Quintessence and Dark Energy

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Abstract. Dark energy accounts for about 70% of the content of our Universe. Perhaps the best candidate to parametrize the dark energy is a scalar field with only gravitational interactions called quintessence.

We first present a generic theoretical approach to quintessence. We show that if the minimum of the scalar potential is at $V|_{min} = 0$ (i.e. there is no arbitrary scale) then the behavior of scalar fields can be determined in a model independent way. Its equation of state parameter w_ϕ takes most of the time the values $w_\phi = 1, -1, w_{\phi o}$. The size of the different regions can also be calculated in a model independent way. The number of free parameters for quintessence models is therefore quite limited.

We show that late time phase transition models are good candidates for quintessence and they could explain why the acceleration of the Universe is at such a late time. We show how these models can be obtained from particle physics and in particular from non-abelian gauge dynamics. The only free parameter for these gauge models is its particle content as it is the case for the standard model of particle physics.

In the second part, we show how a phenomenological approach to the CMB can be implemented. We show that with only four parameters we cover a great number of theoretical models, including quintessence. By varying these parameters and comparing with the CMB we can, in principle, determine the relevant cosmological quantities such as the phase transition scale (when the quintessence field appears) and the present equation of state parameter $w_{\phi o}$.

1 Introduction

In recent time the cosmological observations on the cosmic microwave background radiation ("CMB") [1] and the supernova project SN1a [2] have lead to conclude that the universe is flat and it is expanding with an accelerating velocity. These conclusions show that the universe is now dominated by an energy density with negative pressure with $\Omega_{DE} = 0.7 \pm 0.1$ and $w_{DE} \equiv p_{DE}/\rho_{DE} < -0.78$ [1]. This energy is generically called the dark energy. Structure formation also favors a non-vanishing dark energy [3]. Besides dark energy we also have baryonic matter $\Omega_b \simeq 0.05$ and dark matter $\Omega_{DM} = 0.25 \pm 0.1$, necessary for structure formation, but we still do not know its origin. So, we have a universe which contains only 5% of particles of the well known Standard Model ("SM") of particle physics and 95% of matter unknown to us on earth.

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It is not clear yet what the dark energy is. It could be a true cosmological constant, quintessence (scalar field with gravitationally interaction) [9] or some other kind of exotic energy density. Perhaps the best way of determining the nature of dark energy is trough its equation of state parameter w_{DE}. The survey of redshifts of the different objects should in principle allow as to determine the value of w_{DEo} (o subscript referees to present day quantities) but only at small redshifts $z \leq 2$ with $z_o = 0$. The result from the SN1A project [2] sets an upper limit to $w_{\phi o} < -2/3$ but does not distinguish a true cosmological constant with $w_\Lambda \equiv -1$, quintessence or any other form of exotic energy with $w_{\phi o} < -2/3$. It would be very interesting if in the future the SN1a survey could constrain better the value of $w_{\phi o}$. On the other hand, the CMB could give us information not only on the value of w_{DEo} but also on its evolution during matter domination era, i.e. for a redshift $z \leq 8$.

Energy density of elementary particles of the Standard Model (e.g. quarks, leptons and bosons) have a non-negative pressure $p = w\rho$ with $w = 1/3$ for relativistic and $w = 0$ non-relativistic particles. Therefore, an energy density with negative pressure, the "dark energy", has to be explain from a particle physics point via particles that are not contained in the SM. The only particles that can give a negative pressure are particles with a non trivial self potential V and since fermions and bosons cannot have a vacuum expectation value (these particles transform non-trivially under the Lorentz transformation) the only possibility left are scalar fields. The scalar fields can be fundamental fields, as the Higgs field or the supersymmetric partners of the SM fermion particles, or they could be composite fields as meson fields in QCD.

In order to determine what the nature of the dark energy is we can proceed with two different approaches. On the one hand we can propose models derived from particle physics and see if these models give the correct observable data. On the other hand we could set a model independent analysis on the evolution of the equation of state parameter w_{DE} and determine its impact on the observed CMB spectrum and compare it with the data in order to infer the type of dark energy density.

In Sect. 2 we will discuss the theoretical approaches to obtain a dark energy from field theory. In Sect. 3 we introduce a particle physics model, based on gauge dynamics, that gives a dark energy field in a natural way. In Sect. 4 we study the possibility that the gauge group responsible for giving the dark energy gives at the same time the missing dark matter. In Sect. 5 we analyze a model independent phenomenological approach to dark energy and in Sect. 6 we present our conclusions.

2 Theoretical Approach

2.1 Cosmological Evolution of Quintessence

We will now determine the cosmological evolution of a scalar field ϕ with arbitrary potential $V(\phi)$ and with only gravitational interaction with all other fields. This field is called quintessence.

The cosmological evolution of ϕ with an arbitrary potential $V(\phi)$ can be determined from a system of differential equations describing a spatially flat Friedmann–Robertson–Walker universe in the presence of a barotropic fluid energy density ρ_b that can be either radiation or matter. The equations are

$$\begin{aligned} \dot{H} &= -\frac{1}{2}(\rho_b + p_b + \dot{\phi}^2), \\ \dot{\rho} &= -3H(\rho + p), \\ \ddot{\phi} &= -3H\dot{\phi} - \frac{dV(\phi)}{d\phi}, \end{aligned} \tag{1}$$

where H is the Hubble parameter, $\dot{\phi} = d\phi/dt$, ρ (p) is the total energy density (pressure). We will be working in a flat universe so that $H^2 = \rho/3$ and we use natural units $m_p^2 = G/8\pi \equiv 1$. If is useful to make a change of variables $x \equiv \frac{\dot{\phi}}{\sqrt{6}H}$ and $y \equiv \frac{\sqrt{V}}{\sqrt{3}H}$ and the equations (1) take the following form [27, 26]:

$$\begin{aligned} x_N &= -3x + \sqrt{\frac{3}{2}}\lambda\, y^2 + \frac{3}{2}x[2x^2 + \gamma_b(1 - x^2 - y^2)] \\ y_N &= -\sqrt{\frac{3}{2}}\lambda\, x\, y + \frac{3}{2}y[2x^2 + \gamma_b(1 - x^2 - y^2)] \\ H_N &= -\frac{3}{2}H[2x^2 + \gamma_b(1 - x^2 - y^2)] \end{aligned} \tag{2}$$

where N is the logarithm of the scale factor a, $N \equiv Log(a)$; $f_N \equiv df/dN$ for $f = x, y, H$; $\gamma_b = 1 + w_b$ and $\lambda(N) \equiv -V'/V$ with $V' = dV/d\phi$. In terms of x, y the energy density parameter is $\Omega_\phi = x^2 + y^2$ while the equation of state parameter is given by $\gamma_\phi - 1 = w_\phi \equiv p_\phi/\rho_\phi = \frac{x^2 - y^2}{x^2 + y^2}$. It is clear that $0 \leq x^2, y^2 \leq 1$.

The Friedmann or constraint equation for a flat universe $\Omega_b + \Omega_\phi = 1$ must supplement equations (2) which are valid for any scalar potential as long as the interaction between the scalar field and matter or radiation is gravitational only. This set of differential equations is non-linear and for most cases has no analytical solutions. A general analysis for arbitrary potentials is performed in [25, 26]. All model dependence falls on two quantities: $\lambda(N)$ and the constant parameter $\gamma_b = 1, 4/3$ for matter or radiation, respectively. We will be interested in studying scalar fields that lead to a late time accelerated universe, i.e. to quintessence, and in this case we will have a decreasing $\lambda(N)$

[26] and a late time behavior $\lambda(N) \to 0$. For constant $\lambda(N)$ (exponential potential) one can have an accelerating universe if $\lambda(N) < \sqrt{6}$ but its dynamics would lead to an accelerating universe too rapidly, i.e. not at a late time as ours, unless we fine tune the initial conditions.

It is also useful to have the evolution of $\Omega_\phi = \rho_\phi/3H^2 = x^2 + y^2$ and $\gamma_\phi = 1 + w_\phi$ $(0 \le \gamma_\phi \le 2)$ derived from (2), [21]

$$(\Omega_\phi)_N = 3(\gamma_b - \gamma_\phi)\Omega_\phi(1 - \Omega_\phi) \tag{3}$$

$$(\gamma_\phi)_N = 3\gamma_\phi(2 - \gamma_\phi)\left(\lambda\sqrt{\frac{\Omega_\phi}{3\gamma_\phi}} - 1\right). \tag{4}$$

2.2 Evolution of x, y, and H

We are interested in studying scalar potentials that lead to quintessence, i.e. a late time (present day) acceleration period of the universe. For this to happen one needs $\lambda = -m_{pl}V'/V \to 0$ in the asymptotic limit (or to a constant less then one). An accelerating universe (slow roll conditions) requires $|\lambda| < 1$ and we want this period to be at a late time. We will consider potentials with $V \ge 0$ and since the ϕ field evolves to its minimum $V' < 0$ and $\lambda \ge 0$ where we are assuming, without loss of generality, models with $\phi \ge 0$. We will define the phase transition scale Λ_c in terms of the potential by [35]

$$\Lambda_c = V(\phi_i)^{1/4} \tag{5}$$

where $V(\phi_i) \equiv V_i$ is the initial value of the potential and we will consider models that have an initial value

$$\lambda_i = -m_{pl}\frac{V'(\phi_i)}{V(\phi_i)} \gg 1. \tag{6}$$

From now on the subscript i stands for initial conditions, i.e. at the condensation Λ_c when V appears. From dimensional analysis we expect $\lambda_i = O(m_{pl}/\Lambda_c) \gg 1$. If we have a phase transition at a scale Λ_c which leads to the appearance of the ϕ field (e.g. composite field) then we would also expect $\phi_i \simeq \Lambda_c$ since Λ_c is the relevant scale of the process. We will be working with a late time phase transition but Λ_c could be as large as $10^{16} GeV$ and we would still have $\lambda_i \gg 1$. An interesting general property of these models is the presence of a many e-folds scaling period in which λ is practically constant and $\Omega_\phi \ll 1$.

A semi-analytic approach [18] is useful to study some properties of the differential equation system given by (2). To do this we initially consider only the terms that are proportional to λ, since $\lambda_i \gg 1$, then we follow the evolution of x, y and H so every period has a characteristic set of simplified differential equations. We see from (2) that the leading terms in x and y, for $\lambda \gg 1$, are $x_N = \sqrt{\frac{3}{2}}\lambda\, y^2$ and $y_N = -\sqrt{\frac{3}{2}}\lambda\, x\, y$. Combining these equations we have

$$x_N x = -y_N y \tag{7}$$

with a constant circular solution [18]

$$\Omega_\phi \equiv x^2 + y^2 = x_i^2(N_i) + y_i^2(N_i) \equiv \Omega_{\phi i}(N_i). \tag{8}$$

Since x_N is positive x will grow while y_N is negative giving a decreasing y. This initial period ends at a scale N_{min} with $x^2(N_{min}) \simeq \Omega_{\phi i}(N_i) \gg y_{min}^2$. Since $\lambda_i \gg 1$, the x and y derivatives are quite large and the amount of e-folds between the initial value y_i until y reaches its minimal value y_{min} is very short. An easy estimate can be derived from $y_N/y = -c\lambda \gg 1,\ c = \sqrt{3/2}x$ giving $1 \gg N_{min} - N_i = Log[y_{min}/y_i]/c\lambda_i = O(1/\lambda_i)$, in the assumption $c\lambda_i = cte$.

The minimal value of y, given at N_{min}, can be obtained from (2) with $y_N = 0$. At his point we have [35]

$$\lambda(N_{min}) = -\sqrt{\frac{2}{3}}\frac{H_N}{Hx} = \sqrt{\frac{3}{2}}\frac{[\gamma_b + \Omega_{\phi i}(2-\gamma_b)]}{\sqrt{\Omega_{\phi i}}} \simeq \frac{1}{\sqrt{\Omega_{\phi i}}} \tag{9}$$

where we have taken $x^2(N_{min}) \simeq \Omega_{\phi i}$ and $H_N/H = -3/2(\gamma_b + \Omega_{\phi i}(2-\gamma_b))$ since $y_{min}^2 \ll 1$. We see that λ in (9) is of order $1/\sqrt{\Omega_{\phi i}}$ and we have $\lambda_i/\lambda(N_{min}) \gg 1$.

The value of y_{min} depends on the functional form of $V(\phi)$, which sets the functional form of $\lambda = -V'/V$. In general we have $y_{min}^2 = V(\phi_{min})/(3H_{min}^2)$ but without specifying $V(\phi)$ it is not possible to determine y_{min}. For an inverse power law potential with $V = \Lambda_c^{4+n}\phi^{-n} = 3y^2H^2$ one has

$$\begin{aligned} y_{min} &= \frac{\Lambda_c^{\frac{4+n}{2}}\phi_{min}^{-n/2}}{\sqrt{3}H_{min}} \\ &= y_i\left(\frac{\phi_i}{\phi_{min}}\right)^{\frac{n}{2}} = y_i\left(\frac{1}{\lambda_i\sqrt{\Omega_{\phi i}}}\right)^{\frac{n}{2}} \end{aligned} \tag{10}$$

where we have approximated $H_{min}^2 \simeq H_i^2 = V_i/3y_i^2 = \Lambda_c^{4+n}\phi_i^{-n}/3y_i^2$ in (10) since $N_{min} - N_i \ll 1$ and we have taken from (9) $\phi_{min} = n/\lambda_{min} \simeq n\sqrt{\Omega_{\phi i}}$ and $\phi_i = n/\lambda_i$. Taking the initial value of $\phi_i = n/\lambda_i = n\Lambda_c$ then (10) gives

$$y_{min} = y_i\left(\frac{\Lambda_c}{\sqrt{\Omega_{\phi i}}}\right)^{n/2}. \tag{11}$$

We see that $y_{min} = O(\lambda_i^{-n/2}) \simeq O(\Lambda_c^{n/2}) \ll y_i$ if $\Omega_{\phi i}$ is not too small. At the end of the initial period we have $y^2 \ll 1$ and $\lambda x = O(1)$. Since x_N/x is now negative

$$x_N \simeq (-3 + 3/2\gamma_b)x \tag{12}$$

$|x|$ decreases as

$$x(N) = x(N_{min})e^{(-3+\frac{3}{2}\gamma_b)(N-N_{min})}. \tag{13}$$

leading to the scaling period. The transition between the initial period and the second (scaling) period is short because x decreases rapidly (for $x(N)/x(N_{min}) = 1/10$ one has $N - N_{min} \simeq 1.5$) and we get $1 \gg x \gg y$. The scaling period is defined by the validity of the equation

$$\frac{y_N}{y} = -\frac{H_N}{H}. \tag{14}$$

This period takes place when $\lambda x \ll 1$ as seen from (2). During the scaling period one has $yH = H_{min}y_{min} = cte$ which leads to a constant Hy and potential since $V = 3H^2y^2$. Therefore, λ and ϕ will also be constant during this scaling period [35], i.e.

$$\lambda(N_{min}) \simeq \lambda(N_2) \tag{15}$$

where we have defined the scale N_2 as the end of the scaling period. Neglecting the quadratic terms on x and y in the third equation of system (2) we get the expressions

$$\begin{aligned} H &= H_{min}e^{-\frac{3}{2}\gamma_b(N-N_{min})} \\ y &= y_{min}e^{\frac{3}{2}\gamma_b(N-N_{min})}. \end{aligned} \tag{16}$$

We can take in (16) $N_{min} \simeq N_i$ and $H_{min} \simeq H_i$ as discussed above, but $y_{min} \ll y_i$.

Since during the scaling period y increases as seen from (16) and λ is constant the term λy^2 in x_N will eventually dominate and lead to an increase of x. The end of the scaling period will happen when λx is again of order one and (14) is no longer valid. At this point we have $\lambda x \sim 1$ and $x \sim \lambda y^2$ which leads to an x of the same order of y, i.e. γ_ϕ will be significant larger then zero (say $\gamma_\phi \sim 0.1$). At the end of the scaling period we have $1/x_2 \sim \lambda(N_2) = \lambda(N_{min})$ and [35]

$$\Omega_\phi(N_2) = y^2(N_2) + x^2(N_2) \sim \lambda(N_{min})^{-2} \sim \Omega_{\phi i} \tag{17}$$

as seen from (9) and (15). The value of $\Omega_\phi(N_2)$ depends on the initial $\Omega_{\phi i}$ and can be much smaller than one. After the end of the scaling period $\Omega_\phi(N_2)$ grows to its present day value $\Omega_{\phi o} = 0.7 \pm 0.1$. If $\Omega_\phi(N_2) \ll 1$ then there is enough time for γ_ϕ to grow to its tracker value $\gamma_{\phi tr} = \lambda^2\Omega_\phi/n^2$. However, when $\Omega_\phi(N_2)$ is of the order 0.1 then there is not enough time to allow γ_ϕ to grow to its tracker value and one has at present day $0 < \gamma_{\phi o} \leq \gamma_{\phi tr}$. Finally, the late time behavior has $\lambda \to 0$ and $\Omega_\phi \sim y^2 \to 1$ with $\gamma_\phi \to 0$.

2.3 Parameters and Summary

There are only four independent parameters that fix the cosmological evolution of the models from its initial value to present day. These parameters are

$\Omega_{\phi i}$, Λ_c, y_{min} and the value of $\gamma_{\phi o}$ today. All other quantities can be derived from them.

Let us summarize the evolution of x and y obtained in the previous section [35]

1) Regardless of the value of x_i, y_i we have a very short period ($N_{min} - N_i \ll 1$) with increasing x and decreasing y ending with $x(N_{min})^2 \simeq \Omega_{\phi i}$ and with $y_{min} \ll 1$ model dependent.

2) Shortly afterwards the scaling period starts with $x(N_{min})^2 \gg y_{min}^2$ and $\gamma_\phi \simeq 2$. During this period x decreases while y increases and it finishes when $x \sim y \ll 1$. The size of the period $\gamma_\phi = 2$ depends on how small y_{min} is.

3) After having $x \sim y \ll 1$, we still have a decreasing x and increasing y and the period with $\gamma_\phi = 0$ (with $1 \gg y \gg x$) starts. When λy^2 becomes of order of x, x_N becomes positive and x increases until $\lambda x \sim 1$ making (14) no longer valid and ending the period with $\gamma_\phi \simeq 0$ and the scaling period at N_2. The value of λ remains (almost) constant during all the scaling period which starts at N_{min} and finishes at N_2.

4) The tracking period starts with a increasing $\gamma_\phi \to \gamma_{\phi o}$ and Ω_ϕ.

3 Late Time Phase Transition as Dark Energy

The evolution of the scalar field ϕ depends on the functional form of its potential $V(\phi)$ and a late time accelerating universe constrains the form of the potential and when it appears [18, 30]. A late time appearance of a scalar field is a signal that a phase transition took place and that the scalar field is probably not a fundamental but a composite field. Here, we will present a model where quintessence field appears as a consequence of a phase transition due to a strong gauge coupling constant. This is a very physical assumption since it only requires to have an extra gauge group to the already known gauge groups of the SM. It is well known that the gauge coupling constant of a non-abelian asymptotically free gauge group increases with decreasing energy and the free elementary fields will eventually condense due to the strong interaction, e.g. mesons and baryons in QCD. The scale where the coupling constant becomes strong is called the condensation scale Λ_c and below it there are no more free elementary fields. These condensates, e.g. "mesons", develop a non trivial potential which can be calculated using Affleck's potential [22]. The potential is of the form $V = \Lambda_c^{4+n}\phi^{-n}$, where ϕ represents the "mesons", and depending on the value of n the potential V may lead to an acceptable phenomenology. The final value of $w_{\phi o}$ (from now on the subscript "o" refers to present day quantities) depends n and the initial condition $\Omega_{\phi i}$ [18]. A $w_{\phi o} < -2/3$, which is the upper limit of [6], requires $n < 2.74$ for $\Omega_{\phi i} \geq 0.25$ [18]. For smaller $\Omega_{\phi i}$ one obtains a larger $w_{\phi o}$ for a fixe n. The position of the third CMBR peak favors models with $n < 1$ [7] and for some class of models with $V = M^{4+n}\phi^{-n}e^{\phi^\beta/2}$, with $n \geq 1, \beta \geq 0$, the constraint on the equation of state is even stricter $-1 \leq w_{\phi o} \leq -0.93$

[8]. In this kind of inverse power potential models (i.e. $n < 2$) the tracker solution is not a good approximation to the numerical solution because the scalar field has not reached its tracker value by present day.

Here we focus on a non-abelian asymptotically free gauge group whose gauge coupling constant is unified with the couplings of the standard model ("SM") ones [17, 18]. We will call this group the dark group ("DG"). The cosmological picture in this case is very pleasing. We assume gauge coupling unification at the unification scale Λ_{gut} for all gauge groups (as predicted by string theory) and then let all fields evolve. At the beginning all fields, SM and DG model, are massless and red shift as radiation until we reach the condensation scale Λ_c of DG. Below this scale the fields of the quintessence gauge group will dynamically condense and we use Affleck's potential to study its cosmological evolution. The energy density of the quintessence field Ω_ϕ drops quickly, independently of its initial conditions, and it is close to zero for a long period of time, which includes nucleosynthesis (NS) if Λ_c is larger than the NS energy Λ_{NS} (or temperature $T_{NS} = 0.1-10MeV$), and becomes relevant only until very recently. On the other hand, if $\Lambda_c < \Lambda_{NS}$ than the NS bounds on relativistic degrees of freedom must be imposed on the models. Finally, the energy density of ϕ grows and it dominates at present time the total energy density with the $\Omega_{\phi o} \simeq 0.7$ and a negative pressure $w_{\phi o} < -2/3$ leading to an accelerating universe [5].

The initial conditions at the unification scale and at the condensation scale are fixed by the number of degrees of freedom of the models given in terms of N_c, N_f. Imposing gauge coupling unification fixes N_c, N_f and we do not have any free parameters in the models (but for the susy breaking mechanism which we will comment in Sect. 2). It is surprising that such a simple model works fine. As we will see the restriction on N_c, N_f by gauge unification rules out models with a condensation energy scale between $2 \times 10^{-2}GeV < \Lambda_c < 6 \times 10^3 GeV$ or for models with $2 < n < 4.27$ (the scale Λ_c is given in terms of H_o and n by $\Lambda_c \simeq H_o^{2/(4+n)}$ [23],[18]). Since $w_{\phi o} < -2/3$ requires $n < 2.74$ all models must then have $\Lambda_c < 2 \times 10^{-2} GeV$. The number of models that satisfy gauge coupling unification with a $w_{\phi o} < -2/3$ is quite limited and in fact there are only three different models [18]. All acceptable models have $n \leq 2/3$ which implies that the condensation scale is smaller than the NS scale. The preferred model has $N_c = 3, N_f = 6$, $n = 2/3$ and it gives $w_{\phi o} = -0.90$ with an average value $w_{eff} = -0.93$ agreeing with recent CMBR analysis [6, 7].

3.1 Condensation Scale and Scalar Potential

We start be assuming that the universe has a matter content of the supersymmetric gauge groups $SU(1) \times SU(2) \times SU(3) \times SU(DG)$ where the first three are the SM gauge groups while the last one corresponds to the dark group and that the couplings are unified at Λ_{gut} with $g_1 = g_2 = g_3 = g_{DG} = g_{gut}$.

The condensation scale Λ_c of a gauge group $SU(N_c)$ with N_f (chiral + antichiral) matter fields has in $N = 1$ susy a one-loop renormalization group equation given by

$$\Lambda_c = \Lambda_{gut} e^{-\frac{8\pi^2}{b_o g_{gut}^2}} \tag{18}$$

where $b_o = 3N_c - N_f$ is the one-loop beta function and Λ_{gut}, g_{gut} are the unification energy scale and coupling constant, respectively. From gauge coupling unification we know that $\Lambda_{gut} \simeq 10^{16}$GeV and $g_{gut} \simeq \sqrt{4\pi/25.7}$ [33].

A phase transition takes place at the condensation scale Λ_c, since the elementary fields are free fields above Λ_c and condense at Λ_c. In order to study the cosmological evolution of these condensates, which we will call ϕ, we use Affleck's potential [22]. This potential is non-perturbative and exact [36].

The superpotential for a non-abelian $SU(N_c)$ gauge group with N_f (chiral + antichiral) massless matter fields is [22]

$$W = (N_c - N_f)\left(\frac{\Lambda_c^{b_o}}{det < Q\tilde{Q} >}\right)^{1/(N_c - N_f)} \tag{19}$$

where b_o is the one-loop beta function coefficient. Taking $det < Q\tilde{Q} >= \Pi_{j=1}^{N_f}\phi_j^2$ one has $W = (N_c - N_f)(\Lambda_c^{b_o}\phi^{-2N_f})^{1/(N_c-N_f)}$. The scalar potential in global supersymmetry is $V = |W_\phi|^2$, with $W_\phi = \partial W/\partial\phi$, giving [23, 24, 17, 30]

$$V = c^2 \Lambda_c^{4+n}\phi^{-n} \tag{20}$$

with $c = 2N_f$, $n = 2 + 4\frac{N_f}{N_c - N_f}$ and Λ_c is the condensation scale of the gauge group $SU(N_c)$. The natural initial value for the condensate is $\phi_i = \Lambda_c$ since it is precisely Λ_c the relevant scale of the physical process of the field binding.

In (20) we have taken ϕ canonically normalized, however the full Kahler potential K is not known and for $\phi \simeq 1$ other terms may become relevant [23] and could spoil the runaway and quintessence behavior of ϕ. Expanding the Kahler potential as a series power $K = |\phi|^2 + \Sigma_i a_i |\phi|^{2i}/2i$ the canonically normalized field ϕ' can be approximated[1] by $\phi' = (K_\phi^\phi)^{1/2}\phi$ and (20) would be given by $V = (K_\phi^\phi)^{-1}|W_\phi|^2 = (2N_f)^2\Lambda_c^{4+n}\phi^{-n}(K_\phi^\phi)^{(n/2-1)}$. For $n < 2$ the exponent term of K_ϕ^ϕ is negative so it would not spoil the runaway behavior of ϕ [17, 18].

If we wish to study models with $0 < n < 2$, which are cosmologically favored [18] we need to consider the possibility that not all N_f condensates ϕ_i become dynamical but only a fraction ν are (with $N_f \geq \nu \geq 1$) and we also need $N_f > N_c$ [17, 18]. It is important to point out that even though it has been argued that for $N_f > N_c$ there is no non-perturbative superpotential W generated [22], because the determinant of $Q\tilde{Q}$ in (19) vanishes, this is not necessarily the case [29]. If we consider the elementary quarks $Q_i^\alpha, \tilde{Q}_i^\alpha$ $(i, j = 1, 2, ..., N_f,\ \alpha = 1, 2, ..., N_c)$ to be the relevant degrees of freedom,

[1] The canonically normalized field ϕ' is defined as $\phi' = g(\phi, \bar{\phi})\phi$ with $K_\phi^\phi = (g + \phi g_\phi + \bar{\phi} g_{\bar{\phi}})^2$

then for $N_c < N_f$ the quantity $det(Q^i_\alpha \tilde{Q}^\alpha_j)$ vanishes since, being the sum of dyadics, always has zero eigenvalues. However, we are interested in studying the effective action for the "meson" fields $\phi^i_j =< Q^i_\alpha \tilde{Q}^\alpha_j >$, and the determinant of ϕ^i_j, i.e. $det < Q^i_\alpha \tilde{Q}^\alpha_j >$, being the product of expectation values does not need to vanish when $N_c < N_f$ (the expectation of a product of operators is not equal to the product of the expectations of each operator).

One can have $\nu \neq N_f$ with a gauge group with unmatching number of chiral and anti-chiral fields or if some of the chiral fields are also charged under another gauge group. In this case we have $c = 2\nu, n = 2 + 4\frac{\nu}{N_c - N_f}$ and $N_f - \nu$ condensates fixed at their v.e.v. $< Q\tilde{Q} >= \Lambda_c^2$ [17]. Another possibility is by giving a mass term to $N_f - \nu$ condensates $\varphi =< \bar{Q}_k Q_k >$, $(k = 1, ..., N_f - \nu)$ while leaving ν condensates $\phi^2 =< \bar{Q}_j Q_j >$, $(j = 1, ..., \nu)$ massless. Notice that we have chosen a different parameterization for φ and ϕ. The mass dimension for φ is 2 while for ϕ it is 1. The superpotential now reads [30]

$$W = (N_c - N_f)(\frac{\Lambda_c^{b_o}}{\phi^{2\nu}\varphi^{N_f - \nu}})^{1/(N_c - N_f)} + m\varphi \quad (21)$$

with m the mass of $\bar{Q}_k Q_k$. If we take the natural choice $\phi_i = \Lambda_c$, as discussed above, and $m = \Lambda_c$ [17] and we integrate out the condensates φ using

$$\frac{\partial W}{\partial \varphi} = \varphi^{-1}\left((\nu - N_f)\Lambda_c^{(b_o - 2\nu)/(N_c - N_f)}\varphi^{-(N_f - \nu)/(N_c - N_f)} + m\varphi\right) = 0 \quad (22)$$

we obtain $\varphi = (N_f - \nu)^{(N_c - N_f)/(N_c - \nu)}\Lambda_c^2$. By integrating out the φ field the second terms in (21), which is proportional to the first term, can be eliminated. Substituting the solution of (22) into (21) one finds

$$W = (N_c - \nu)(N_f - \nu)^{(N_f - \nu)/(N_c - \nu)}\Lambda_c^{3+a}\phi^{-a} \quad (23)$$

with $a = 2\nu/(N_c - N_f)$.

The scalar potential $V = |\partial W|^2$ is now given by [30]

$$V = c'^2 \Lambda_c^{4+n'}\phi^{-n'} \quad (24)$$

with $c'^2 = 4\nu^2(\frac{N_c - \nu}{N_c - N_f})^2(N_f - \nu)^{(N_f - \nu)/(N_c - \nu)}$ and $n' = 2 + 4\nu/(N_c - N_f)$. Notice that for $\nu = N_f$ we recover (20). From now on we will work with (24) and we will drop the quotation on n'.

The radiative corrections to the scalar potential (24) are $V \sim \Lambda_c^{4+n}\phi^{-n}(1 + O(\Lambda_c^2\phi^{-2}))$ [28]. They are not important because we have $\phi \geq \Lambda_c$ and are negligible at late times when $\phi \gg \Lambda_c$.

3.2 Gauge Unification Condition

In order to have a model with gauge coupling unification the scale Λ_c given in (20) or (24) must be identified with the energy scale in (18). However,

Table 1. We show the matter content for the three different models and we give the number of degrees of freedom for the susy and non susy Q group in the last two columns, respectively. Notice that the condensation scale and b_o is the same for all models.

Num	N_c	N_f	ν	n	b_o	$\Lambda_c(eV)$	g_{Qs}
I	3	6	1	2/3	3	42	97.5
II	6	15	3	2/3	3	42	468.5
III	7	18	4	6/11	3	42	652.5

not all values of Λ_c will give an acceptable cosmology. The correct values of Λ_c depend on the cosmological evolution of the scalar condensate ϕ which is determined by the power n in (24). The Λ_c scale can be expressed in terms of present day quantities from (20) by [18, 30]

$$\begin{aligned}\Lambda_c &= \left(3H_o^2 yo^2 \phi_o^n c^{-2}\right)^{\frac{1}{4+n}} \\ &\simeq \left(3H_o^2 \Omega_{\phi o}\right)^{\frac{1}{4+n}}\end{aligned} \tag{25}$$

where $y^2 \equiv V/3H^2 \simeq \Omega_\phi$, with $\Omega_{\phi o} = 0.7$. A rough estimate of (25) gives $\Lambda_c \simeq H_o^{2/(4+n)}$ since we also expect $\phi_o = O(1)$ [18] today (we are living at the beginning of an accelerating universe). The number of models that satisfy the unification and cosmological constraints of having $\Omega_{\phi o} = 0.7, h_o = 0.7$ (with the Hubble constant given by $H_o = 100 h_o$ km/Mpc sec) and $w_{\phi o} < -2/3$ [5] is quite limited [18]. In fact there are only three models given in Table 1. These models are obtained by equating Λ_c from (18), which is a function of N_c, N_f through b_o, and (24), which is also a function of N_c, N_f, ν through n. The exact value of y_o, ϕ_o must be determined by the cosmological evolution of ϕ (c.f. (1)) starting at Λ_c until present day. For an acceptable model the parameters N_c, N_f and ν must take integer values. We consider an acceptable model when Λ_c in (18) and (25) do not differ by more than 50%. With this assumption there are only 3 models, given in Table 1, that have (almost) integer values for N_f. In all these models one has $n \leq 2/3$ and the quantum corrections to the Kahler potential are, therefore, not dangerous. All other combinations of N_c, N_f, ν do not lead to an acceptable cosmological model. From (25) one has for $n \leq 4.27$ a scale $\Lambda_c \leq 6.5 \times 10^3 GeV$ and from (18) this implies that $b_o \leq 5.7$. Since $b_o = 3N_c - N_f = 2N_c + 4\nu/(n-2)$ and the minimum acceptable value for N_c is two one finds $b_o \geq 4 + 4\nu/(n-2)$. Taking $2 < n \leq 4.27$ gives a value of $b_o \geq 5.7$. The value of $n = 4.27$ gives the upper limiting value for which we can find a solution of (18) and (25). We see that it is not possible to have quintessence models with gauge coupling unification with $2 < n < 4.27$. In terms of the condensation scale the restriction for models with $2 \times 10^{-2} GeV < \Lambda_c < 6 \times 10^3 GeV$.

Using $n = 2 + 4\nu/(N_c - N_f)$ or equivalently $N_f = N_c + 4\nu/(n-2)$ with $b_o = 3N_c - N_f = 2N_c + 4\nu/(n-2)$ we can write from (18) as $b_o =$

$8\pi^2/g_{gut}^2(Log(\frac{\Lambda_{gut}}{\Lambda_c}))^{-1}$ and N_c [30]

$$\begin{aligned} N_c &= \frac{1}{2}b_o + \frac{2\nu}{2-n} \\ &= \frac{4\pi^2}{g_{gut}^2}(Log[\frac{\Lambda_{gut}}{\Lambda_c}])^{-1} + \frac{2\nu}{2-n} \end{aligned} \tag{26}$$

From (25) we have Λ_c as a function of n (with the approximation of $y_o^2\phi_o^n = 1$) and N_c in (26) becomes a function of n and ν only. For $2\times10^{-2}GeV < \Lambda_c < 6.5\times10^3 GeV$ we have a $N_c < 2$ and therefore are ruled out. In terms of n the condition is that models with $2 < n < 4.27$ are not viable. In deriving these conditions, we have taken $\nu = 1$ which gives the smallest constraint to N_c as seen from (26).

The upper limit $\Lambda_c > 6.5\times10^3 GeV$ has $n > 4.27$ (c.f. (25)). As mentioned in the introduction, the value of $w_{\phi o}$ depends on the initial condition $\Omega_{\phi i}$ and on n [18]. The larger n the larger $w_{\phi o}$ will be (same is true for the tracker value $w_{tr} = -2/(2+n)$). It has been shown that assuming an initial value of $\Omega_{\phi i}$ no smaller than 0.25 then the value of $w_{\phi o}$ will be less then $w_{\phi o} < -2/3$ only if $n < 2.74$ [18]. Therefore, the models with $n > 4.27$ are not phenomenological acceptable and since $4.27 > n > 2$ are also ruled out by the constraint on gauge coupling unification, we are left with models with [30]

$$\Lambda_c < 2\times10^{-2}GeV \qquad or \qquad n < 2. \tag{27}$$

So, only models with a cosmological late time phase transition are allowed.

3.3 Thermodynamics, Nucleosynthesis Bounds, and Initial Conditions

Before determining the evolution of ϕ we must analyze the initial conditions for the $SU(DG)$ gauge group. The general picture is the following: The "DG" gauge group is by hypothesis, unified with the SM gauge groups at the unification energy Λ_{gut}. For energies scales between the unification and condensation scale, i.e. $\Lambda_c < \Lambda < \Lambda_{gut}$, the elementary fields of $SU(DG)$ are massless and weakly coupled and interact with the SM only gravitationally. The DG gauge interaction becomes strong at Λ_c and condense the elementary fields leading to the potential in (24).

Since for energies above Λ_{gut} we have a single gauge group it is naturally to assume that all fields (SM and DG) are in thermal equilibrium. However, at temperatures $T < T_{gut}$ the gauge group DG is decoupled since it interacts with the SM only via gravity.

The energy density at the unification scale is given by $\rho_{Tot} = \frac{\pi^2}{30}g_{Tot}T^4$, where $g_{Tot} = \Sigma Bosons + 7/8\Sigma Fermions$ is the total number of degrees of freedom at the temperature T. The minimal models have $g_{Tot} = g_{smi} + g_{DGi}$, with $g_{smi} = 228.75$ and $g_{DGi} = (1+7/8)(2(N_c^2-1)+2N_fN_c)$ for the minimal supersymmetric standard model MSSM and for the $SU(DG)$ supersymmetric gauge group with N_c colors and N_f (chiral + antichiral) massless fields, respectively. The initial energy density at the unification scale for each group is simply given in terms of number of degrees of freedom, $\Omega = \rho/\rho_c$,

$$\Omega_{DGi}(\Lambda_{gut}) = \frac{g_{DGi}}{g_{Tot}}, \qquad \Omega_{smi}(\Lambda_{gut}) = \frac{g_{smi}}{g_{Tot}} \tag{28}$$

with $\Omega = \Omega_{DG} + \Omega_{sm} = 1$. Since the SM and DG gauge groups are decoupled below Λ_{gut}, their respective entropy, $S_k = g_k a^3 T^3$ with g_k the degrees of freedom of the k group and a the scale factor of the universe (see (2)), will be independently conserved. The total energy density ρ as a function of the photon's temperature T above Λ_c (i.e. $\Lambda_c < \Lambda < \Lambda_{gut}$), with the DG fields still massless and redshifting as radiation, is given by

$$\rho = \frac{\pi^2}{30} g_* T^4 \tag{29}$$

with

$$g_* = g_{smf} + g_{DGf}\left(\frac{T_{DG}}{T}\right)^4 = g_{smf} + g_{DGf}\left(\frac{g_{smf}\,g_{DGi}}{g_{smi}\,g_{DGf}}\right)^{4/3} \tag{30}$$

and $g_{smi}, g_{smf}, g_{DGi}, g_{DGf}$ are the initial (i.e. at decoupling) and final standard model and DG model relativistic degrees of freedom, respectively. From the entropy conservation, we know that the relative temperature between the standard model and the DG model is given by

$$\frac{T_{DG}}{T} = \left(\frac{g_{smf}\,g_{DGdec}}{g_{smdec}\,g_{DGf}}\right)^{1/3} \tag{31}$$

where g_{smdec} stands for the degrees of freedom when the DG-particle decouple from the SM. It is clear that the energy density for the DG model $\rho_{DG} = \pi^2/30 g_{DG} T_{DG}^4$ in terms of the photon's temperature T is fixed by the number of degrees of freedom,

$$\begin{aligned} \Omega_{DGf} &= \frac{g_{DGf} T_{DG}^4}{g_* T^4} \\ &= \frac{g_{DGf}(T_{DG}/T)^{4/3}}{g_{smf} + g_{DGf}(T_{DG}/T)^{4/3}}. \end{aligned} \tag{32}$$

Equation (32) permits us to determine the energy density of the DG group at any temperature above the condensation scale.

3.4 Energy Density at the Condensation Scale

We would like now to determine the energy density at the condensation scale which will set the initial energy density for the scalar composite field ϕ.

Just above the condensation scale Λ_c we take, for simplicity of argument, that all particles in the DG group are still massless and we can use (32) to determine the $\Omega_{DG}(\Lambda_c)$ with $g_{DGi} = g_{DGf}$ giving [30]

$$\Omega_{DGf} = \frac{g_{DGf}(T_{DG}/T)^{4/3}}{g_{smf} + g_{DGf}(T_{DG}/T)^{4/3}}. \tag{33}$$

If the decoupling of DG particles is above neutrino decoupling (around $1MeV$) then for temperatures below $1MeV$ one has $T_{DG}/T = (43/11/g_{smdec})^{1/3}$. At Λ_c we have a phase transition and we no longer have elementary free particles in the DG group. They are bind together through the strong gauge interaction and the acquire a non-perturbative potential and mass given by (24). In other words, below the condensation scale there are no free "quarks" DG and we have "meson" and "baryon" fields.

If we consider only the SM and the DG group, the energy density within the particles of the DG group must be conserved since they are decoupled from the SM (the interaction is by hypothesis only gravitational). All the energy density of the DG group is transmitted into dark energy (and possible dark matter, see Sect. 4) at the condensation scale Λ_c. This is a natural assumption from a particle point of view but is not crucial from a cosmological point of view, in the sense that any "reasonable" fraction of the energy density in the DG group would give a correct cosmological evolution of the ϕ field. We would like to stress out that the initial condition for ϕ is no longer a free parameter but it is given in terms of the degrees of freedom of the MSSM and the DG group.

3.5 Nucleosynthesis Constraint on the Energy Density

The big-bang nucleosynthesis (NS) bound on the energy density from non SM fields, relativistic or non-relativistic, is quite stringent $\Omega_{DG} < 0.1 - 0.2$ [31, 32].

If the DG gauge group condense at temperatures much higher than NS then, the evolution of the condensates will be given by (2) with the potential of (20) and we must check that Ω_{DG} at NS is no larger than 0.1-0.2. This will be, in general, no problem since it was shown that even for a large initial Ω_{DG} at the condensation scale the evolution of ϕ is such that Ω_{DG} decreases quite rapidly and remains small for a long period of time (see figure 2) [17, 18].

On the other hand, if the gauge group condenses after NS we must determine if the DG energy density is smaller than $\Omega_{DG} < 0.1 - 0.2$ at NS. Since the condensations scale Λ_c is smaller than the NS scale, all fields in the DG group are still massless and the energy density is given in terms of the relativistic degrees of freedom and from (32) to set a limit on g_{DGf} and g_{DGi},

$$\Delta g_{DG} \equiv g_{DGf}^{-1/3} g_{DGi}^{4/3} = \frac{\Omega_{DG}}{1 - \Omega_{DG}} g_{smf}^{-1/3} g_{smi}^{4/3} \tag{34}$$

and for $g_{DGf} = g_{DGi} = g_{DG}$

$$\Delta g_{DG} = g_{DG} = \frac{\Omega_{DG}}{1-\Omega_{DG}} g_{smf}^{-1/3} g_{smi}^{4/3} \tag{35}$$

where we should take $g_{smf} = 10.75$ at the final stage (i.e. NS scale) and $g_{smi} = 228.75$ at the initial stage (i.e. at unification) for the minimal supersymmetric standard model MSSM. For $\Omega_{DG} \leq 0.1, 0.2$ (34) gives un upper limit on the number of relativistic degrees of freedom $\Delta g_{DG} \leq 70, 158$ respectively (or $g_{DG} \leq 70, 158$ if $g_{DGf} = g_{DGi} = g_{DG}$).

The l.h.s. of (34) depends on the initial (i.e. at unification) and final (at NS) number of degrees of freedom of the gauge group DG. The smaller (larger) the initial (final) degrees of freedom of DG the smaller Δg_{DG} and Ω_{DG} will be.

4 Dark Matter

We would like to see now if the dark group responsible for giving the dark energy through its scalar condensates can at the same time account for the missing dark matter [34].

The restrictions on DM is that it must have $\Omega_{DM} = 0.25 \pm 0.1$ and it should allow for structure formation at scales larger than Mpc. As we will see later our models have a warm DM with a mass of the order of keV. There are still problems with cold and warm DM models. Cold DM have an overproduction of substructure of galactic halos which a warm DM model does not have [11]. On the other hand, recent observations on the reionization redshift [1] seem to indicate that warm dark matter is not a good candidate. However, the value of the parameters used are still not well established which makes the conclusion not definite [10]. So, we believe that further studies need to be done in order to fully set the nature of dark matter.

If we have a dark gauge group with $N_c < N_f$ then on top of the gauge singlet meson fields we can have gauge singlet dark baryons $B^{i,\dots,N_c} = \prod_i^{N_c} Q^i$ and anti–baryons. These particles get a non-vanishing mass due to non-perturbative effects (like protons and neutrons in QCD). These baryons could be degenerated in mass or there could be a lightest massive stable baryon. The order of magnitude of the mass of the DM particle can be estimated by dimensional analysis and it must be given by the condensation scale

$$m = k\,\Lambda_c \tag{36}$$

with $k = O(1)$ a constant. In this picture we have at high energies $E > \Lambda_c$ a DG with massless particles. At Λ_c non-perturbative effects, due to a strong coupling, generate a mass for dark baryons and a scalar potential for dark meson. The DM is the massive stable particle with mass given by (36) while the quintessence with potential (20) gives the DE.

Before studying the dynamics of the DG let us determine the constraint on the temperature and mass for DM in order to agree with structure formation. The relative temperature of the decoupled particle compared to the photon temperature T is given by (31). For a neutrino one has at decoupling $g_{smdec} = 11/2, g_{smf} = 2$ and with $g_{DGf} = g_{DGdec}$ one has $T_\nu = T(4/11)^{1/3} = (1/1.76)T$. However, if the decoupling is at a high energy scale, say $T \gg 10^3 GeV$, then all particles of the standard model are still relativistic and $g_{dec} = 106.75, 228.75$ for the non-susy and susy standard model respectively giving a temperature $T_D = T(43/11/g_{smdec})^{1/3}$ (below 1 MeV with $g_{smf} = 43/11$ that takes into account neutrino decoupling) with $T_D \simeq (1/3)T$ for non-susy SM and $T_D \simeq (1/3.88)T$ for the susy-SM. The temperature of DG is in these cases $3-4$ times smaller then the photon temperature and $2-3$ times smaller then T_ν. If there are more relativistic degrees of freedom coupled to the susy-SM (could be Kaluza-Klein states or other gauge groups, e.g. gauge group responsible for susy breaking [30]) at decoupling then T_D would be even smaller.

We can set an upper and lower limit to Ω_{DG}. The smallest number of degrees of freedom would be for a gauge group with $N_c = 2, N_f = 1$ giving $g_{DG} = 18.75$. While the upper limit on g_{DG} comes from Nucleosynthesis "NS" bounds which requires an upper limit to any extra energy density. This limit is $\Omega_{DG}(NS) \leq 0.1 - 0.2$ [31]. Since from (34) $g_{DG}/g_{dec}^{4/3} = (10.75)^{-1/3}\Omega_{DG}(NS)/(1-\Omega_{DG}(NS))$ the NS bound sets un upper limit $g_{DG} \leq 0.05 g_{dec}^{4/3}, 0.113 g_{dec}^{4/3}$ for $\Omega_{DG}(NS) \leq 0.1, 0.2$, respectively. Taking $g_{DG} \leq 0.113 g_{dec}^{4/3} \sim 158(g_{dec}/228.75)^{4/3}$ we obtain un upper limit $\Omega_{DGc} \leq 0.17$ at any condensation scale below $1 MeV$.

The free streaming scale λ_{fs} gives the minimum size at which perturbations survive. For scales smaller than the λ_{fs} the perturbations are wiped out. For structure formation it is required that $\lambda_{fs} \leq O(1) Mpc$. One has [12]

$$\begin{aligned} \lambda_{fs} &\simeq 0.2(\Omega_{DM}h^2)^{1/3}(1.5/g'_{DM})^{1/3}(keV/m)^{4/3} \\ &= 0.079(\Omega_{DM}h^2)^{-1}(g'_{DM}/1.5)(228.75/g_{dec})^{4/3} \end{aligned} \tag{37}$$

where $g'_{DM} = g_{bDM} + 3/4 g_{fDM}$ with g_{bDM} the bosonic, g_{bfDM} the fermion degrees of freedom of DM and we used (38) in the second equality of (37).

The energy density of the DG will be divided in DE (quintessence) and DM. For DM the entropy conservation gives $n_{DM}/n_\gamma = (g'_{DM}/2)(T_D/T)^3$ where $n_{DM}, n_\gamma = 2(\zeta(3)/\pi^2)T^3$ are the number density for DM and photon respectively. Since the energy density for matter is $\rho_m = nm$ and using $\rho_\gamma = n_\gamma(\pi^4/30\zeta(3))T$ we can write $\Omega_{DMo} = \Omega_{\gamma o}(\zeta(3)30/\pi^4)(n_{DM}/n_\gamma)(m/T_{\gamma o}) = \Omega_{\gamma o}(\zeta(3)30/\pi^4)(g'_{DM}/2)(m/T_{\gamma o})(T_D/T)^3$ giving [34]

$$\Omega_{DMo} = 0.25 \left(\frac{0.71}{h_o}\right)^2 \left(\frac{g'_{DM} m}{g_{dec} 1.66\, eV}\right) \tag{38}$$

where we have used in the last equation the present day quantities $h_o^2 \Omega_\gamma = 2.47 \times 10^{-5}$, $T_{\gamma o} = 2.37 \times 10^{-13} GeV$. Equation (38) is valid for all DM that decouples at temperature $T_i \gg 10^3 GeV$ from the susy-SM. Taking the central values of wmap [1] $\Omega_{DMo}h_o^2 = 0.135-0.0224 = 0.1126$ (where $\Omega_b h^2 = 0.0224$) one gets a neutrino mass $m = 12\, eV$ and $\lambda_{fs} = 36\, Mpc$ giving the usual hot DM problem. It cannot form structure at small scales. For a model that decouples from the susy-SM at $T \gg 10^3 GeV$ one has $T_D/T \leq 1/3.88$ with $g_{dec} \geq 228.75$, a mass $m \geq 381(254)\, eV$ for $g'_{DM} = 1(1.5)$, respectively, and (37) gives $\lambda_{fs} \simeq 0.62(0.41)\, Mpc$. Allowing for a more conservative variation of $\Omega_{DMo} = 0.25 \pm 0.1$ and $h_o = 0.7 \pm 0.05$ the constraint on $g'_{DM}\, m/g_{dec}$ from (38) is $0.83 g_{dec} eV \leq g'_{DM}\, m \leq 2.59 g_{dec} eV$. The number of degrees of freedom g'_{DM} is not arbitrary since $0.113 g_{dec}^{4/3} \geq g_{DG} > g'_{DM} \geq 1$, as discussed above. This bound implies that the mass of the DM particle must be [34]

$$1.2(228.75/g_{dec})^{1/3}\, eV \leq m \leq 593(g_{dec}/228.75)\, eV. \tag{39}$$

For $g_{dec} \leq 228.75$ one has $m \leq 593\, eV$ while for $m \geq 750\, eV, 1keV$ one requires $g_{dec} \geq 450(676), 600(901)$ for $g'_{DM} = 1(1.5)$, respectively.

If we do not want to relay on having the same initial temperature between the SM and DG we can estimate the amount of DM by the backward evolution of DM from present day to the phase transition scale Λ_c where the particles acquire a mass. The evolution of the DM is $\rho_{DMo} = \rho_{DM}(a/a_o)^3$ where $a(t)$ is the scale factor. In terms of $\Omega_{DM} = 3H^2\rho_{DM}$ (we have taken $8\pi G = 1/m_{pl}^2 = 1$) we can write the DM energy density as

$$\Omega_{DMo} = \Omega_{DMc}(\Omega_{ro}/\Omega_{rc})^{\frac{3}{4}}(H_c^2/H_o^2)^{\frac{1}{4}} \tag{40}$$

where we have expressed $a_c/a_o = (\Omega_{ro}H_o^2/\Omega_{rc}H_c^2)^{1/4}$. The evolution of the DE depends on the specific potential. However, the non-abelian gauge dynamics leads to an inverse power potential of the form [23, 17, 30]

$$V = \Lambda_c^{4+n}\phi^{-n} \tag{41}$$

where $\phi =< \bar{Q}Q >$ is the condensate of the elementary fields. Here we will treat n as a free parameter but it can be related to N_c, N_f by $n = 2 + 4\nu/(N_c - N_f)$ and ν counts the number of light condensates [17, 30]. When the kinetic term is much smaller than the potential energy one has $\Omega_{DE} \simeq \Lambda_c^{4+n}\phi^{-n}/3H^2$. This is certainly valid for present day since we require ρ_{DE} to accelerate the universe and the slow roll condition $E_k \ll V$ must be satisfied. Since the beginning of an accelerated epoch is very recently one has $\phi_o \simeq 1$ [23]. Of course, a numerical analysis must be performed [17, 30] in order to obtain the precise values of $\phi_o, w_{\phi o}$ but the analytic solution is a reasonable approximation. At the condensation scale Λ_c the initial value of the condensate ϕ_c must be giving by Λ_c and taking $\phi_c = \Lambda_c$ [17] we have

$$\Omega_{DEc} = \frac{\Lambda_c^4}{3H_c^2}, \qquad \Omega_{DEo} = \frac{\Lambda_c^{n+4}}{3H_o^2}. \tag{42}$$

Using (40) and (25) we can write [34]

$$\Omega_{DMo} = \Omega_{DMc}(\Omega_{ro}/\Omega_{rc})^{\frac{3}{4}}(\Omega_{DEo}/\Omega_{DEc})^{\frac{1}{4}}\Lambda_c^{-\frac{n}{4}} \tag{43}$$

where we have used $H_o^2/H_c^2 = (\Omega_{DEc}/\Omega_{DEo})\Lambda_c^n$. Using $\Lambda_c = (3\Omega_{DEo}H_o^2)^{\frac{1}{4+n}}$ in (43) we can determine the allowed range of values of n and Λ_c. The allowed range is quite limited. Taking the central wmap values $h_o = 0.71$, $\Omega_{DMo} = 0.25$ [1] with $g'_{DM} = 1.5$ (i.e. $g_{DM} = 7/4$) and taking as examples $g_{DG} = 97.5(160)$ we get for $g_{dec} = 228.75, 676, 901$ an inverse power $n = 0.78(0.79), 0.87(0.88), 0.90(0.91)$ and $\Lambda_c = 189(214), 559(634), 745(845)\, eV$, respectively. If we allow for a conservative variation $\Omega_{DMo} = 0.25 \pm 0.1$ and $h_o = 0.7 \pm 0.05$ and taking $g_{DGc} = 97.5$ and the upper value $g_{DGc} = 0.113 g_{dec}^{4/3}$ (results in parenthesis) then the range for n and Λ_c for $g_{dec} = 228.75, 676, 901$ is 0.34, 0.42, 0.44 (0.31, 0.29, 0.28) $\leq n \leq$ 0.87,0.96,0.98 (0.88, 1.0, 1.04) and 0.55, 1.63, 2.17 (0.34, 0.23, 0.21) $eV \leq \Lambda_c \leq$518, 1530, 2040 (585, 2484, 3645) eV, respectively, where the lower limit has $g_{DMc} = g_{DGc} - 1, h_o = 0.65, \Omega_{DMo} = 0.15$ and the upper limit has $g_{DMc} = 1, h_o = 0.75, \Omega_{DMo} = 0.35$. We see that the allowed range is [34]

$$0.28 \leq n \leq 1.04 \quad \Leftrightarrow \quad 0.21\, eV \leq \Lambda_c \leq 3645\, eV \tag{44}$$

given for $g_{dec} \leq 901$. Increasing g_{dec} would enlarge the range of n, Λ_c but not significantly. In Fig. 1 we show the behavior of Ω_{DMo} as a function of n for different values of $g_{DGc} = 0.113 g_{dec}^{4/3}$ with $g_{dec} = 228.75, 676, 901$ (solid,dashed and dotted lines, respectively) for the extreme values of g_{DMc} given by $g_{DG} - 1 \geq g_{DMc} \geq 1$. The allowed region is in between the horizontal lines $\Omega_{DMo} = 0.15 - 0.35$. From (44) we see that there is only a limited range of condensation energy scales and IPL parameter n that allows for a gauge group to give the correct DM and DE densities. It is also interesting to note that the lower limit on Λ_c is very similar to the one obtain by CMB analysis [30] where the minimum scale was $\Lambda_c = 0.2\, eV$. On the other hand, the evolution of quintessence requires for $\Omega_{DEc} < 0.17$ an IPL parameter n to be smaller than $n \leq 1.6$ for $w_{DEo} \leq -0.78$ which is the upper value of wmap. For smaller Ω_{DEc} we will need a smaller n, e.g. $\Omega_{DEc} = 0.05$ requires $n \leq 1.05$. So, once again there is a consistency within the acceptable values of n coming form different considerations (amount of DM and observable w_{DEo}). The constraint on Λ_c is very similar to the constraint obtained for the DM particle mass m obtained in (39). The similarity $m \sim \Lambda_c$ is required by non-abelian gauge dynamics and it is indeed satisfied as can be verified using (38), (25) and (43) [34]

$$k \equiv \frac{m}{\Lambda_c} = \frac{\pi^4}{\zeta(3)30} \, \frac{g_{DMc}}{g_{DEc}^{1/4} g'_{DM}} \tag{45}$$

with $\pi^4/(\zeta(3)30) \simeq 2.7$. Equation (36) should be compared with (45). There is a subtle point on the values of $g_{DMc}, g_{DEc}, g'_{DM}$. The "true" degrees of freedom of the dark matter particles (i.e. the lightest field of the dark gauge

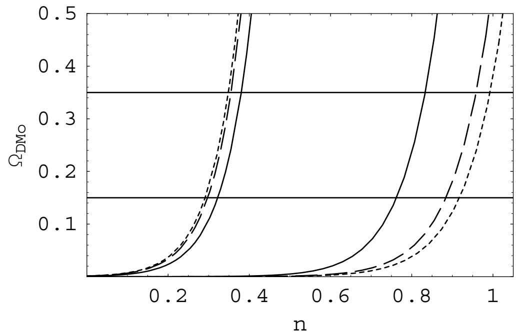

Fig. 1. We plot Ω_{DMo} as a function of the IPL parameter n. The allowed region is the one between the horizontal lines $\Omega_{DMo} = 0.15 - 0.35$ and the curves with the limiting values of $g_{DM} = 1.5$ and $g_{DG} - 1$ for $g_{dec} = 228.75, 676, 901$ (solid,dashed and dotted lines, respectively).

group) are given by g'_{DM} while g_{DMc} and g_{DEc} represent the proportion of energy density that goes into Ω_{DMc} and Ω_{DEc}. It is reasonable to assume that the particles of the dark group will decay into the lightest state. Therefore we expect $g_{DMc} > g'_{DM}$ and $m > \Lambda_c$.

5 Phenomenological Approach

The best way to determine what kind of energy is the dark energy is trough the equation of state parameter w_{DE} we and through its imprint on the CMB. This presentation is based on [35]. We will analyze the contribution to the CMB from a dark energy with a $\gamma_{DE} = w_{DE} + 1$ that takes four different values [35]. It will have a $w_{DE} = 1/3$ for energies above a certain scale Λ_c, which we will call the phase transition scale. Starting at Λ_c we will have a region with $w_{DE} = 1$ and duration ΔN_1, where N is the logarithm of the scale factor a ($N = Log[a]$). Thirdly we will have $w_{DE} = -1$ for almost the same amount of time as in the previous period, $\Delta N_2 \simeq \Delta N_1$, and finally we will end up in a region with $-1 \leq w_{DEo} = cte \leq -2/3$ for a duration of ΔN_o. The cosmological evolution and the resulting CMB will have only four new parameters $\Delta N_1, \Delta N_2, \Delta N_o$ and w_{DEo}. By varying these parameters we will cover a wide range of models. In particular we will cover all quintessence models.

The analysis of the CMB with this kind of dark energy does not depend on its nature, it could be a scalar field (quintessence) or any other form of dark energy that gives the four sectors described above. In Fig 2 we show an example with an IPL potential with $n = 1$ and $\Omega_{DEi} = 0.05$ and we see that the $w = 1/3, 1, -1, w_{tr} = cte$ approximation fits well with the numerical w_{DE}. The strategy is to analyze the spectra of CMB, using a modified version

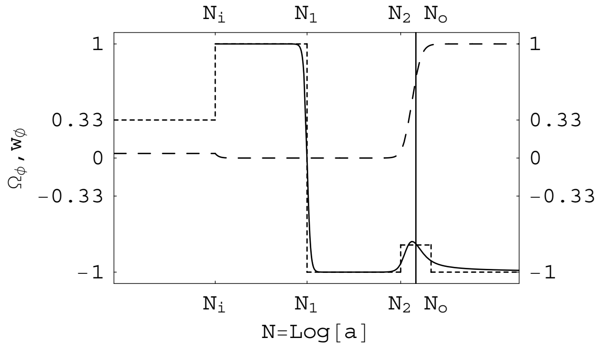

Fig. 2. We show the evolution of w_{DE} and Ω_{DE}, solid and dashed lines respectively for an IPL potential with $n = 1$ and $\Omega_{DEi} = 0.05$ as a function of $N = Log[a]$, where a is the scale factor. The dotted line represents the theoretical approximation w_{DE} and we see that it makes a good fit to the numerical solution. N_i is given at the initial scale Λ_c and N_1, N_2 give the end of the regions with $w_{DE} = 1, -1$ respectively while the solid vertical line at N_o denotes present day. Notice that for $N < N_i$ we are assuming that the energy density ρ_{DE} redshifts as radiation with $w_{DE} = 1/3$.

[13, 14] of CMBFAST [15] to include an energy density with a varying w, in a model independent way and see from its result if we can distinguish between different quintessence models, tracker, cosmological constant or other kinds of exotic energy densities.

In the case of a scalar field, we will assume that the scalar field appears at a scale Λ_c with an energy density $\Omega_{DE}(\Lambda_c)$. The late time appearance of the ϕ field suggests that a phase transition takes place creating the scalar field. We are not concern with the precise mechanism of its appearance (see [17, 30]). However, energy conservation would suggest that the energy density of the ϕ field after the phase transition would be given in terms of the energy density of the system before the phase transition and we will take them to be equal. It is natural to assume that all the energy density before the phase transition, in this sector, was relativistic. If the phase transition takes place after nucleosynthesis "NS" then the primordial creation of nuclei puts an upper limit to the relativistic energy density to be less than 0.1-0.2 of the critical energy density [31, 32]. If Λ_c is larger than the NS scale then we do not need to worry about the NS bound since independent of its initial value, Ω_{DE} will drop rapidly and remain small for a long period of time (covering NS).

In a chronological order, we would start with a universe filled with the SM particles and a DG sector (could be another gauge group) and with gravitational interaction between the two sectors only. In both sectors all fields start massless, i.e. they redshift as radiation. The evolution of the SM is the standard one and we have nothing new to say. However, the DG sector will have a phase transition at Λ_c leading to the appearance of a scalar field

ϕ with a potential $V(\phi)$, the quintessence field. Above Λ_c the fields in this sector behave as radiation. The evolution of ϕ for energies below Λ_c is that of a scalar field with given potential V. However, the precise form of V is unknown. In Table 1 we show the different model independent regions that we consider. The model dependence lies only on the size of these different periods and on the value of γ_{DE} in the last region.

From a cosmological point of view we have only 4 free parameters $\Delta N_1 = N_i - N_1, \Delta N_2 = N_1 - N_2$, $\Delta N_o = N_2 - N_o$ and γ_{DEo} (the value of γ_{DE} during the third period), where $N \equiv log[a]$ with a the scale factor. With these parameters we cover all models. The cosmological parameters can be expressed in terms of the field theoretical parameters $\Omega_{DEi}, \Lambda_c, y_{min}$ and γ_{DEo}.

Table 2. We show the different regions, its duration and the value of γ_{DE} in each region with $N = Log[a]$ and N_o is at present day.

Sector	Energy	Duration	$\gamma_{DE} = w_{DE} + 1$
Radiation	$E > \Lambda_c$	$N_i < N$	$4/3$
First	$E_1 < \Lambda_c$	$N_i < N < N_1$	2
Second	$E_2 < E_1$	$N_1 < N < N_2$	0
Third	$E_3 < E_2$	$N_o < N < N_2$	$\lambda^2 \Omega_{DE}/3$

5.1 Evolution of w

We have seen the evolution of x, y, H in the preceding subsection and we would like now to show how w_{DE} evolves in a general framework.

The evolution of the equation of state parameter, $\gamma_{DE} = 1 + w_{DE}$, as given by (4) has a generic behavior for all scalar fields independent of its potential. We see that $(\gamma_{DE})_N = 0$ has three solutions, $\gamma_{DE} = 2, 0$ and $\lambda^2 \Omega_{DE}/3$ [35] (or $w_{DE} = 1, -1$ and $\lambda^2 \Omega_{DE}/3 - 1$). The parameter γ_{DE} will be most of the time in either of the three critical points. Independent of its initial value it will go quite rapidly to $\gamma_{DE} = 2$ and remain there for a long period of time. The fast increase in γ_{DE} is because $\lambda_i \gg 1$. This stage represents a scalar field whose kinetic energy density dominates ($E_k \gg V$), it is called the kinetic region, and the energy density redshifts as $\rho_{DE} \sim a^{-6} = e^{-6N}$. Afterwards it will sharply go to $\gamma_{DE} = 0$ and stay there during almost the same amount of time as in the first stage. This second period is valid when the potential energy dominates ($E_k \ll V$) and the energy density redshifts as a cosmological constant with $\rho_{DE} \sim a^0 \sim cte$. Finally it will go to its last period given by the tracker value $\gamma_{tr} = \lambda^2 \Omega_{DE}/3$ where it will remain. This last critical value $\gamma_{tr} = \lambda^2 \Omega_{DE}/3$ is not necessarily constant (λ, Ω_{DE} evolve in time). The energy density redshifts as a tracker field $\rho_{DE} \sim a^{-3\gamma_{tr}} = e^{-3N\gamma_{tr}}$.

Let us define the quantity

$$A \equiv \lambda \sqrt{\frac{\Omega_{DE}}{3\gamma_{DE}}}. \tag{46}$$

We see from (4) that the sign of $\gamma_{\phi N}$ depends if $A > 1$ or $A < 1$. For $A > 1$ we have $\gamma_{\phi N} \geq 0$ and the value $\gamma_{\phi max} = 2$ or $w_{DE} = 1$, which is the maximum value for γ_{DE}, is a stable point. For $A < 1$ we have $\gamma_{\phi N} \leq 0$ and the value $\gamma_{DE} = w_{DE} + 1 = 0$ will be a stable point also.

We will denote the beginning of the kinetic term by N_i and the end by N_1. The second period ($\gamma_{DE} = 0$) starts at N_1 and finishes at N_2 while the last period starts at N_2 and lasts until present day N_o. We have then, $\Delta N_1 \equiv N_1 - N_i$ and $\Delta N_2 \equiv N_2 - N_1$, the amount of e-folds during the $\gamma_{DE} = 2$ and $\gamma_{DE} = 0$ periods, respectively, and $\Delta N_o \equiv N_o - N_2$ the size of the tracking period.

First Period, w=1

At the initial time since $\lambda_i \gg 1$ we have $A > \lambda\sqrt{\Omega_{DEi}/6} \gg 1$ since $\gamma_{DE} \leq 2$. From (4) we see that the derivative $(\gamma_{DE})_N \gg 1$ and γ_{DE} will rapidly go to its maximum value 2. The period of $\gamma_{DE} \simeq 2$ remains valid for a long period of time since for $x(N_{min})^2 = \Omega_{DEi} \gg y^2_{min}$ one has $\gamma_{DE}(N_{min}) = 2x(N_{min})^2/(x(N_{min})^2 + y^2_{min}) \simeq 2(1 - y^2_{min}/x(N_{min})^2) \ll 1$. So we expect γ_{DE} to be close to two until $y \sim x$. We will have at the end of the period $N = N_1$, $\gamma_{DE} \sim 2$ and $\Omega_{DE}(N_1) = r_1/(1 + r_1) \ll 1$ with

$$r_1 \equiv \frac{\rho_{DE}(N_1)}{\rho_b(N_1)} = \frac{\rho_{DE}(N_i)}{\rho_b(N_i)} \; e^{-3(N_1 - N_i)(2-\gamma_b)}. \tag{47}$$

Second Period, w=-1

The second stage starts when $1 \gg x \sim y$ and $\gamma_{DE} \sim 0$. The transition time between $\gamma_{DE} = 1.9$ and $\gamma_{DE} = 0.1$ is quite short, about $\Delta N = 1 - 1.5$, so we do not take it into account. In this second region we are still in the scaling regime with $yH = cte$ and since we have $\Omega_{DE} \ll 1$ we have $A \ll 1$ and $(\gamma_{DE})_N < 0$. The quantity γ_{DE} has been decreasing and it will arrive at its minimum possible value $\gamma_{DE} \simeq 0$ or $w_{DE} \simeq -1$. As long as $A < 1$ the value of $\gamma_{DE} \sim 0$ will remain constant.

During the second period we have, $\gamma_{DE} \sim 0, \phi \sim cte, \lambda = \lambda_{min}$ and the evolution of $\Omega_{DE}(N_2) = r_2/(1 + r_2)$ is given by

$$r_2 \equiv \frac{\rho_{DE}(N_2)}{\rho_b(N_2)} = \frac{\rho_{DE}(N_1)}{\rho_b(N_1)} \; e^{3(N_2 - N_1)\gamma_b}. \tag{48}$$

Since in this period ρ_{DE} redshifts much slower than radiation or matter, Ω_{DE} will increase and A will eventually become larger than one again. The second period ends (as the scaling period) when (14) is no longer valid and the first term in the equation y_N of (2) cannot be neglected. This happens for $x(N_2) \sim \lambda(N_2)^{-1}$ (c.f. discussion below (16)).

Third Period, w=wtr

The third period starts when γ_{DE} is not too small (i.e. x is comparable to y and $\gamma_{DE} = O(1/10)$). During this region we will have $A > 1$ and a growing γ_{DE}. However, in this case γ_{DE} will not arrive at the maximum value $\gamma_{DE} = 2$ since λ is not very large and γ_{DE} will be stabilized at the critical point $\gamma_{\phi N} \simeq 0$ with

$$\gamma_{tr} = \lambda^2 \Omega_{DE}/3. \tag{49}$$

We will have $\Omega_{DE}(N_o) = r_3/(1 + r_3)$ with

$$r_3 \equiv \frac{\rho_{DE}(N_o)}{\rho_b(N_o)} = \frac{\rho_{DE}(N_2)}{\rho_b(N_2)} \; e^{-3(N_o - N_2)(\gamma_{tr} - \gamma_b)} \tag{50}$$

If $\gamma_{tr} < \gamma_b$ then $\Omega_{DE} \rightarrow 1$. While $\lambda^2 \Omega_{DE}$ remains constant we have the constant tracker value for γ_{DE} or w_{DE}. A constant γ_{DE} is possible when $\Omega_{DE} \ll 1$. However, at late times the attractor value will be $\gamma_{tr} \rightarrow 0$ and $\Omega_{DE} \rightarrow 1$ since Ω_{DE} is constrained to be smaller than one and $\lambda \rightarrow 0$. But, even for γ_{tr} not constant the evolution of γ_{tr} in (49) is valid and the value generalizes the tracker behavior. For an inverse power law potential $V = V_o \phi^{-n}$ we have $\lambda = n/\phi$ and $\gamma_{tr} = n^2 \Omega_{DE}/3\phi^2$ which is the valued obtained by [9],[18].

5.2 Duration of the Periods and Relation to the Field Parameters

In order to know the relative size of the different periods we can use (47) and (48). Combining both (47) and (48) we have

$$\frac{r_2}{r_i} = \frac{\rho_{DE}(N_2)\rho_b(N_i)}{\rho_b(N_2)\rho_{DE}(N_i)} = e^{-3\Delta N_1(2-\gamma_b) + 3\Delta N_2 \gamma_b} \tag{51}$$

Solving for ΔN_2 in (51) we obtain [35]

$$\Delta N_2 = \Delta N_1 \left(\frac{2}{\gamma_b} - 1 \right) + \frac{1}{3\gamma_b} Log \left[\frac{r_2}{r_i} \right] \tag{52}$$

If we use the result of quintessence evolution at the beginning and end of the scaling period $\Omega_{DE}(N_2) = \Omega_{DEi}(N_i)$ given in (17) we have $r_2 = r_i$. For matter, $\gamma_b = 1$, and (52) gives $\Delta N_2 = \Delta N_1$ while for radiation, $\gamma_b = 4/3$, and $\Delta N_2 = \Delta N_1/2$. The universe has been dominated by matter for a period of $N_o - N_{rm} \simeq 8$, where N_o stands for present day value and N_{rm} for the scale at radiation-matter equivalence.

Including the third period we have from (47), (48) and (50) [35]

$$\begin{aligned} \frac{r_3}{r_i} &= \frac{\rho_{DE}(N_o)\rho_b(N_i)}{\rho_b(N_o)\rho_{DE}(N_i)} = e^{-3\Delta N_1(2-\gamma_b(N_1)) + 3\Delta N_2 \gamma_b(N_2) - 3\Delta N_o(\gamma_{tr} - 1)} \\ &= \frac{r_2}{r_i} \; e^{-3\Delta N_o(\gamma_{tr} - 1)}, \end{aligned} \tag{53}$$

$$\Delta N_o = \frac{3}{1-\gamma_{tr}} Log\left[\frac{r_3}{r_2}\right] = \frac{3}{1-\gamma_{tr}} Log\left[\frac{\Omega_{DEo}}{1-\Omega_{DEo}}\frac{1-\Omega_{DE}(N_2)}{\Omega_{DE}(N_2)}\right] \quad (54)$$

where we have assumed that the third period is already at the matter dominated epoch, $\gamma_b(N_o)=1$. If we take in (54) the equality $\Omega_{DEi}(N_i)=\Omega_{DE}(N_2)$ the size ΔN_o and the value of γ_{tr} will set the initial energy density Ω_{DEi}. Of course, on the other hand, if we know Ω_{DEi} then we can determine $\Delta N_o \gamma_{tr}$.

Let us now relate the four field parameters $\Lambda_c, \Omega_{DEi}, y_{min}, \gamma_{DEo}$ to the size of the different periods. The amount of e-folds between the initial time N_i at Λ_c and N_1, the scale where w goes from $w=1$ to $w=-1$ is set by the condition $x \sim y \ll 1$. We use the evolution of x from (16) and (13) to get

$$\Delta N_1 \equiv N_1 - N_i = \frac{1}{3} Log\left[\frac{x(N_{min})}{y_{min}}\right] \quad (55)$$

were we have assumed $N_i \simeq N_{min}$. Equation (55) is independent of γ_b. We can take $x(N_{min})=\sqrt{\Omega_{DEi}}$, $y_i \simeq \sqrt{\Omega_{DEi}}$ and for an IPL model we have $y_{min} \simeq y_i(\Lambda_c/\sqrt{\Omega_{DEi}})^{n/2}$ and (55) gives

$$\Delta N_1 = \frac{n}{6} Log\left[\frac{\sqrt{\Omega_{DEi}}}{\Lambda_c}\right]. \quad (56)$$

The amount of e-folds between the initial time N_i at Λ_c and the end of the scaling period N_2 is given by (16), (9) and (17) with $y \sim x$ but this time we have with $x = 1/\lambda(N_{min}) \sim \sqrt{\Omega_{DEi}}, y_i \sim \sqrt{\Omega_{DEi}}$ giving

$$\Delta N_1 + \Delta N_2 = N_2 - N_i = \frac{2}{3\gamma_b} Log\left[\frac{y_2}{y_{min}}\right] = \frac{n}{3\gamma_b} Log[\frac{\sqrt{\Omega_{DEi}}}{\Lambda_c}]. \quad (57)$$

Notice that (57) minus (56) gives ΔN_2 in (52). Summing (57) and (54) we have [35]

$$\Delta N_T \equiv N_o - N_i = \frac{n}{3\gamma_b} Log[\frac{\sqrt{\Omega_{DEi}}}{\Lambda_c}] + \frac{3}{1-\gamma_{tr}} Log\left[\frac{\Omega_{DEo}}{1-\Omega_{DEo}}\frac{1-\Omega_{DEi}}{\Omega_{DEi}}\right] \quad (58)$$

which gives the total scale ΔN_T between the initial time at Λ_c and present day and we taken $\Omega_{DEi}=\Omega_{DE}(N_2)$ in (58). Alternatively we can estimate the magnitude of the phase transition scale Λ_c. From $\Lambda_c \equiv V_i^{1/4} = (3\Omega_{DEi}H_i^2)^{1/4}$ and using the approximation that $\Omega_{DE} \ll 1$ during almost all the time between present day and initial time (at Λ_c) we have

$$H_i = H_o \, e^{3\gamma_b \Delta N_T/2} \quad (59)$$

giving a scale

$$\Lambda_c = (3\Omega_{DEi}H_o^2)^{1/4} \, e^{3\gamma_b \Delta N_T/4}. \quad (60)$$

The scale Λ_c increases with larger ΔN_T. From (58) and (60) we can derive the order of magnitude for Λ_c in terms of n and H_o giving $\Lambda_c \simeq H_o^{2/(4+n)}$

which is the well known result for IPL potentials [18]. If we know ΔN_T then we can determine Λ_c and the power n for IPL models. We see that the size of the different regions can be determine by the four parameters $\Lambda_c, \Omega_{DEi}, y_{min}$ and γ_{DEo}.

How long do the periods last depends on the models and by varying the size of $\Delta N_2, \Delta N_o$ and γ_{tr} we cover all models. If $\Delta N_o = 0$ and $\Delta N_2 > \Delta N_{rm} = N_o - N_{rm}$ then the model would be undistinguishable from a true cosmological constant $\gamma_{DE} = w_{DE} + 1 = 0$ since during all the matter domination era the equation of state would be $\gamma_{DE} = 0$. If we have $\Delta N_o > \Delta N_{rm}$ then the model reduces to tracker models with a constant γ_{DEo} during all the matter domination era. So, our model contains the tracker and cosmological constant as limiting cases.

More interesting is to see if we can determine the nature and scale of the dark energy. For this to happen a late time phase transition must take place such that Λ_c is at $\Delta N_T = O(\Delta N_{rm})$.

5.3 Analysis of CMB spectra

We will now analyze the generic behavior of a fluid with equation of state divided in four different regions with $w_{DE} = 1/3, 1, -1, w_{DEo}$ [35]. We will vary the sizes of the regions and we will determine the effect of having regions with $w_{DE} = 1/3$ or $w_{DE} = 1$ in contrast to a cosmological constant or a tracker field (with $-1 < w_{DEo} = cte < -2/3$). We compare the CMB spectra with the data given in RADPACK [16]. This analysis is valid for all kinds of fluids with the specific equation of state and it is also the generic behavior of scalar fields. We will compare to the model $w_{tr} = -0.82$ which was found to be a better fit to CMB than a true cosmological constant [6].

5.4 Effect of Radiation Period, w=1/3

The first section we have $w_{DE} = 1/3$ and the fluid (scalar field) redshifts as radiation. As long as the fluid has $w_{DE} = 1/3$ its energy density will remain the same compared to radiation. If during nucleosynthesis the fluid has $w_{DE} = 1/3$ then the BBNS bound requires the $\Omega_{DE}(NS) < 0.1 - 0.2$ [31, 32].

In Fig. 3 we show the different CMB for $w_{DE} = 1/3, 0, -1$ for $\Delta N_T = N_o - N_i = 9, \Delta N_2 = N_2 - N_1 = 4.5$ and $\Delta N_o = N_o - N_2 = 0$. We have chosen $\Delta N_T = 9$ because it is the smallest value satisfying the condition $\Delta N_1 = \Delta N_2, \Delta N_o = 0$ and giving the correct CMB spectrum. We have taken $w_{DE} = 1, -1$ for $N_i < N < N_1$ and $N_1 < N < N_2 = N_o$, respectively.

We see that the first and second peaks are suppressed for $w_{DE} = 1/3$ compared to $w_{DE} = -1$ while the third peak is enhanced. The positions of the first two peaks is basically the same and the position of the third peak is moved from 868 to 864 (0.4%), for $w = 1/3, -1$ respectively. For smaller N_i, i.e. more distant from present day, the effect is suppressed. It is not surprising

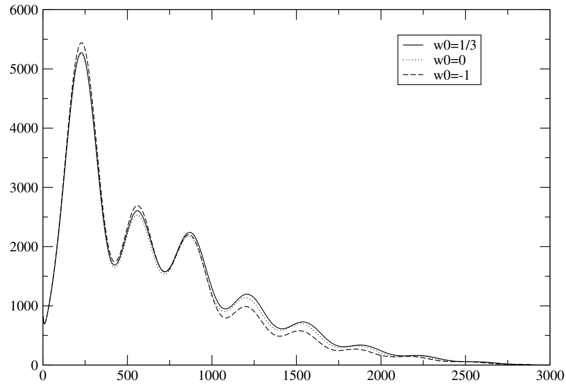

Fig. 3. We show the effect of the radiation on the CMB era for $N < N_i$ by changing $w_{DE} = 1/3, 0, -1$ with $\Delta N_T = \Delta N_1/2 \Delta N_2/2 = 9$. The vertical axes is $l(l+1)c_l/2\pi(\mu K^2)$.

since the N_i would be further way from energy-matter equality and its effect on CMB would be less important. The total χ^2 obtained by comparing the CMB spectrum with the data [16] gives $\chi^2 = 75, 74, 80$ for $w_{DE} = 1/3, 0, -1$ respectively.

5.5 Effect of First Period, w=1

In Fig. 4 we show the CMB for different values of $w_{DE} = 1, 0, -1$ during $N_i < N < N_1$ and take $w = 1/3$ for $N < N_i$ while $w_{DE} = -1$ for $N_1 < N_2 = N_o$. The effect of having a kinetic period enhances the first three peaks and shifts the spectrum to higher modes, i.e. higher l. The curve for $w_{DE} = 0$ is indi-

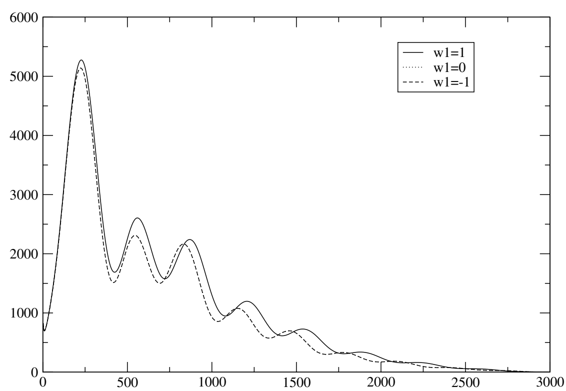

Fig. 4. We show the effect of the kinetic era with $N_i < N < N_1$ by varying $w_{DE} = 1, 0, -1$ with $\Delta N_T = \Delta N_1/2 = \Delta N_2/2 = 9$. The curves with $w_{DE} = 0$ and $w_{DE} = -1$ cannot be distinguished. The vertical axes is $l(l+1)c_l/2\pi(\mu K^2)$.

stinguishable from the $w_{DE} = -1$ one. The position and height of the peaks are $p_1 = (227, 5275), p_2 = (559, 2605), p_3 = (868, 2240)$ for $w_{DE} = 1$ while for $w_{DE} = -1$ we have $p_1 = (224, 5138), p_2 = (545, 2310), p_3 = (832, 2165)$ giving a percentage difference $p_1 = (1.3\%, 2.6\%), p_2 = (2.5\%, 12.7\%), p_3 = (4.3\%, 3.4\%)$. We see that the largest discrepancy is the altitude of the second peak. The total χ^2 gives $75, 78, 78$ for $w = 1, 0, -1$ respectively.

The difference in height and positions may in principle distinguish between a cosmological constant and a scalar field, or any fluid with the specific equation of state behavior.

5.6 Equal Length Periods

We have studied the case with $\Delta N_1 = \Delta N_2,\ \Delta N_o = 0$. In Fig. 5 we show the behavior for different values of $\Delta N_T = 2\Delta N_2 = 6, 8, 9, 12, 16$ giving a total χ^2 of $1685, 465, 75, 75, 78$, respectively.

There is a lower limit of ΔN_T that gives an acceptable CMB spectrum. The lower limit is $\Delta N_T \geq 9$. For smaller ΔN_T the peaks move to the right of the spectrum and the height increases giving a spectrum not consistent with the CMB data.

For larger $\Delta N_T > 9$ the curves tend to the cosmological constant. It is not surprising since for large $\Delta N_T = 2\Delta N_2$ it means that we have a larger time with $w = -1$ and in the case that $\Delta N_2 > N_o - N_{rm}$ the universe content, after matter radiation equality, would have been given by matter and a fluid with $w_{DE} = -1$, i.e. a cosmological constant.

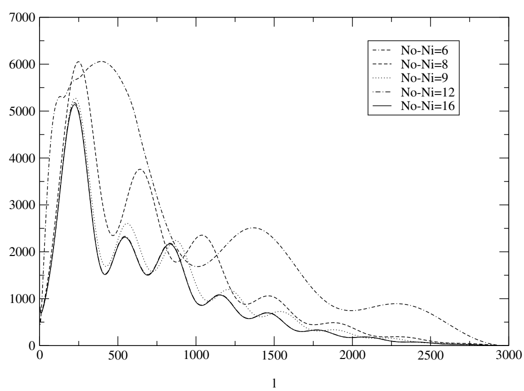

Fig. 5. We show the effect on the CMB by varying the $\Delta N_T = \Delta N_1/2 = \Delta N_2/2 = 6, 8, 9, 12, 16$ with $\Delta N_o = 0$. The curves with $\Delta N_T = 12, 16$ cannot be easily distinguished. The vertical axes is $l(l+1)c_l/2\pi(\mu K^2)$.

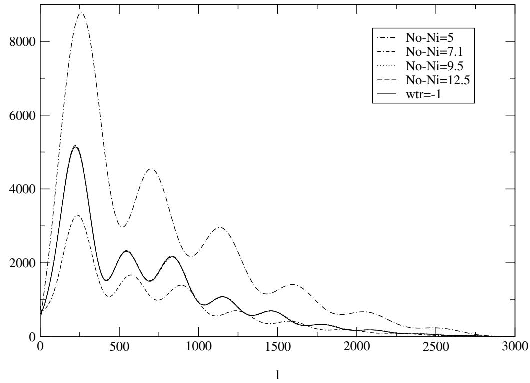

Fig. 6. We show the effect on the CMB by varying the $\Delta N_T = N_o - N_i = 5, 7.1, 9.5, 12.5$, with the constraint $\Omega_{DEi}(N_i) = \Omega_{DE}(N_2) = 0.1$ and $w_{DEo} = -1, \Delta N_2 = 1.03$ and we include the cosmological constant $w_{tr} \equiv -1$, for comparison. The curves with $\Delta N_T = 9.5, 12.5$ cannot be easily distinguished from the cosmological constant. The vertical axes is $l(l+1)c_l/2\pi(\mu K^2)$.

5.7 Scaling Condition

Following the discussion in Sect. 2.2, we now that a scalar field will end up its scaling period with a Ω_{DE} equal to its starting value, i.e. $\Omega_{DE}(N_i) = \Omega_{DE}(N_2) = 0.1$. We have taken this value of Ω_{DE} since for $N > N_i$ the energy density behaved as radiation and we have to impose the nucleosynthesis bound on relativistic degrees of freedom $\Omega_{DE}(NS) \leq 0.1 - 0.2$. Imposing this condition we have determined the evolution of the CMB for three different values of $w_{DEo} = -1, -0.82, -0.7$. We have chosen to analyze the $w_{DEo} = -0.82$ because it was found to be the best fit tracker model by [6] and we take the other two cases as the extreme ones. We have $w_{DE} = 1/3$ for $N \leq N_i$, $w = 1$ for $N_i \leq N \leq N_1$, $w_{DE} = -1$ for $N_1 \leq N \leq N_2$ and $w_{DEo} = w_{tr}$ for $N_2 \leq N \leq N_o$. The value of N_2 is determined so that the energy density grows from $\Omega_{DE}(N_2) = 0.1$ to $\Omega_{DE} = 0.7$ today. This conditions sets the range of the period to $N_o - N_2 = 1.03, 1.25, 1.47$ for $w_{DEo} = -1, -0.82, -0.7$ respectively.

In Figs. 6 and 7 we show the curves for different values of N_i with the restriction that $\Omega_{DE}(N_i) = \Omega_{DE}(N_2) = 0.1$ and for $w_{DEo} = -1, -0.82$, respectively. In the case of $w_{DEo} = -1$ we have $\Delta N_o = 1.03$ and that the smallest acceptable model has $\Delta N_T = 8.5$, $\Delta N_2 = 3.6$, see Fig. 6. The best model has $\Delta N_T = 8.88, N_1 - N_o = 3.7$ and peaks $p_1 = (224, 5133), p_2 = (549, 2363), p_3 = (840, 2178)$ with $\chi^2 = 75$. The total χ^2 obtained gives $697, 597, 77.3, 75.3$ for $\Delta N_T = 5, 7.1, 9.5, 12.5$, respectively, and $\chi^2 = 78$ for a cosmological constant.

For $w_{DEo} = -0.82$, Fig. 7 we have $\Delta N_o = N_o - N_2 = 1.25$ and the minimum acceptable distance is $\Delta N_T = N_i - N_o = 7.19, \Delta N_2 = 3.9$. Smal-

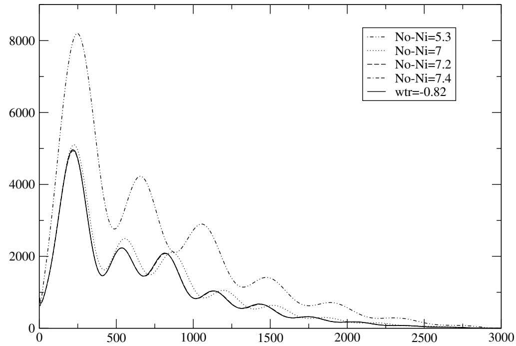

Fig. 7. We show the effect on the CMB by varying the $\Delta N_T = N_o - N_i = 5.3, 7, 7.2, 7.4$, with the constraint $\Omega_{DEi}(N_i) = \Omega_{DE}(N_2) = 0.1$ and $w_{DEo} = -0.82, \Delta N_2 = 1.25$. We also include the tracker with constant $w_{tr} \equiv -0.82$. The curves with $\Delta N_1 = 7.2, 7.4$ cannot be distinguished from the tracker one. The vertical axes is $l(l+1)c_l/2\pi(\mu K^2)$.

ler values of ΔN_T give a spectrum with peaks too large and second and third peaks moved to the right (high l modes). For large ΔN_T the spectrum tends to the tracker spectrum $w_{tr} = -0.82$. The total χ^2 obtained gives $1004, 1285, 78, 75$ for $\Delta N_T = 5.3, 7, 7.2, 7.4$, respectively, and $\chi^2 = 98$ for the tracker $w_{tr} = -0.82$.

The best model has $\Delta N_T = N_1 - N_o = 7.4, \Delta N_2 = 3$ with peaks and position $p_1 = (225, 5101), p_2 = (555, 2493), p_3 = (860, 2110)$ and it has a better fit than the tracker model with constant $w_{tr} = -0.82$ which was found to be the best tracker fit [6]. We see that having a dynamical w_{DE} is not only more reasonable from a theoretical point of view but it fits the data better.

Finally, we consider $w_{DEo} = -0.7$ for $N > N_2$. In this case we have $\Delta N_o = N_o - N_2 = 1.47$ and the minimum acceptable model has $\Delta N_T = 6.8, \Delta N_2 = 3.6$, while the best model has $\Delta N_T = 7.3, \Delta N_2 = 3.8$ with peaks $p_1 = (222, 4954), p_2 = (550, 2422), p_3 = (853, 2035)$. The total χ^2 obtained gives $711, 218, 82$ for $\Delta N_T = 5.5, 6.8, 7.3$, respectively, and $\chi^2 = 144$ for the tracker $w_{tr} = -0.7$

We see that in all three cases $w_{DEo} = -1, -0.82, -0.7$, with condition $\Omega_{DE}(N_i) = \Omega_{DE}(N_2) = 0.1$ we have a minimum acceptable value of ΔN_T and for smaller ΔN_T the peaks move to the right of the spectrum and the height of the peaks increases considerably. This conclusion is generic and sets a lower limit to ΔN_T, the distance to the phase transition scale Λ_c, or equivalently it sets a lower limit to Λ_c.

The smallest ΔN_T is set by the largest acceptable w_{DEo} (here we have taken it to be $w_{DEo} = -0.7$) giving in our case a $\Delta N_T = 6.8$ for $\Omega_{DEi} = 0.1$. This result puts a constraint on how late the phase transition can take place. In terms of the energy $\Lambda_c = \rho_{DEi}^{1/4} = [\Omega_{DEi} 3H_i^2]^{1/4}$ we can set a lower value for the transition scale. Using (60) with $\Omega_{DEi} = 0.1$ and$\Delta N_T = 6.82$ we get

$$\Lambda_c = \rho_{DEi}^{1/4} = 2 \times 10^{-10}\, GeV = 0.2\, eV \tag{61}$$

i.e. for models with a phase transition below (61) the CMB will not agree with the observations. This result is independent of the type of potential.

Furthermore, we now that for inverse power potential there is an upper limit to Λ_c coming by requiring that $w_{DEo} < -2/3$. The limiting value assuming $\Omega_{DEi} \leq 0.1$, for $V = \Lambda^{4+n}\phi^{-n}$, is $n < 1.8$ giving $\Lambda_c = 4\, MeV \simeq H_o^{2/(4+n)}$. Therefore, for IPL potentials the only acceptable models have phase transition scale

$$4\; MeV > \Lambda_c > 0.2\; eV. \tag{62}$$

6 Conclusions

We have studied the dark energy necessary for explaining the positive acceleration and flatness of the universe and structure formation. We have derived the model independent evolution of quintessence,i.e. a scalar field with only gravitational coupling with the SM particles.

We proposed a quintessence model based on a non-abelian asymptotically free gauge group. This group forms dynamically gauge invariant particles below the condensation scale (as mesons and baryons in QCD) and it is these scalar condensates that acquire a non trivial potential V and parameterize the quintessence field. We have shown that an unification scheme, where all coupling constants are unified, as predicted by string theory, leads to an acceptable dark energy parameterized in terms of the condensates of a non-abelian gauge group. Above the unification scale we have all fields in thermal equilibrium and the number of degrees of freedom for the SM and DG model determines the initial conditions for each group. Below Λ_{gut} the DG group decouples, since it interacts with the SM only through gravity. For temperatures above the condensation scale of the DG group its fields are relativistic and redshift as radiation. Below Λ_c we have the gauge invariant condensates.

We have also studied the possibility that the dark gauge group contains the dark matter and energy. The allowed values of the different parameters are severely restricted by different considerations. However, the constraints on the dark energy and dark matter overlap allowing for the possibility of having a gauge group containing both dark energy and dark matter. The NS constraint on g_{DG} sets a limit to the dark energy density at Λ_c of $\Omega_{DGc} \leq 0.17$. The evolution and acceptable values of DM and DE leads to a constraint of Λ_c and n giving $0.21\, eV \leq \Lambda_c \leq 3645\, eV$ and $0.24 \leq n \leq 0.104$ for $g_{dec} \leq 901$. The evolution of the quintessence field requires also a small n in order to have a small w_{DEo}. For $\Omega_{DE} \leq 0.17$ and $w_{DEo} \leq -0.78$ one needs $n < 1.6$. On the other hand, the analysis of the CMB spectrum sets also a lower scale for the condensation scale $\Lambda_c > 0.2\; eV$ with $n > 0.27$. So, from three different analysis (quintessence, dark matter and CMB spectrum) we are led to conclude that the most acceptable models have a low condensation scale Λ_c of the order of $1 - 10^3\, eV$. The fact that the condensation is low explains why the acceleration of the universe is at such a late time.

Finally, we have analyzed the CMB spectra for a fluid with an equation of state that takes different values. The values are $w_{DE} = 1/3, 1, -1, w_{DEo}$ for N in the regions $N_i - N_{Planck}, N_1 - N_i, N_2 - N_1, N_o - N_2$, respectively. The results are independent of the type of fluid we have. The cosmological constant and the tracker models are special cases of our general set up.

We have shown that the evolution of a scalar field, for any potential that leads to an accelerating universe at late times, has exactly the kind of behavior described above. It starts at the condensation scale Λ_c and enters a period with $w_{DE} = 1$, then it undergoes a period with $w_{DE} = -1$ and finally ends up in a region with $-1 \leq w_{DEo} \leq -2/3$. We have shown that the energy density at the end of the scaling period (end of $w_{DE} = -1$ region) has the same energy ratio as in the beginning, i.e. $\Omega_{DE}(N_i) = \Omega_{DE}(N_2)$. The time it spends on the last region depends on the value of $\Omega_{DE}(N_2)$ and on w_{DEo} during this time. Before the phase transition scale Λ_c we are assuming that all particles were at thermal equilibrium and massless in the quintessence sector. At the phase transition scale Λ_c the particles acquire a mass and a non trivial potential.

We have shown that models with $w_{DE} = 1/3, 1, -1, w_{DEo}$ have a better fit to the data then tracker or cosmological constant. Furthermore, we have determined the effect on the CMB of the first two periods $w_{DE} = 1/3$ and $w_{DE} = 1$ compared to a cosmological constant and even though the effect is small it is nonetheless observable.

In general, the CMB spectrum sets a lower limit to ΔN_T, which implies a lower limit to the phase transition scale Λ_c. For smaller ΔN_T the CMB peaks are moved to the right of the spectrum and the height increases considerably. For any Ω_{DEi} the CMB sets a lower limit to the phase transition scale. In the case of $\Omega_{DEi}(N_i) = 0.1$ the limit is $\Lambda_c = 0.2eV$ for any scalar potential. We do not take Ω_{DEi} much larger because we should comply with the NS bound on relativistic degrees of freedom $\Omega_{DEi} \leq 0.1 - 0.2$. If we take $\Omega_{DEi} \ll 0.1$ then the constraint on the phase transition scale will be less stringent since the effect of the scalar field is only relevant recently ($\Omega_{DE} \ll 1$ during all the time before present time). For inverse power law potentials we can also set an upper limit to Λ_c and for $\Omega_{DEi} \leq 0.1$ it gives an inverse power $n \leq 1.8$ and $\Lambda_c \leq 4 \times\ MeV$. In this class of potentials only models with $4\, MeV > \Lambda_c > 0.2\, eV$ would give the correct w_{DEo} and CMB spectrum.

This work allows for the possibility of distinguishing the kind of physical process that gives rise to the dark energy.

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