SLAC-PUB-1151
(TH) and (EXP)
November 1972

REGGE PHENOMENOLOGY OF TWO PARTICLE INCLUSIVE PROCESSES*

Richard C. Brower

Lauritsen Laboratory of Physics
California Institute of Technology, Pasadena, California 91109

Robert N. Cahn

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

John Ellis

Lauritsen Laboratory of Physics
California Institute of Technology, Pasadena, California 91109

(Submitted to Phys. Rev.)

*Work supported in part by the U. S. Atomic Energy Commission.

ABSTRACT

Using the Regge analysis of Mueller and assuming factorization we calculate the leading meson Regge corrections to scaling in several two particle inclusive processes, $a + b \rightarrow c + d + X$. We consider the limit where c is a fragment of a and d is a fragment of b, denoted by $(a \rightarrow c \,|\, d \leftarrow b)$. Inclusive Reggeon vertices are extracted from one particle inclusive data, using exchange degeneracy assumptions motivated by experiment and conservative theoretical prejudices about early scaling. These vertices are then combined (i) to predict that the following processes should scale early: $(p \rightarrow \pi^- \,|\, \pi^- \leftarrow \pi^+)$ $(p \rightarrow \pi^- \,|\, \pi^- \leftarrow p)$, $(p \rightarrow \pi^- \,|\, \pi^- \leftarrow K^+)$, $(p \rightarrow \pi^+ \,|\, \pi^- \leftarrow K^+)$; (ii) to predict large Regge corrections to scaling in the processes $(p \rightarrow \pi^- \,|\, \pi^+ \leftarrow \bar{p})$, and $(p \rightarrow \pi^- \,|\, \pi^+ \leftarrow K^-)$; and (iii) to make qualitative predictions about other inclusive processes. Comparisons with experiment agree with the predictions for $(p \rightarrow \pi^- \,|\, \pi^- \leftarrow p)$, $(p \rightarrow \pi^- \,|\, \pi^- \leftarrow K^+)$ and $(p \rightarrow \pi^- \,|\, \pi^+ \leftarrow K^-)$, and are consistent with our expectations for $(p \rightarrow \pi^- \,|\, \pi^- \leftarrow \pi^+)$ and $(p \rightarrow \pi^- \,|\, \pi^- \leftarrow \pi^-)$.

I. INTRODUCTION

The Mueller[1,2] method of combining Regge theory with unitarity[3] is a powerful means for analyzing inclusive processes. While not as explicit as the multiperipheral[4] and diffractive excitation models,[5] it gives a simple derivation of hadronic scaling laws proposed by Feynman, Yang and others.[6] Mueller analysis also makes predictions for the way in which scaling is approached,[7] and for the structure of correlations between produced particles.[2,8] So far, Mueller theory has usually been applied to single particle inclusive processes.[9] In this paper we discuss the Regge phenomenology of two particle inclusive processes. Starting from existing data on one particle inclusive processes and using theoretical assumptions which are supported phenomenologically, we make predictions for two particle inclusive processes at intermediate energies,[10] and compare them with existing data as far as possible.

It is generally assumed[1,2,7,8] that the same Regge singularities appear in inclusive cross sections as in exclusive processes. Thus the Pomeron is believed to be responsible for hadronic scaling, and the approach to limiting fragmentation is believed to be as $s^{-1/2}$, and to be controlled by the leading meson trajectories (ρ, ω, f, A_2). Experimental evidence has been presented in favor of this latter prediction.[11] Many more predictions follow from the Mueller picture if the leading Regge singularities factorize. Evidence has been presented for the factorization of the Pomeron in scaled inclusive cross sections,[10,12] and for the factorization of the leading nonscaling terms at intermediate energies.[13] Theoretically, Regge singularities should include cuts, and factorization should not be exact. However, phenomenologically factorization seems not to break down by more than 10-20%.[14] Nonleading

Regge corrections to scaling are generally considerably greater at intermediate energies so that factorizability seems a reasonable first approximation.

In the Mueller picture,[1,2] Regge corrections to scaling in one particle distributions are given by diagrams of Fig. 2b, just as Regge corrections to asymptotic total cross sections are given by the diagrams of Fig. 1b. If factorization were correct, the same inclusive Reggeon vertices would control the approach to scaling in limits where one final state particle was in the fragmentation region of the target, and one in the fragmentation region of the projectile. Two particle correlations could in principle be calculated entirely in terms of single particle distributions and total cross sections, from the diagrams of Fig. 3b. In practice, quite apart from problems with factorization, the experimental data on single particle processes are too crude to permit accurate determination of inclusive Reggeon vertices. However, a large number of qualitative and semiquantitative predictions can already be made on the basis of existing data, and some of these can be compared with experimental two particle distributions.

It is well-known[12] that certain single particle inclusive distributions seem to approach scaling rapidly (e.g., $K^+p \rightarrow \pi^- X$, $pp \rightarrow \pi^- X$, $\pi^+ p \rightarrow \pi^- X$ in the target fragmentation region). This means that phenomenologically certain inclusive Reggeon vertices are approximately exchange degenerate. On the other hand, certain reactions seem to approach scaling very slowly (e.g., $\pi^- p \rightarrow \pi^- X$,[11] $K^- p \rightarrow \pi^+ X$[15]) so that at least some inclusive Reggeon vertices are large. These facts, combined with a bare minimum of conservative theoretical prejudices about early scaling in unmeasured single particle inclusive process,[16] lead to striking predictions for the Reggeon contributions to different two particle distributions. In cases where one or more sets of inclusive Reggeon vertices are exchange degenerate, there are a number of predictions of zero Regge contributions. Examples are

$$pp \to \pi^{\pm}\pi^{\pm}X \ ,$$

$$K^{+}p \to \pi^{-}\pi^{\pm}X \ ,$$

$$\pi^{+}p \to \pi^{-}\pi^{\pm}X \ .$$

In cases where two sets of large inclusive Reggeon vertices are involved, Regge contributions of the order of more than one hundred percent can be expected in the 10 - 20 GeV/c range of initial lab momenta. Examples are

$$\bar{p}p \to \pi^{+}\pi^{-}X \ ,$$

$$K^{-}p \to \pi^{+}\pi^{-}X \ .$$

There are other cases where single particle data are inadequate for making definite predictions but consistency of the one and two particle data with theory can be checked. Wherever possible our predictions are confronted with experiment, and agreement seems fair, bearing in mind the necessarily approximate nature of our predictions and of the data.

The layout of the paper is as follows: In Section II we discuss notation and kinematics, and review the theory behind our predictions. In Section III we make a phenomenological analysis of cross section and single particle distribution data extracting from experiment inclusive Reggeon vertices. In Section IV we make predictions for the Reggeon contribution to a number of different two particle inclusive processes and discuss their reliability. In Section V we review the results and discuss which two particle processes would provide the most interesting experimental tests of our predictions. Finally, in a theoretical Appendix we outline the exchange degeneracy patterns of inclusive

Reggeon vertices which follow from our conservative theoretical prejudices about early scaling. It should be emphasized that we have used few of the results of the Appendix in our phenomenological analysis, and that these results and our predictions for two particle inclusive processes are largely independent.

II. NOTATION

The rapidity variable is particularly convenient for our considerations. Consider a single particle inclusive process, $a+b \rightarrow c+X$, in the rest frame of particle a. Then we can write

$$
\begin{aligned}
p_a &= (m_a, 0, 0, 0) \\
p_b &= (m_b \cosh Y,\ 0, 0,\ m_b \sinh Y) \qquad (2.1) \\
p_c &= (\omega_c \cosh y, p_{cx}, p_{cy}, \omega_c \sinh y)
\end{aligned}
$$

where $\omega_c^2 = p_{cx}^2 + p_{cy}^2 + m_c^2 = p_\perp^2 + m_c^2$. Thus $s = (p_a+p_b)^2 = m_a^2+m_b^2+2m_a m_b \cosh Y \approx m_a m_b e^Y$. The variable y is the lab rapidity of particle c. If we fix y and $p_\perp$ and let s increase to infinity, we expect the Lorentz invariant cross section, $E(d\sigma/d^3p)/\sigma_{tot}$ to become a function of $p_\perp$ and y only. This follows from assuming a leading pole in the $b\bar{b}$ channel when we view the process as a discontinuity of the six-point amplitude. More generally we can decompose the cross section for the process in this kinematical limit, which we denote $(a \rightarrow c\,|b)$ as (see Fig. 2):

$$
E\frac{d\sigma}{d^3p}(a \rightarrow c\,|b) = \sum_j \beta_j^b F_j^{a\rightarrow c}(\delta y_c, p_\perp)\ \tau_j (s/s_0)^{\alpha_j - 1} \qquad (2.2)
$$

where δy_c is the distance in rapidity from the kinematical boundary, $\delta y_c = y_c - y_{min} = y_c - \ln(m_c/m_a)$, and where the sum is over j-plane singularities in the $b\bar{b}$ channel with intercepts α_j. Each non-Pomeron we shall assume has an intercept of one-half, while the Pomeron is assumed to have unit intercept. In writing (2.2) we have assumed factorization for all the Reggeons. The signature of the j Reggeon is given by τ_j. We take $s_0 = 1\ \mathrm{GeV}^2$. In this notation the total cross section is given by (see Fig. 1):

$$
\sigma_{ab}(s) = \beta_{\mathbb{P}}^a \beta_{\mathbb{P}}^b + \sum_{j \neq \mathbb{P}} \beta_j^a \beta_j^b \tau_j \left(\frac{s}{s_0}\right)^{-1/2} \qquad (2.3)
$$

When $\delta y_c = y_c - y_{min}$ becomes large, we go over to the double Regge behavior of the central region with the boundary condition

$$F_j^{a\to c}(\delta y_c, p_\perp) \to \beta_{\mathbb{P}}^a f_j^c(p_\perp)\, e^{(1-\alpha_j)\delta y_c} \tag{2.4}$$

as $\delta y_c \to 0(Y/2)$.[17]

Consider next a process like $a+b \to c+d+X$. There is a variety of possible limits. We shall be concerned with the limit in which δy_c, δy_d, $\underset{\sim}{p}_{\perp c}$, and $\underset{\sim}{p}_{\perp d}$ are fixed while $Y\to\infty$ (i.e., $s\to\infty$). In this case, we can write an expansion for the cross section for the process, $(a\to c\,|\,d\leftarrow b)$, as (see Fig. 3)

$$E_c E_d \frac{d\sigma}{d^3p_c d^3p_d}(a\to c\,|\,d\leftarrow b) = \sum_j F_j^{a\to c}(\delta y_c, p_{\perp c})$$

$$\times\, F_j^{b\to d}(\delta y_d, p_{\perp d})\, \tau_j \left(\frac{s}{s_0}\right)^{\alpha_j - 1} \tag{2.5}$$

where the sum is over j-plane singularities and where δy_d is again the distance from the kinematic boundary: $\delta y_d = Y - \ln(m_c/m_b) - y_d$. As we shall see, the signature plays a vital role, analogous to the role it plays in differentiating pp and $p\bar{p}$ total cross sections.

The correlation function is defined by

$$g = E_c E_d \frac{d\sigma}{d^3p_c d^3p_d} - \frac{1}{\sigma_{ab}}\left(E_c \frac{d\sigma}{d^3p_c}\right)\left(E_d \frac{d\sigma}{d^3p_d}\right) \tag{2.6}$$

Thus if our sums in Eqs. (2.2 - 2.5) included only Pomerons, there would result a zero correlation function. By direct computation, one finds that including terms up to order $s^{-1/2}$ but neglecting terms of order s^{-1},

$$\sigma_{ab}\, g = \left(\frac{s}{s_0}\right)^{-1/2} \sum_{j\neq\mathbb{P}} \left(\beta_{\mathbb{P}}^a F_j^{a\to c} - \beta_j^a F_{\mathbb{P}}^{a\to c}\right)\left(\beta_{\mathbb{P}}^b F_j^{b\to d} - \beta_j^b F_{\mathbb{P}}^{b\to d}\right)\tau_j \tag{2.7}$$

where the arguments of the F's are the same as in Eq. (2.5), and where the sum is over the Reggeons with intercept one-half. The correlation function factorizes just as the cross sections do. The s dependent portion of the two particle cross section (Eq. (2.5)) and the correlation function (Eq. (2.7)) are quite similar, although the former is somewhat simpler. As a consequence we shall frequently state our predictions in terms of the s-dependence of the two-particle inclusive cross section.

The formula (2.7) that we use for the Regge correlations in the limit $s \to \infty$, δy_c and δy_d fixed can be related to the Regge prediction of a correlation length of two units in $(y_c - y_d)$ by using the boundary condition (2.4). For large rapidity separations $\delta y_c, \delta y_d$, and $\Delta y = y_d - y_c$, we get the form

$$\sigma_{ab}\, g \approx \frac{1}{\sqrt{2}\,\omega_c} \sum_{j \neq \mathbb{P}} \left(\beta^a_{\mathbb{P}}\right)^2 f^c_j(p_{\perp c})\, e^{-\frac{1}{2}\Delta y}\, \tau_j\, f^d_j(p_{\perp d}) \left(\beta^b_{\mathbb{P}}\right)^2 \tag{2.8}$$

since $F^{a\to c}_{\mathbb{P}}/F^{a\to c}_j$ and $F^{b\to d}_{\mathbb{P}}/F^{b\to d}_j \to 0$ as δy_c and $\delta y_d \to \infty$. Thus we get a smooth transition to the limit $s \to \infty$, δy_c, $\delta y_d \to \infty$, where the correlation in the central region is a constant independent of s and where the correlation has a characteristic length of about two units.

III. ESTIMATES OF INCLUSIVE REGGEON VERTICES

It is a fundamental prediction of the Mueller picture of inclusive reactions that the single particle cross section for $(a \to c\,|\,b)$ should approach its asymptotic value as $s^{-1/2}$. By performing such an experiment at a number of energies and with a variety of particles in place of particle b, in principle the various Reggeon contributions, f, ω, A_2, and ρ, could be isolated. At present few reactions have been studied at many energies and in most cases the fragmentation $(a \to c)$ has been studied for only one or two projectiles b. These limitations are compounded by the need for very good statistics if the nonasymptotic terms are to be determined, and by the likelihood of contributions from nonfactorizable cuts.

As a consequence, we can make only very crude estimates at this time for the value of the inclusive Reggeon vertices. We will only consider data integrated over $p_\perp$. We will normally be concerned with the ratios of the inclusive Reggeon and Pomeron vertices: $F_j^{a\to c}/F_{\mathbb{P}}^{a\to c}$. It is consistent with the crude nature of present data to assume that these ratios are independent of y, at least for values of y not more than a few rapidity units from the kinematic boundary, and we shall do so hereafter. Moreover, we are forced to rely on certain theoretical prejudices to fill in the lacunae in the data. We shall assume that certain single particle cross sections attain their asymptotic values at low energies. Data indicate that this is the case for $(p \to \pi^-\,|\,p)$, $(p \to \pi^-\,|\,K^+)$, and $(p \to \pi^-\,|\,\pi^+)$, for example.[18] Early scaling requires exchange degeneracy of certain inclusive Reggeon vertices in analogy with two body Regge phenomenology. A more detailed presentation of our assumptions about exchange degeneracy is given in the Appendix. These assumptions are not fundamental to our presentation, but are made simply because of the lack of high quality data. Of course, as better data become available, our estimates could be refined and the theoretical prejudices replaced by experimental results.

With these caveats we proceed to the estimation of the inclusive Reggeon vertices, $F_j^{a\to c}$, for j=f, ω, ρ, and A_2. We shall need certain two body residues, which in our normalization, we take to have the following nominal values, in $mb^{1/2}$

$$\begin{aligned} &\beta_{\mathbb{P}}^{p} = 6.1 \qquad \beta_{\mathbb{P}}^{\pi^+} = 3.6 \qquad \beta_{\mathbb{P}}^{K^+} = 2.9 \\ &\beta_f^p = \beta_\omega^p = 6.3 \\ &\beta_\rho^p = \beta_{A_2}^p = 1.4 \\ &\beta_\rho^{\pi^+} = \beta_f^{\pi^+} = 2.9 \\ &\beta_\rho^{K^+} = \beta_{A_2}^{K^+} = \beta_f^{K^+} = \beta_\omega^{K^+} = 1.5 \end{aligned} \tag{3.1}$$

A. $(p \to \pi^-)$

Alston-Garnjost et al.[19] have presented data showing that $(p \to \pi^- \mid \pi^+)$ scales early, at least approximately. Thus we have

$$\beta_f^{\pi^+} F_f^{p\to\pi^-} - \beta_\rho^{\pi^+} F_\rho^{p\to\pi^-} \simeq 0$$

so that $F_f^{p\to\pi^-} \simeq F_\rho^{p\to\pi^-}$. The data presented by Stroynowski[20] at 16 GeV/c show that

$$\frac{(p \to \pi^- \mid \pi^-)}{(p \to \pi^- \mid \pi^+)} \simeq 1.5$$

This ratio of cross sections can be expressed as

$$\frac{(p\to\pi^-|\pi^-)}{(p\to\pi^-|\pi^+)} = \frac{\beta_{I\!P}^{\pi^+} F_{I\!P}^{p\to\pi^-} + \left(\frac{s}{s_0}\right)^{1/2}\left[\beta_f^{\pi^+} F_f^{p\to\pi^-} + \beta_\rho^{\pi^+} F_\rho^{p\to\pi^-}\right]}{\beta_{I\!P}^{\pi^+} F_{I\!P}^{p\to\pi^-} + \left(\frac{s}{s_0}\right)^{1/2}\left[\beta_f^{\pi^+} F_f^{p\to\pi^-} - \beta_\rho^{\pi^+} F_\rho^{p\to\pi^-}\right]}$$

We find directly that

$$\frac{F_f^{p\to\pi^-}}{F_{I\!P}^{p\to\pi^-}} \simeq 1.7 \qquad (3.2)$$

We can check this ratio against other single particle inclusive data. We assume, as is suggested by the data,[18] that $(p\to\pi^-|K^+)$ and $(p\to\pi^-|p)$ scale early so that

$$\beta_f^{K^+} F_f^{p\to\pi^-} - \beta_\omega^{K^+} F_\omega^{p\to\pi^-} \simeq 0$$

$$\beta_\rho^{K^+} F_\rho^{p\to\pi^-} - \beta_{A_2}^{K^+} F_{A_2}^{p\to\pi^-} \simeq 0$$

Using the exchange degeneracy of the K^+ residues we conclude that

$$\begin{aligned} F_f^{p\to\pi^-} &\simeq F_\omega^{p\to\pi^-} \\ F_\rho^{p\to\pi^-} &\simeq F_{A_2}^{p\to\pi^-} \end{aligned} \qquad (3.3)$$

Thus all four $(p\to\pi^-)$ residues must be equal. By direct computation we find that

$$\frac{(p\to\pi^-|K^-)}{(p\to\pi^-|K^+)} \approx 1 + 4\left(\frac{s}{s_0}\right)^{-1/2} \frac{\beta_f^{K^+}}{\beta_{I\!P}^{K^+}} \frac{F_f^{p\to\pi^-}}{F_{I\!P}^{p\to\pi^-}}$$

$$\approx 1 + 3.5\left(\frac{s}{s_0}\right)^{-1/2} \qquad (3.4)$$

For 9 GeV/c lab momentum, we find the ratio of the cross sections to be about two, in agreement with the data of Foster et al.[15]

Using these residues, we can also make predictions for $(p \to \pi^- | \bar{p})$:

$$\frac{(p \to \pi^- | \bar{p})}{(p \to \pi^- | \bar{p})_{s=\infty}} = \frac{(p \to \pi^- | \bar{p})}{(p \to \pi^- | p)} = 1 + 2\left(\frac{s}{s_0}\right)^{-1/2} \frac{\left(\beta_f^p + \beta_\rho^p\right)}{\beta_{\mathbb{P}}^p} \frac{F_f^{p \to \pi^-}}{F_{\mathbb{P}}^{p \to \pi^-}}$$

$$\approx 1 + 4\left(\frac{s}{s_0}\right)^{-1/2} \tag{3.5}$$

Unfortunately data are not yet available on this reaction for comparison with this prediction. Assuming[21] that $\beta_f^\gamma \approx \beta_{A_2}^\gamma$ and $\left(\beta_f^\gamma / \beta_{\mathbb{P}}^\gamma\right) = \left(\beta_f^\pi / \beta_{\mathbb{P}}^\pi\right)$, we can also calculate $(p \to \pi^- | \gamma)$:

$$\frac{(p \to \pi^- | \gamma)}{(p \to \pi^- | \gamma)_{s=\infty}} = 1 + 2\left(\frac{s}{s_0}\right)^{-1/2} \frac{\beta_f^\pi}{\beta_{\mathbb{P}}^\pi} \frac{F_f^{p \to \pi^-}}{F_{\mathbb{P}}^{p \to \pi^-}}$$

$$\approx 1 + 3\left(\frac{s}{s_0}\right)^{-1/2} \tag{3.6}$$

This implies that at 9 GeV/c lab momentum, $(p \to \pi^- | \gamma)$ should be a little less than twice its expected asymptotic value: experimentally[22] the ratio is about two.

B. <u>$(K^+ \to \pi^-)$</u>

Assuming $(K^+ \to \pi^- | K^+)$ and $(K^+ \to \pi^- | \pi^+)$ scale early, we find

$$\begin{aligned} F_f^{K^+ \to \pi^-} &\simeq F_\rho^{K^+ \to \pi^-} \\ F_\omega^{K^+ \to \pi^-} &\simeq F_{A_2}^{K^+ \to \pi^-} \end{aligned} \tag{3.7}$$

If in addition $(K^+ \to \pi^- | p)$ scales early, all four residues are equal. As a consequence we can express the ratio of cross sections for $(K^+ \to \pi^- | p)$ and $(K^- \to \pi^+ | p)$ as

$$\frac{(K^- \to \pi^+ | p)}{(K^+ \to \pi^- | p)} = 1 + 2\left(\frac{s}{s_0}\right)^{-1/2} \left(\frac{\beta_f^p + \beta_\rho^p}{\beta_{\mathbb{P}}^p}\right) \frac{F_f^{K^+ \to \pi^-}}{F_{\mathbb{P}}^{K^+ \to \pi^-}} \quad . \tag{3.8}$$

At 9 GeV/c, Foster et al.[15] find this ratio to be about 2. Inserting this value we find,

$$F_f^{K^+ \to \pi^-} / F_{\mathbb{P}}^{K^+ \to \pi^-} \approx 1.7 \quad .$$

C. $(\pi^+ \to \pi^-)$

By G-parity conservation, there are only Reggeon vertices for f and ρ . If $(\pi^+ \to \pi^- | \pi^+)$ scales early, then

$$F_f^{\pi^+ \to \pi^-} \simeq F_\rho^{\pi^+ \to \pi^-} \tag{3.9}$$

It is difficult to estimate the size of these residues. One approach is to compare $(\pi^+ \to \pi^- | p)$ with $(\pi^- \to \pi^+ | p)$. Since these differ only in the ρ contribution, their difference is small because $\beta_\rho^p / \beta_f^p \simeq \frac{1}{4.5}$. On the other hand, looking at the energy dependence of $(\pi^+ \to \pi^- | p)$ is not reliable because quasi-elastic processes, e.g., $\pi^+ p \to \rho^0 \Delta^{++}$ are known to be important (see, for example, Alston-Garnjost et al.[19]). At 16 GeV/c, Stroynowski[20] shows that the cross section for $(\pi^- \to \pi^+ | p)$ is greater than that for $(\pi^+ \to \pi^- | p)$. This would indicate that $F_\rho^{\pi^+ \to \pi^-} > 0$, since we have

$$(\pi^\pm \to \pi^\mp | p) = \beta_{\mathbb{P}}^p \, F_{\mathbb{P}}^{\pi^+ \to \pi^-} + \left(\frac{s}{s_0}\right)^{-1/2} \left[\beta_f^p F_f^{\pi^+ \to \pi^-} \mp \beta_\rho^p F_\rho^{\pi^+ \to \pi^-}\right]$$

Of course contamination from lower lying Regge singularities may invalidate even this tentative statement.

D. $(p \to \pi^+)$

Data presented by Lander[12] indicate that $(p \to \pi^+ | p)$ and $(p \to \pi^+ | K^+)$ scale early. If this is so, then

$$F_f^{p\to\pi^+} \simeq F_\omega^{p\to\pi^+} \quad ,$$
$$F_\rho^{p\to\pi^+} \simeq F_{A_2}^{p\to\pi^+} \quad . \qquad (3.10)$$

On the other hand, Lander[12] finds that $(p \to \pi^+ | \pi^+)$ does not scale early and exceeds its presumed asymptotic limit. This indicates that $F_f^{p\to\pi^+} > F_\rho^{p\to\pi^+}$. More precise conclusions cannot be drawn at this time.

E. $(\pi^+ \to \pi^+)$

The data of Stroynowski[20] and Alston-Garnjost et al.[19] show that the cross section for $(\pi^+ \to \pi^+ | p)$ falls by about 15% between a lab momentum of 8 GeV/c and 16 GeV/c. Thus if we ignore the ρ contribution because of its small coupling to the nucleon, we have

$$1.15 = \frac{1 + (16)^{-1/2}\left[\dfrac{\beta_f^p}{\beta_{\mathbb{P}}^p}\,\dfrac{F_f^{\pi^+\to\pi^+}}{F_{\mathbb{P}}^{\pi^+\to\pi^+}}\right]}{1 + (31)^{-1/2}\left[\dfrac{\beta_f^p}{\beta_{\mathbb{P}}^p}\,\dfrac{F_f^{\pi^+\to\pi^+}}{F_{\mathbb{P}}^{\pi^+\to\pi^+}}\right]} \quad ,$$

implying $$F_f^{\pi^+\to\pi^+}/F_{\mathbb{P}}^{\pi^+\to\pi^+} \simeq 3.8 \quad .$$

The residue $F_{\rho}^{\pi^+\to\pi^+}$ cannot be determined at this time because β_{ρ}^{p} is so small. At 16 GeV/c the $(\pi^+ \to \pi^+ | p)$ and $(\pi^- \to \pi^- | p)$ cross sections differ by less than 10%.[20] This gives us a crude bound on $F_{\rho}^{\pi^+\to\pi^+}$, to wit

$$\left| \frac{F_{\rho}^{\pi^+\to\pi^+}}{F_{\mathbb{P}}^{\pi^+\to\pi^+}} \right| \lesssim 2 \quad . \tag{3.11}$$

IV. PREDICTIONS AND COMPARISON WITH DATA

Having developed the theory in Section II and analyzed the single particle data and extracted Reggeon inclusive vertices in Section III, we can now make predictions about two particle distributions. As discussed above, these results apply when one of the detected particles is in the fragmentation region of the target and the other is in the fragmentation region of the projectile. Also, the energy and the distances of the observed particles from the kinematic boundaries should be such that the difference between the rapidities of the observed particles are larger than two. Otherwise, the contributions of lower nonscaling exchanges might well not be negligible.

First we discuss which two particle distributions may be expected to have small Regge corrections to scaling. As discussed in the previous section, the following approximate exchange degeneracies between Reggeon inclusive vertices are suggested by experiment and/or are theoretically plausible:

$$
\begin{aligned}
&F_{\rho}^{p\to\pi^-} \simeq F_{\omega}^{p\to\pi^-} \simeq F_{f}^{p\to\pi^-} \simeq F_{A_2}^{p\to\pi^-} \\
&F_{\rho}^{\pi^+\to\pi^-} \simeq F_{f}^{\pi^+\to\pi^-} \\
&F_{\rho}^{K^+\to\pi^-} \simeq F_{\omega}^{K^+\to\pi^-} \simeq F_{f}^{K^+\to\pi^-} \simeq F_{A_2}^{K^+\to\pi^-} \\
&F_{f}^{p\to\pi^+} \simeq F_{\omega}^{p\to\pi^+}, \quad F_{\rho}^{p\to\pi^+} \simeq F_{A_2}^{p\to\pi^+}
\end{aligned}
\qquad (4.1)
$$

It can be seen from Eq. (6) that the signature factors (τ=+1 for f and A_2; τ=-1 for ρ and ω) will induce cancellations between the vertices (4.1) when suitable two particle inclusive distributions are selected. For example

$$(p \to \pi^- \mid \pi^- \leftarrow \pi^+) = F_{\mathbb{P}}^{p\to\pi^-} F_{\mathbb{P}}^{p\to\pi^-} + \left(\frac{s}{s_0}\right)^{-1/2} \left[\tau_f F_f^{p\to\pi^-} F_f^{\pi^+\to\pi^-} + \tau_\rho F_\rho^{p\to\pi^-} F_\rho^{\pi^+\to\pi^-}\right]$$

$$\simeq F_{\mathbb{P}}^{p\to\pi^-} F_{\mathbb{P}}^{\pi^+\to\pi^-} + O(s^{-1}) \tag{4.2}$$

Other two particle distributions which are expected to have zero or small Regge corrections include:

$$\begin{aligned} &(p \to \pi^- \mid \pi^- \leftarrow p) \\ &(p \to \pi^- \mid \pi^- \leftarrow K^+) \\ &(p \to \pi^- \mid \pi^+ \leftarrow p) \\ &(p \to \pi^+ \mid \pi^- \leftarrow K^+) \end{aligned} \tag{4.3}$$

Similar predictions for other less readily measurable processes can easily be constructed from the exchange degeneracy patterns set out in the Appendix. The predictions (4.2) and (4.3) probably provide the most conservative suggestions about early scaling in two particle inclusive processes.

As discussed in Section III, it is possible to give approximate estimates, based on intermediate energy single particle inclusive data, of some of the vertices (4.1)

$$\begin{aligned} F_f^{p\to\pi^-}/F_{\mathbb{P}}^{p\to\pi^-} &\simeq 1.7 \quad , \\ F_f^{K^+\to\pi^-}/F_{\mathbb{P}}^{K^+\to\pi^-} &\simeq 1.7 \quad . \end{aligned} \tag{4.4}$$

It should be emphasized again that because of systematic and statistical uncertainties in the data, possible breakdowns of factorization and $0(s^{-1})$ corrections to scaling, the values (4.4) are necessarily approximate. However, using (4.4) it is possible to make some predictions for the approach to scaling in two inclusive processes, for example

$$\frac{(p\to\pi^- \mid \pi^+ \leftarrow \bar{p})}{(p\to\pi^- \mid \pi^+ \leftarrow \bar{p})_{s=\infty}} = 1+\left(\frac{s}{s_0}\right)^{-1/2} \sum_{i=\rho,\omega,f,A_2} \frac{\tau_i F_i^{p\to\pi^-} F_i^{\bar{p}\to\pi^+}}{F_{\mathbb{P}}^{p\to\pi^-} F_{\mathbb{P}}^{\bar{p}\to\pi^+}} \tag{4.5}$$

By charge conjugation,

$$F_{\mathbb{P}}^{p\to\pi^-} = F_{\mathbb{P}}^{\bar{p}\to\pi^+}$$

$$F_{f,A_2}^{p\to\pi^-} = F_{f,A_2}^{\bar{p}\to\pi^+} \qquad\qquad F_{\rho,\omega}^{p\to\pi^-} = -F_{\rho,\omega}^{\bar{p}\to\pi^+} \tag{4.6}$$

It is apparent that because of the sign changes in Eqs. (4.6), the Regge corrections in Eq. (4.5) add rather than cancel:

$$\frac{(p\to\pi^- \mid \pi^+ \leftarrow \bar{p})}{(p\to\pi^- \mid \pi^+ \leftarrow \bar{p})_{s=\infty}} \simeq 1+4\left(\frac{s}{s_0}\right)^{-1/2} \left(\frac{F_f^{p\to\pi^-}}{F_{\mathbb{P}}^{p\to\pi^-}}\right)^2 \tag{4.7}$$

Substituting the value (4.4) into Eq. (4.7) we infer

$$\frac{(p\to\pi^- \mid \pi^+ \leftarrow \bar{p})}{(p\to\pi^- \mid \pi^+ \leftarrow \bar{p})_{s=\infty}} \simeq 1+12\left(\frac{s}{s_0}\right)^{-1/2} \tag{4.8}$$

A precisely analogous discussion yields

$$\frac{(p\to\pi^- \mid \pi^+ \leftarrow K^-)}{(p\to\pi^- \mid \pi^+ \leftarrow K^-)_{s=\infty}} \simeq 1+12\left(\frac{s}{s_0}\right)^{-1/2} \tag{4.9}$$

The predictions (4.8) and (4.9) deserve some comments. Because of the errors in estimating the ratios (4.4) of inclusive vertices, the coefficients of $s^{-1/2}$ in Eqs. (4.8) and (4.9) may well be in error by a factor of 2. However, we may believe that the Regge corrections are larger than is usual in total cross sections, for comparison using the two body residues (3.1) we obtain

$$\frac{\sigma(p\bar{p})}{\sigma(p\bar{p})_{s=\infty}} \simeq 1+2\left(\frac{s}{s_0}\right)^{-1/2}$$

$$\frac{\sigma(K^-p)}{\sigma(K^-p)_{s=\infty}} \simeq 1+1\left(\frac{s}{s_0}\right)^{-1/2} \tag{4.10}$$

The reason for the large Regge corrections is clear: the Regge corrections to one particle distributions are often larger than those to total cross sections (e.g., $(p\to\pi^-|\pi^-)$). This means that the ratios $F_f/F_{\mathbb{P}}$ are in general larger than the corresponding two body residues ratios $\beta_f/\beta_{\mathbb{P}}$. Finally, the two particle process corrections are proportional to $(F_f/F_{\mathbb{P}})^2$. The predictions (4.8) and (4.9) also give meaning to the statements that the processes (4.2) and (4.3) should exhibit small Regge corrections.

Because of the problems mentioned in the previous section in extracting the Reggeon inclusive vertices $F_i^{\pi^+\to\pi^-}$, $F_i^{\pi^+\to\pi^+}$ and $F_i^{p\to\pi^+}$, it is not possible to make numerical estimates of two particle inclusive processes involving them. However, the estimates (4.2), (4.3), (4.8) and (4.9) probably place bounds on the likely energy dependence of such processes.

Since inclusive processes are often not measured at more than one energy in the same experiment, it may be useful to give alternative expressions for the predictions (4.8) and (4.9). By charge conjugation invariance of the Pomeron,

$$(p\to\pi^-|\pi^+\leftarrow\bar{p})_{s=\infty} = (p\to\pi^-|\pi^-\leftarrow p)_{s=\infty}$$

$$(p\to\pi^-|\pi^+\leftarrow K^-)_{s=\infty} = (p\to\pi^-|\pi^-\leftarrow K^+)_{s=\infty} \tag{4.11}$$

Also, by Eqs. (4.3) the charge conjugated reactions should be energy independent. By factorization and using the energy independence of $(p\,|\,\pi^- \leftarrow p)$ and $(K^+\,|\,\pi^- \leftarrow p)$:

$$(p \to \pi^-\,|\,\pi^- \leftarrow p) \simeq \frac{(p\to\pi^-\,|\,p)(p\,|\,\pi^-\leftarrow p)}{\sigma\,(pp)}$$

$$(p \to \pi^-\,|\,\pi^- \leftarrow K^+) \simeq \frac{(p\to\pi^-\,|\,K^+)(p\,|\,\pi^-\leftarrow K^+)}{\sigma(K^+p)} \tag{4.12}$$

so that Eqs. (4.8) and (4.9) could be written entirely in terms of cross sections at intermediate energies:

$$\frac{(p\to\pi^-\,|\,\pi^+\leftarrow\bar{p})}{(p\to\pi^-\,|\,\pi^+\leftarrow\bar{p})_{s=\infty}} \simeq \frac{(p\to\pi^-\,|\,\pi^+\leftarrow\bar{p})}{(p\to\pi^+\,|\,\pi^-\leftarrow p)} \simeq \frac{(p\to\pi^-\,|\,\pi^+\leftarrow\bar{p})\,\sigma(pp)}{(p\to\pi^-\,|\,p)(p\,|\,\pi^-\leftarrow p)}$$

$$\simeq 1+12\left(\frac{s}{s_0}\right)^{-1/2} \tag{4.13}$$

$$\frac{(p\to\pi^-\,|\,\pi^+\leftarrow K^-)}{(p\to\pi^-\,|\,\pi^+\leftarrow K^-)_{s=\infty}} \simeq \frac{(p\to\pi^-\,|\,\pi^+\leftarrow K^-)}{(p\to\pi^-\,|\,\pi^-\leftarrow K^+)} \simeq \frac{(p\to\pi^-\,|\,\pi^+\leftarrow K^-)\,\sigma(K^+p)}{(p\to\pi^-\,|\,K^+)(p\,|\,\pi^-\leftarrow K^+)}$$

$$\simeq 1+12\left(\frac{s}{s_0}\right)^{-1/2} \tag{4.14}$$

Quantities even more accessible to experiment are the two particle correlations

$$g = (a\to c\,|\,d\leftarrow b) - \frac{(a\to c\,|\,b)(a\,|\,d\leftarrow b)}{\sigma(ab)} \tag{4.15}$$

Equation (13) gives an expression for $g\sigma(ab)$ in terms of Regge residues and inclusive vertices

$$\sigma(ab)g = \left(\frac{s}{s_0}\right)^{-1/2} \sum_{i=\rho,\,\omega,\,f,\,A_2} \tau_i \left(\beta^a_{\mathbb{P}} F_i^{a\to c} - \beta^a_i F^{a\to c}_{\mathbb{P}}\right)\left(\beta^b_{\mathbb{P}} F_i^{b\to d} - \beta^b_i F^{b\to d}_{\mathbb{P}}\right) \tag{4.16}$$

Inserting the vertex ratios (4.4) and the two body residues (3.1) we can calculate some correlations in the limit δy_c, δy_d fixed, $s \to \infty$:

$$\begin{aligned}
&g(p \to \pi^- | \pi^- \leftarrow p) \approx 0 \\
&g(p \to \pi^- | \pi^+ \leftarrow \bar{p}) \approx 5 \left(\frac{s}{s_0}\right)^{-1/2} (p \to \pi^- | \pi^+ \leftarrow \bar{p})_{s=\infty} \\
&g(K^+ \to \pi^- | \pi^- \leftarrow p) \approx 0 \\
&g(K^- \to \pi^+ | \pi^- \leftarrow p) \approx 6 \left(\frac{s}{s_0}\right)^{-1/2} (K^- \to \pi^+ | \pi^- \leftarrow p)_{s=\infty}
\end{aligned} \tag{4.17}$$

It is apparent that the correlation function has less energy dependence than the two particle cross section. Note also that the correlation function, being proportional to products of quantities like

$$\left(\frac{F_i}{F_{I\!P}} - \frac{\beta_i}{\beta_{I\!P}}\right) \tag{4.18}$$

is more sensitive to errors in estimating $F_i/F_{I\!P}$ than is the two particle distribution.

We now discuss what comparisons can be made between data and the predictions (4.2), (4.3), (4.8), and (4.9). Data have been published[28] on the inclusive process $(p \to \pi^- | \pi^- \leftarrow p)$ at an energy of 21 GeV and with the rapidities of the observed pions in a suitable range for our prediction (4.3) to apply. In Fig. 4 we have plotted the two particle distribution in the appropriate kinematic range and $(p \to \pi^- | p)(p | \pi^- \leftarrow p)/\sigma(pp)$ for comparison. It will be seen that the two sets of quantities are very similar. Since $(p \to \pi^- | p)$ is essentially energy independent, this means that $(p \to \pi^- | \pi^- \leftarrow p)$ scales early as predicted.

Data have also been presented[24] on the reaction $(p \to \pi^- | \pi^- \leftarrow K^+)$ at 11.8 GeV. For this process we predict a small correlation since $(p \to \pi^- | K^+)$, $(p | \pi^- \leftarrow K^+)$ and $\sigma(K^+ p)$ are either known or believed to have small Regge corrections to scaling. The data have been presented as a function of the difference, Δy, in the rapidities of the two pions, integrated over the remaining rapidity variable. Thus comparison with our prediction is not direct, as our results are meant to apply when the two pions are in opposite fragmentation regions, as well as having a rapidity separation greater than two. However the data do indicate that for $0.4 < |\Delta y| < 4$, the correlation function is between 10% and -50%. Bearing in mind the fact that some of the data are not in the region where our prediction (4.3) is meant to apply, and remembering the expected sizes of Regge corrections to scaling other inclusive reactions (4.8) and (4.9), we regard these data as consistent with our prediction (4.3).

More interesting would be comparisons of the data with our non-null predictions (4.8) and (4.9). Unfortunately, a conclusive comparison is not yet possible. No data on the process $(p \rightarrow \pi^- | \pi^+ \leftarrow \bar{p})$ are available. There are data available[25] on $(p \rightarrow \pi^- | \pi^+ \leftarrow K^-)$, but the energy is 9 GeV and we do not feel very confident about our predictions at such a relatively low energy. Nevertheless, since this is the only process which has been measured for which we expect a strikingly large energy dependence, we give a crude comparison. For data points with a π^- in the range $-0.2 > x^- > -0.4$ and a π^+ in the range $0.2 < x^+ < 0.4$, we can compare the two particle cross sections with the product of the single particle cross sections. We find the following range of values for the ratio at 9 GeV/c:

$$1.0 < \frac{(K^- \rightarrow \pi^+ | \pi^- \leftarrow p)\, \sigma(K^+ p)}{(K^- \rightarrow \pi^+ | p)(K^- | \pi^- \leftarrow p)} < 1.5 \tag{4.19}$$

Now we have as usual

$$\sigma(K^- p)_{s=\infty} (K^- \rightarrow \pi^+ | \pi^- \leftarrow p)_{s=\infty} = (K^- \rightarrow \pi^+ | p)_{s=\infty} (K^- | \pi^- \leftarrow p)_{s=\infty} \tag{4.20}$$

Again using the data of Foster et al.,[15]

$$(K^- \to \pi^+ | p)_{P_L = 9\ \mathrm{GeV}} \simeq 2\,(K^- \to \pi^+ | p)_{s=\infty}$$

$$(K^- | \pi^- \leftarrow p)_{P_L = 9\ \mathrm{GeV}} \simeq 2\,(K^- | \pi^- \leftarrow p)_{s=\infty} \tag{4.21}$$

Now if we suppose that

$$\frac{(K^- \to \pi^+ | \pi^- \leftarrow p)}{(K^- \to \pi^+ | \pi^- \leftarrow p)_{s=\infty}} = 1 + \lambda \left(\frac{s}{s_0}\right)^{-1/2} \tag{4.22}$$

we find $9 \lesssim \lambda \lesssim 14$ in agreement with our prediction $\lambda = 12$.

Data have also been published[26] on the reactions $(p \to \pi^- | \pi^- \leftarrow \pi^\pm)$ at 18.5 GeV/c. We are not able to make predictions for these reactions because of the difficulty discussed in Section III of extracting the inclusive Reggeon vertices $F_i^{\pi^\pm \to \pi^-}$ from single particle inclusive data. However it is possible to work the other way, assuming that Regge exchanges are dominant in the appropriate kinematic regions of $(p \to \pi^- | \pi^- \leftarrow \pi^\pm)$ and investigating whether this assumption is consistent with what little is known about $F_i^{\pi^\pm \to \pi^-}$ from single particle inclusive data.

In Fig. 5 we have plotted $(p \to \pi^- | \pi^- \leftarrow \pi^-)$ and for comparison, $(p \to \pi^- | \pi^-)(p | \pi^- \leftarrow \pi^-)/\sigma(\pi^- p)$ where all the quantities are evaluated at 18.5 GeV. It will be seen that for $-1.8 > y > -2.2$ and $0.2 < y < 3$, the two are equal to within about 25%. From the analysis of Section III we have

$$\frac{\sigma(\pi^- p)}{\sigma(\pi^- p)_{s=\infty}} \approx 1 + 1 \left(\frac{s}{s_0}\right)^{-1/2}$$

$$\frac{(p \to \pi^- | \pi^-)}{(p \to \pi^- | \pi^-)_{s=\infty}} \approx 1 + 3 \left(\frac{s}{s_0}\right)^{-1/2} \tag{4.23}$$

Hence at an energy of 18.5 GeV we deduce

$$\frac{(p\to\pi^-|\pi^-\leftarrow\pi^-)}{(p\to\pi^-|\pi^-\leftarrow\pi^-)_{S=\infty}} \approx \frac{(p\to\pi^-|\pi^-)}{(p\to\pi^-|\pi^-)_{S=\infty}}\;\frac{(p|\pi^-\leftarrow\pi^-)}{(p|\pi^-\leftarrow\pi^-)_{S=\infty}}\;\frac{\sigma(\pi^-p)_{S=\infty}}{\sigma(\pi^-p)}$$

$$\approx 1.4\;\frac{(p|\pi^-\leftarrow\pi^-)}{(p|\pi^-\leftarrow\pi^-)_{S=\infty}} \tag{4.24}$$

where we have used

$$(p\to\pi^-|\pi^-\leftarrow\pi^-)_{S=\infty} = \frac{(p\to\pi^-|\pi^-)_{S=\infty}\,(p|\pi^-\leftarrow\pi^-)_{S=\infty}}{\sigma(\pi^-p)_{S=\infty}} \tag{4.25}$$

which follows from Pomeron factorization. Also we have

$$\frac{(p\to\pi^-|\pi^-\leftarrow\pi^-)}{(p\to\pi^-|\pi^-\leftarrow\pi^-)_{S=\infty}} = 1+\left(\frac{s}{s_0}\right)^{-1/2}\left[\frac{F_\rho^{\pi^-\to\pi^-}}{F_{\mathbb{P}}^{\pi^-\to\pi^-}}\;\frac{F_\rho^{p\to\pi^-}}{F_{\mathbb{P}}^{p\to\pi^-}} + (\rho\longleftrightarrow f)\right]$$

$$\simeq 1+1.7\left(\frac{s}{s_0}\right)^{-1/2}\left[\frac{F_\rho^{\pi^-\to\pi^-}}{F_{\mathbb{P}}^{\pi^-\to\pi^-}} + 3.8\right] \tag{4.26}$$

$$\frac{(p|\pi^-\leftarrow\pi^-)}{(p|\pi^-\leftarrow\pi^-)_{S=\infty}} = 1+\left(\frac{s}{s_0}\right)^{-1/2}\left[\frac{\beta_\rho^p}{\beta_{\mathbb{P}}^p}\;\frac{F_\rho^{\pi^-\to\pi^-}}{F_{\mathbb{P}}^{\pi^-\to\pi^-}} + \frac{\beta_f^p}{\beta_{\mathbb{P}}^p}\;\frac{F_f^{\pi^-\to\pi^-}}{F_{\mathbb{P}}^{\pi^-\to\pi^-}}\right]$$

$$\simeq 1+\left(\frac{s}{s_0}\right)^{-1/2}\left[0.2\,\frac{F_\rho^{\pi^-\to\pi^-}}{F_{\mathbb{P}}^{\pi^-\to\pi^-}} + 3.9\right] \tag{4.27}$$

where we have inserted numerical values for quantities estimated in Section III. Comparing Eqs. (4.24), (4.26), (4.27) we find that $F_\rho^{\pi^-\to\pi^-}/F_{\mathbb{P}}^{\pi^-\to\pi^-} \approx 0$ which is consistent with the bound $|F_\rho^{\pi^- \to \pi^-}/F_{I\!P}^{\pi^- \to \pi^-}| \lesssim 2$ that we deduced in Section III. Hence we conclude that the data on correlations in $(p \to \pi^- | \pi^- \leftarrow \pi^-)$ at 18.5 GeV are consistent with being dominated by Regge exchanges.

Finally we should note that data also exist[26] on $(p \to \pi^- | \pi^- \leftarrow \pi^+)$ at 18.5 GeV/c, which we now compare with the single particle data.

In Section III we concluded that $F_\rho^{\pi^+ \to \pi^-} \simeq F_f^{\pi^+ \to \pi^-}$ and $F_\rho^{\pi^+ \to \pi^-} > 0$. For the process $(p | \pi^- \leftarrow \pi^+)$ we have

$$(p | \pi^- \leftarrow \pi^+) = \beta_{I\!P}^p F_{I\!P}^{\pi^+ \to \pi^-} + \left(\frac{s}{s_0}\right)^{-1/2} \left[\beta_f^p F_f^{\pi^+ \to \pi^-} - \beta_\rho^p F_\rho^{\pi^+ \to \pi^-}\right] \tag{4.28}$$

Since $\beta_f^p/\beta_\rho^p \approx 4.5$, we can anticipate that the cross section for $(p | \pi^- \leftarrow \pi^+)$ is a decreasing function of s:

$$\frac{(p | \pi^- \to \pi^+)}{(p | \pi^- \to \pi^+)_{s=\infty}} = 1 + \left(\frac{s}{s_0}\right)^{-1/2} \frac{F_f^{\pi^+ \to \pi^-}}{F_{I\!P}^{\pi^+ \to \pi^-}} \; \frac{\beta_f^p - \beta_\rho^p}{\beta_{I\!P}^p}$$

$$\approx 1 + \left(\frac{s}{s_0}\right)^{-1/2} \frac{F_f^{\pi^+ \to \pi^-}}{F_{I\!P}^{\pi^+ \to \pi^-}} \; (0.8) \tag{4.29}$$

On the other hand, we believe[18] that $(p \to \pi^- | \pi^+)$ scales early, as should $(p \to \pi^- | \pi^- \leftarrow \pi^+)$ (Eq. (4.2)). The scaling for the two particle inclusive process follows from the exchange degeneracy pattern for $(p \to \pi^-)$ and $(\pi^+ \to \pi^-)$. Thus we see that

$$(\pi^+ \to \pi^- | \pi^- \leftarrow p) = (\pi^+ \to \pi^- | \pi^- \leftarrow p)_{s=\infty} = \frac{(\pi^+ \to \pi^- | p)_{s=\infty} (\pi^+ | \pi^- \leftarrow p)_{s=\infty}}{\sigma(\pi^+ p)_{s=\infty}}$$

$$= \frac{(\pi^+ \to \pi^- | p)(\pi^+ | \pi^- \leftarrow p)}{\sigma(\pi^+ p)} \times \left(1 + 0.8 \left(\frac{s}{s_0}\right)^{-1/2} \frac{F_f^{\pi^+ \to \pi^-}}{F_{I\!P}^{\pi^+ \to \pi^-}}\right)^{-1} \frac{\sigma(\pi^+ p)}{\sigma(\pi^+ p)_{s=\infty}} \tag{4.30}$$

In Fig. 6 we show data[26] for $(\pi^+ \to \pi_1^- | \pi_2^- \leftarrow p)$ with $y(\pi_1^-) \approx 2.4$. Our calculations might be expected to work for $y(\pi_2^-) \lesssim -1$. Shown in the same figure are data for $(\pi^+ \to \pi^- | p)\ (\pi^+ | \pi^- \leftarrow p)/\sigma(\pi^+ p)$ at the same energy, 18.5 GeV. The factor $\sigma(\pi^+ p)/\sigma(\pi^+ p)_{s=\infty}$ is close to unity by comparison with the uncertainties in the data. Since in Section III we found $F_f^{\pi^+ \to \pi^-} > 0$, we expect the product of the one particle data to be above the two particle data. In the relevant domain, the ratio is about 1.5, which would give $F_f^{\pi^+ \to \pi^-}/F_{\mathbb{P}}^{\pi^+ \to \pi^-} \approx 3.5$. This ratio would yield

$$\frac{(p | \pi^- \leftarrow \pi^+)}{(p | \pi^+ \leftarrow \pi^-)} \approx \frac{1 + 3\left(\frac{s}{s_0}\right)^{-1/2}}{1 + 5\left(\frac{s}{s_0}\right)^{-1/2}} \tag{4.31}$$

At a lab momentum of 16 GeV/c, this ratio would be ≈ 0.8, which is completely consistent with the data of Stroynowski.[20]

V. DISCUSSION

In this section we examine to what extent the comparisons with data support the assumptions we used in making our theoretical predictions. We also discuss what experiments would be able to test our assumptions and predictions more stringently.

If the applicability of the Mueller analysis in general is accepted, there are three major assumptions we made in our work: that the dominant corrections to scaling in the energy range 10-20 GeV come from leading meson (ρ, ω, A_2, and f) exchanges, that the Pomeron and these trajectories approximately factorize, and that many of the inclusive Reggeon vertices are approximately exchange degenerate. To test the assumption on energy dependence independently of other assumptions, it is necessary to study accurately the same inclusive process at three different energies. This has not been done in a single experiment[27] for a one particle inclusive process except at very high energies, let alone for a two particle process. We believe such studies are essential if Regge dominance of the approach to scaling is to be confirmed, and if inclusive Reggeon vertices are to be extracted reliably. However, energy dependence may be tested if other assumptions are made: inclusive Reggeon vertices extracted from data at one energy may be used (assuming factorization) to predict the subasymptotic values of these processes at different energies. This we have done[29] in calculating $(p \to \pi^- | K^-)$ and $(p \to \pi^- | \gamma)$ using data on $(p \to \pi^- | \pi^-)$ at 16 GeV and comparing successfully with data on the first two interactions at 9 GeV/c. However, because of experimental and theoretical uncertainties, these are not very stringent tests.

Because in general four meson trajectories may contribute to the approach to scaling, it is difficult to test Regge factorization except by assuming in addition that inclusive Regge vertices often have approximate exchange degeneracy. This was done in calculating $(p \to \pi^- | K^-)$ and $(p \to \pi^- | \gamma)$. The combined assumptions of factorization and exchange degeneracy were found to imply that certain two particle inclusive processes should have small Regge correction (4.2), (4.3). These predictions seem to work for $(p \to \pi^- | \pi^- \leftarrow p)$ and $(p \to \pi^- | \pi^- \leftarrow K^+)$ especially when one recalls the expected magnitude of corrections to scaling in other processes (4.8), (4.9). Experimental studies of the other reactions (4.2), (4.3) would be useful, particularly of processes involving the fragmentation $(p \to \pi^+)$ for which exchange degeneracy is less firmly established than for $p \to \pi^-$. It may be objected that as $\sigma(pp)$, $\sigma(K^+p)$, $(p \to \pi^- | p)$ and $(p \to \pi^- | K^+)$ all scale early, it does not require much theory to predict that $(p \to \pi^- | \pi^- \leftarrow p)$ and $(p \to \pi^- | \pi^- \leftarrow K^+)$ should also scale early. This amounts to an argument that factorization and exchange degeneracy should work because of general principles: It is encouraging to see that experimentally they do work.

The best checks of factorization and exchange degeneracy in two particle inclusive processes would be studies of $(p \to \pi^- | \pi^+ \leftarrow K^-)$ and $(p \to \pi^- | \pi^+ \leftarrow \bar{p})$ at energies above 10-15 GeV, to check the predictions (4.8) and (4.9). In fact, experimental studies of one particle inclusive $\bar{p}$ induced processes seem to be sorely lacking — for $(p \to \pi^- | \bar{p})$ we have the prediction (3.5).

To expand the range of predictions of Mueller theory, more experimental information on inclusive Regge vertices is necessary. Accurate data at energies > 16 GeV would surely help determine the $\pi^\pm \to \pi^\pm$ inclusive Reggeon vertices.

We would emphasize the utility of standardizing the presentation of experimental data. It would be desirable to have data available as functions of rapidity or the Feynman x variable, integrated over $p_\perp$, for every experiment.

Finally we should comment on the relationship of our work to calculations made[30] with the nova model. Our predictions apply to a smaller kinematic range in x or y, but are determined completely (in principle) by one particle inclusive data, whereas in the nova model the relative proportions of single and double nova production are a priori unknown. Our results should be relevant to experiment up to energies where a naive factorizing Regge picture breaks down. In our case this is probably when the $s^{-1/2}$ Regge corrections are comparable to the probable 10-20% breakdown of factorization of the Pomeron. For a process (4.8), (4.9) with a cross section $1+12\ (s/s_0)^{-1/2}$, this energy would be over 1500 GeV. Thus our predictions may apply from present accelerator energies up to and including energies available at NAL and ISR. By contrast, the nova model is vague about the energies at which multiple nova production may become significant and introduce new undetermined parameters into its predictions. Note however that just as Regge behavior for one particle inclusive processes seems to set in at a higher energy than in total cross sections so Regge behavior may start dominating at still higher energies in two particle inclusive processes. Thus we would only believe our results qualitatively below 10-15 GeV.

ACKNOWLEDGEMENTS

Most of this work was done while the authors were at the Stanford Linear Accelerator Center. One of us (RCB) would like to thank the Theoretical Group for the hospitality extended to him while he was a guest at SLAC.

APPENDIX: EXCHANGE DEGENERACY

The contributions of f, ρ, ω, and A_2 to total cross sections are expected nearly to cancel for reactions in which the s-channel is exotic. This requires the standard exchange degeneracy conditions for the usual Regge residues:

$$\beta_f^{\pi^+} = \beta_\rho^{\pi^+}$$

$$\beta_f^{K^+} = \beta_\omega^{K^+} = \beta_\rho^{K^+} = \beta_{A_2}^{K^+}$$

$$\beta_f^{p} = \beta_\omega^{p}$$

$$\beta_\rho^{p} = \beta_{A_2}^{p}$$

With considerably less experimental and theoretical certainty, similar exchange degeneracy constraints may be placed on the inclusive Reggeon vertices, $F_i^{a\to c}$. We shall assume that inclusive processes $(a\to c|b)$ reach their asymptotic cross sections early if they fall into either of two categories:

1) Reggeon — particle exotic (RPE): a nonexotic Reggeon (i.e., $a\bar{c}$ nonexotic) scatters off particle b and $ab\bar{c}$ is exotic.

2) Super exotic (SE): for purely mesonic reactions, when ab, $ab\bar{c}$, $a\bar{c}$, and $b\bar{c}$ are exotic (e.g., $(K^+\to \pi^-|\pi^+)$).

We believe these are conservative assumptions: Reactions which fall into neither of these categories may scale early as well. We find that these assumptions enable us to derive many useful constraints while being weak enough to avoid the areas of major controversy.[16] In particular, (i) we make no assumptions about behavior in the central region. (ii) Our assumption for baryonic reactions is a natural extension of particle-particle exoticity to Reggeon-particle exoticity. Some experimental corroboration has been mentioned in the text. We avoid estimating corrections to the smaller cross sections with $a\bar{c}$ exotic. (iii) For mesons, SE is an assumption weaker than all other proposed criteria. Again we avoid the baryon problem posed by reactions such as $(p \rightarrow \bar{p} | p)$.

Even with these conservative assumptions, we find a plethora of exchange degeneracy conditions which should hold in the fragmentation region (i.e., away from the central region). These conclusions can be drawn prior to a complete resolution of the exoticity puzzle for single particle inclusive processes. The results of this analysis are given in Table I.

FOOTNOTES AND REFERENCES

1. A. H. Mueller, Phys. Rev. D2, 2963 (1970).

2. A. H. Mueller, Phys. Rev. D4, 150 (1971).

3. C-I. Tan, Phys. Rev. D4, 2412 (1971).

   H. P. Stapp, Phys. Rev. D4, 3177 (1971).

   J. C. Polkinghorne, Nuovo Cimento 7A, 555 (1972).

4. See for example, C. E. De Tar, Phys. Rev. D3, 128 (1971).

5. See for example, J. Benecke, T. T. Chou, C. N. Yang, and E. Yen, Phys. Rev. 188, 2159 (1969); R. Hwa and C. S. Lam, Phys. Rev. D5, 766 (1972); M. Jacob and R. Slansky, Phys. Rev. D5, 1847 (1972).

6. R. P. Feynman, High Energy Collisions (Gordon and Breach, New York, 1969), p. 237.

   T. T. Chou and C. N. Yang, Phys. Rev. Letters 25, 1072 (1970), and J. Benecke et al., Ref. 5.

7. H. D. I. Abarbanel, Phys. Letters 34B, 69 (1971).

8. H. D. I. Abarbanel, Phys. Rev. D3, 227 (1971).

9. For a recent review, see H.-M. Chan, Rutherford Laboratory Preprint RPP/T/17 (1972).

10. Pomeron factorization in two particle inclusive processes has been discussed by W. S. Lam, Phys. Letters 40B, 466 (1972). For a shorter account of this work see R. C. Brower, R. N. Cahn, and J. Ellis, Report No. SLAC-PUB-1153, (1972).

11. See for example, W. D. Shephard et al., Phys. Rev. Letters 27, 1164 (1971); M. Alston-Garnjost et al., Phys. Letters 39B, 402 (1972); K. C. Moffeit et al., Report No. SLAC-PUB-1004 (LBL-572), Stanford Linear Accelerator Center (1971).

12. For a comparison between $pp \rightarrow \pi^- X$ and $K^+ p \rightarrow \pi^- X$, see M. S. Chen et al., Phys. Rev. Letters 26, 1585 (1971). For a comparison between $pp \rightarrow \pi^+ X$ and $K^+ p \rightarrow \pi^+ X$, see R. L. Lander, Proceedings of the APS Division of Particles and Fields meeting, Rochester, 30 August - 2 September 1971 (American Institute of Physics, New York, 1971).

13. H. I. Miettinen, Phys. Letters 38B, 431 (1972).
H.-M. Chan, H. I. Miettinen and W. S. Lam, Phys. Letters 40B, 112 (1972).

14. The data quoted in Ref. 12 give no evidence of factorization breaking down. However, it has been argued by K. Wilson, Cornell preprint CLNS-131, unpublished (1970) and by M. Le Bellac, Phys. Letters 37B, 413 (1971) that there should be long range correlations in inclusive processes, which are generated by exclusive diffractive processes. Numerical estimates of this effect by J. Ellis, J. Finkelstein, and R. D. Peccei, Report No. SLAC-PUB-1020 to be published in Nuovo Cimento A (1972) indicate that these correlations may be of order 20%. These correlations correspond to a breakdown of Pomeron factorization, though in a different kinematic limit from that considered in this paper.

15. M. Foster et al., Phys. Rev. Letters 27, 1312 (1971).

16. Our specific assumptions are set out in detail in the Appendix. We are definitely not using any of the controversial early scaling criteria:
H.-M. Chan, C. S. Hsue, C. Quigg, and J.-M. Wang, Phys. Rev. Letters 26, 672 (1971);
J. Ellis, J. Finkelstein, P. H. Frampton, and M. Jacob, Phys. Letters 35B, 227 (1971);

M. B. Einhorn, M. B. Green, and M. A. Virasoro, Phys. Letters 37B, 292 (1971);

S.-H. H. Tye and G. Veneziano, Phys. Letters 38B, 30 (1972).

17. R. C. Brower and J. Ellis, Phys. Rev. D5, 2253 (1972).

18. M. S. Chen *et al.*, Ref. 12.

19. M. Alston-Garnjost *et al.*, Ref. 11.

20. R. Stroynowski, Multiparticle Phenomena and Inclusive Reactions (C.N.R.S., 1972), p. 77.

21. V. Barger and R. J. N. Phillips, Nucl. Phys. B32, 93 (1971).

H.-M. Chan, H. I. Miettinen and W. S. Lam, Ref. 13.

22. K. C. Moffeit *et al.*, Ref. 11.

23. E. L. Berger, B. Y. Oh, and G. A. Smith, preprint ANL/HEP 7209 (1972).

24. R. L. Lander, Multiparticle Phenomena and Inclusive Reactions (C.N.R.S., 1972), p. 257.

25. M. C. Foster and C. Quigg, S.U.N.Y. Stonybrook preprint (1972) and unpublished data (private communication).

26. W. D. Shephard, Proceedings of the International Conference on Inclusive Reactions at Davis (U. C. Davis, 1972), p. 272.

27. Comparison between different experiments is always made difficult by possible systematic errors.

28. There are data available from the ISR at different energies, but at energies too high to be useful for determining energy dependence, given the precision of these measurements. Data from NAL on $pp \to \gamma X$ (J. Pilcher *et al.*, Harvard preprint (1972) indicate that $(p \to \gamma | p)$ is essentially energy independent between 30 and 200 GeV/c.

29. See also H.-M. Chan et al., Ref. 13.

30. E. L. Berger, M. Jacob and R. Slansky, preprint NAL/THY-45, Yale-3075/4, ANL/HEP-7211 (1972).

TABLE CAPTION

The constraints on the vertices $F_i^{a \to c}$ are given in the first column assuming early scaling for the reaction in the second column on the basis of SE or RPE. Many vertices not listed are given trivially by charge conjugation or time reversal. No constraints are found for cases like $F_i^{\pi^+ \to \pi^+}$, $F_i^{K^+ \to K^+}$, or $F_i^{p \to p}$ where $a\bar{c}$ is the vacuum channel. In the table, when the charge label is absent then all charges are included: $\pi = (\pi^+, \pi^o, \pi^-)$, $K = (K^+, K^o)$, $N = (p, n)$: such cases are marked by asterisks (*).

TABLE I

Exchange Degeneracy Relations		Criterion
$F_f^{\pi^+\to\pi^-} = F_\rho^{\pi^+\to\pi^-}$	$(\pi^+\to\pi^- \mid \pi^+)$	SE
$F_f^{\pi^+\to\pi^o} = F_\rho^{\pi^+\to\pi^o}$	$(\pi^+\to\pi^o \mid \pi^+)$	RPE
$F_f^{K^+\to\pi^-} = F_\rho^{K^+\to\pi^-}$	$(K^+\to\pi^- \mid \pi^+)$	SE
$F_\omega^{K^+\to\pi^-} = F_{A_2}^{K^+\to\pi^-}$	$(K^+\to\pi^- \mid K^+)$	SE
$F_f^{K^+\to\pi^{+,o}} = F_\omega^{K^+\to\pi^{+,o}}$	$(K^+\to\pi^{+,o} \mid K)$	RPE *
$F_\rho^{K^+\to\pi^{+,o}} = F_{A_2}^{K^+\to\pi^{+,o}}$	$(K^+\to\pi^{+,o} \mid N)$	RPE *
$F_f^{K^+\to\pi^o} = F_\rho^{K^+\to\pi^o}$	$(K^+\to\pi^o \mid \pi^+)$	RPE
$F_f^{K^+\to K^-} = F_\rho^{K^+\to K^-}$	$(K^+\to K^- \mid \pi^+)$	SE
$= F_\omega^{K^+\to K^-} = F_{A_2}^{K^+\to K^-}$	$(K^+\to K^- \mid K)$	SE *
$\{ F_f^{p\to\pi} = F_\omega^{p\to\pi}$	$(p\to\pi \mid K^+)$	RPE *
$F_\rho^{p\to\pi} = F_{A_2}^{p\to\pi}$	$(p\to\pi \mid N)$	RPE *
$F_\rho^{p\to\pi^-} = F_f^{p\to\pi^-}$	$(p\to\pi^- \mid \pi^+)$	RPE
$F_f^{p\to K^+} = F_\omega^{p\to K^+}$	$(p\to K^+ \mid N)$	RPE *
$F_\rho^{p\to K^+} = F_{A_2}^{p\to K^+}$	$(p\to K^+ \mid N)$	RPE *
$F_f^{p\to n} = F_\rho^{p\to n}$	$(p\to n \mid \pi^+)$	RPE
$F_\omega^{p\to n} = F_{A_2}^{p\to n}$	$(p\to n \mid K^+)$	RPE

FIGURE CAPTIONS

1. (a) The Pomeron contribution to the total cross section, $\sigma(ab)$.

   (b) Reggeon contributions to the total cross section, $\sigma(ab)$.

2. (a) The Pomeron contribution to the fragmentation $(a \to c)$ in the process $a+b \to c+\text{anything}$.

   (b) Reggeon contributions to the fragmentation $(a \to c)$ in the process $a+b \to c+\text{anything}$.

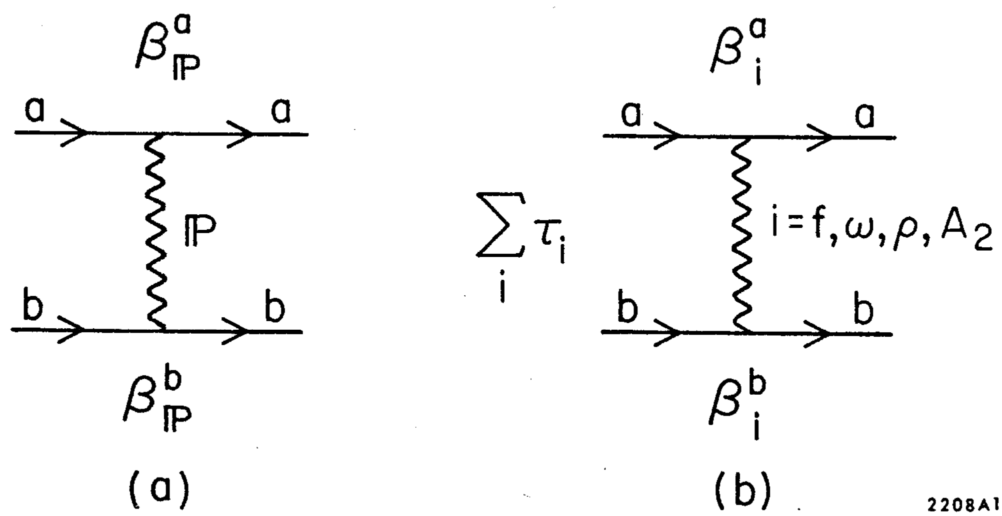
3. (a) The Pomeron contribution to the double fragmentation $(a \to c)$, $(b \to d)$ in the process $a+b \to c+d+\text{anything}$.

   (b) Reggeon contributions to the double fragmentation $(a \to c)$, $(b \to d)$ in the process $a+b \to c+d+\text{anything}$.

4. Comparison of the invariant cross sections $(p \to \pi^- | \pi^- \leftarrow p)$ (shown as x's) and $(p \to \pi^- | p)(p | \pi^- \leftarrow p)/\sigma(pp)$ (shown as dots) using data[22,29] at 21 GeV/c. The predictions of this paper concern the region $y_2 < -1$. Figure based on Ref. 30.

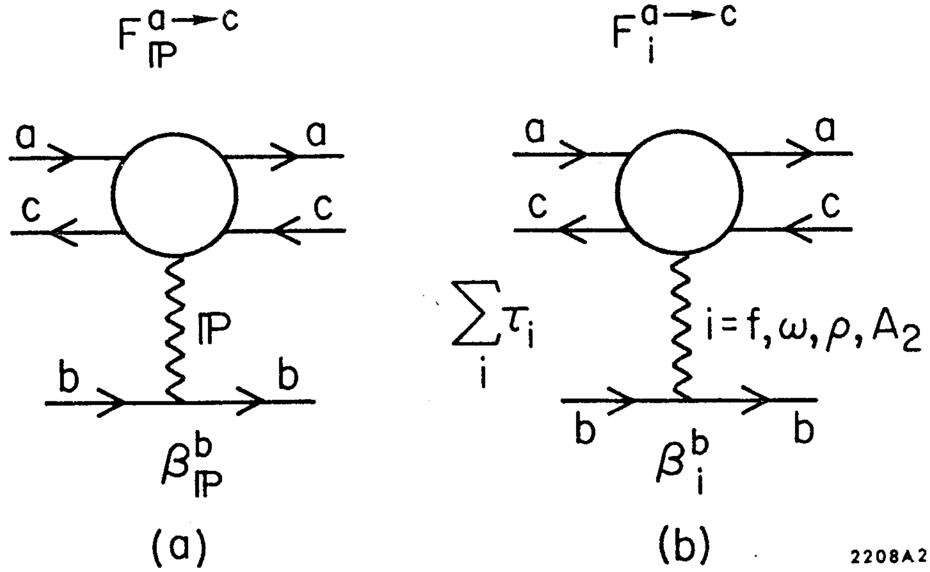
5. Comparison of the invariant cross sections $(p \to \pi^- | \pi^- \leftarrow \pi^-)$ (shown as dots) and $(p \to \pi^- | \pi^-)(p | \pi^- \leftarrow \pi^-)/\sigma(\pi^- p)$ (shown as triangles) using data[26,30] at 18.5 GeV/c. The predictions of this paper concern the region $y_2 > 1$.

6. Comparison of the invariant cross sections $(p \to \pi^- | \pi^- \leftarrow \pi^+)$ (shown as dots) and $(p \to \pi^- | \pi^+)(p | \pi^- \leftarrow \pi^+)/\sigma(\pi^+ p)$ (shown as triangles) using data[26,30] at 18.5 GeV/c. The predictions of this paper concern the region $y_2 < -0.5$.



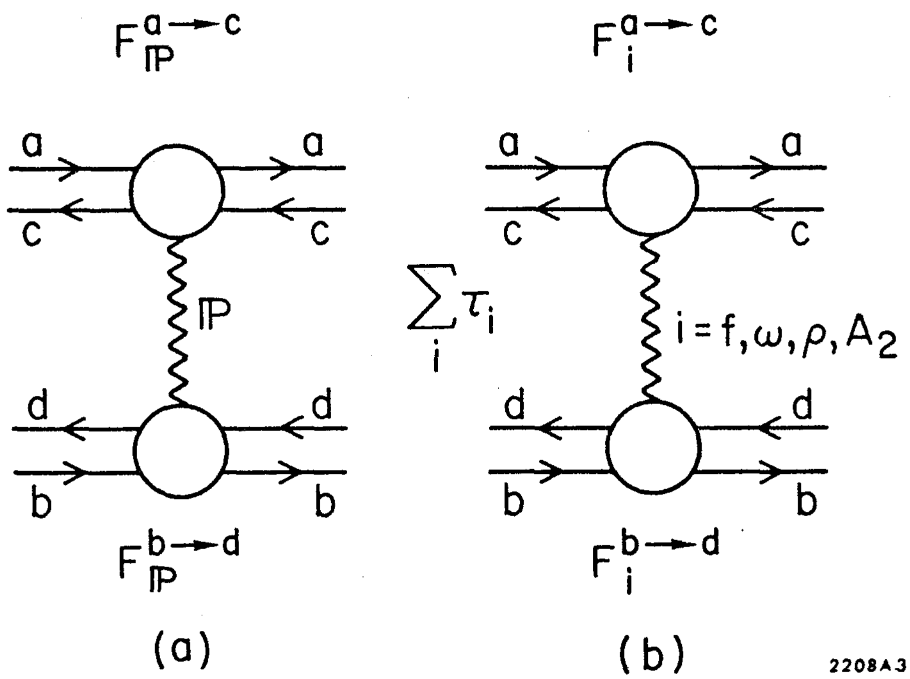
β^a_{IP}
a
a
IP
b
b
β^b_{IP}
(a)
$\sum_i \tau_i$
β^a_i
a
a
$i = f, \omega, \rho, A_2$
b
b
β^b_i
(b)
2208A1

Fig. 1



$F_{\mathbb{P}}^{a\to c}$
a
a
c
c
$\mathbb{P}$
b
b
$\beta_{\mathbb{P}}^{b}$
(a)
$F_{i}^{a\to c}$
a
a
c
c
$\sum_i \tau_i$
$i=f,\omega,\rho,A_2$
b
b
β_{i}^{b}
(b)
2208A2

Fig. 2



$F_{\mathbb{P}}^{a \to c}$
a
a
c
c
$\mathbb{P}$
d
d
b
b
$F_{\mathbb{P}}^{b \to d}$
(a)
$\sum_i \tau_i$
$F_i^{a \to c}$
a
a
c
c
$i = f, \omega, \rho, A_2$
d
d
b
b
$F_i^{b \to d}$
(b)
2208A3

Fig. 3

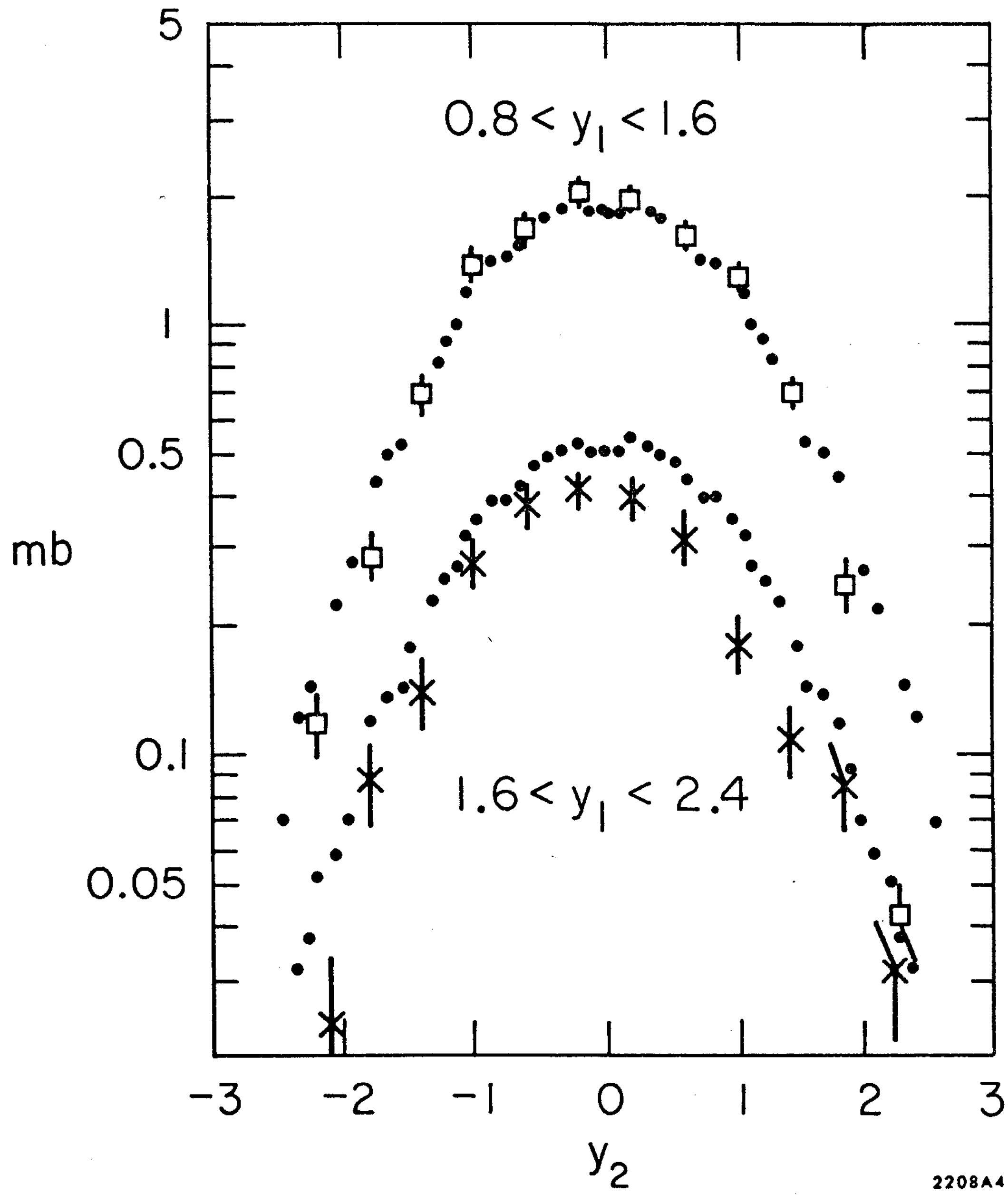


Fig. 4

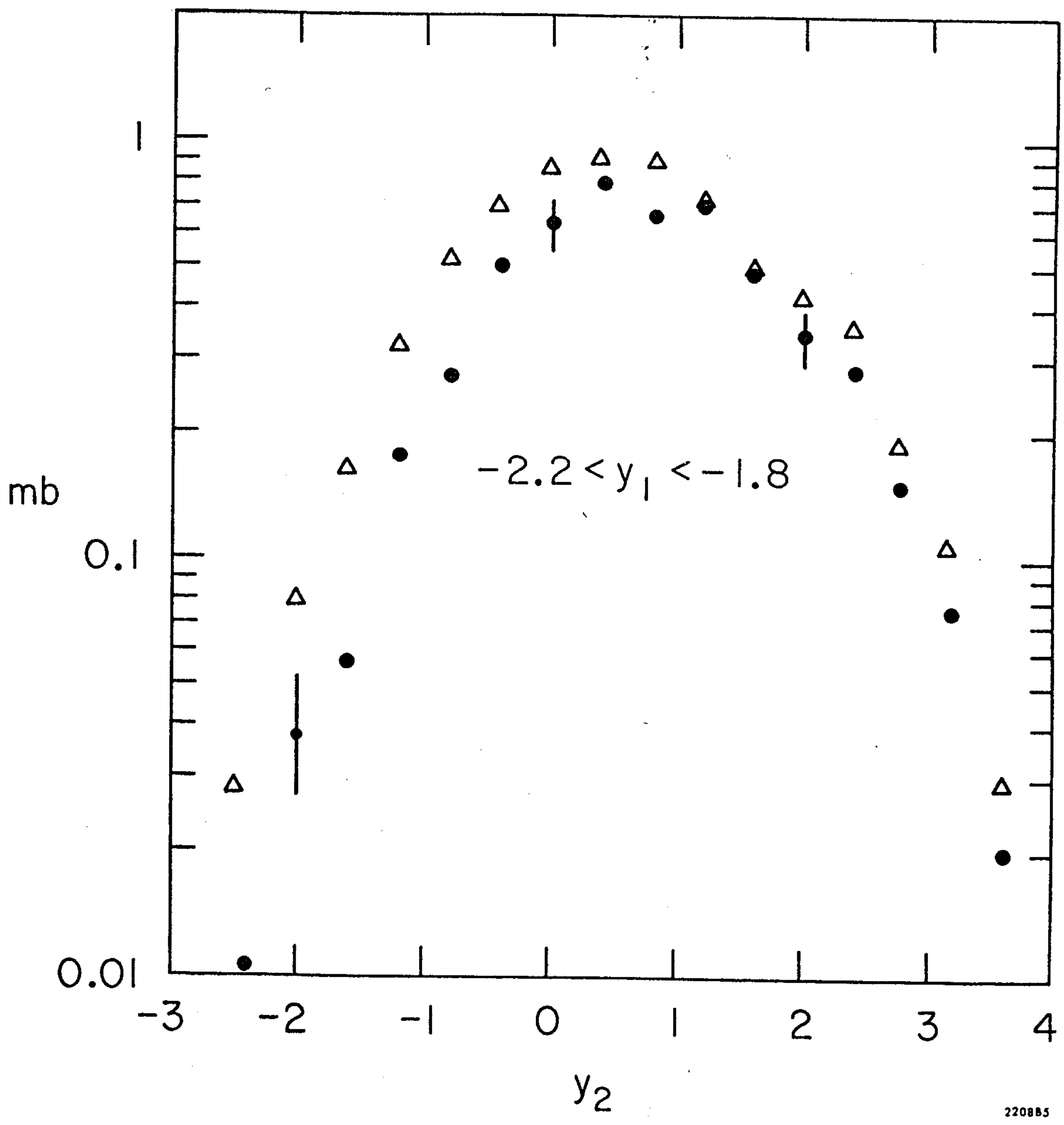


Fig. 5

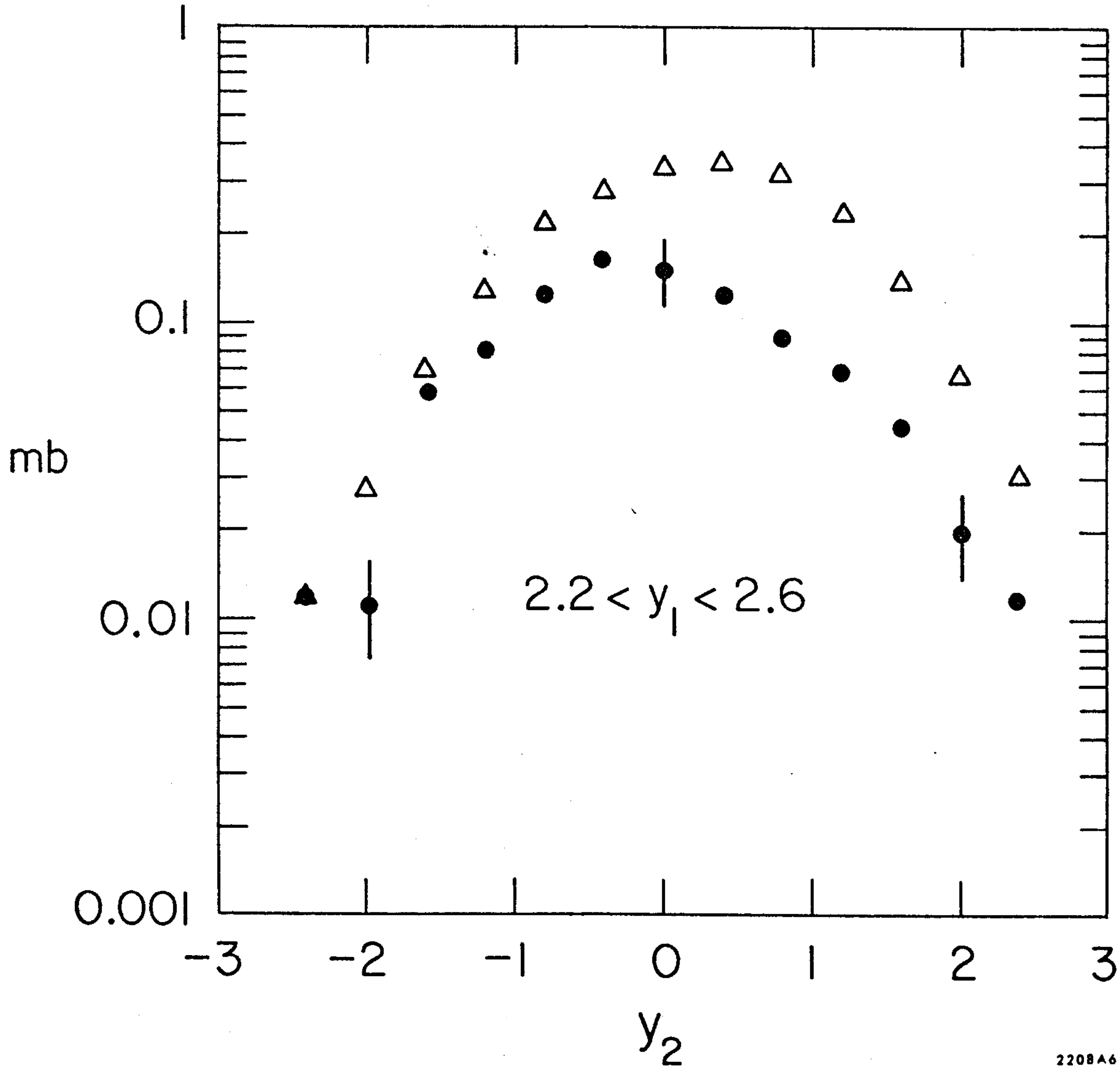


Fig. 6