REFLECTION OF ELECTROMAGNETIC WAVES FROM MOVING SURFACES

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1. The reflection of an electromagnetic wave from a moving surface involves a double Doppler effect: first, the frequency of the signal is changed when the latter reaches the reflecting surface, and then again it occurs on reflection. The amplitude of the signal is changed simultaneously with changing frequency. Consequently this effect may be used for the development of new methods of generation of microwaves and their frequency multiplication. It should be noted that in fact we are concerned here with a conversion of the known Fermi effect: there, matter gets energy from a moving field; here, the field gets energy from matter.

Since the amplitude-and-frequency multiplication factor for a single reflection is

$$\delta = \frac{1+v/v_p}{1-v/v_p}$$

the single reflection may lead to considerable effects (at the small speeds of reflecting surfaces) only in the case when these speeds are close to the phase velocity of a wave in a given medium. This circumstance was first pointed out by Ya. B. Fajnberg; for the case when the reflecting matter is plasma moving along a magnetic field in a medium where the phase velocity of the wave is $v_p < c$, investigations were carried out by Ya. B. Fajnberg and V. S. Tkalich [2]. The case without a magnetic field was investigated by M. Lampert [3]. Recently this effect was detected experimentally [4].

The effects of multiplying the frequency and the amplitude of the wave may be increased considerably by means of multiple reflection. This possibility was pointed out by Ya. B. Fajnberg and by J. G. Linhart [5].

The present paper is concerned with the quantitative analysis of the effects relative to multiple reflection.

2. As a model of an arrangement for multiple reflection of fields, we consider two plasma semi-spaces, moving towards each other at a constant speed v in a medium with a given ε and μ. At the initial time $t = 0$, the semi-spaces are at a distance $2a$ from each other, and the fields between them are characterized by the functions $E(x)$ and $H(x)$ $|x| \leq a$; we consider the case of normal incidence).

As the fields $E(x)$ and $H(x)$ are superpositions of the waves travelling in both directions, it is evident that, in the course of time, each of the waves being reflected successively from each surface will increase its frequency and amplitude.

The problem of the determination of the field resultants $\mathcal{E}\ (x, t)$ and $\mathfrak{H}\ (x, t)$ in the space between the moving surfaces at any time ($|x| \leq a - vt$); $0 \leq t \leq a/v$) is reduced to finding the solutions of the Maxwell equations

$$\frac{\partial \mathcal{E}}{\partial x} = -\mu/c \frac{\partial \mathfrak{H}}{\partial t}; \quad -\frac{\partial \mathfrak{H}}{\partial x} = \varepsilon/c \frac{\partial \mathcal{E}}{\partial t} \tag{1}$$

which at $t = 0$ have the known values:

$$\mathcal{E}(x,0) = 2E(x)\,; \ \mathfrak{H}(x,0) = 2(\varepsilon/\mu)^{\frac{1}{2}} \cdot H(x) \tag{2}$$

Two cases are possible.

(i). $v \geq vp$. In this case the fields between the surfaces are completely determined by Equation (1) and the conditions (2):

$$\mathcal{E}(x,t) = E(z_+) + E(z_-) - H(z_+) + H(z_-)$$

$$\mathfrak{H}(x,t) = \{H(z_+) + H(z_-) - E(z_+) + E(z_-)\}(\varepsilon/\mu)^{\frac{1}{2}} \tag{3}$$

$$z_{\pm} = x \pm \bar{v}_p t$$

(ii). $v < v_p$. In this case, which is more interesting than the previous one, the formulae (3) do not give a solution, as $z_{\pm}$ can have, for $v_p > v$, values greater than a, while E and H are known only for $|z_{\pm}| \leq a$. Substituting $t = a/v$ for $z_{\pm}$, we obtain as the region of dependence for the given process $|x| \leq av_p/v$. Consequently, for the determination of the fields $\mathcal{E}(x, t)$ and $\mathcal{H}(x, t)$ we must know $E(x)$ in the range $|x| \leq av_p/v$.

At $a \leq |x| \leq av_p/v$ the missing values $E(x)$ and $H(x)$ are found to be automatically derived from $E(x)$ and $H(x)$ at $|x| \leq a$ when solution (3) is required to satisfy the boundary conditions. These boundary conditions are derived from the first equation of the system 1:

$$\left| \mathcal{E}(x,t) \pm \mu \cdot v/c \cdot \mathcal{H}(x,t) \right|_{x = \pm(a - vt)} = 0 \qquad (4)$$

(the limiting surfaces are assumed to have an infinite conductivity).

Thus, by subjecting the solutions (3) to the boundary conditions (4), if $v < v_p$ we obtain

$$\begin{aligned} E[\pm|x|] &= (-1)^n \delta^n E[\pm(-1)^n(a - \Delta\delta^n)] \\ H[\pm|x|] &= \delta^n \cdot H[\pm(-1)^n(a - \Delta\delta^n)] \end{aligned} \qquad (5)$$

where $\Delta = |x| - x_n$; $x_n = x_{n-1} + 2a\delta^{-n}$; $x_0 = a$ and n is the maximum number for which $\Delta > 0$. Physically n corresponds to the number of reflections of the field which was at the point x at the moment t, starting from $t = 0$.

Therefore, the formulae (3) and (5) permit one to determine the fields at any point between the surfaces at any moment, that is they solve the given problem.

If $v = 0$, the problem is reduced to the classical Cauchy problem for the one-dimensional wave equation; the formulae (3) are turned into the d'Alembert formulae, and the formulae (5) are turned into the corresponding formulae of the periodic continuation of the functions E and H for the whole axis $-\infty < x < \infty$ with the period $\mathfrak{D} = 4a$.

3. The main conclusions, derived from this analysis are as follows:

(i) As the distances where the fields are substantially changed are reduced in the course of time, the characteristic wave numbers, as well as the frequencies of the contracted field, are increased according to Eqs. (5) and (3). If, for example, at $t = 0$ the wave was monochromatic with the frequency ω_0, then its frequency after m reflections will be $\omega_0 \delta^m$.

(ii) In the course of time, the amplitudes of the field between the surfaces are also being increased, Eq. (5). This is due partly to the decrease of the volume of the field and partly to the action of the outside forces driving the surfaces, which counteract the pressure of the compressed field.

Under real conditions, plasma layers having a thickness greater than a skin-depth may act as surfaces. As a result of the variation of the reflection coefficient, the increase of the amplitude of the field will take place only until the frequencies become critical[2, 3]:

$$\omega_c \sim (4\pi n_0 e^2 / m\delta\varepsilon)^{\frac{1}{2}}.$$

Since the change of the frequency of the signal and the variation of the reflection coefficient are known, it is easy to trace the course of the compression for any values of the parameters δ, ω_0, n_0, a.

In conclusion I want to express my acknowledgments to Ya. B. Fajnberg for proposing and continuously stimulating this work and to G. J. Lyubarskij for several valuable discussions.

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