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DISCUSSION

O'NEILL: Have you had any trouble maintaining these fairly tight tolerances off the median plane of the machine?

MARTIN: Maintaining the median plane (spacing) tolerances is the most difficult part of constructing the machine. Much of that work will be done by hand fitting. We hope to get the field accurate enough to get a beam through the machine even if at somewhat higher accelerating voltage then the lowest we plan to use. Then, by measuring the phase of the electron we will be able to make appropriate changes in the trimming coil currents to achieve the desired degree of isochronism. The tolerance requirements are considerably less stringent with respect to the azimuthal variations in the magnetic field.

CYCLOTRON WITH SPACE VARIATION OF THE MAGNETIC FIELD (**)

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I. INTRODUCTION

The first ideas of the application, in circular accelerators, of the magnetic field space variation date back to 1938 when L. Thomas suggested a variation of the field with azimuth in a cyclotron [1]. At that time these ideas were not properly developed due to the fact that the restriction on energy for a cyclotron was caused by the ion phase motion, and this restriction was removed by the suggested method only in a narrow energy region of accelerated ions. No less an essential obstacle to the development of this problem was the relatively low level of both measuring and calculating techniques. The principle of phase-stable acceleration suggested in 1944-1945 by V. I. Veksler [2] and McMillan [3] made it possible to

(*) See note on reports, p. 696.

(**) Mention of the starting up of this accelerator has been made by Vasilevskaya et al. See the reference [6] to their paper on p. 210.

remove the restrictions on maximum energy for circular accelerators with azimuthally symmetrical structure of the magnetic field. However, serious difficulties of technical and economical character arose in designing accelerators of 10-15 GeV and higher energies. These difficulties were partially overcome in alternating gradient accelerators [4] where the magnetic field was needed in a quite smaller volume.

For the time being, proton accelerators of this type are being designed and constructed in several countries [5, 6, 7] for energies of up to a few 10^{10} eV.

The pulsed operation of these accelerators limits the possible mean current of accelerated particles and considerably reduces the methodical possibilities of nuclear investigation.

The suggestion of using intersecting particle beams for the analysis of nuclear reactions, the great importance of investigation of nuclear reactions induced by secondary particles (π, μ, K, $\tilde{p}$, Σ etc.), and the increasing requirements of experimental accuracy make it necessary to rise the intensity of particle beams obtained in accelerators. Due to this, it became quite necessary to study in detail all the new acceleration techniques [8, 9] which can be found in non-uniform structures of fixed fields [(*)].

In 1955 it was suggested to use magnetic fields with azimuthal and radial periodic variations of strength [12]. The theoretical analyses of particle motion in such fields show that they had considerable advantages over the magnetic fields structure suggested by L. Thomas. For circular accelerators, this means that the maximum energy of the accelerated particles increases and that the required flutter field considerably decreases. For accelerators of the synchro-cyclotron type, such field structures make it possible to obtain stable orbits during the whole accelerating cycle and to have momentum compaction in the narrow ring of magnetic field.

In 1955-1958, at the Laboratory of Nuclear Problems of the Joint Institute for Nuclear Research, a circular accelerator has been designed and constructed on which investigations on two structures of magnetic field have been carried out. This model was used to test the linear theory of space stability developed in Dubna [13, 14, 15] and Harwell [16, 17, 18], to determine the limits of its applicability, to investigate some linear resonance effects and non-linear resonances in the centre of the accelerator.

With this accelerator, the problems of ion phase motion were experimentally investigated and the methods of calculation and production of the required structures of the magnetic field were tested.

II. LINEAR THEORY

The equations describing the motion of a charged particle in a magnetic field, when written in cylindrical co-ordinates, have the form

$$\begin{aligned} \ddot{r}-r\dot{\phi}^2 &= \frac{e}{mc}(r\dot{\phi}H_z-\dot{z}H_\phi)\,, \\ r\ddot{\phi}+2\dot{r}\dot{\phi} &= \frac{e}{mc}(\dot{z}H_r-\dot{r}H_z)\,, \\ \ddot{z} &= -\frac{e}{mc}(r\dot{\phi}H_r-\dot{r}H_\phi)\,. \end{aligned} \tag{1}$$

Since for this system $\dot{r}^2+r^2\dot{\phi}^2+\dot{z}^2 = v^2 = \text{const}$, we can introduce an independent variable ϕ:

$$\begin{aligned} r''-\frac{2r'^2}{r}-r &= \frac{e}{mc}\frac{\sqrt{r'^2+r^2+z'^2}}{v}\left[rH_z-z'H_\phi-\frac{z'r'}{r}H_r+\frac{r'^2}{r}H_z\right], \\ z''-\frac{2r'z'}{r} &= -\frac{e}{mc}\frac{\sqrt{r'^2+r^2+z'^2}}{v}\left[r'H_\phi-rH_r-\frac{z'^2}{r}H_r+\frac{z'r'}{r}H_z\right], \end{aligned} \tag{2}$$

where r', z' denotes the derivatives with respect to ϕ; H_z, H_r, H_ϕ are the corresponding components of the magnetic field, mv is the particle momentum.

The frame of reference is chosen so that in the median plane the total magnetic field is directed along the axis. The function characterizing the distribu-

(*) We discuss here neither the problems of using the properties of relativistic plasma [10] in accelerators nor the possibilities of coherent acceleration [11], as they definitely are out of the scope of this report.

tion of the magnetic field in this plane defines the particle motion. This function can be written:

$$H_z = H(r)[1+\varepsilon f(r,\phi)]\,, \tag{3}$$

where ε is the field flutter parameter, $f(r, \phi)$ is a periodic function with respect to both variables with mean value equal to zero.

After subsdituting the function (3) in (2) we obtain the following set of equations which describes the motion of particles with momenta

$$p = mv = \frac{e}{c}H(R)\cdot R \tag{4}$$

(In these equations, terms up to the third order have been retained):

$$\rho''+\left[1+n+\varepsilon R\frac{\partial f}{\partial r}+(2+n)\varepsilon f\right]\rho+\left[\frac{d}{R}+\frac{3}{2R}+\frac{2n}{R}+\frac{\varepsilon}{R}\left(\frac{3}{2}+2n+\frac{d}{2}\right)f+\varepsilon(2+n)\frac{\partial f}{\partial r}+\frac{\varepsilon R}{2}\frac{\partial^2 f}{\partial r^2}\right]\rho^2-$$

$$-\left[\frac{1}{2R}-\frac{3}{2}\frac{\varepsilon}{R}f\right]\rho'^2+\frac{1}{2}\left[\frac{d}{R}+\frac{d}{R}\varepsilon f+2n\varepsilon\frac{\partial f}{\partial r}+\frac{\varepsilon}{R}\frac{\partial^2 f}{\partial \phi^2}+\varepsilon R\frac{\partial^2 f}{\partial r^2}\right]z^2-\frac{\varepsilon}{R}\frac{\partial f}{\partial \phi}zz'+\frac{1}{2R}(1+\varepsilon f)z'^2 = -\varepsilon Rf, \tag{5}$$

$$z''-\left[n+\varepsilon nf+\varepsilon R\frac{\partial f}{\partial r}\right]z-\frac{2}{R}(n+d)+\frac{2\varepsilon}{R}(n+d)f+2\varepsilon(1+n)\frac{\partial f}{\partial r}+\varepsilon R\frac{\partial^2 f}{\partial r^2}\Big]z\rho+\frac{\varepsilon}{R}\frac{\partial f}{\partial \phi}z\rho'-\left[\frac{1}{R}-\frac{\varepsilon}{R}f\right]z'\rho'=0$$

where

$$\rho = r-R, \quad n = \frac{R}{H(R)}\left(\frac{dH(r)}{dr}\right)_{r=R}, \qquad d = \frac{1}{2}\frac{R^2}{H(R)}\left(\frac{d^2H(r)}{dr^2}\right)_{r=R}$$

the function f, and its partial derivatives being taken at $r = R$.

It follows from the consideration of the first equation that in the chosen frame of reference, in the plane $z = 0$, there are forced oscillations which indicate the lack of closed circular orbits in the field under consideration. Therefore it is worthwhile for the analysis of the solutions of Eq. (5) to find a closed orbit and to consider free oscillations around this orbit. The equation of the closed orbit in a linear approximation has the form

$$\bar{\rho}''+\left[1+n+\varepsilon R\frac{\partial f}{\partial r}+(2+n)\varepsilon f\right]\bar{\rho} = -\varepsilon Rf. \tag{6}$$

Denoting by $\bar{\rho}$ a particular solution of the inhomogeneous equation (6) we obtain the following equations, in the linear approximation, for the oscillations around the closed orbit

$$\rho''+\left[1+n+\varepsilon R\frac{\partial f}{\partial r}+(2+n)\varepsilon f+\frac{1}{R}(3+4n+2d)\bar{\rho}+2\varepsilon(2+n)\bar{\rho}\frac{\partial f}{\partial r}+\varepsilon\bar{\rho}\frac{\partial^2 f}{\partial r^2}\right]\rho-\frac{1}{R}(1-3\varepsilon f)\bar{\rho}'\rho' = 0\,, \tag{7}$$

$$z''-\left[n+\varepsilon nf+\varepsilon R\frac{\partial f}{\partial r}+\frac{2}{R}(n+d)\bar{\rho}+\frac{2\varepsilon}{R}(n+d)\bar{\rho}f+2\varepsilon(1+n)\bar{\rho}\frac{\partial f}{\partial r}+\varepsilon R\bar{\rho}\frac{\partial^2 f}{\partial r^2}-\frac{\varepsilon}{R}\frac{\partial f}{\partial r}\bar{\rho}'\right]z-\frac{1}{R}(1-\varepsilon f)\bar{\rho}'z' = 0\,.$$

To determine the main possibilities of the cyclotron method of acceleration in a magnetic field of the type in Eq. (3), we shall consider a practically realized case where the loci of the extremum values of H_z are Archimedes spirals

$$f = \sin\left(\frac{r}{\lambda}-N\phi\right), \tag{8}$$

where $2\pi\lambda$ is the radial pitch and N is the periodicity of the magnetic field structure.

Since for the cyclotron (ω_0 = const) the magnetic field parameter " n " must change as $\frac{\beta^2}{1-\beta^2}$, the choice of a magnetic field structure in which extremum values are located on logarithmic spirals [16], is not reasonable.

As will be shown below, the flutter factor for cyclotrons satisfies the condition $\varepsilon<1$; therefore, from Eq. (6), the closed orbit is described by the following function

$$\rho = \frac{\varepsilon R}{N^2-(1+n)} \sin\left(\frac{R}{\lambda} - N\phi\right) - \frac{\varepsilon^2 R(2+n)}{2[N^2-(1+n)](1+n)} - \frac{\varepsilon^2 R\left[\left(\frac{R}{\lambda}\right)^2 + (2+n)^2\right]^{\frac{1}{2}}}{2[N^2-(1+n)]} \cos\left[2\left(\frac{R}{\lambda} - N\phi\right) + \phi_o\right], \qquad (9)$$

$$\text{where} \quad \mathrm{tg}\,\phi_0 = \frac{R}{(2+n)\lambda}$$

After substituting Eq. (9) in Eq. (7) and changing the variables in order to suppress " periodical friction " terms (z', p'), each equation is transformed into the form

$$\rho'' + \left\{1 + n - \frac{\varepsilon^2 R^2}{2\lambda^2[N^2-(1+n)]} - \frac{\varepsilon^2(2+n)}{2(1+n)} \cdot \frac{3+4n+2d}{N^2-(1+n)} - \frac{\varepsilon^2 N^2}{8[N^2-(1+n)]} + \frac{\varepsilon R}{\lambda} \cos\left(\frac{R}{\lambda} - N\phi\right) + \right.$$
$$\left. + \left[(2+n)\varepsilon + \frac{\varepsilon(3+4n+2d)}{N^2-(1+n)} - \frac{1}{2} \frac{\varepsilon^2 N^2}{[N^2-(1+n)]}\right] \sin\left(\frac{R}{\lambda} - N\phi\right)\right\}\rho = 0, \qquad (10)$$

$$z'' + \left\{-n + \frac{\varepsilon^2}{N^2-(1+n)}\left[\frac{N^2}{2} + \frac{n+d}{n+1}\right] + \frac{\varepsilon^2 R^2}{2\lambda^2[N^2-(1+n)]} - \frac{\varepsilon^2 N^2}{8[N^2-(1+n)]^2} - \frac{\varepsilon R}{\lambda} \cos\left(\frac{R}{\lambda} - N\phi\right) - \right.$$
$$\left. - \left[\varepsilon n + \frac{2\varepsilon n}{N^2-(1+n)} + \frac{1}{2} \frac{\varepsilon N^2}{N^2-(1+n)}\right] \sin\left(\frac{R}{\lambda} - N\phi\right)\right\} z = 0 .$$

For the cyclotron under consideration the main focusing effect is determined by terms containing the ratio R/λ, which, for a non-conservative choice of parameters [19] exceeds unity for all radii except very close to the centre of the accelerator, where the linear theory is not applicable.

Neglecting small terms the system in Eq. (10) can be written in the canonical form as

$$\rho'' + (a_r + 2q \cos 2\xi)\rho = 0 ,$$
$$z'' + (a_z - 2q \cos 2\xi)z = 0 , \qquad (11)$$

where

$$a_r = \frac{4}{N^2}\left\{1 + n - \frac{\varepsilon^2 R^2}{2\lambda^2[N^2-(1+n)]}\right\},$$

$$a_z = -\frac{4}{N^2}\left\{n - \frac{\varepsilon^2 R^2}{2\lambda^2[N^2-(1+n)]}\right\},$$

$$q = \frac{2}{N^2}\frac{\varepsilon R}{\lambda}, \qquad 2\xi = \frac{R}{\lambda} - N\phi .$$

From Eq. (11) it follows that for cyclotrons the initial coefficients of Mathieu's equation are respectively equal to: $a_r = 4/N^2$, $a_z = 0$, $q = 0$ that is, the working point is in the first stable region [20]. The width of this region at $q<1$ is determined with an accuracy of some per cent by

$$-\tfrac{1}{2}q^2 \leq a_{r,z} \leq 1 - q - \tfrac{1}{8}q^2. \qquad (12)$$

From inequality, Eq. (12), it follows that for vertical oscillation the stability region has only a lower boundary $-\frac{1}{2}q^2$ and for radial ones only an upper boundary $1-q-\frac{1}{8}q^2$.

The first condition is written as

$$\varepsilon \geq \frac{N\lambda}{R}\sqrt{n}\sqrt{\frac{N^2-(1+n)}{N^2-\frac{1}{2}(1+n)}}, \qquad (13)$$

the second

$$\varepsilon \leq \frac{N^2\lambda}{2R}\left\{\frac{4[N^2-(1+n)]}{3N^2+1+n} - \sqrt{\frac{16[N^2-(1+n)]^2}{(3N^2+n+1)^2} - \frac{8[N^2-(1+n)]}{3N^2+n+1} + \frac{32(1+n)[N^2-(1+n)]}{N^2(3N^2+n+1)}}\right\} \qquad (14)$$

If one takes $a_z = e$, $a_r = \frac{4}{N^2}$ the necessary variation will be equal to

$$\varepsilon = \sqrt{2}\frac{eH(r)}{E_o}N\lambda\cdot\sqrt{1-\frac{1+n}{N^2}}. \quad (15)$$

From the equality of Eq. (13) and Eq. (14) for a given value N, one can determine the limiting values of n and consequently the maximum energy which can be obtained in a cyclotron with the magnetic field of the type in Eq. (8)

$$E_{\text{kin}} = E_0[\sqrt{n+1}-1]. \quad (16)$$

The maximum energy increases with an increasing periodicity of the magnetic field (N) and for practically realized structures it is given in Table I.

TABLE I

N	4	6	8
E_{kin} (MeV)	500	1130	1780

However, for the accelerator under consideration and for the known types of accelerators, the frequency at which the acceleration process is limited is determined by resonant values of the betatron oscillation frequencies but not by the stability region boundaries.

From Eq. (11) the frequency of natural oscillations is equal to

$$Q_{z,r} = \frac{N}{2}\mu_{z,r} \quad (17)$$

where μ_z and μ_r for Mathieu's equation are determined from the expression

$$\cos\mu\pi = \cos\pi\sqrt{a} - \frac{\pi^2}{4}\frac{\sin\pi\sqrt{a}}{\pi\sqrt{a}}\cdot\frac{q^2}{1-a}. \quad (18)$$

This expression gives a satisfactory accuracy for practical calculations in all the region of free oscillations frequency variation. If $q \ll 1$, then from Eqs. (11), (17) and (18) it follows that

$$Q_r = \sqrt{1+n} + \frac{3}{4}\frac{1}{N^3}\left(\frac{\varepsilon R}{\lambda N}\right)^2 \frac{1-\frac{3}{4}\frac{1+n}{N^2}}{1-\frac{1+n}{N^2}}, \quad (19)$$

at $N \ll N^2$

$$Q_z = \sqrt{\left(\frac{\varepsilon R}{N\lambda}\right)^2 - n}. \quad (20)$$

From Eqs. (19) and (20) it follows that in cyclotrons the initial frequencies of betatron oscillation are

$$Q_z = 0, \quad Q_r = 1.$$

In the process of acceleration, these frequencies increase. If the resonant excitation of oscillations in the central region of the accelerator is not taken into account, the first linear resonant excitation of oscillations of the first and the third harmonics of the magnetic field is possible in areas where $Q_z = 0.5$, $Q_r = 1.5$, respectively.

Let us consider a quasi-static method of amplitude estimation at the parametric resonance. If in the structure of the magnetic field the first harmonic exists, one has, in Eq. (3)

$$f(r,\phi) = \sin\left(\frac{2}{\lambda} - N\phi\right) + \frac{\varepsilon_1}{\varepsilon}\sin\left[\alpha_1(r)-\phi\right], \quad (21)$$

where ε_1 is the amplitude of the first harmonic ($\varepsilon_1 \ll \varepsilon$).

From Eq. (7) provided $R\left|\frac{d\alpha_1(r)}{dr}\right|_{r=R} \ll n$ for $Q_z = 0.5$ we obtain

$$z'' + \{0.25 + n\varepsilon_1 \sin\left[\alpha_1(R)-\phi\right]\}z = 0. \quad (22)$$

From Eq. (22), the resonance band is symmetric around the frequency $Q_z = 0.5$ and its width is

$$\Delta Q_z = \frac{n\varepsilon_1}{2}, \quad (23)$$

and the maximum index of exponential growth of the amplitude within this band is equal to

$$\mu_{\max} = n\varepsilon_1. \quad (24)$$

If $R\left|\frac{d\alpha_1}{dr}\right|_{r=R} \gg \eta$, in the expressions Eqs. (23) and (24) the value of "n" must be replaced by $R\left|\frac{d\alpha_1}{dr}\right|_{r=R}$ and the maximum index of the amplitude oscillation increase will be equal to

$$\mu_{\max} = \varepsilon_1 R\left|\frac{d\alpha_1}{dr}\right|_{r=R}. \quad (25)$$

The number of ion turns in the resonant bandwidth depends on the chosen regime of the working point motion through the stability region. More detailed information on the passage through a parametric resonance is given in the paper by Kol'ga [15]. The calculation of the oscillation amplitude increase near the resonant regions $Q_r = 2$ and $Q_z = I^{(*)}$ indicates the possible restriction of the maximum energy Eq. (16) to the following values

TABLE II

N	4	6	8	∞
E_{kin} (MeV)	500	790	850	938

III. NON-LINEAR EFFECTS

In analogy with systems considered in the paper by Bogolyubov and Mitropolskij [21] one should expect the investigated system Eqs. (2), (3), (8) to be excited at frequencies $Q_{r,z} = \frac{p}{q}N$ where p and q are integers. Thus in the central region of the accelerator $Q_r = 1$, $p = 1$, $q = N$) there exists a possibility of non-linear resonant effect, if the initial amplitude is greater than a value which is determined by the parameters of the chosen magnetic field structure. To find this amplitude it is necessary to solve the Equation (2) at $z = 0$

$$r'' - \frac{2r'^2}{r} - r = -\frac{e}{pc}\frac{(r'^2+r^2)^{3/2}}{r}H_z(r,\phi), \qquad (26)$$

where

$$H_z(r,\phi) = H_0(1+\alpha r^2)\left[1+\varepsilon\sin\left(\frac{r}{\lambda} - N\phi\right)\right],$$

$$\alpha = \frac{1}{2r_\infty^2}, \quad r_\infty = \frac{E_0}{eH_0}.$$

For the central region of the accelerator $\alpha r^2 \ll 1$ and $\varepsilon \ll 1$; therefore one can take as a primary solution of Eq. (26)

$$r = \cos(\phi-\psi) + \sqrt{R^2 - S^2\sin^2(\phi-\psi)}, \qquad (27)$$

where S, ψ are the co-ordinates of the centre of curvature of the trajectory, whose radius of curvature is

$$R = \frac{pc}{eH_0}. \qquad (28)$$

Since the non-linear resonant effects at $\varepsilon \ll 1$ are expressed as a shift of the instantaneous centre of curvature of the orbit it is natural to look for a solution of Eq. (26) in the form Eq. (27) where $S = S(\phi)$ and $\psi = \psi(\phi)$.

The relation between the co-ordinates of a particle (r, ϕ) and co-ordinates of the centre of curvature (S, ψ) in the magnetic field $H_z(r, \phi)$ can be written as

$$\frac{dS}{d\phi} = \frac{pc}{e}\frac{H_z'}{H_z^2\sqrt{r'^2+r^2}}[r\cos(\phi-\psi) + r'\sin(\phi-\psi)],$$

$$S\frac{d\psi}{d\phi} = \frac{pc}{e}\frac{H_z'}{H_z^2\sqrt{r'^2+r^2}}[r\sin(\phi-\psi) - r'\cos(\phi-\psi)], \qquad (29)$$

where H_z' denotes the total derivative with respect to ϕ. Substituting Eq. (27) in Eq. (29) and using the method [21] of averaging over ϕ provided $S_{\max} < \lambda \ll R$ we obtain for even structures ($N = 2k$)

$$\frac{dS}{d\phi} = (-1)^{N/2}\varepsilon R\frac{1}{(N-1)!\,2^N}\left(\frac{S}{\lambda}\right)^{N-1}\sin\left(N\psi - \frac{R}{\lambda}\right),$$

$$\frac{d\psi}{d\phi} = -\alpha R^2 + (-1)^{N/2}\frac{\varepsilon R}{\lambda}\frac{1}{(N-1)!\,2^N}\left(\frac{S}{\lambda}\right)^{N-2} \times \cos\left(N\psi - \frac{R}{\lambda}\right). \qquad (30)$$

From the analysis of Eq. (30), it follows that the boundary between the precession regime of the centre of curvature and the regime at which the azimuthal motion of the centre of curvature is limited by an angle π/N is characterised by the inequality

$$\alpha R^2 > \frac{\varepsilon}{2^N(N-1)!}\frac{R}{\lambda}\left(\frac{S_{\max}}{\lambda}\right)^{N-2}. \qquad (31)$$

Experimental investigations of non-linear resonance were carried out on a model having the following parameters: $N = 4$, $\lambda = 1.34$ cm, $\varepsilon = 0.02$. The observed shift of instantaneous orbit centres for different radii is seen on Fig. 1. Here, the points denote the orbit centres and the figures are the radii

(*) The resonance $Q_r = I$ and resonances of linear interaction $Q_r + Q_z = 1$, $Q_r + Q_z = 2$ take place only in the case of violation of the magnetic field smoothness.

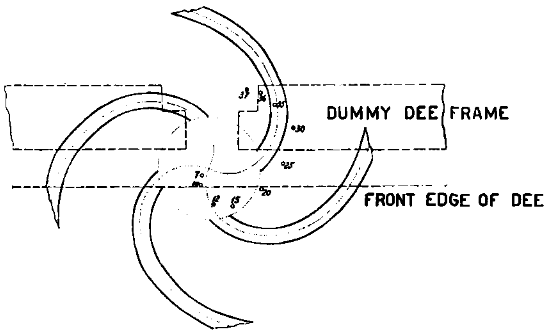

Fig. 1 Position of the orbits centres for N = 4.

of curvature. The theoretically calculated maximum deviations of the centres for the case $s > \lambda$ are in agreement with the experiment.

For a proper choice of the parameters of the magnetic field structure for which the non-linear resonant effects will not be present, it is sufficient to use the expression Eq. (31) where instead of S_{max} one must insert the maximum initial oscillation amplitude for the given accelerator.

On the basis of the above analysis, a structure of the magnetic field corresponding to the following parameters has been realized: $N = 6, \lambda = 2.7$cm, $\varepsilon = 0.066$.

Using these parameters, one can see that inequality Eq. (31) is satisfied beginning with a radius $R = 2$ cm. The experimental determination of the orbit centres of curvature for this case show that the orbit shift does not exceed the deviations due to lower harmonics in the magnetic field structure; in absolute value these shifts do not exceed 2 cm.

The motion of the working point through the stability region characterizing the change in frequencies of the free oscillations in the acceleration process are given in Fig. 2.

In designing an accelerator of this type for synchrocyclotron energies, the choice of the regime of the motion of the working point through the stability region will essentially depend upon the results of the investigation of particle motion through non-linear resonance regions $\frac{N}{N-1}, \frac{N}{N-2}, \ldots$ to $\frac{N}{N-0.5N} = 2$ for radial oscillations, as well as through the area of coupled oscillations.

IV. PHASE SHIFT

If the average magnetic field strength changes according to the law

$$H(r) = \frac{H_0}{\sqrt{1 - \left(\frac{r}{r_\infty}\right)^2}}, \tag{32}$$

where

$$r_\infty = \frac{E_0}{eH_0}.$$

then for particles of momentum Eq. (4) on the orbit $R + \bar{\rho}(\phi)$ the phase shift can be due to two reasons:

a) deviations of the magnetic field from the law, Eq. (32), connected with an insufficient accuracy of shimming and stabilization;

b) deviations of the closed orbit form from the circle.

The required stabilization of the magnetic field must be better than

$$\frac{\Delta H}{H} = \frac{1}{4A}, \tag{33}$$

where A is the number of turns of an ion during the acceleration period.

Errors of the magnetic field measurements should also be less than this value [22].

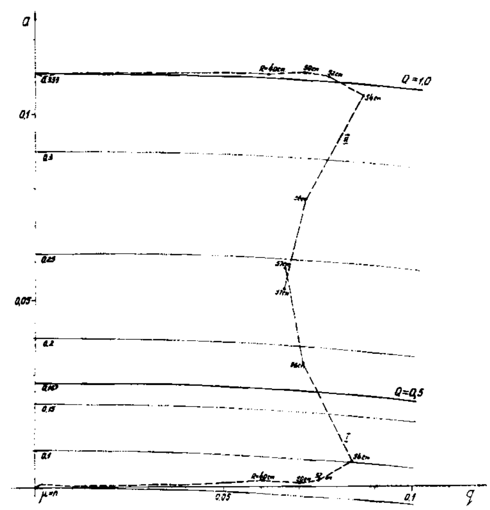

Fig. 2 Motion of the representing point through the stability region, I—for vertical oscillations, II—for radial oscillations.

The phase shift due to the change of the particle revolution period on a closed orbit (Eq. (9)) when the energy changes, can be calculated:

$$\Delta\omega = \frac{v}{R} - \frac{2\pi v}{\int_0^{2\pi} \sqrt{r'^2 + r^2}\, d\phi}. \tag{34}$$

Using Eq. (9) we obtain

$$\frac{\Delta\omega}{\omega} = \frac{\varepsilon^2}{2}\left\{\frac{2+n}{(1+n)[N^2-(1+n)]} - \frac{N^2}{2[N^2-(1+n)]^2}\right\}. \tag{35}$$

From Eq. (35) it follows that the correction to the law, (Eq. (32)) is of the order of $(\varepsilon/N)^2$. For $\varepsilon > 0.1$ one should introduce corrections to the law, Eq. (32).

The phase shift regime in the cyclotron was tested with the above six-spirals structure of the magnetic field when deuterons were accelerated to 13 MeV. The minimum accelerating voltage on the dee for a magnetic field $H(r)$ given in Fig. 3, was found to be 5 kV. The ions make approximately 2 500 turns.

The accelerated particle energy on the maximum radius (54 cm) was measured by two methods:

a) by measuring the mean radius of curvature of the orbit by means of three probes,

b) by measuring the accelerated deuteron ranges in aluminium foils.

The experiments were carried out when the intensity of the internal beam did not exceed $1\mu A$. Due to this, a low activity only was induced in the chamber.

Fig. 4 represents the internal beam intensity as a function of accelerator radius when the accelerating voltage on the dee is 11 kV. The beam at all the radii is well focused and the halfwidth of its vertical distribution is less than 1 cm.

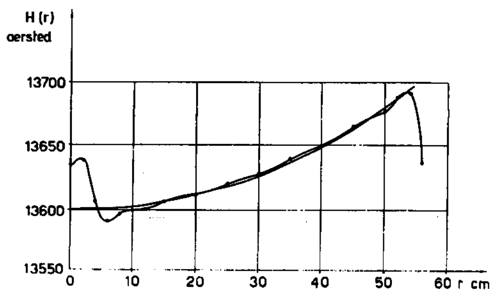

Fig. 3 Average magnetic field vs. radius for N = 6.

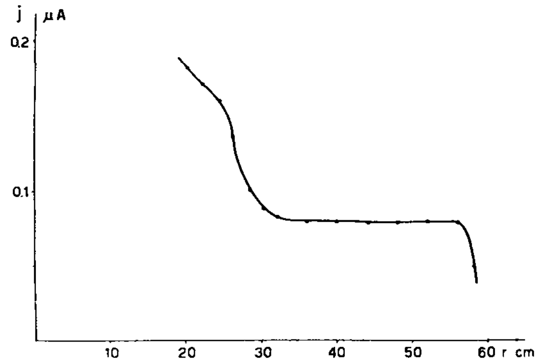

Fig. 4 Beam current at different radii (V_0 = 11 kV).

V. CALCULATION AND SHIMMING OF THE MAGNETIC FIELD

A magnetic field of the type Eq. (3) was formed in the middle plane of the electromagnet (diameter 120 cm, aperture $2h_M = 22$ cm) with the aid of shims of rectangular cross-section having the form of Archimedes spirals $r = \lambda N\phi$, and a system of ring-shaped shims. The parameters of the spiral shims (the ratio between width and height) were taken in the assumption that the iron of the shims is close to saturation along the axis z, since in the useful region of the gap $H_z \gg H_r$. Then the obtaining of the necessary magnetic field law can be divided into two practically independent problems. The first consists in forming the necessary field variation (ε), the second involves the formation of the azimuthally-symmetrical part of the field.

The field components of shims of arbitrary configuration are found from

$$\psi = \mu \frac{\partial}{\partial z} \int_{V'} \frac{1}{\rho_i} dV', \tag{36}$$

$$\vec{H} = -\operatorname{grad}\psi,$$

where ϕ is the scalar magnetic potential, $\mu = \mu_z$ is the mean magnetization of a pattern along the axis z, ρ_i is the distance from the observation point in the field (r, ϕ, z) to a variable point (r', ϕ', z') of elementary volume dV'.

From Eq. (36) using the known integral representation [23]) be equal to the amplitudes of the harmonics in the field series of an unlimited system of straight shims of infinite length having the same cross-section.

The field of such shims, in a co-ordinate system indicated in Fig. 5, can be represented provided $\delta/h_1 \ll 1$ in the plane $z = 0$, in the form

$$H_z(y) = \frac{2\mu h_1 \delta}{(2\pi\lambda)^2} \sum_{s=-\infty}^{\infty} \left[\frac{1}{\left(\frac{h_1}{2\pi\lambda}\right)^2 + \left(\frac{y}{2\pi\lambda}+s\right)^2} - \frac{1}{\left(\frac{h_1+2b}{2\pi\lambda}\right)^2 + \left(\frac{y}{2\pi\lambda}+s\right)^2} \right]. \quad (46)$$

Summing Eq. (46) over s and expanding the obtained result into Fourier series, we shall find the following expression for the amplitudes[26] of the harmonic

$$\overset{*}{H}_K = 4\mu \frac{\delta}{\lambda} e^{-\frac{Kh_1}{\gamma}} \left[1 - e^{-K\frac{2b}{\gamma}} \right]. \quad (47)$$

The formula Eq. (47) is convenient for a preliminary choice of the parameters of spiral shims systems.

The effect of a pole piece on a shim field was taken into account assuming an infinite dimension of the magnet poles on the basis of reflection theory[27].

If the permeability of the magnet poles $\mu \gg 1$, one obtains for the amplitudes of the harmonics

$$\overset{*}{H}_K = 4\mu \frac{\delta}{\lambda} e^{-K\frac{h_1}{\gamma}} \frac{1 - e^{-K\frac{4b}{\lambda}}}{1 - e^{-K\frac{2h_\mu}{\lambda}}} \quad (48)$$

where $h_\mu = (h_1 + 2b)$.

It is seen from Eq. (48) that if $h_\mu > \frac{2\pi\lambda}{4}$ it is possible to compute with a sufficient practical accuracy the harmonic amplitudes of a field without taking into account the pole piece effect, considering that the shims are infinite in height. This reduces considerably the calculations involved by the formulas Eq. (41).

It should be noted that these conclusions do not refer to the calculation of the average field of a spiral shims system for which, in order to reach the necessary accuracy, one must take into account the effect of pole pieces.

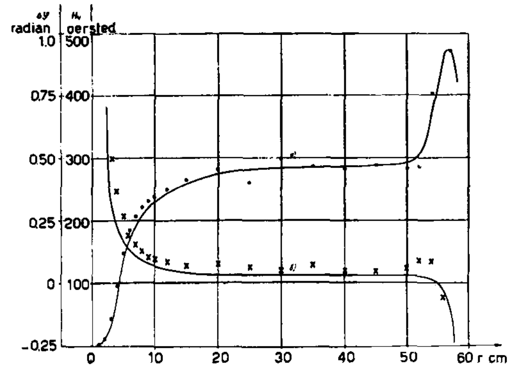

Fig. 7 a) Amplitude of the fourth harmonic of the magnetic field vs. radius,
b) Deviation of the 4th harmonic phase from the spiral.

The investigated structures of the magnetic field were made out of spiral shims having the following parameters:

1) $N = 4$, $\lambda = 1.34$ cm, $\delta = 1.2$ cm, $h_1 = 4$ cm, $2b = 4$ cm;

2) $N = 6$, $\lambda = 2{,}7$ cm, $\delta = 2.5$ cm, $h_1 = 4$ cm, $2b = 3$ cm.

The pole piece for $N = 4$ is shown in Fig. 6. The calculated and experimental values of the amplitude of the main harmonic for this version are represented in Fig. 7. It is seen from this dependence that the amplitude starts from zero at the centre and increases with radius, reaching already 90% of the maximum amplitude at $r \approx 15$ cm. In the same figure the deviations of the main harmonic phase β_4 from the

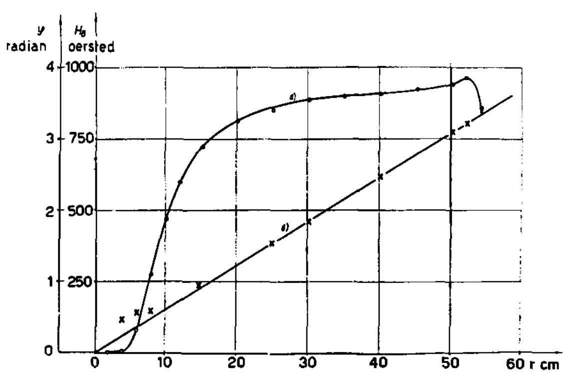

Fig. 8 Amplitude and phase of the 6th harmonic vs. radius.
a) Amplitude of the magnetic field 6th harmonic H_6 vs. radius;
b) The 6th harmonic phase vs. radius.

$$\frac{1}{\rho_i}=\int_0^{\infty} e^{\lambda(z-z')}J_0(\lambda r)\cdot J_0(\lambda r')d\lambda+$$

$$+2\sum_{m=1}^{\infty}\cos m(\phi-\phi')\int_0^{\infty} e^{-\lambda(z-z')}J_m(\lambda r')d\lambda\,, \quad (37)$$

one can find the magnetic field given by an arbitrary system of curvilinear shims in the form of a Fourier series:

$$H_z(r,\phi,z)=H_z(r,z)+$$

$$+\sum_{m=1}^{\infty}H_{mN}(r,z)\sin\left[\beta_{mN}(r,z)-mN\phi\right]. \quad (38)$$

Here $2\pi/N$ is the period of the field structure, $H_N=\varepsilon H(r)$ is the amplitude of the main harmonic, which is chosen in accordance with conditions Eqs. (13), (14). It should be noted that if the amplitude of higher harmonics in the magnetic field is not great, the conditions of stability are not violated [24].

The system of shims giving a distribution of H_z corresponding to Eq. (38) consists of $2N$ identical shims located symmetrically with respect to the mean plane of the electromagnetic gap ($z=0$) and shifted with respect to each other by an angle $2\pi/N$. For a system of spiral shims of infinite height whose thickness is small compared to any other dimensions, the average field and the harmonic amplitudes in series Eq. (38) at $z=0$ can be represented [25] from Eqs. (36) and (37), by

$$H(r)=\frac{2\mu\delta Nh_1}{\pi r\sqrt{r\lambda N}}\int_{\phi_H}^{\phi_K}\sqrt{\frac{1+\phi'^2}{\phi'^3}}\left[-\frac{d}{dx}Q_{-\frac{1}{2}}(x)\right]d\phi'\,, \quad (39)$$

$$H_K(z)=\sqrt{[H_K^c(r)]^2+[H_K^s(r)]^2}\,, \quad (40)$$

where $K=mN \quad (m=1,2,3,\ldots)$,

$$H_K^c(r)=\frac{4\mu\delta Nh_1}{\pi r\sqrt{r\lambda N}}\int_{\phi_H}^{\phi_K}\sqrt{\frac{1+\phi'^2}{\phi'^3}}\cos K\phi'\left[-\frac{d}{dx}Q_{K-\frac{1}{2}}(x)\right]d\phi', \quad (41)$$

$$H_K^s(r)=\frac{4\mu\delta Nh_1}{\pi r\sqrt{r\lambda N}}\int_{\phi_H}^{\phi_K}\sqrt{\frac{1+\phi'^2}{\phi'^3}}\sin K\phi'\left[-\frac{d}{dx}Q_{K-\frac{1}{2}}(x)\right]d\phi'.$$

$Q_{k-\frac{1}{2}}(X)$ is the spherical Legendre function of the second type of argument $x=\dfrac{h_1^2+r^2+\lambda^2N^2\phi'^2}{2r\lambda N\phi'}$, $\phi_K-\phi_H$ is the azimuthal dimension, δ is the shim thickness, $2h_1$ is the vertical gap between shims, and $\mu=\dfrac{21000}{4\pi}$ Oe.

The phases of the harmonics are found from the relation

$$\beta_K(r)=\operatorname{arc\,tg}\frac{H_K^s(r)}{H_K^c(r)}. \quad (42)$$

The field of a system of shims limited in height is easily obtained from Eqs. (39) and (40) as the difference of the fields produced by 2 systems of shims of infinite height with h_1 and h_2 (the shim height $2b=h_2-h_1$). Here the amplitudes and phases of the harmonics are found from

$$H(r,2b)=H(r,h_1)-H(r,h_2)\,, \quad (43)$$

$$H_K(r,2b)= \quad (44)$$

$$=\sqrt{[H_K^c(r,h_1)-H_K^s(r,h_2)]^2+[H_K^s(r,h_1)-H_K^s(r,h_2)]^2}\,,$$

$$\beta_K(r,2b)=\operatorname{arc\,tg}\frac{H_K^s(r,h_1)-H_K^s(r,h_2)}{H_K^c(r,h_1)-H_K^c(r,h_2)}, \quad (45)$$

where $H(r,h)$, $H_k^c(r,h)$, $H_k^s(r,h)$ are determined from Eqs. (39) and (41). From the analysis of Eq. (40) it is seen that the amplitudes of the harmonics are equal to zero at $r=0$ and increase with the radius according to the law

$$H_K(r)=r^K[\gamma_0^{(K)}+\gamma_2^{(K)}r^2+\gamma_4^{(K)}r^4+\ldots]\,,$$

where $\gamma_{2n}^{(K)}$ are coefficients depending upon the shim parameters.

If the spiral shim curvature $K=\dfrac{1}{\lambda N}\dfrac{\phi'^2+2}{(\phi'^2+1)^{3/2}}$ vanishes, the amplitudes of the harmonic Eq. (38) reach their maximum values. These maximum values will

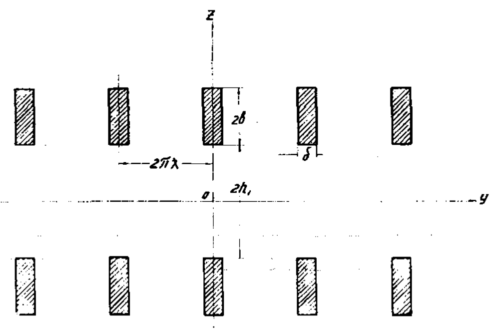

Fig. 5 The system of straight shims.

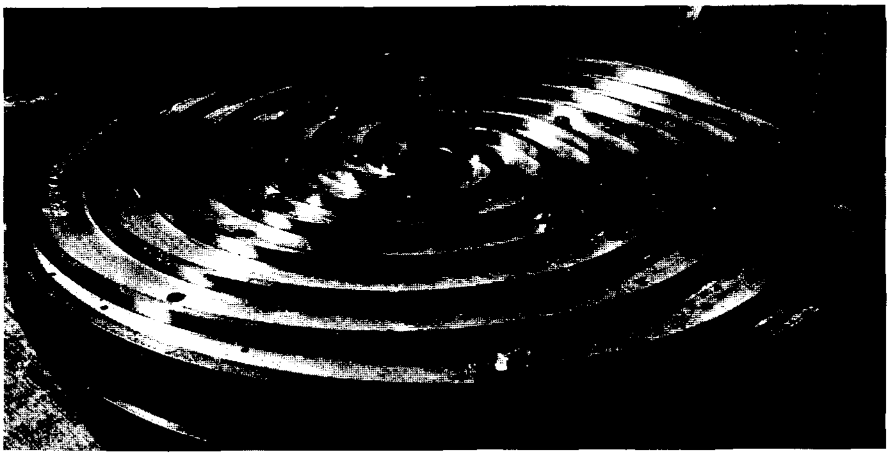

Fig. 6 Pole piece with spiral shims for N = 4.

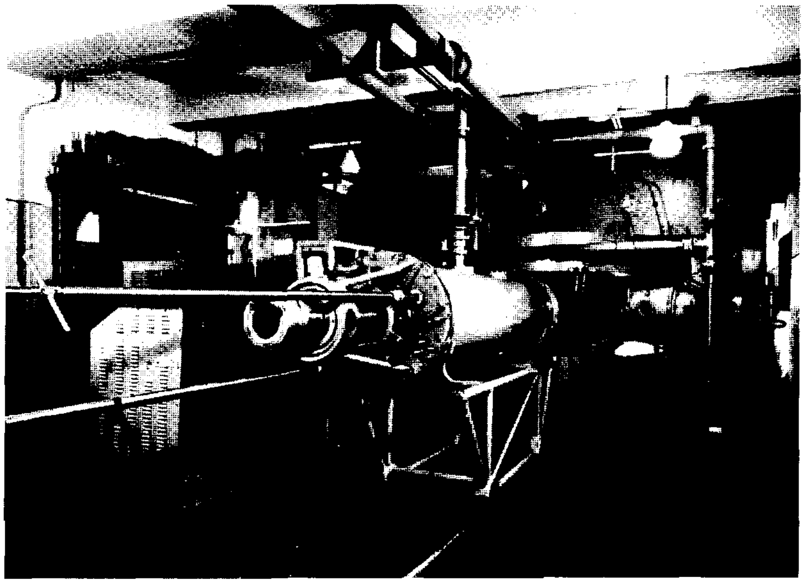

Fig. 11 General view of the accelerator on the side of the resonant line.

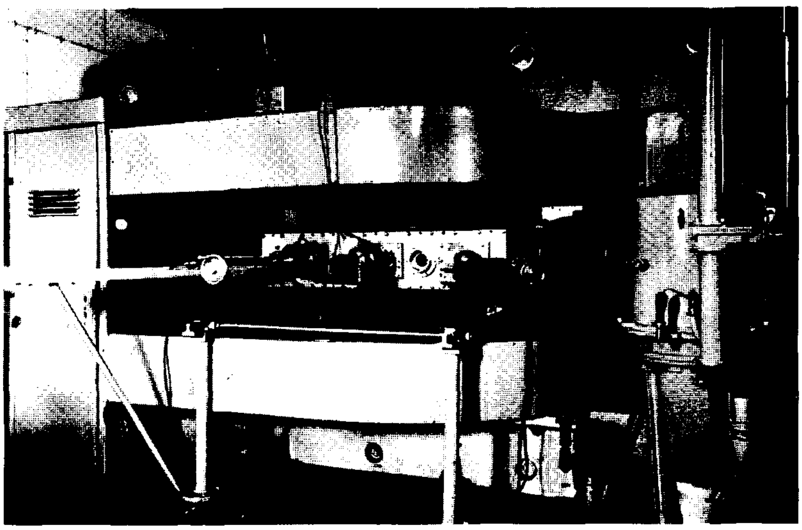

Fig. 12 General view of the accelerator on the side of the ion source.

ideal phase r/λ are shown. The variations of these deviations lead to an effective increase of λ; this locally increases the amplitude at these radii. For the optimum λ (under the condition of amplitude maximum) this effect appreciably increases. This is seen from the analysis of the data given in Fig. 8 which shows the dependence of the amplitude and phase of the main harmonic upon the radius for $N = 6$, λ being taken close to the optimum. The diagram of the pole piece with spiral shims for $N = 6$ is given in Fig. 9. The ratios of the harmonics amplitudes in the field series corresponds, in the radius interval 20-50 cm, to that calculated from relation Eq. (47).

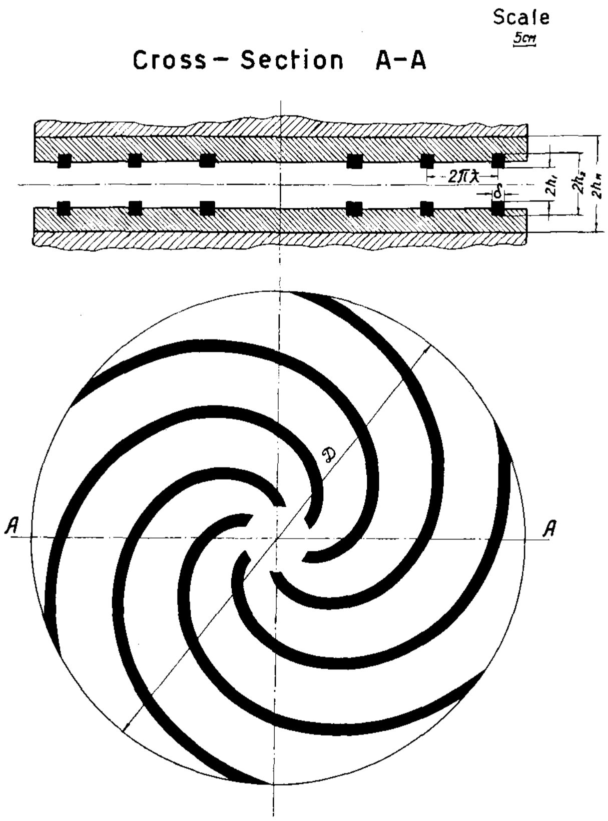

Fig. 9 Pole piece with spiral shims for N = 6 (diagram).

The most complicated problem in realizing the magnetic field of the cyclotron was the production of the average field, Eq. (32). The average magnetic field was produced by the ring shims; the magnetic field was calculated on the assumption of uniform axial magnetization from the expression (36). The magnetic field component from thin ring shims of radial dimension $\Delta R = R_2 - R_1$ and height Δh is described, if $\frac{\Delta h}{h_1} \ll 1$, by [28)]

$$H_z(r) = 4\pi\mu \cdot \Delta h \{\psi(r,h_1,R_2) - \psi(r,h_1,R_1)\}, \quad (49)$$

where

$$\psi(r,h_1,R) = \frac{1}{2}\frac{1}{\sqrt{(R+r)^2+h_1^2}}\left[F_0(\alpha) + \frac{R^2-r^2-h_1^2}{(R-r)^2+h_1^2}E_0(\alpha)\right],$$

F_0 and E_0 are normalized total elliptical integrals of the first and the second type;

$$\alpha = \arcsin\sqrt{\frac{4Rr}{(R+r)^2+h_1^2}}.$$

The numerical value of the magnetization μ of the ring shims was found from the magnetization curves of the substance [29)] and from the demagnetization factor which is determined by [28)]

$$N_V = \frac{1}{V}\int_V f_z(r,z,\Delta h,R_1,R_2) r dr \cdot d\phi \cdot dz,$$

where $f_z = H_z/M$ is a function describing the distribution of the vertical magnetic field component inside the pattern considered. The demagnetization factor is a dimensionless function of shim parameters and it changes from 0 (at $\frac{\Delta h}{\Delta R} \to \infty$) to 4π (at $\frac{\Delta h}{\Delta R} \to 0$).

Preliminary experiments carried out with thin ring shims show that the difference between the calculated and experimental curves does not exceed 10% of the maximum shim field and that the principle of superposition of fields produced by single shims is valid within this precision.

Steel cylinders of small diameter (0.8 cm) were used for more accurate shimming of the average magnetic field. Re-distribution of cylinders in azimuth was used to reduce the amplitudes of the first and second harmonics, the appearance of which in the field series can be explained by errors in the geometry of the shims which are equal to 0.01 cm in the manufactured set of spiral shims, and by the distortion of the pole pieces which did not exceed 0.05 cm. The amplitudes of the lower harmonics as a function of radius are given in Fig. 10. The amplitudes of the

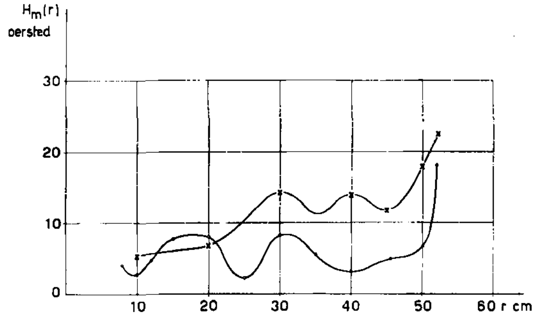

Fig. 10 Amplitudes of the low harmonics of the magnetic field vs. radius for N = 6 (—o—o— first harmonic, —x—x— second one).

first and second harmonics for $0<r<50$ cm do not exceed 15 Oe.

To improve the conditions of the initial beam formation in the central region of the accelerator, the average field has been slightly increased. In the radius interval 8-52 cm, the deviation of the average field from the resonant one does not exceed 2×10^{-2} % (see Fig. 3, where the experimental average field is given by the dotted curve).

VI. MEASUREMENT AND STABILIZATION OF THE MAGNETIC FIELD

The absolute value of the inhomogeneous magnetic field of the accelerator was measured by a specially developed magnetometer based on the Hall effect [30] and nuclear magnetic resonance [31, 32].

By means of the above nuclear magnetometer, the absolute value of the magnetic field can be measured within 250-24 000 Oe, the gradient of H_0 being smaller than 10%, with an accuracy of $\pm$ 0.01%. These measurements are practically " dotted " as the sample volume in which the magnetic resonance is observed at the mean value of the magnetic field, is equal to 2×10^{-4} cm^3. Simultaneously, the gradient of the magnetic field at the measuring point and its direction can be determined with an accuracy of ±1%.

To measure the magnetic field in the magnet gap, the probes were mounted on a special device which permitted them to be shifted along the radius, vertically and in azimuth with an accuracy of $\pm$0.01 cm, $\pm$0.01 cm and $\pm0.1°$ respectively.

In the nuclear magnetometer, one used a system of remote tuning of the oscillating circuit of the regenerative detector of the nuclear magnetic resonance signal, and a system of semi-automatic remote control of the probe azimuth shifting. This permitted the time necessary for measuring the magnetic field in the shimming process to be reduced.

The required stability of the magnetic field (0.01%) can be obtained only by means of a device which directly responds to the magnetic field change in the electromagnetic gap. The method based on nuclear magnetic resonance is the most accurate and convenient. The stability of the resonant magnetic field is determined practically only by the frequency stability of the RF excitation field. The use of generators stabilized by quartz makes it possible to stabilize the magnetic field with an accuracy of 0.01 to 0.001%.

In the cyclotron, the magnetic field was stabilized with an accuracy of 0.005% by a nuclear stabilizer in which the resonant signal was observed by the method of nuclear induction [33].

VII. HIGH FREQUENCY SYSTEM

The RF system of the cyclotron is a co-axial quarter wavelength resonant line ($W = 64\ \Omega$), closed at one side by a movable shorting plug and loaded at the other side by the capacity of the dee [34].

By moving the shorting plug, the system can be tuned in the region 7.6 to 12 mc/s. The working frequency is 10.5 mc/s.

The characteristic feature of the RF system is the presence of only one dee and of small gaps between the dee and the chamber (1.5—2 cm). High-frequency breakdown between the dee and the chamber limits the maximum possible accelerating voltage to 40 kV. A considerable detuning of the system is caused by heating due to the small gaps; this leads to the necessity of taking special care for keeping the natural frequency unchanged.

A general view of the resonant line is given in Fig. 11. The outside conductor of the line is the steel vacuum tank covered inside with copper sheets. The diameters of the outer and inner electrodes are equal to 58.4 and 20.0 cm, respectively.

The radius of the dee is 57.5 cm, the aperture 4 cm. To increase the working aperture, the cooling tubes are placed along the side edge of the dee, outside the working radius.

The RF system is fed by a seven stage generator. The master oscillator uses a triode 6 with grounded anode and capacitive feedback; to guarantee a high frequency stability, thermocompensated condensers are used, the anode supply is stabilized and the inductances are wound on ceramic formers. The frequency stability of such a generator is 4×10^{-5} (after two hours of operation).

The output stage uses two lamps ГЧ-12A in parallel. In order to suppress auto-oscillations, the tubes were mounted with grounded grid. The power stage is placed near the resonant line (Fig. 11). The RF system is linked to the anode of the power stage by means of a short co-axial feeder having a characteristic impedance of 64 Ω. The accelerating voltage can change due to a drift of the oscillating frequency of the RF system as a result of heating. The frequency is kept constant (and so the amplitude of the accelerating voltage) by means of an automatic system of stabilization which changes the capacity between the dee and the additional electrode inserted into the accelerator chamber. The stabilizing system keeps the amplitude of the accelerating voltage constant with an accuracy of 1.5%.

VIII. VACUUM CHAMBER AND ION SOURCE

The vacuum chamber having the form of a rectangular parallelepiped of $158 \times 154 \times 33.5$ cm was designed so that it might provide experimental conveniences, simplicity and reliability of exploitation. The chamber, apart from cylindrical steel covers, is made out of an alloy of the " avial " type to reduce the harmful radioactive background of long-lived isotopes accumulated in the chamber. The chamber and the outer tube of the resonant line form a common vacuum volume pumped out by three oil diffusion pumps of the H-5T type. The pumps give an operating vacuum of $(1\text{-}2) \times 10^{-5}$ mm Hg when admission of gas into the source is about 2-5 cm^3/min.

Fore-vacuum pumping out of the chamber and of the diffusion pumps is provided by two mechanical pumps BH-I.

The use of a Penning discharge as an ion source made it possible to avoid considerable constructional difficulties connected with heating the cathode and cooling other parts of the ion source. The ion source is constructed in such a way that is is possible to move it in all directions while in vacuum. The chamber is supplied by three probes with quartz targets as beam indicators. A thin tungsten wire wound around the quartz targets makes it possible to measure the current of the accelerated particle beam simultaneously with visual observation.

A general view of the accelerator is given in Fig. 12.

IX. CONCLUSION

From an investigation of accelerators with space variation of the magnetic field, it is possible to draw the following conclusions :

a) the linear theory of space stability developed for these accelerators was experimentally confirmed;

b) non-linear resonance effects in the centre of the accelerator were theoretically and experimentally investigated and regions of the magnetic field parameters were found for which these effects are practically absent;

c) the methods of computation of the magnetic field provide the necessary practical accuracy and can be applied in constructing accelerators of this type;

d) all the indicated theoretical and experimental investigations as well as the equipment developed for measuring and stabilizing the magnetic fields of complicated configuration and the methods of shimming the average field, allow the design of cyclotrons of energies comparable to those obtained in synchrocyclotrons, producing beam intensities of the order of 100 μA.

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DISCUSSION

KHOE KONG TAT: Is the 5 kV on the dee the lowest voltage at which the machine will work, or is it a working voltage?

DMITRIEVSKIJ: This is the threshold voltage below which the machine does not work. It corresponds to the shims we have used.

MARTIN: I should like to ask what is the magnetic gap at the centre of your machine?

DMITRIEVSKIJ: 8 cm.

LAWSON: I should like to ask what the maximum current is, and what is the dee voltage under these circumstances?

DMITRIEVSKIJ: In order to avoid the radioactivation of the chamber, the machine was operated at a current of 1-5 μA, implying a source power of about 7 W. The dee voltage was then 11 kV, and the current depended on the radius. At the voltage of 25-30 kV this dependence was rather weak.

WELTON: I have been curious as to why the number of sectors, N, was chosen as 6. The vertical stability can be achieved much more easily with a lower sector number. At first sight the resonance might be a problem. Was this the reason for the choice?

DMITRIEVSKIJ: Yes, it was. The value $N = 6$ was chosen because there would be a non-linear resonance at $N = 4$, and so we have to take large λ, and so the R/λ factor tends to zero and one attains the normal Thomas machine.

KHOE KONG TAT: I should like to ask what is the first harmonic content in the azimuthal variation of the magnetic field?

DMITRIEVSKIJ: In order to overcome the $Q_z = 1/2$ resonance, it was necessary to have 1.5 Oe in the first harmonic. The amplitude of the first harmonic on the mean radii was between 5 and 10 Oe.

1.5 METRE CYCLOTRON WITH AZIMUTHALLY VARYING MAGNETIC FIELD

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(presented by B. J. Zamolodchikov)

INTRODUCTION

Considerable difficulties are met in an attempt to accelerate ions to energies above 22-24 MeV in the conventional cyclotron because of an increase in the mass of ions as the resonance condition between the revolution frequency of a particle and the acceleration voltage frequency is violated. Frequency modulation of the dee voltage permits an increase in the attainment of higher energies though the intensity is unavoidably lowered.

Another method of compensating the relativistic mass increase is to produce a magnetic field increasing with orbital radius. This method would lead to defocusing and as a result to the loss of all accelerated particles in a conventional cyclotron. In