileptons are not sensitive to $\text{Re}(z)$. The parameter $\text{Re}(z)$ has been determined by *BABAR* and by Belle with comparable uncertainties; the 2012 result of Belle with $535 \times 10^6 \Upsilon(4S)$ events [31] is $\text{Re}(z) = 0.019 \pm 0.037 \pm 0.033$. To my knowledge, there are no additional measurements of $\text{Re}(z)$ in the $B^0\text{-}\bar{B}^0$ system. LHCb gives only a result for the $B_s\text{-}\bar{B}_s$ system [32], $\text{Re}(z, B_s) = -0.022 \pm 0.033 \pm 0.005$, together with $\text{Im}(z, B_s) = 0.004 \pm 0.011 \pm 0.002$.

Acknowledgments

I would like to thank the organizers, especially J Bernab   and A. Di Domenico, for inviting me to present this review at DISCRETE2016. I also thank all members of the *BABAR* Collaboration for numerous discussions on tests of discrete symmetries during the last 20 years. In addition, I especially thank T. Ruf and H.-J. Gerber (CPLEAR Collaboration) for extended discussions on T -symmetry violation and its parametrizations.

References

- [1] Dalitz R H 1953 *Phil. Mag.* **44**, 1068
- [2] Lee T D and Yang C N 1956 *Phys. Rev.* **104**, 254
- [3] Garwin R L, Lederman L M and Weinrich M 1957 *Phys. Rev.* **105**, 1415
- [4] Wu C S *et al* 1957 *Phys. Rev.* **105**, 1413
- [5] Christenson J H, Cronin J W, Fitch V L and Turlay R 1964 *Phys. Rev. Lett.* **13**, 138
- [6] Bell J S and Steinberger J 1966 in Walsh T R *et al* eds., *Oxford International Conference on Elementary Particles* Didcot Chilton, p. 195
- [7] Chollet J C *et al* 1970 *Phys. Lett. B* **31**, 658
- [8] Schubert K R *et al* 1970 *Phys. Lett. B* **31**, 662
- [9] Antonelli M and D'Ambrosio G 2013 CPT invariance tests in neutral Kaon decay, review in ref. [10]
- [10] Patrignani C *et al* (Particle Data Group) 2016 *Chin. Phys. C*, **40**, 100001
- [11] Aubert B *et al* (*BABAR* Collaboration) 2001 *Phys. Rev. Lett.* **87**, 091801
- [12] Abe K *et al* (Belle Collaboration) 2001 *Phys. Rev. Lett.* **87**, 091802
- [13] Fidecaro M, Gerber H-J and Ruf T 2015 *Eur. Phys. J. C* **75**:73
- [14] Lee T D, Oehme R and Yang C N 1957 *Phys. Rev.* **106**, 340
- [15] Lloyd S P 1951 *Phys. Rev.* **81**, 161
- [16] Jackson J D, Treiman S B and Wyld H W Jr. 1957 *Phys. Rev.* **106**, 517
- [17] Enz C P and Lewis R R 1965 *Helv. Phys. Acta* **38**, 860
- [18] Applebaum E *et al* 2014 *Phys. Rev. D* **89**, 076011
- [19] Aubert B *et al* (*BABAR* Collaboration) 2006 *Phys. Rev. Lett.* **96**, 251802
- [20] Lees J P *et al* (*BABAR* Collaboration) 2013 *Phys. Rev. Lett.* **111**, 101802
- [21] Amhis Y *et al* (Heavy Flavor Averaging Group) 2016 arXiv:1612.07233v1 [hep-ex]
- [22] Lipkin H J 1968 *Phys. Rev.* **176**, 1715
- [23] Aubert B *et al* (*BABAR* Collaboration) 2004 *Phys. Rev. D* **70**, 012007, inserting $\text{Re}(\lambda) = -0.73$
- [24] Lees J P *et al* (*BABAR* Collaboration) 2016 *Phys. Rev. D* **94**, 011101
- [25] Aubert B *et al* (*BABAR* Collaboration) 2009 *Phys. Rev. D* **79**, 072009
- [26] Ba  uls M C and Bernab   J 1999 *Phys. Lett. B* **464**, 117
- [27] Bernab   J 2016 these Proceedings
- [28] Lees J P *et al* (*BABAR* Collaboration) 2012 *Phys. Rev. Lett.* **109**, 211801
- [29] von Witsch W, Richter A and von Brentano P 1968 *Phys. Rev.* **169**, 923
- [30] See Ref. [24] and citations 19-22 therein.
- [31] Higuchi T *et al* (Belle Collaboration) 2012 *Phys. Rev. D* **85**, 071105
- [32] Aaij R *et al* (LHCb Collaboration) 2016 *Phys. Rev. Lett.* **116**, 241601