$Sp(2)$-Symmetric Realization of the Ghost Spectrum in Gauge Theories

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Abstract

$Sp(2)$-covariant BRST-approach to general closed gauge algebra theories is developed. Ordinary and supersymmetric Yang-Mills theories, quantum gravity and Chapline-Manton model are considered as examples. In the first three cases Landau gauge properties are analysed. It is shown that conformal Ward identities for Yang-Mills theory are essentially simplified in $Sp(2)$-symmetric gauges.

BRST-symmetry produced powerful method of the covariant quantization of the dynamical systems with constraints (see reviews [1,2]). Usually this approach considers ghost and anti-ghost fields in the asymmetrical manner. In some gauges, however, these fields turn out to be on equal rights and form doublet of the $Sp(2, R)$-group [3-5]. Then BRST and anti-BRST charges also form doublet of this group. Such situation holds e.g. for the Landau gauge condition. This paper is devoted to the $Sp(2)$-symmetric realization of the ghost spectrum for the general closed gauge algebras. Applications to the (super-) Yang-Mills theory, quantum gravity, Chapline-Manton model are considered.

Let's denote gauge fields φ_p, generators of gauge transformations $R^p{}_a(\varphi)$; $R^p{}_a \delta S_{\rm cl}/\delta\varphi_p = 0$, $S_{\rm cl}$–classical action. For the sake of simplicity we assume that φ_p are bosonic fields and R^p_a are not degenerated (no "ghosts for ghosts"). The closure of the gauge algebra is expressed by the condition

$$R^q{}_a R^p{}_{b,q} - R^q{}_b R^p{}_{a,q} = f^c{}_{ab} R^p{}_c\,,$$

where $R_{,q} = \delta R/\delta\varphi_q$. In general case Jacobi identities look like

$$f^d{}_{ae} f^e{}_{bc} + R^p{}_a f^d{}_{bc,p} + (abc\,\mathrm{perm}) = 0.$$

The ghost and anti-ghost fields will be denoted $c^a_i(x)$, where $i = 1,2$-index of $Sp(2)$. Basic requirement for the extended BRST-transformations $s_i = i[Q_i, \ldots]_\pm$ (Q_i–BRST charges) is [1]

$$s_i s_k + s_k s_i = 0 \tag{1}$$

Then we easily find

$$s_i\varphi_p = R^p{}_a c^a_i, \qquad s_i c^a_k = \epsilon_{ik} B^a - \tfrac{1}{2} f^a{}_{bc} c^b_i c^c_k, \qquad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{2}$$

$$s_i B^a = \tfrac{1}{2} f^a{}_{bc} B^b c^c_i - \frac{1}{12}(f^a{}_{be} f^b{}_{dc} + 2 f^a{}_{dc,p} R^p{}_e) c^d_i c^c_n \epsilon_{nm} c^e_m.$$

The field B^a is auxiliary and it will be used as Lagrange multiplier for gauge condition. Due to (1) there is only $s_i\epsilon_{ik}s_k$-variations of the fields

$$\begin{aligned} s\epsilon s\varphi_p &= 2B^a R^p{}_a + R^q{}_a R^p{}_{b,q} c^a \epsilon c^b, \\ s\epsilon s c_i^a &= -4 s_i B^a, \qquad s\epsilon s B^q = 0. \end{aligned} \tag{3}$$

Gauge fixing and ghost parts of the action can be written as $S_{qu} = s\epsilon s\, f(\varphi, c, B)$. Landau gauge condition corresponds to $f \sim \varphi_p^2$ and for ordinary Yang-Mills fields we have

$$\begin{aligned} S_{\rm tot} &= S_{\rm cl} + S_{\rm qu}, \qquad S_{\rm cl} = -\frac{1}{4g^2}F_{\mu\nu}^2, \\ S_{\rm qu} &= \frac{i}{4} s\epsilon s \int d^4x A_\mu^a A_\mu^a = i\int d^4x (A_\mu^a \partial_\mu B^a + \tfrac{1}{2}\partial_\mu c^a \epsilon D_\mu^{ab} c^b), \end{aligned} \tag{4}$$

with the hermiticity assignment: $A_\mu^+ = A_\mu, c_i^+ = c_i, B^+ = -B$. Action $S_{\rm tot}$ is invariant under the following global transformations [3,6]

1. gauge transformation (generators G^a)

$$\varphi^a \to \varphi^a + f^{abc}\theta^b\varphi^c, \qquad \varphi = (A_\mu, c_i, B) \tag{5}$$

2. $Sp(2)$-rotations (generators J^α)

$$c_i \to c_i + \tfrac{1}{2}\theta_\alpha \sigma_{ik}^\alpha c_k, \qquad A_\mu \to A_\mu, \qquad B \to B, \tag{6}$$

for the simplicity we put σ^α equal to Pauli matrices

3. extended BRST-transformations (2) (generators Q_i)
4. shifts of the auxiliary field (generators P^a)

$$B^a \to B^a + \beta^a, \qquad \beta^a = \text{ const}, \tag{7}$$

5. shifts of the ghost and auxiliary fields (generators $S^a{}_i$)

$$\begin{aligned} c_i^a &\to c_i^a + \theta^a{}_i, \quad \theta^a{}_i = \text{ const}, \\ B^a &\to B^a + \tfrac{1}{2} f^{abc} c^b \epsilon \theta^c. \end{aligned} \tag{8}$$

One can verify that symmetries (5)-(8) uniquely pick out the Landau gauge from the renormalizable covariant gauges.

Ward identities for the effective action Γ coming from (8) are found in [6] and look like

$$\int d^4x \frac{\delta}{\delta c_i(x)}(\Gamma - S_{\rm tot}) = 0, \tag{9}$$

i.e. ghost fields enter Γ via combination $\partial_\mu c_i$ beginning from 1-loop order. That is why in the Landau gauge ghost-ghost-gluon vertex need no renormalization.

Commutation relations of the conserved charges $G^a, J^\alpha, Q^i, P^a, S^a{}_i$ are

$$\begin{aligned} [G^a, G^b] &= if^{abc}G^c, & [J^\alpha, J^\beta] &= \epsilon^{\alpha\beta\gamma}J^\gamma, \\ [G^a, P^b] &= if^{abc}P^c, & [J^\alpha, S^a{}_i] &= \tfrac{1}{2}\sigma_{ik}^\alpha S^a{}_k, \\ [G^a, S^b{}_i] &= if^{abc}S^c{}_i, & [J^\alpha, Q_i] &= \tfrac{1}{2}\sigma_{ik}^\alpha Q_k, \\ [G^a, Q_i] &= 0, & [J^\alpha, G^a] &= [J^\alpha, P^a] = 0, \end{aligned} \tag{10}$$

$$[P^a, P^b] = [P^a, S^b{}_i] = \{Q_i, Q_k\} = 0, \tag{11}$$

$$\{S^a{}_i, S^b{}_k\} = i\epsilon_{ik} f^{abc} P^c, \quad [P^a, Q_i] = iS^a{}_i, \tag{12}$$

$$\{Q_i, S^a{}_k\} = i\epsilon_{ik} G^a. \tag{13}$$

This superalgebra will be denoted **sym**. All the surface terms $\sim \int d^3x \partial_\alpha f^\alpha(\varphi)$ in (10)-(13) are dropped and their relevance will be discussed elsewhere. What is the structure of **sym**? First of all there is nilpotent subalgebra of shift symmetries generated by P^a and $S^a{}_i$. Denote it **sh**. It is easy to see that $sp(2)$ subalgebra enters **sym** via the semidirect sum with ideal $I_1 = (G^a, Q_i, P^a, S^a{}_i)$. I_1 is also splitted into the semidirect sum of subalgebra of BRST-charges $\underline{q}$ and $I_2 = (G^a, P^a, S^a{}_i)$. Therefore we have

$$\mathbf{sym} = sp(2) \Subset I_1, \quad I_1 = \underline{q} \Subset I_2, \quad I_2 = \underline{g} \Subset \mathbf{sh}, \tag{14}$$

where $\underline{g}$ is gauge subalgebra. It should be noted that due to (13) **sh** isn't an ideal of **sym** in contrast with I_1 and I_2. Untrivial unification of BRST and gauge algebras (14) allows to suppose that there must be geometrical interpretation of the ghost fields different from that based on supergroup $OSp(4,2)$, which unifies Lorentz group with $Sp(2)$ [4,7].

Physical states are defined by the condition [8,9]

$$Q_i \,|\mathrm{phys}\rangle = 0, \quad \langle \mathrm{phys}\,|\mathrm{phys}\rangle \neq 0. \tag{15}$$

If all charges are well defined then from (13) we have

$$i\epsilon_{ik} G^a \,|\mathrm{phys}\rangle = Q_i S^a{}_k \,|\mathrm{phys}\rangle = 0, \quad \|\, Q_i \,|\psi\rangle \,\| = \langle \psi |\, Q_i^2 \,|\psi\rangle = 0. \tag{16}$$

This is Kogu-Ojima confinement criteria [8], but we cannot say that it is really justified. The fact is that symmetries from **sh** are spontaneously broken in the infinite space volume and charges P^a and $S^a{}_i$ are defined badly. Goldstone modes appear in the fields B, c_i and in the longitudinal part of gauge field. For QED these modes coincide with the fields. For example, the same phenomena occurs with a free scalar massless field. If we neglect **sh** then superalgebra of symmetries is reduced to $\underline{g} \oplus (sp(2) \Subset q)$, which is much less interesting than (14).

$Sp(2)$-symmetric treatment of ghost fields has interesting conformal properties. Infinitesimal dilatational and special conformal transformations are

$$[iD, \varphi] = (d_\varphi + x_\mu \partial_\mu)\varphi, \tag{17}$$

$$[iK_\lambda, \varphi] = (x^2 \partial_\lambda - 2x_\lambda x_\nu \partial_\nu - 2d_\varphi x_\lambda)\varphi + 2x_\mu A_\lambda - 2g_{\mu\lambda} x_\nu A_\nu, \tag{18}$$

where d_φ-canonical dimensions of the fields, $d_{A_\mu} = d_{c_i} = 1$, $d_B = 2$. The last term in (18) is present only for A_μ. If we neglect conformal anomaly then action (4) is invariant under (17) and under (18) transforms as follows

$$\delta S_{\mathrm{tot}} = [ia_\lambda K_\lambda, S_{\mathrm{tot}}] = -4ia_\lambda \int d^4x B^a A^a_\lambda = -2ia_\lambda \int d^4x s\epsilon(cA_\lambda). \tag{19}$$

Two remarks are in order. First, due to the condition (15) physical states will not suffer from conformal symmetry breaking (19). Second, ghost part of the action appeared to be conformally invariant and non-invariant gauge fixing part produces only term bilinear in fields. In the gauges without $Sp(2)$-symmetry there will appear interaction term and conformal Ward identities will be essentially complicated. In our case term (19) can easily be taken into account by the redefinition of the gauge field propagator

$$\langle T\, A_\mu^a(x) A_\nu^b(y)\rangle = \frac{\delta^{ab}}{i(2\pi)^4}\int \frac{d^4p\, e^{-ip(x-y)}}{p^2+i0}\left\{g_{\mu\nu} - \frac{p_\mu p_\nu + 4i(a_\mu p_\nu - a_\nu p_\mu)}{p^2+i0}\right\}. \tag{20}$$

Therefore effective action calculated with (20) will not depend on a_μ after the conformal transformation of the effective fields (if conformal anomaly is absent). In general α-gauge, where ghosts $\bar{c}^a$ and c^a are inequivalent, conformal non-invariance can be compensated by non-local BRST-transformation only if $d_{\bar{c}} = 2$, $d_c = 0$ [10]. Here we have shown that when $\alpha = 0$ one can choose another prescription $d_{\bar{c}} = d_c = 1$. It can be checked that (19) is compensated by nonlocal extended BRST-transformations with parameters $\epsilon_i \sim a_\mu \int d^4x\, c_i A_\mu$.

Let's consider $N = 1$ supersymmetric Yang-Mills theory in the superfield formulation. Ghost part of the action in the Landau gauge is similar to (4)

$$S_{\rm qu} = \frac{1}{4} s\epsilon s \int d^8z V^2(z) = \int d^8z \left\{(B-\overline{B})V + \tfrac{1}{2}(c-\bar{c})L_{\nu/2}\left(c+\bar{c}+{\rm cth}L_{\nu/2}(c-\bar{c})\right)\right\}, \tag{21}$$

where $z = (x,\theta,\bar{\theta})$, $L_A B = [A,B]$, V-vector superfield, c_i and B-chiral superfields. Additional shift symmetry analogous to (8)

$$\begin{array}{lll} c_i \to c_i + \theta_i, & B \to B + \frac{1}{2}\epsilon_{ik}[c_i,\theta_k], & \theta_i{}^+ = \theta_i, \\ \bar{c}_i \to \bar{c}_i + \theta_i, & \overline{B} \to \overline{B} + \frac{1}{2}\epsilon_{ik}[\bar{c}_i,\theta_k] & \end{array} \tag{22}$$

gives the following Ward identity [6]

$$\int d^4x d^2\theta \frac{\delta(\Gamma - S)}{\delta c_i} + \int d^4x d^2\bar{\theta}\frac{\delta(\Gamma - S)}{\delta \bar{c}_i} = 0 \tag{23}$$

As a result of (23) superfield ghost-ghost-gluon vertex should not be renormalized in parallel with the ordinary Yang-Mills case. Analogous result for the lowest orders of perturbation series was obtained earlier in [11] by means of the explicit resolution of BRST and anti-BRST Slavnov-Taylor identities.

In quantum gravity Landau gauge corresponds to the harmonic gauge

$$S_{\rm qu} = \frac{1}{4} s\epsilon s \int d^4x \eta_{\mu\nu}\tilde{g}^{\mu\nu} = \int d^4x \tilde{g}^{\mu\nu}(\partial_\mu B_\nu + \tfrac{1}{2}\partial_\mu c^\lambda \epsilon \partial_\nu c_\lambda), \tag{24}$$

where surface terms are omitted, $\tilde{g}^{\mu\nu} = \sqrt{-g}\, g^{\mu\nu}$, $\eta_{\mu\nu}$-flat metric. In Ref.[12] global symmetries of the harmonic gauge were analysed but not in a $Sp(2)$-symmetric manner. The use of c^μ and $\bar{c}_\mu$ ghosts with contravariant and covariant indices allowed to preserve there $GL(4)$ invariance. Here c_i^μ and B^μ have flat indices $c_{\mu i} = \eta_{\mu\nu}c_i^\nu$, $B_\mu = \eta_{\mu\nu}B^\nu$ and $GL(4)$ is manifestly broken by $S_{\rm qu}$ to Lorentz-invariance.

Action (24) is invariant under the various $c_{\mu i}$ and B_μ constant shift symmetries. The consequences of the corresponding Ward identities were formulated in [13] before the explicit description of thesesymmetries in [12].

Next example concerns Chapline-Manton model [14]. Classical action is invariant under the mixed abelian and non-abelian gauge transformations

$$\delta A_\mu^a = D_\mu^{ab}\theta^b(x), \quad \delta B_{\mu\nu} = \partial_{[\mu}\theta_{\nu]}(x) + \theta^a(x)\partial_{[\mu}A_{\nu]}^a, \tag{25}$$

where $B_{\mu\nu}$ is skew tensor entering supergravity multiplet. In this case gauge generators $R^p{}_a$ are degenerated and "ghosts for ghosts" appear. Correct ghost spectrum was found in [15]. Let's represent it in a more compact $Sp(2)$-way. For the fields A_μ all looks standartly (2), for $B_{\mu\nu}$ we need ghosts $c_{\mu i}$ and auxiliary fields $B_{\mu i}$, $M_{ik} = M_{ki}$ (bosonic) and N_i (fermionic)

$$\begin{aligned}
S_i B_{\mu\nu} &= \partial_{[\mu}c_{\nu i]} + c_i^a\partial_{[\mu}A_{\nu]}^a, \\
S_i c_{\mu\kappa} &= \epsilon_{ik}B_\mu + \partial_\mu M_{ik} - \tfrac{1}{2}f^{abc}A_\mu^a c_i^b c_k^c, \\
S_i B_\mu &= \partial_\mu N_i + \tfrac{1}{2}f^{abc}A_\mu^a\left(B^b c_i^c - \frac{1}{6}f^{bde}c_i^d c_k^e\epsilon_{kn}c_n^c\right) \\
&\quad + \frac{1}{6}f^{abc}c_i^a\left(D_\mu^{bd}c_k^d\right)\epsilon_{kn}c_n^c, \\
S_i M_{kj} &= -\epsilon_{ik}N_j - \epsilon_{ij}N_k + \frac{1}{6}f^{abc}c_i^a c_k^b c_j^c, \\
S_i N_k &= \frac{1}{6}f^{abc}B^a c_i^b c_k^c.
\end{aligned} \tag{26}$$

The simplest $Sp(2)$-symmetric not degenerated action S_{qu} is rather cumbersome and we omit it.

Instead of the Lagrangian formalism used hitherto one may work within the Hamiltonian BRST-approach [2,16]. In the latter main problem lies in the construction of the conserved nilpotent BRST-charge. Let's look for the $Sp(2)$-covariant case. Let ψ_a be bosonic constraints of the first class $i[\psi_a, \psi_b] = U^c{}_{ab}\psi_c$, v^a-Lagrange multipliers for the constraints and $\mathcal{P}^a$-corresponding conjugated momenta, $[v^a, \mathcal{P}^b] = i\delta^{ab}$. Ghost fields and their canonical momenta are doublets c_i^a and π^{ai}, $\{c_i^a, \pi^{bk}\} = -i\delta^{ab}\delta_i^k$. If structure coefficients $U^a{}_{bc}$ are independent of fields (first range theories) then extended BRST-charges are written in such a way

$$\begin{aligned}
Q_i &= \psi_a c_i^a - \tfrac{1}{2}\pi_a^n U^a{}_{bc}c_n^b c_i^c + \epsilon_{in}\pi_a^n\mathcal{P}^a \\
&\quad - \tfrac{1}{2}v_a U^a{}_{bc}\left(\mathcal{P}^b c_i^c - \frac{1}{6}U^b{}_{de}c_i^d c_n^e\epsilon_{nm}c_m^c\right).
\end{aligned} \tag{27}$$

They satisfy basic relation $\{Q_i, Q_k\} = 0$. Generalization to the higher rang theories also can be done but then Q_i involve much more complicated structures.

To conclude, in this paper $Sp(2,R)$-covariant realization of the ghost spectrum for the general closed gauge algebra theories is considered. Extended BRST-transformations are described. Properties of the simplest $Sp(2)$-symmetric Landau gauge are briefly analysed for the (super-) Yang-Mills theory and quantum gravity. In this gauge there are additional constant shift symmetries with interesting Ward identities. For the ordinary Yang-Mills theory conformal properties of the ghost fields are investigated. Essential simplification of the conformal Ward identities is achieved in $Sp(2)$-symmetric gauges. In summary we can say that $Sp(2)$-symmetry is useful additional tool in quantizing of gauge theories.

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