



On RR couplings and bulk singularity structures of non-BPS branes



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ABSTRACT

We compute the five point world sheet scattering amplitude of a symmetric closed string Ramond–Ramond, a transverse scalar field, a world volume gauge field and a real tachyon in both world volume and transverse directions of brane in type IIA and IIB superstring theory. We provide the complete analysis of $\langle C^{-1}\phi^0 A^0 T^{-1} \rangle$ S-matrix and show that both $u' = u + \frac{1}{4}$ and t channel bulk singularity structures can also be examined by this S-matrix. Various remarks about new restricted Bianchi identities on world volume for the other pictures have also been made.

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1. Introduction

No matter we talk about stable BPS or unstable (non-BPS) branes, D_p -branes are supposed to be thought of sources for closed string Ramond–Ramond (RR) field [1]. It has been proven that making use of RR couplings, one can investigate or try to address various remarkable issues about string theory, whereas we highlight some of the most crucial ones as follows. The so called brane embeddings [2], through D-brane language the K-theory [3], also the so called Dielectric effect or Myers effect [4] and in particular the way of looking for all order α' higher derivative corrections to BPS or non-BPS branes [5,6] are explored. Note that for the definitions of non-BPS branes, D_p -branes with p odd (even) in IIA (IIB) are taken in which p stands for the spatial dimension of branes. Based on various symmetries, a universal conjecture for all order α' higher derivative corrections to both BPS and non-BPS branes was proposed in [7] and naturally it has been applied at practical levels to various higher point fermionic S-matrices [8] as well.

It was argued in [9] in detail that, to deal with effective theory of unstable branes after integrating out all the massive modes, one is just left with massless and tachyon states, and also to work out the dynamics of branes not only DBI action but also Wess–Zumino terms are indeed needed. To obtain effective actions, either Boundary String Field theory (BSFT) [10] or S-matrix formalism should be employed where the latter has a very strong potential to be taken so that upon applying that, all the coefficients of higher derivative corrections for all orders in α' can be found.

The Wess–Zumino effective action has been given in [11] as

$$S_{WZ} = \mu'_p \int_{\Sigma_{(p+1)}} C \wedge \text{Str} e^{i2\pi\alpha'\mathcal{F}}, \quad (1)$$

with β' being normalisation constant and with the so called the curvature of super connection as follows

$$i\mathcal{F} = \begin{pmatrix} iF - \beta'^2 T^2 & \beta' DT \\ \beta' DT & iF - \beta'^2 T^2 \end{pmatrix}. \quad (2)$$

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Now if we expand the exponential, one produces various couplings such as

$$S_{WZ} = 2\beta' \mu'_p (2\pi\alpha') \text{Tr} \left(C_p \wedge DT + (2\pi\alpha') C_{p-2} \wedge DT \wedge F + \frac{(2\pi\alpha')^2}{2} C_{p-4} \wedge F \wedge F \wedge DT \right) \quad (3)$$

Based on the internal Chan–Paton matrix some partial selection rules for superstring amplitudes have been released in [12], and to make sense of S-matrix computations, one needs to keep track of internal CP matrix of tachyons around unstable point of tachyon DBI action. The final form of tachyon DBI up to some orders with its all ingredients is demonstrated in [11] to be as follows

$$S_{DBI} \sim \int d^{p+1} \sigma \text{Str} \left(V(T^i T^i) \sqrt{1 + \frac{1}{2} [T^i, T^j] [T^j, T^i]} \sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab} + 2\pi\alpha' D_a T^i (Q^{-1})^{ij} D_b T^j)} \right), \quad (4)$$

with

$$V(T^i T^i) = e^{-\pi T^i T^i / 2}, \quad Q^{ij} = I \delta^{ij} - i [T^i, T^j], \quad (5)$$

where $i, j = 1, 2$, i.e., $T^1 = T\sigma_1$, $T^2 = T\sigma_2$. The entire information about this effective action is given in [12]. This action (4) is also consistent with the entire S-matrix calculations of $\langle V_C V_T V_\phi V_\phi \rangle$ of [11].

Around the stable point of tachyon potential this action gets reduced to tachyon DBI action [13] with $T^4 V(T^2)$ potential and by taking the limit of the tachyon to infinity, the term $T^4 V(TT)$ is sent to zero and this can be well described from condensation of an unstable brane.

By making use of the S-matrix of $\langle V_C V_\phi V_T V_T \rangle$ in [14] and $\langle V_C V_A V_T V_T \rangle$ in [15], we have also explored not only the entire form of D-brane -anti D-brane effective actions but also its all order α' corrections of two scalar (two gauge field)-two tachyon couplings.

Various applications in the literature in favour of those higher derivative string couplings have been pointed out. For instance, employing either some new BPS (non-BPS) couplings, their corrections or Myers terms to M-theory [16] one can actually interpret and get to the phenomenon of N^3 entropy growth of M5 branes or discuss various combinations of M2, M5 branes [17]. Evaluating some of (non)-BPS couplings, one derives (AdS)-dS brane world solutions [18] and could further elaborate on the point that despite the fact that we are dealing with non-supersymmetric case, as long as the EFT holds, all the large volume scenario minima will become stable [19]. Applying CFT techniques [20] the tachyonic DBI supersymmetrised action was suggested [21] to be as follows

$$L = -T_p V(T) \sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab} - 2\pi\alpha' \tilde{\Psi} \gamma_b \partial_a \Psi + \pi^2 \alpha'^2 \tilde{\Psi} \gamma^\mu \partial_a \Psi \tilde{\Psi} \gamma_\mu \partial_b \Psi + 2\pi\alpha' \partial_a T \partial_b T)} \quad (6)$$

One of the reasons we work with non-BPS branes is due to its direct relationship with realising the properties IIA (IIB) string theory in almost time-dependent backgrounds [22].

The source of instability in flat empty space is tight to the presence of tachyons and by using their effective action one hopes to get diverse results, such as the evaluation of these non-BPS branes and indeed tachyonic action can well describe decay properties of unstable D_p -branes.

The cosmological applications through these effective actions can also be addressed, for example if one considers, the action of D-brane anti D-brane then one is able to explain inflation through string theory [23] whereas several other applications to non-BPS branes have also been released [24]. Tachyons and their corrections can also be employed for models such as holographic like QCD [25] and finally brane anti brane system as a background was used in Sakai–Sugimoto models [26].

The outline of the paper is as follows.

In the next section, we first introduce all the vertex operators with their CP matrix, apply CFT techniques to actually obtain the entire form of a symmetric RR, a scalar, a gauge and a tachyon of $\langle V_{C^{-1}} V_{\phi^0} V_{A^0} V_{T^{-1}} \rangle$ S-matrix in both transverse and world volume directions of non-BPS branes of IIA(IIB) superstring theory. We then make use of the expansion for non-BPS branes and reveal that apart from an infinite $u' = u + \frac{1}{4}$ and (t) channel tachyon (scalar) singularities, this S-matrix clearly does involve an infinite u' , (t) channel singularity structures in the bulk as well. Indeed we show that $\mathcal{A}_3, \mathcal{A}_4$ of this S-matrix do carry an infinite p, ξ_1 singular terms, whose momenta are located in the bulk directions and from now on due to the presence of p^i , we are going to call them bulk singularities and start producing them in effective field theory (EFT) as well.

We also write down the results for symmetric $\langle V_{C^{-1}} V_{\phi^{-1}} V_{A^0} V_{T^0} \rangle$, $\langle V_{C^{-1}} V_{\phi^0} V_{A^{-1}} V_{T^0} \rangle$ and asymmetric $\langle V_{C^{-2}} V_{\phi^0} V_{A^0} V_{T^0} \rangle$ S-matrix and start comparing them which leads to finding out various generalised Bianchi Identities in the presence of non-BPS branes. In section 2 and 3 we produce an infinite number of u' , t channel tachyon, scalar singularities. At the end we apply all order α' higher derivative corrections properly to Chern–Simons couplings in such a way that even we are able to produce an infinite number of u' , t channel bulk singularity structures. This obviously confirms that those bulk singularities are also needed in the entire S-matrix as they even can be constructed in EFT side as well.

Note that based on various results of this paper, we reveal the important fact as follows. A-priori without knowing any restricted Bianchi identities on world volume of D-branes for RR, there seems to be no chance to even see all the needed bulk singularity structures of $\mathcal{A}_3$ and $\mathcal{A}_4$ of (18) from the other symmetric analysis (unlike asymmetric analysis) as we go through them in detail in the next sections.

1.1. The entire $\langle C^{-1} \phi^0 A^0 T^{-1} \rangle$ S-matrix

Here we would like to explore the entire form of the S-matrix elements (both in transverse and world volume direction) of an RR, a real massless scalar field, a gauge field and a real tachyon of type IIA (IIB) superstring theory, which is indeed a five point non-BPS S-matrix from the world sheet point of view. First we are going to explain our notations so that all $\mu, \nu, \dots$ show the entire ten dimensional space-time, on the other hand all $a, b, c, \dots$ and $i, j, k, \dots$ are taken to be employed for world volume and transverse directions appropriately.

It is discussed in [11] that in the presence of non-BPS branes one needs to consider the Chan–Paton matrices inside the vertices as follows

$$\begin{aligned}
V_T^{(0)}(x) &= \alpha' ik \cdot \psi(x) e^{\alpha' ik \cdot X(x)} \lambda \otimes \sigma_1, \\
V_T^{(-1)}(x) &= e^{-\phi(x)} e^{\alpha' ik \cdot X(x)} \lambda \otimes \sigma_2 \\
V_\phi^{(-1)}(x) &= e^{-\phi(x)} \xi_i \psi^i(x) e^{\alpha' iq X(x)} \lambda \otimes \sigma_3 \\
V_A^{(-1)}(x) &= e^{-\phi(x)} \xi_a \psi^a(x) e^{\alpha' iq X(x)} \lambda \otimes \sigma_3 \\
V_\phi^{(0)}(x) &= \xi_{1i} (\partial X^i(x) + i \alpha' k \cdot \psi \psi^i(x)) e^{\alpha' ik \cdot X(x)} \otimes I \\
V_\phi^{(-2)}(x) &= e^{-2\phi(x)} V_\phi^{(0)}(x) \otimes I \\
V_A^{(0)}(x) &= \xi_{1a} (\partial X^a(x) + i \alpha' k \cdot \psi \psi^a(x)) e^{\alpha' ik \cdot X(x)} \otimes I \\
V_C^{(-\frac{1}{2}, -\frac{1}{2})}(z, \bar{z}) &= (P_- \mathbb{H}_{(n)} M_p)^{\alpha\beta} e^{-\phi(z)/2} S_\alpha(z) e^{i \frac{\alpha'}{2} p \cdot X(z)} e^{-\phi(\bar{z})/2} S_\beta(\bar{z}) e^{i \frac{\alpha'}{2} p \cdot D \cdot X(\bar{z})} \lambda \otimes \sigma_3 \sigma_1, \\
V_C^{(-\frac{3}{2}, -\frac{1}{2})}(z, \bar{z}) &= (P_- \mathbb{C}_{(n-1)} M_p)^{\alpha\beta} e^{-3\phi(z)/2} S_\alpha(z) e^{i \frac{\alpha'}{2} p \cdot X(z)} e^{-\phi(\bar{z})/2} S_\beta(\bar{z}) e^{i \frac{\alpha'}{2} p \cdot D \cdot X(\bar{z})} \lambda \otimes \sigma_1
\end{aligned} \tag{7}$$

Note that the CP factors of RR in the presence of $D\bar{D}$ system for symmetric and asymmetric pictures are σ_3 and I accordingly. Hence $\langle C^{-1} \phi^0 A^0 T^{-1} \rangle$ S-matrix shall be taken as follows

$$\mathcal{A}^{\langle C^{-1} \phi^0 A^0 T^{-1} \rangle} \sim \int dx_1 dx_2 dx_3 dz d\bar{z} \langle V_\phi^{(0)}(x_1) V_A^{(0)}(x_2) V_T^{(-1)}(x_3) V_{RR}^{(-\frac{1}{2}, -\frac{1}{2})}(z, \bar{z}) \rangle, \tag{8}$$

where we are dealing with the disk level amplitude and mass-shell conditions for k_1, k_2, p, k_3 are

$$k_1^2 = k_2^2 = p^2 = 0, \quad k_3^2 = \frac{1}{2\alpha'}, \quad k_2 \cdot \xi_2 = k_2 \cdot \xi_1 = k_1 \cdot \xi_1 = k_3 \cdot \xi_1 = 0 \tag{9}$$

where we set $\alpha' = 2$. In order to just use the holomorphic correlators, one needs to keep track of the following notations for projection, RR field strength and for spinor as well.

$$P_- = \frac{1}{2}(1 - \gamma^{11}), \quad \mathbb{H}_{(n)} = \frac{a_n}{n!} H_{\mu_1 \dots \mu_n} \gamma^{\mu_1} \dots \gamma^{\mu_n}, \quad (P_- \mathbb{H}_{(n)})^{\alpha\beta} = C^{\alpha\delta} (P_- \mathbb{H}_{(n)})_{\delta}{}^{\beta}.$$

For type IIA (type IIB) $n = 2, 4$, $a_n = i$ ($n = 1, 3, 5$, $a_n = 1$).

We also apply the doubling trick to make use of just holomorphic parts of all the world sheet fields as below

$$\tilde{X}^\mu(\bar{z}) \rightarrow D_\nu^\mu X^\nu(\bar{z}), \quad \tilde{\psi}^\mu(\bar{z}) \rightarrow D_\nu^\mu \psi^\nu(\bar{z}), \quad \tilde{\phi}(\bar{z}) \rightarrow \phi(\bar{z}), \quad \text{and} \quad \tilde{S}_\alpha(\bar{z}) \rightarrow M_\alpha{}^\beta S_\beta(\bar{z}),$$

with the following definitions for the aforementioned matrices

$$D = \begin{pmatrix} -19^{-p} & 0 \\ 0 & 1_{p+1} \end{pmatrix}, \quad \text{and} \quad M_p = \begin{cases} \frac{\pm i}{(p+1)!} \gamma^{i_1} \gamma^{i_2} \dots \gamma^{i_{p+1}} \epsilon_{i_1 \dots i_{p+1}} & \text{for } p \text{ even} \\ \frac{\pm 1}{(p+1)!} \gamma^{i_1} \gamma^{i_2} \dots \gamma^{i_{p+1}} \gamma_{11} \epsilon_{i_1 \dots i_{p+1}} & \text{for } p \text{ odd} \end{cases}$$

Having set that, we can now go ahead with the correct form of the correlations for all X^μ, ψ^μ, ϕ fields, as follows

$$\begin{aligned}
\langle X^\mu(z) X^\nu(w) \rangle &= -\frac{\alpha'}{2} \eta^{\mu\nu} \log(z - w), \\
\langle \psi^\mu(z) \psi^\nu(w) \rangle &= -\frac{\alpha'}{2} \eta^{\mu\nu} (z - w)^{-1}, \\
\langle \phi(z) \phi(w) \rangle &= -\log(z - w).
\end{aligned} \tag{10}$$

Substituting the above vertex operators into the S-matrix, the amplitude reads off

$$\begin{aligned}
\mathcal{A}^{\langle C^{-1} \phi^0 A^0 T^{-1} \rangle} &\sim \int dx_1 dx_2 dx_3 dx_4 dx_5 (P_- \mathbb{H}_{(n)} M_p)^{\alpha\beta} \xi_{1i} \xi_{2a} x_{45}^{-1/4} (x_{34} x_{35})^{-1/2} \\
&\quad \times (I_1 + I_2 + I_3 + I_4) \text{Tr}(\lambda_1 \lambda_2 \lambda_3) \text{Tr}(\sigma_3 \sigma_1 \sigma_2),
\end{aligned} \tag{11}$$

so that one needs to go over the following correlation functions

$$\begin{aligned}
I_1 &= \langle : \partial X^i(x_1) e^{\alpha' ik_1 \cdot X(x_1)} : \partial X^a(x_2) e^{\alpha' ik_2 \cdot X(x_2)} : e^{\alpha' ik_3 \cdot X(x_3)} : e^{\frac{\alpha'}{2} ip \cdot X(x_4)} : e^{\frac{\alpha'}{2} ip \cdot D \cdot X(x_5)} : \rangle \langle : S_\alpha(x_4) : S_\beta(x_5) : \rangle, \\
I_2 &= \langle : \partial X^i(x_1) e^{\alpha' ik_1 \cdot X(x_1)} : e^{\alpha' ik_2 \cdot X(x_2)} : e^{\alpha' ik_3 \cdot X(x_3)} : e^{\frac{\alpha'}{2} ip \cdot X(x_4)} : e^{\frac{\alpha'}{2} ip \cdot D \cdot X(x_5)} : \rangle \langle : \alpha' ik_{2b} : S_\alpha(x_4) : S_\beta(x_5) : \psi^b \psi^a(x_2) : \rangle \\
I_3 &= \langle : e^{\alpha' ik_1 \cdot X(x_1)} : \partial X^a(x_2) e^{\alpha' ik_2 \cdot X(x_2)} : e^{\alpha' ik_3 \cdot X(x_3)} : e^{\frac{\alpha'}{2} ip \cdot X(x_4)} : e^{\frac{\alpha'}{2} ip \cdot D \cdot X(x_5)} : \rangle \langle : \alpha' ik_{1c} : S_\alpha(x_4) : S_\beta(x_5) : \psi^c \psi^i(x_1) : \rangle \\
I_4 &= \langle : e^{\alpha' ik_1 \cdot X(x_1)} : e^{\alpha' ik_2 \cdot X(x_2)} : e^{\alpha' ik_3 \cdot X(x_3)} : e^{\frac{\alpha'}{2} ip \cdot X(x_4)} : e^{\frac{\alpha'}{2} ip \cdot D \cdot X(x_5)} : \rangle \\
&\quad \times \langle : (-\alpha')^2 k_{1c} k_{2b} : S_\alpha(x_4) : S_\beta(x_5) : \psi^c \psi^i(x_1) : \psi^b \psi^a(x_2) : \rangle
\end{aligned} \tag{12}$$

Having taken the Wick-theorem and (10), we were able to compute all the correlators of X . However, to get to fermionic correlations, involving the spin operators, one employs the so called Wick-like rule [27] as follows

$$I_2^{ic} = \langle : S_\alpha(x_4) : S_\beta(x_5) : \psi^c \psi^i(x_1) : \rangle = 2^{-1} x_{45}^{-1/4} (x_{14} x_{15})^{-1} (\Gamma^{ic} C^{-1})_{\alpha\beta}$$

$$I_3^{ab} = \langle : S_\alpha(x_4) : S_\beta(x_5) : \psi^b \psi^a(x_2) : \rangle = 2^{-1} x_{45}^{-1/4} (x_{24} x_{25})^{-1} (\Gamma^{ab} C^{-1})_{\alpha\beta}, \quad (13)$$

where $x_{ij} = x_i - x_j$, $x_4 = z$, $x_5 = \bar{z}$. For the calculations of two spin operators and two currents one applies the generalisation of Wick-like rule [28] to indeed find out the fermionic correlations as follows

$$I_4^{abic} = \langle : S_\alpha(x_4) : S_\beta(x_5) : \psi^c \psi^i(x_1) : \psi^b \psi^a(x_2) : \rangle$$

$$= \left\{ (\Gamma^{abic} C^{-1})_{\alpha\beta} + \alpha' \frac{Re[x_{14} x_{25}]}{x_{12} x_{45}} (\eta^{cb} (\Gamma^{ai} C^{-1})_{\alpha\beta} - \eta^{ac} (\Gamma^{bi} C^{-1})_{\alpha\beta}) \right\} 2^{-2} x_{45}^{3/4} (x_{14} x_{15} x_{24} x_{25})^{-1} \quad (14)$$

Considering all the bosonic and fermionic correlators into (11), one reveals the whole closed part of the amplitude as follows

$$\mathcal{A}^{<C^{-1}\phi^0 A^0 T^{-1}>} \sim \int dx_1 dx_2 dx_3 dx_4 dx_5 (P_- \mathcal{H}_{(n)} M_p)^{\alpha\beta} I_{\xi_{1i} \xi_{2a}}(2i) x_{45}^{-1/4} (x_{34} x_{35})^{-1/2}$$

$$\times \left(a_1^i a_2^a x_{45}^{-5/4} C_{\alpha\beta}^{-1} + \alpha' i k_{2b} a_1^i I_3^{ab} + \alpha' i k_{1c} a_2^a I_2^{ic} - (\alpha')^2 k_{1c} k_{2b} I_4^{abic} \right) \text{Tr}(\lambda_1 \lambda_2 \lambda_3)$$

such that

$$I = |x_{12}|^{\alpha'/2 k_1 k_2} |x_{13}|^{\alpha'/2 k_1 k_3} |x_{14} x_{15}|^{\alpha'/2 k_1 p} |x_{23}|^{\alpha'/2 k_2 k_3} |x_{24} x_{25}|^{\alpha'/2 k_2 p} |x_{34} x_{35}|^{\alpha'/2 k_3 p} |x_{45}|^{\alpha'/2 p.D.p},$$

$$a_1^i = i p^i \frac{x_{54}}{x_{14} x_{15}},$$

$$a_2^a = i k_1^a \left(\frac{x_{14}}{x_{12} x_{24}} + \frac{x_{15}}{x_{12} x_{25}} \right) + i k_3^a \left(\frac{x_{43}}{x_{23} x_{24}} + \frac{x_{53}}{x_{23} x_{25}} \right). \quad (15)$$

One is able now to precisely check out the $SL(2, \mathbb{R})$ invariance of the above S-matrix, and we do gauge fixing by fixing all the positions of open strings as $x_1 = 0$, $x_2 = 1$, $x_3 \rightarrow \infty$ so that at the end, one has to come over the following sort of integration on the upper half complex plane

$$\int d^2 z |1 - z|^a |z|^b (z - \bar{z})^c (z + \bar{z})^d. \quad (16)$$

Note that all a, b, c are some combinatoric parts of the defined Mandelstam variables as follows

$$s = -(k_1 + k_3)^2, \quad t = -(k_1 + k_2)^2, \quad u = -(k_2 + k_3)^2.$$

Results of integrations for both $d = 0, 1$ and for $d = 2$ have been appropriately explored in [29,11], so that the compact and ultimate results for the entire S-matrix in both transverse and world volume directions of brane are discovered as

$$\mathcal{A}^{<C^{-1}\phi^0 A^0 T^{-1}>} = \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 + \mathcal{A}_4 + \mathcal{A}_5 + \mathcal{A}_6, \quad (17)$$

where

$$\mathcal{A}_1 \sim 2i \left(-\xi_{1i} \xi_{2a} k_{1c} k_{2b} \text{Tr}(P_- \mathcal{H}_{(n)} M_p \Gamma^{abic}) + \xi_{1i} p \xi_{2a} k_{2b} \text{Tr}(P_- \mathcal{H}_{(n)} M_p \Gamma^{ab}) \right) L_1,$$

$$\mathcal{A}_2 \sim 2i \xi_{1i} \xi_{2a} \text{Tr}(P_- \mathcal{H}_{(n)} M_p \Gamma^{ai})(t)(u + \frac{1}{4}) L_3,$$

$$\mathcal{A}_3 \sim 4i k_1 \cdot \xi_2 \xi_1 \cdot p \text{Tr}(P_- \mathcal{H}_{(n)} M_p)(u + \frac{1}{4}) L_3,$$

$$\mathcal{A}_4 \sim -4i k_3 \cdot \xi_2 \xi_1 \cdot p \text{Tr}(P_- \mathcal{H}_{(n)} M_p)(t) L_3,$$

$$\mathcal{A}_5 \sim 4i k_3 \cdot \xi_2 \xi_{1i} k_{1c} \text{Tr}(P_- \mathcal{H}_{(n)} M_p \Gamma^{ic})(t) L_3,$$

$$\mathcal{A}_6 \sim 4i k_1 \cdot \xi_2 (k_{1c} + k_{2c}) \xi_{1i} \text{Tr}(P_- \mathcal{H}_{(n)} M_p \Gamma^{ic})(u + \frac{1}{4}) L_3, \quad (18)$$

where

$$L_1 = (2)^{-2(t+s+u)} \pi \frac{\Gamma(-u + \frac{1}{4}) \Gamma(-s + \frac{1}{4}) \Gamma(-t + \frac{1}{2}) \Gamma(-t - s - u + \frac{1}{2})}{\Gamma(-u - t + \frac{3}{4}) \Gamma(-t - s + \frac{3}{4}) \Gamma(-s - u + \frac{1}{2})},$$

$$L_3 = (2)^{-2(t+s+u)-1} \pi \frac{\Gamma(-u - \frac{1}{4}) \Gamma(-s + \frac{3}{4}) \Gamma(-t) \Gamma(-t - s - u)}{\Gamma(-u - t + \frac{3}{4}) \Gamma(-t - s + \frac{3}{4}) \Gamma(-s - u + \frac{1}{2})}.$$

This S-matrix does satisfy the Ward identity associated to the gauge field, so that by replacing $\xi_{2a} \rightarrow k_{2a}$ the whole S-matrix vanishes. One can also use $(k_1 + k_2)_c = (k_3 + p)_c$ in $\mathcal{A}_6$.

Note that, if we just change the picture of scalar field in the presence of a symmetric RR, one ends up having the final form of the S-matrix of $< C^{-1} \phi^{-1} A^0 T^0 >$ [12] as follows

$$\mathcal{A}^{<C^{-1}\phi^{-1}A^0T^0>} = \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3, \quad (19)$$

where

$$\begin{aligned} \mathcal{A}_1 &\sim 2\xi_{1i}\xi_{2a}k_{3c}k_{2d}\text{Tr}(P_{-}\mathbb{H}_{(n)}M_p\Gamma^{cadi})L_1, \\ \mathcal{A}_2 &\sim \left\{ -\text{Tr}(P_{-}\mathbb{H}_{(n)}M_p\gamma.\xi_2\gamma.\xi_1)(u + \frac{1}{4}) - 2k_3.\xi_2\text{Tr}(P_{-}\mathbb{H}_{(n)}M_p\gamma.k_2\gamma.\xi_1) \right\} L_3(2t) \\ &\quad + \text{Tr}(P_{-}\mathbb{H}_{(n)}M_p\gamma.k_3\gamma.\xi_1) \left\{ 2t(k_3.\xi_2) + 2(-u - \frac{1}{4})k_1.\xi_2 \right\} (-2L_3). \end{aligned} \quad (20)$$

If one applies the momentum conservation along the world volume of brane, one then realises the fact that $\mathcal{A}_1$ of (20) produces the first term $\mathcal{A}_1$ of (18). The first term $\mathcal{A}_2$ of (20) exactly generates $\mathcal{A}_2$ of (18), the sum of the second and third term $\mathcal{A}_2$ of (20), reconstructs $\mathcal{A}_5$ of (18) and finally the last term $\mathcal{A}_2$ of (20) generates $\mathcal{A}_6$ of (18). Thus there seems to be no chance to produce all the needed bulk singularities $\mathcal{A}_3$ and $\mathcal{A}_4$ of (18). It is also important to stress that the second term $\mathcal{A}_1$ of (18) was also overlooked in (20).

Note that since the momentum along the brane is just conserved, from now on instead of Bianchi identities, we use Bianchi identities restricted on D-brane directions or restricted Bianchi identities on world volume. Consider the following restricted Bianchi identity on world volume as follows

$$p^i \epsilon^{a_0 \dots a_p} H_{a_0 \dots a_p} - p_c \epsilon^{a_0 \dots a_{p-1} c} H_{a_0 \dots a_{p-1}}^i = 0 \quad (21)$$

Note that H in (21) is $(p+1)$ form field strength of C_p form and this is obviously true from the traces of gamma matrices which appear in all $\mathcal{A}_2$ terms of (20), basically all the traces for $\mathcal{A}_2$ terms of (20) are non-zero just for $n = p+1$ case.

Upon taking into account the above restricted Bianchi identity and applying momentum conservation along the world volume of brane $(k_1 + k_2 + k_3 + p)^a = 0$ to the sum of the 2nd and 3rd term of $\mathcal{A}_2$ (and also to the last term of $\mathcal{A}_2$) of (20), one is able to actually produce precisely all infinite $u'(t)$ channel bulk singularities $\mathcal{A}_4$ (and $\mathcal{A}_3$) of (18) accordingly.

While a priori without knowing any restricted Bianchi identity on world volume for RR, there seems to be no chance to even see all the needed bulk singularities of $\mathcal{A}_3$ and $\mathcal{A}_4$ of (18) from $<C^{-1}\phi^{-1}A^0T^0>$ S-matrix.

Let us see what happens in the other picture of the S-matrix. One reads off the S-matrix $<C^{-1}\phi^0A^{-1}T^0>$ [30] as follows

$$\mathcal{A}^{<C^{-1}\phi^0A^{-1}T^0>} = \mathcal{A}_1 + \mathcal{A}_2, \quad (22)$$

with

$$\begin{aligned} \mathcal{A}_1 &\sim \left(2\xi_{1i}\xi_{2a}k_{3c}k_{1d}\text{Tr}(P_{-}\mathbb{H}_{(n)}M_p\Gamma^{caid}) - \xi_1.p(2k_{3c}\xi_{2a})\text{Tr}(P_{-}\mathbb{H}_{(n)}M_p\Gamma^{ca}) \right) L_1, \\ \mathcal{A}_2 &\sim \left\{ t\xi_1.p(4k_3.\xi_2)\text{Tr}(P_{-}\mathbb{H}_{(n)}M_p) + 4(u + \frac{1}{4})k_{3c}\xi_{1i}\text{Tr}(P_{-}\mathbb{H}_{(n)}M_p\Gamma^{ci})k_1.\xi_2 \right. \\ &\quad \left. - 4tk_3.\xi_2k_{1b}\xi_{1i}\text{Tr}(P_{-}\mathbb{H}_{(n)}M_p\Gamma^{ib}) - 2t(u + \frac{1}{4})\xi_{1i}\xi_{2a}\text{Tr}(P_{-}\mathbb{H}_{(n)}M_p\Gamma^{ai}) \right\} L_3 \end{aligned} \quad (23)$$

By comparisons of the elements of (23) with (18), we are able to produce all the terms inside (18) except its $\mathcal{A}_3$. In the other words again in this $<C^{-1}\phi^0A^{-1}T^0>$ S-matrix, there seems to be no chance to produce $\mathcal{A}_3$ bulk singularities of (18).

Taking into account the above restricted Bianchi identity (21) and applying momentum conservation along the world volume of brane to the 2nd term $\mathcal{A}_2$ of (23), one is able to indeed construct exactly all infinite t channel bulk singularities $\mathcal{A}_3$ of (18). Meanwhile in this particular picture of S-matrix ($<C^{-1}\phi^0A^{-1}T^0>$) one could already see that the infinite u' channel bulk singularities $\mathcal{A}_4$ of (18) have been shown up in the entire S-matrix.

While a priori without knowing any restricted Bianchi identity for RR, there seems to be no chance to even observe all the needed t channel bulk singularities $\mathcal{A}_3$ of (18) from $<C^{-1}\phi^0A^{-1}T^0>$ S-matrix.

Note that if we would consider the Ward identity associated to the gauge field ($\xi_{2a} \rightarrow k_{2a}$), we would reveal that due to presence of the 2nd term of $\mathcal{A}_1$ of (23) and 1st term $\mathcal{A}_2$ of (23), the S-matrix is not gauge invariant any more. In order to restore gauge invariance, one needs to consider further remarks. Basically if we replace ($\xi_{2a} \rightarrow k_{2a}$) in all four terms $\mathcal{A}_2$ of (23), apply momentum conservation along the world volume of brane as well as simultaneously consider the restricted Bianchi identity (21), then one observes that all four terms $\mathcal{A}_2$ of (23) respect Ward identity.

Finally if one replaces ($\xi_{2a} \rightarrow k_{2a}$) in all two terms $\mathcal{A}_1$ of (23), apply momentum conservation along the world volume of brane as well as take into account the following restricted Bianchi identity on world volume directions

$$\xi_{1i}k_{3c}k_{2a}(-p_a \epsilon^{a_0 \dots a_{p-3} cad} H_{a_0 \dots a_{p-3}}^i + p^i \epsilon^{a_0 \dots a_{p-2} ac} H_{a_0 \dots a_{p-2}}) = 0 \quad (24)$$

then one obviously clarifies that all two terms $\mathcal{A}_1$ of (23) are also now respecting Ward identity associated to gauge field, therefore based on applying those restricted Bianchi identities on world volume, now the whole S-matrix respects Ward identity.

Note that the form that appears in (24) is H of $(p-1)$ form field strength of C_{p-2} form and this is true from the traces of gamma matrices which appear in all $\mathcal{A}_1$ terms of (23), basically all the traces for $\mathcal{A}_1$ terms of (23) are non-zero just for $n+1 = p$ case. Hence, the terms in (21) and (24) do correspond to different RR field couplings.

It is worth mentioning that unlike the symmetric picture, in the asymmetric picture of RR and in the presence of non-BPS branes, without using any further restricted Bianchi identity the ultimate result of amplitude does satisfy Ward identity associated to the gauge

field. Eventually one can compute the same S-matrix but in asymmetric picture of RR so that the result of $\langle C^{-2}\phi^0 A^0 T^0 \rangle$ is found in [30] to be

$$\mathcal{A}^{\langle C^{-2}\phi^0 A^0 T^0 \rangle} = \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 + \mathcal{A}_4 \quad (25)$$

where

$$\begin{aligned} \mathcal{A}_1 &\sim 2^{3/2} i \xi_{1i} \xi_{2a} k_{3c} k_{2b} L_1 \left(p^i \text{Tr}(P_{-(n-1)} M_p \Gamma^{cab}) - k_{1d} \text{Tr}(P_{-(n-1)} M_p \Gamma^{cabid}) \right) \\ \mathcal{A}_2 &\sim 2^{3/2} i \xi_{1i} \cdot p L_3 \text{Tr}(P_{-(n-1)} M_p \gamma^c) \left(2tk_3 \cdot \xi_2 [-k_{3c} - k_{2c}] + 2k_1 \cdot \xi_2 u' k_{3c} - tu' \xi_{2c} \right) \\ \mathcal{A}_3 &\sim 2^{3/2} i \xi_{1i} L_3 \text{Tr}(P_{-(n-1)} M_p \Gamma^{cid}) \left[-2k_1 \cdot \xi_2 u' k_{3c} (k_{1d} + k_{2d}) + 2tk_3 \cdot \xi_2 k_{1d} (k_{3c} + k_{2c}) \right] \\ \mathcal{A}_4 &\sim 2^{3/2} i \xi_{1i} L_3 tu' \xi_{2a} \text{Tr}(P_{-(n-1)} M_p \Gamma^{cai}) (k_{3c} + k_{1c} + k_{2c}) \end{aligned} \quad (26)$$

As we can see in this asymmetric picture we can precisely produce even all the bulk singularities $\mathcal{A}_3$ and $\mathcal{A}_4$ of (18).

Considering all the definitions of $\mathcal{H}_{(n)}$, M_p , Γ^{cadi} , one knows that the S-matrix is non-zero just for $p = n + 1$ and $p + 1 = n$ cases. Keeping in mind the momentum conservation along the world volume of brane, one obtains

$$s + t + u = -p^a p_a - \frac{1}{4}, \quad (27)$$

Standard scattering of RR on BPS branes, with three massless open string states, does not restrict $p^a p_a$ momentum invariant of the closed string state to a specific value. Obviously for non-BPS branes due to the presence of the tachyon the kinematic invariants are more restricted. Indeed for an RR and a tachyon momentum conservation leads to $p^a p_a = k^2 = \frac{1}{4}$. It is also discussed in [11] that for an RR, two massless open strings and a tachyon, using the on-shell relations $k_1^2 = k_2^2 = 0$ and $k_3^2 = \frac{1}{4}$, we are able to rewrite the momentum expansion

$$(k_1 + k_2)^2 \rightarrow 0, \quad k_1 \cdot k_3 \rightarrow 0, \quad k_2 \cdot k_3 \rightarrow 0, \quad (28)$$

just in terms of $t \rightarrow 0$, $s \rightarrow -\frac{1}{4}$, $u \rightarrow -\frac{1}{4}$. In fact the constraint (27) clearly confirms that $p^a p_a$ must be sent to $\frac{1}{4}$ or $p^a p_a \rightarrow \frac{1}{4}$ and this just makes sense only for euclidean brane. This is also consistent with the observation that has been pointed out in [31], which means that on-shell conditions do impose to us the fact that the amplitude must be carried out just for non-BPS SD-branes [32]. The other point which is worth mentioning is as follows. It is shown in [11] that the constraint $p^a p_a \rightarrow \frac{1}{4}$ is valid for all three, four and five point non-BPS functions and more importantly it is checked that by using the constraint $p^a p_a = \frac{1}{4}$, one is able to precisely produce all infinite $(t + s + u + \frac{1}{2})$ tachyon poles of an RR, two scalar fields and a tachyon of [11] as the final form of amplitude clearly involves the factor $\Gamma(-t - s - u - \frac{1}{2})$ (for more information see equation (20) and section 4 of [11]).

Now using $p^a p_a \rightarrow \frac{1}{4}$ for non-BPS branes, also taking into account the non-zero vertex operator of two scalars and a gauge field, one immediately gets to know that $t \rightarrow 0$, $s \rightarrow -\frac{1}{4}$, $u \rightarrow -\frac{1}{4}$ is the only unique expansion of the S-matrix. Given the facts that the S-matrix does not include the coefficients of $\Gamma(-s - \frac{1}{4})$, $\Gamma(-t - s - u - \frac{1}{2})$, and also $\langle V_{A^0} V_{T^0} V_{\phi^{-1}} V_{\phi^{-1}} \rangle$, $\langle V_{A^0} V_{T^0} V_{A^{-1}} V_{\phi^{-1}} \rangle$ have zero contribution (based on applying CP matrices, as $\text{Tr}(I\sigma_1\sigma_3\sigma_3) = 0$), we understand that, this S-matrix does not have any $s' = s + 1/4$, $(s' + t + u')$ poles at all, thus we are left over with an infinite number of u' , t channel poles.

Let us analyse all infinite tachyon u' channel poles, then we reconstruct all infinite t-channel scalar poles accordingly. Indeed, by taking the momentum expansion into considerations, we realise that $\mathcal{A}_1, \mathcal{A}_2$ in (17) are all contact interactions, while $\mathcal{A}_3, \mathcal{A}_6$ and $\mathcal{A}_4, \mathcal{A}_5$ are related to all infinite t and u' singularities of string amplitude appropriately. Also note that $\mathcal{A}_3, \mathcal{A}_4$ do carry $p \cdot \xi_1$ singular terms, whose momenta are located in the bulk directions and from now on due to the presence of p^i , we are going to call them bulk singularities and start producing them in effective field theory (EFT) as well.

2. An infinite u' channel tachyon singularities

In order to deal with all singularities of S-matrix, we first need to have the entire expansion of Gamma functions. The expansion of tL_3 around $t \rightarrow 0$, $s \rightarrow -\frac{1}{4}$, $u \rightarrow -\frac{1}{4}$ is given by

$$tL_3 = \pi^{3/2} \left(\frac{1}{u'} \sum_{n=-1}^{\infty} c_n (s' + t)^{n+1} + \sum_{p,n,m=0}^{\infty} f_{p,n,m} (u')^p (ts')^n (t + s')^m \right), \quad (29)$$

with some of the coefficients as

$$c_{-1} = 1, \quad c_0 = 0, \quad c_1 = \frac{1}{6}\pi^2, \quad f_{0,0,1} = \frac{1}{3}\pi^2, \quad f_{1,0,1} = f_{0,0,2} = 6\zeta(3).$$

The results for the trace that include γ^{11} can also be true for the following

$$p > 3, \quad H_n = *H_{10-n}, \quad n \geq 5.$$

We first extract the trace in $\mathcal{A}_5$ and write down all tachyon singularities as follows

$$(4ik_3 \cdot \xi_2) k_{1c} \xi_{1i} \frac{16}{(p+1)!} (\pi^{3/2}) (\mu'_p \beta' \pi^{1/2}) H_{a_0 \dots a_{p-1}}^i \epsilon^{a_0 \dots a_{p-1} c} \sum_{n=-1}^{\infty} c_n \frac{1}{u'} (s' + t)^{n+1} \text{Tr}(\lambda_1 \lambda_2 \lambda_3) \quad (30)$$

where we used $(\mu'_p \beta' \pi^{1/2})$ as normalisation constant to the S-matrix. To reconstruct all infinite tachyon singularities, one has to consider the following sub amplitude in field theory

$$\mathcal{A} = V^\alpha(C_p, \phi_1, T) G^{\alpha\beta}(T) V^\beta(T, T_3, A_2). \quad (31)$$

Note that tachyon kinetic term $2\pi\alpha' D^a T D_a T$ has already been fixed in the DBI action and it has no correction, also $V^\beta(T, T_3, A_2)$ vertex operator comes from tachyon kinetic term by taking $(D_a T = \partial_a T - i[A^a, T])$ so it has no correction either. Both Taylor expansion and pull-back of $C_{a_0 \dots a_{p-1}}$ are needed. A field strength in the string amplitude $H_{a_0 \dots a_{p-1}}^i$ being reproduced by pull-back and Taylor expansion of the $C_{a_0 \dots a_{p-1}}$. Consider the following coupling

$$2i\beta' \mu'_p (2\pi\alpha')^2 \int_{\Sigma_{p+1}} \text{Tr}(\partial_i C_p \wedge DT \phi^i) \quad (32)$$

Above coupling is the mixing Chern–Simons and Taylor expansion of scalar field. We need to take into account $2i\beta' \mu'_p (2\pi\alpha')^2 \int_{\Sigma_{p+1}} \epsilon^{a_0 \dots a_p} C_{ia_0 \dots a_{p-2}} D_{a_{p-1}} \phi^i D_{a_p} T$ coupling as well. If we take integration by parts $D_{a_{p-1}}$ can just act on C-field. Because of the ϵ tensor it gives zero result if it acts on $D_{a_p} T$. Now if we take into account $\epsilon^{a_0 \dots a_p} \partial_{[a_{p-1}} C_{a_0 \dots a_{p-2}]i} \phi^i D_{a_p} T$ where the bracket corresponds to antisymmetrisation of the corresponding world-volume indices and use the definition $H_{a_0 \dots a_{p-1}}^i = p \partial_i C_{a_0 \dots a_{p-1}} + \partial_{[a_{p-1}} C_{a_0 \dots a_{p-2}]i}$, then we clarify that both terms are needed to reproduce the string amplitude term $H_{a_0 \dots a_{p-1}}^i k_{ap} \xi^i$.

Note that the contributions from Taylor expansion and the other coupling in momentum space would be $\epsilon^{a_0 \dots a_p} p_i C_{a_0 \dots a_{p-1}} k_{ap} \xi^i$ and $(-\epsilon^{a_0 \dots a_p} p_{a_{p-1}} C_{a_0 \dots a_{p-2}}^i k_{ap} \xi^i)$ accordingly, thus one obtains the following counterparts for the vertices in EFT

$$\begin{aligned} V^\beta(T, T_3, A_2) &= 2T_p (2\pi\alpha') k_3 \cdot \xi_2 \text{Tr}(\lambda_2 \lambda_3 \Lambda^\beta), \\ V^\alpha(C_p, \phi_1, T) &= 2\mu'_p \beta' \frac{(2\pi\alpha')^2}{(p+1)!} \epsilon^{a_0 \dots a_p} H_{a_0 \dots a_{p-1}}^i k_{ap} \xi_{1i} \text{Tr}(\lambda_1 \Lambda^\alpha), \\ G^{\alpha\beta}(T) &= \frac{-i\delta^{\alpha\beta}}{(2\pi\alpha') T_p (k^2 + m^2)} = \frac{-i\delta^{\alpha\beta}}{(2\pi\alpha') T_p(u')}, \end{aligned} \quad (33)$$

where k in the above is momentum of off-shell tachyon $k = k_2 + k_3 = -(p + k_1)$. Now by replacing (33) inside (31) one obtains

$$(4ik_3 \cdot \xi_2) \xi_{1i} \frac{16}{(p+1)!} \pi^2 \mu'_p \beta' H_{a_0 \dots a_{p-1}}^i (p + k_1)_{ap} \epsilon^{a_0 \dots a_p} \frac{1}{u'} \quad (34)$$

where if one uses the identity $p_{ap} \epsilon^{a_0 \dots a_p} = 0$,¹ then one reveals that the first u' channel tachyon pole of (30) can be precisely produced. However, in (30) we do have infinite poles and the only way to regenerate them is to induce the higher derivative corrections as follows

$$\frac{2i\beta' \mu'_p}{p!} (2\pi\alpha')^2 \partial_i C_p \wedge \text{Tr} \left(\sum_{n=-1}^{\infty} c_n (\alpha')^{n+1} D_{a_1} \dots D_{a_{n+1}} D T D^{a_1} \dots D^{a_{n+1}} \phi^i \right) \quad (35)$$

One needs to be reminded about the facts that the tachyon propagator and the vertex of two tachyon and one gauge field, do not receive any correction as they have been derived from the kinetic term of tachyons (as it has been already fixed), that is why we claim that, taking (35) is the only way of reconstructing all the singularities.

Having set (35), we were able to actually obtain the extension of the vertex operator to all orders as follows

$$V^\alpha(C_p, T, \phi_1) = \frac{2\mu'_p \beta' (2\pi\alpha')^2 k_{ap} \xi_{1i}}{(p)!} \epsilon^{a_0 \dots a_p} H_{a_0 \dots a_{p-1}}^i \sum_{m=-1}^{\infty} c_m (\alpha' k_1 \cdot k)^{m+1} \text{Tr}(\lambda_1 \Lambda^\alpha). \quad (36)$$

Substituting (36) to (31), considering the fixed propagator (the fixed two tachyons, a gauge field vertex operator) and making use of the following identity $\sum_{m=-1}^{\infty} c_m (\alpha' k_1 \cdot k)^{m+1} = \sum_{m=-1}^{\infty} c_m (t + s')^{m+1}$, one exactly reconstructs all infinite u' tachyon poles of (30) in the effective field theory side as well. It is also worth pointing out the fact that by comparisons we get to know that there was no residual contact interactions to be left over in the EFT. Now we turn to infinite t channel scalar singularities.

¹ Note that, the derivation of the identity $p_a \epsilon^{a_0 \dots a_{p-1} a} = 0$ can be found from various equations of [30], for instance it is clarified in formula (9) of [30], namely, to actually obtain the same result for the amplitude of a three point function of one RR and a scalar field in both symmetric and asymmetric S-matrix, this identity $p_a \epsilon^{a_0 \dots a_{p-1} a} = 0$ should be true. The other example to prove the above identity is as follows. We have just shown it in section 5 of [30], basically to get to the same result for the S-matrix of a four point function of $\langle C^{-1} T^0 \phi^{-1} \rangle$ and $\langle C^{-2} T^0 \phi^0 \rangle$, that identity should be employed (notice to the footnote 19 of [30]). Finally, we have shown that if and only if that identity holds, then certainly the S-matrix of $\langle C^{-1} A^{-1} T^0 T^0 \rangle$ of [14] does satisfy Ward identity related to the gauge field.

3. An infinite t channel scalar singularities

Having expanded $u' L_3$ around the same $t \rightarrow 0$, $s \rightarrow -\frac{1}{4}$, $u \rightarrow -\frac{1}{4}$, we get

$$u' L_3 = \pi^{3/2} \left(\frac{1}{t} \sum_{n=-1}^{\infty} c_n (u' + s')^{n+1} + \sum_{p,n,m=0}^{\infty} f_{p,n,m} t^p (u' s')^n (u' + s')^m \right), \quad (37)$$

where the coefficients of $c_n, f_{p,n,m}$ are read.² After extracting the trace in $\mathcal{A}_6$ one writes down all scalar t -channel singularities of the S-matrix as follows

$$4ik_1 \cdot \xi_2 k_3 c \xi_{1i} \frac{16\mu'_p \beta' \pi^2}{(p+1)!} \sum_{n=-1}^{\infty} c_n \frac{1}{t} (s' + u')^{n+1} H_{a_0 \dots a_{p-1}}^i \epsilon^{a_0 \dots a_{p-1} c} \text{Tr}(\lambda_1 \lambda_2 \lambda_3). \quad (38)$$

Later on we consider the restricted Bianchi identity to actually write down $p_c H_{a_0 \dots a_{p-1}}^i \epsilon^{a_0 \dots a_{p-1} c}$ in terms of $p^i H_{a_0 \dots a_p} \epsilon^{a_0 \dots a_p}$ and generate all the bulk singularities of the S-matrix in EFT as well. To construct all infinite scalar singularities, one must consider the following sub amplitude in field theory

$$\mathcal{A} = V_i^\alpha(C_p, T_3, \phi) G_{ij}^{\alpha\beta}(\phi) V_j^\beta(\phi, \phi_1, A_2). \quad (39)$$

Note that the scalar kinetic term has already been fixed in the DBI action and it has no correction, also $V_j^\beta(\phi, \phi_1, A_2)$ can be derived just from scalar kinetic term by extracting the covariant derivative of scalar field ($D_a \phi^i = \partial_a \phi^i - i[A_a, \phi^i]$) so it has no correction either. One derives the following vertices in EFT

$$V_j^\beta(\phi, \phi_1, A_2) = -2(2\pi\alpha')^2 T_p k_1 \cdot \xi_2 \xi_{1j} \text{Tr}(\lambda_1 \lambda_2 \Lambda^\beta), \quad (40)$$

$$\begin{aligned} V_i^\alpha(C_p, \phi, T_3) &= 2\mu'_p \beta' \frac{(2\pi\alpha')^2}{(p+1)!} \epsilon^{a_0 \dots a_p} H_{a_0 \dots a_{p-1}}^i k_{3a_p} \text{Tr}(\lambda_3 \Lambda^\alpha), \\ G_{ij}^{\alpha\beta}(\phi) &= \frac{-i\delta^{\alpha\beta} \delta^{ij}}{(2\pi\alpha')^2 T_p (k^2)} = \frac{-i\delta^{\alpha\beta} \delta^{ij}}{(2\pi\alpha')^2 T_p(t)}, \end{aligned} \quad (41)$$

where k_3 in the above is momentum of on-shell tachyon and we used momentum conservation along the world volume direction as well. Now by replacing (41) inside (39) one concludes that the first t -channel scalar pole of (38) can be precisely produced. However, as it is clear in (38) we do have infinite poles and the only way to reproduce them is to propose the higher derivative corrections to the actions.

Taking (35) into account one can get the all order extensions of the vertex operator as

$$V_i^\alpha(C_p, \phi, T_3) = 2\mu'_p \beta' \frac{(2\pi\alpha')^2}{(p+1)!} \epsilon^{a_0 \dots a_p} H_{a_0 \dots a_{p-1}}^i k_{3a_p} \sum_{m=-1}^{\infty} c_m (\alpha' k_3 \cdot k)^{m+1} \text{Tr}(\lambda_3 \Lambda^\alpha). \quad (42)$$

Using momentum conservation, one gets, $(\alpha' k_3 \cdot k) = (u' + s')$. If we substitute (42) inside (39) and keeping fixed all the other vertices, one clarifies the fact that all the infinite t -channel poles of (38) are precisely gained in the effective field theory side as well, so the higher derivative corrections of (35) are exact.

To generate all the contact interactions, we just highlight the following references [11,7]. Also notice to the point that for this S-matrix we do have external gauge field as well as an external scalar and a real tachyon therefore in the action of (32), one needs to first consider the presence of commutator inside the covariant derivative of tachyon, so that external gauge field shows up and then try to apply higher derivative corrections properly. Lets turn to the main point of the paper which is indeed dealing with all the bulk singularity structures of the S-matrix.

4. An infinite u' bulk singularity structures

Apart from an infinite number of u' channel tachyon poles, we have an infinite number of bulk singularity structures that can be accommodated in EFT as well.

Consider the expansion of (29) and do replace it into the entire $\mathcal{A}_4$ of (18), extract the related trace as well as normalise this fourth part of S-matrix so that the whole bulk singularities are now found out as follows

$$p \cdot \xi_1 (-4ik_3 \cdot \xi_2) \frac{16\mu'_p \beta' \pi^2}{(p+1)!} H_{a_0 \dots a_p} \epsilon^{a_0 \dots a_p} \sum_{n=-1}^{\infty} c_n \frac{1}{u'} (s' + t)^{n+1} \text{Tr}(\lambda_1 \lambda_2 \lambda_3) \quad (43)$$

$$\begin{aligned} c_2 &= 2\zeta(3), f_{2,0,0} = f_{0,1,0} = 2\zeta(3), f_{1,0,0} = \frac{1}{6}\pi^2, f_{1,0,2} = \frac{19}{60}\pi^4, \\ f_{0,0,1} &= \frac{1}{3}\pi^2, f_{0,0,3} = f_{2,0,1} = \frac{19}{90}\pi^4, f_{1,1,0} = f_{0,1,1} = \frac{1}{30}\pi^4. \end{aligned}$$

We call them bulk singularities, because they do involve all the infinite momenta of RR in the bulk directions (due to an infinite number of $p.\xi_1$ terms). In the other words, we can keep track of these terms that carry the scalar product of momentum of RR in the bulk and scalar polarisation and claim that these bulk singularities are needed in the entire S-matrix as we are going to reconstruct them in EFT as well.

To reconstruct all these infinite bulk u' channel singularities, one has to once more, consider the following sub amplitude in field theory side

$$\mathcal{A} = V^\alpha(C_p, \phi_1, T) G^{\alpha\beta}(T) V^\beta(T, T_3, A_2). \quad (44)$$

It is worth highlighting the remark that both tachyon propagator and $V^\beta(T, T_3, A_2)$ vertex operator, will not receive any correction. To actually produce $p.\xi_1$ terms in EFT one needs to consider the following integrations

$$2i\beta'\mu'_p(2\pi\alpha')^2 \int_{\Sigma_{p+1}} \left(-\text{Tr}(\partial_i d_{a_p} C_{a_0 \dots a_{p-1}} T \phi^i) - \text{Tr}(\partial_i C_{a_0 \dots a_{p-1}} T d_{a_p} \phi^i) \right), \quad (45)$$

where we suppose all the fields are zero at infinity. We have already taken into account the contribution of the second term of the above action and were able to produce all infinite u' channel tachyon singularities, so here we just need to consider the contribution of the first term of (45) to actually derive the following vertex operator

$$V^\alpha(C_p, \phi_1, T) = 2\mu'_p \beta' \frac{(2\pi\alpha')^2}{(p+1)!} \epsilon^{a_0 \dots a_p} H_{a_0 \dots a_p} p.\xi_1 \text{Tr}(\lambda_1 \Lambda^\alpha) \quad (46)$$

Keeping fixed $V^\beta(T, T_3, A_2)$ and the tachyon propagator and replacing (46) inside (44) one gets

$$(-4ik_3.\xi_2) p.\xi_1 \frac{16\mu'_p \beta' \pi^2}{u'(p+1)!} H_{a_0 \dots a_p} \epsilon^{a_0 \dots a_p} \text{Tr}(\lambda_1 \lambda_2 \lambda_3) \quad (47)$$

One can observe the fact that (47) is indeed the first bulk singularity of the S-matrix (consider $n = -1$ inside (43)). To actually even produce all infinite bulk singularities, one needs to apply all higher derivative corrections to the first part of the above action as follows

$$\frac{-2i\beta'\mu'_p}{(p+1)!} (2\pi\alpha')^2 \int_{\Sigma_{p+1}} \left(\text{Tr} \sum_{n=-1}^{\infty} c_n(\alpha')^{n+1} \partial_i d_{a_p} C_{a_0 \dots a_{p-1}} D_{a_1} \dots D_{a_{n+1}} T D^{a_1} \dots D^{a_{n+1}} \phi^i \right) \quad (48)$$

Having set (48), we are able to precisely reconstruct all order extensions of the above vertex operator as follows

$$V^\alpha(C_p, T, \phi_1) = \frac{2\mu'_p \beta' (2\pi\alpha')^2 p.\xi_1}{(p+1)!} \epsilon^{a_0 \dots a_p} H_{a_0 \dots a_p} \sum_{m=-1}^{\infty} c_m(t+s')^{m+1} \text{Tr}(\lambda_1 \Lambda^\alpha). \quad (49)$$

Substituting (49) to (44), considering the fixed propagator (and fixed two tachyons, a gauge field vertex operator), one is able to exactly regenerate all infinite u' bulk poles of (43) in the effective field theory side as well. Eventually in the next section, we try to produce an infinite t channel bulk Singularity structures in the effective field theory side too.

5. An infinite t channel bulk singularity structures

Apart from an infinite number of t channel scalar poles, we do have an infinite number of t channel bulk singularity structures that can be found out in EFT as well.

Consider the expansion of (37) and substitute it into the whole $\mathcal{A}_3$ of (18), extract the trace as well as normalise this third part of S-matrix, to indeed get to the whole bulk t -channel singularities from the string theory point of view as follows

$$p.\xi_1 (4ik_1.\xi_2) \frac{16\mu'_p \beta' \pi^2}{(p+1)!} H_{a_0 \dots a_p} \epsilon^{a_0 \dots a_p} \sum_{n=-1}^{\infty} c_n \frac{1}{t} (s' + u')^{n+1} \text{Tr}(\lambda_1 \lambda_2 \lambda_3) \quad (50)$$

These are also bulk singularities, because all the infinite singularity terms that involve momentum of RR in the bulk have been embedded into the S-matrix and in below we want to show that they even can be reconstructed in EFT as well. To regenerate all these infinite bulk t channel scalar singularities, one needs to deal with the following sub amplitude in field theory side

$$\mathcal{A} = V_i^\alpha(C_p, T_3, \phi) G_{ij}^{\alpha\beta}(\phi) V_j^\beta(\phi, \phi_1, A_2), \quad (51)$$

Note that both scalar propagator and $V_j^\beta(\phi, \phi_1, A_2)$ vertex operator, do not get corrected. To actually produce $p.\xi_1$ terms in EFT one should make use of Taylor expansion and take integration by parts as well. Indeed in (41), we have considered the contribution of the second term of (45) and precisely produced all infinite t channel scalar singularities of $\mathcal{A}_6$, in the meantime, here we just need to consider the contribution from the first term of (45) to actually explore

$$V_i^\alpha(C_p, \phi, T_3) = -2p_i \mu'_p \beta' \frac{(2\pi\alpha')^2}{(p+1)!} \epsilon^{a_0 \dots a_p} H_{a_0 \dots a_p} \text{Tr}(\lambda_3 \Lambda^\alpha) \quad (52)$$

Keeping fixed $V_f^\beta(\phi, \phi_1, A_2)$ and the scalar propagator and replacing (52) inside (51) one gains

$$(4ik_1 \cdot \xi_2) p \cdot \xi_1 \frac{16\mu'_p \beta' \pi^2}{t(p+1)!} H_{a_0 \dots a_p} \epsilon^{a_0 \dots a_p} \text{Tr}(\lambda_1 \lambda_2 \lambda_3) \quad (53)$$

One can observe the fact that (53) is indeed the first bulk singularity of the S-matrix (consider $n = -1$ inside (50)). To be able to produce all infinite bulk singularities, one needs to apply all higher derivative corrections properly. Having set (48), we are able to precisely reconstruct all order extensions of $V_i^\alpha(C_p, \phi, T_3)$ vertex operator as follows

$$V_i^\alpha(C_p, \phi, T_3) = -2p_i \mu'_p \beta' \frac{(2\pi\alpha')^2}{(p+1)!} \epsilon^{a_0 \dots a_p} H_{a_0 \dots a_p} \sum_{m=-1}^{\infty} c_m (u' + s')^{m+1} \text{Tr}(\lambda_3 \Lambda^\alpha). \quad (54)$$

Substituting (54) to (51), considering the fixed scalar propagator (and the fixed two scalars, a gauge field vertex operator), one is able to precisely regenerate all infinite t channel bulk singularities of (50) in the effective field theory side as well. Therefore, we were able to even produce all the infinite bulk singularities u' , t in the EFT and that evidently confirms that the presence of bulk singularities is needed inside the entire S-matrix. Note that, it also has the important consequences for which one of them is the essential appearances of new restricted Bianchi identities on world volume directions of D-branes that we got in this particular S-matrix.

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