

## Status of calculations of the nuclear matrix elements for double beta decay

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**Abstract.** The present status of calculations of the nuclear matrix elements for neutrinoless double beta decay is reviewed. Advantages and disadvantages of different nuclear structure models used for the calculations are discussed in detail.

A study of nuclear neutrinoless double beta ( $0\nu\beta\beta$ ) decay  ${}^A_Z\text{El}_N \longrightarrow {}^A_{Z+2}\text{El}_{N-2} + 2e^-$  offers the only feasible way to test the charge-conjugation property of neutrinos. The existence of  $0\nu\beta\beta$  decay would immediately prove the neutrino to be a Majorana particle [1, 2, 3, 4], i.e., identical with its own antiparticle.

Also, the study of  $0\nu\beta\beta$  decay may provide one of a few ways to probe the absolute neutrino mass scale with a high sensitivity. The next generation of  $0\nu\beta\beta$ -decay experiments (GERDA, MAJORANA, CUORE, SuperNEMO etc., see, e.g., Ref. [3] for a recent review and corresponding contributions to these proceedings) has a great discovery potential. Provided the corresponding decay lifetimes are accurately measured, knowledge of the relevant nuclear matrix elements (NME)  $M^{0\nu}$  will become indispensable to reliably deduce fundamental information about neutrino properties from the future experimental data.

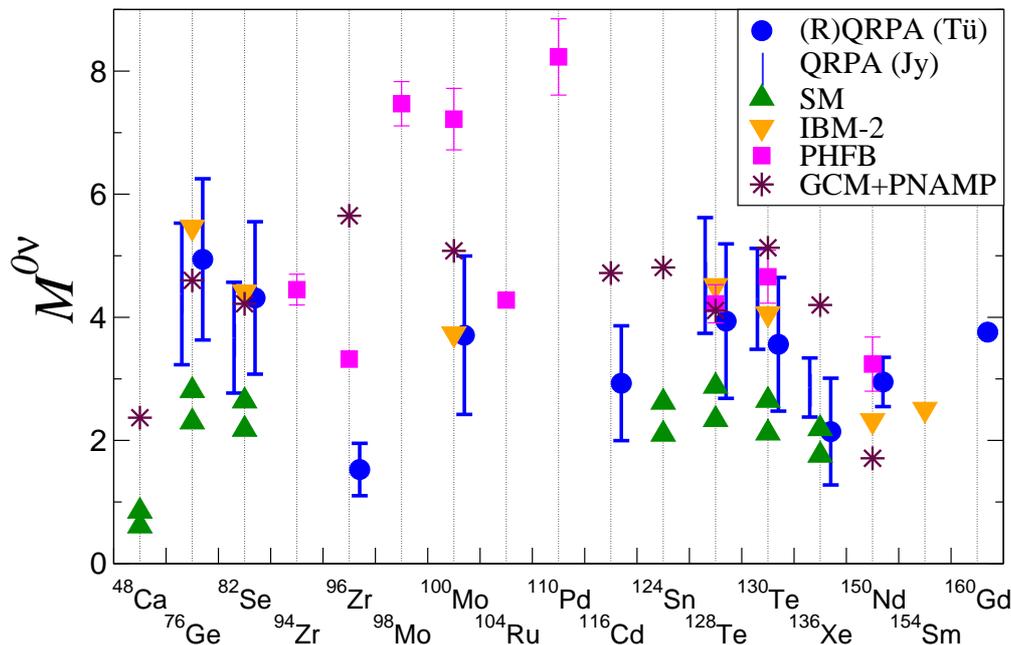
The light Majorana neutrino exchange mechanism of  $0\nu\beta\beta$  decay is the most popular one since the only condition for the neutrino to be a Majorana particle suffices for it. However, more exotic mechanisms can also trigger  $0\nu\beta\beta$  decay. The inverse  $0\nu\beta\beta$ -decay lifetime for is given as a product of three factors,

$$\left(T_{1/2}^{0\nu}\right)^{-1} = G^{0\nu} \left|M^{0\nu}\right|^2 m_{\beta\beta}^2, \quad (1)$$

where  $G^{0\nu}$  is an accurately calculable phase space factor,  $M^{0\nu}$  is the  $0\nu\beta\beta$  NME, and  $m_{\beta\beta}$  is the (nucleus-independent) effective Majorana neutrino mass, which is the quantity of great interest in neutrino physics. Thus, accurate values of  $M^{0\nu}$  are needed to extract  $m_{\beta\beta}$  out of the experimental  $0\nu\beta\beta$ -decay rates. The total matrix element is a sum of the Gamow-Teller (GT), Fermi (F) and Tensor (T) parts:  $M^{0\nu} = M_{GT}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_F^{0\nu} - M_T^{0\nu}$ , with  $M_{GT}^{0\nu}$  contributing most (appr. 70 %) to the total  $M^{0\nu}$ .

Several theoretical approaches have been used to evaluate  $M^{0\nu}$ . The present world status of the results on  $M^{0\nu}$  for the light neutrino mass mechanism is shown in Fig. 1.

Over the last decade, there has been great progress in the calculations of  $M^{0\nu}$  within the quasiparticle random phase approximation (QRPA), mostly performed by the groups in Tübingen [5, 6] and Jyväskylä [7, 8]. Now, the QRPA  $0\nu\beta\beta$  NME of different groups seem to converge. Along with the standard spherical QRPA, very recently its modification accounting



**Figure 1.** Neutrinoless double beta decay transition matrix elements  $M^{0\nu}$  calculated in different approaches: the quasiparticle random phase approximation (QRPA Tü) [5] (the results for  $^{150}\text{Nd}$  and  $^{160}\text{Gd}$  obtained in the version of the QRPA accounting for deformation [10]) and (QRPA Jy) [7], the nuclear shell model (SM) [13], the projected Hartree-Fock-Bogoliubov method (PHFB) [16], the Interacting Boson Model (IBM-2) [17], and the Generating Coordinate Method with particle number and angular momentum projection (GCM+PNAMP) [18]. See text for an explanation for the error bars in the QRPA and PHFB results, as well as for different treatments of the short range correlations.

for nuclear deformation has been developed and successfully applied to describe  $0\nu\beta\beta$  decay of  $^{150}\text{Nd}$  and  $^{160}\text{Gd}$  [10].

At present, the most elaborate analysis of the uncertainties in the calculated  $M^{0\nu}$  was also performed within the QRPA [5, 6]. The experimental  $2\nu\beta\beta$  decay rates were systematically used in the QRPA calculations [5, 6, 7, 8] to adjust the most relevant parameter, the renormalization strength  $g_{pp}$  of a realistic particle-particle interaction. The major observation of Refs. [5, 6] is that such a procedure makes the calculated  $M^{0\nu}$  essentially independent of the size of the single-particle (s.p.) basis of the QRPA. Furthermore, the matrix elements were also demonstrated to become rather stable with respect to the possible quenching of the axial vector coupling constant  $g_A$ . Thus, to visualize the degree of variation in the results, the error bars in the QRPA results in Fig. 1 were calculated from the highest and the lowest values of  $M^{0\nu}$  obtained in the calculations with different single-particle basis sets, two different axial vector coupling constants  $g_A = 1.25$  and  $g_A = 1.00$  (the quenched value), and two different treatments of the short range correlations (SRC, Jastrow-like [11] and the UCOM [12]).

The nuclear shell model (NSM) has been applied by the Strasbourg-Madrid collaboration to describe  $0\nu\beta\beta$  decay [13, 14]. It makes use of the closure relation with an averaged energy denominator in the neutrino potential, and in this way avoids explicit calculation of the states in the odd-odd intermediate nuclei. The quality of the results depends then on the description of the  $0^+$  ground states in the initial and final nuclei of the double beta decay system, e.g.  $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ . In Fig. 1 the NSM results are shown for Jastrow and UCOM SRC, lower and upper

triangles for each nucleus, respectively. One sees that  $M^{0\nu}$  of the NSM are systematically and substantially (up to a factor of 2) smaller than the other results.

Though the NSM tries to pursue an *ab initio* description of nuclear structure, severe truncation of the s.p. model space is needed for realistic calculations, since the number of many-body configurations increases drastically with the size of the s.p. basis. So, for  $A = 76$  and  $A = 82$  systems one is forced to restrict the s.p. basis to the  $0\hbar\omega$  set of levels:  $1p_{3/2}, 0f_{5/2}, 1p_{1/2}$  and  $0g_{9/2}$ . For the mass region around  $A = 130$  the NSM basis is restricted to  $0g_{7/2}, 1d_{3/2}, 1d_{5/2}, 2s_{1/2}$  and  $0h_{11/2}$  levels. The problem with these small basis sets is that the spin-orbit partners  $0f_{7/2}$  and  $0g_{7/2}$  are missing. The SM results then automatically violate the Ikeda Sum Rule, while the QRPA satisfies it exactly.

In Refs. [15, 8] the importance of having all the spin-orbit partners in the model space for description of  $M^{0\nu}$  was demonstrated. By using the same truncated basis of the NSM, the QRPA obtains roughly the same results as the NSM, which however get increased by about a factor of two, once the missing spin-orbit partners are added to the model space. Thus, inclusion of all spin-orbit partners is crucial for a realistic calculation of  $M^{0\nu}$ , that substitutes the main challenge for future NSM calculations of  $M^{0\nu}$ .

Now, we are going to briefly discuss the other nuclear models applied to calculate  $M^{0\nu}$  in the closure approximation. These are the projected angular-momentum Hartree-Fock-Bogoliubov method (PHFB) [16], the Interacting Boson Model (IBM-2) [17], and the Generating Coordinate Method with particle number and angular momentum projection (GCM+PNAMP) [18]. Note, that all of them were originally devised to describe the states of even-even nuclei, and, in contrast to the QRPA and the NSM, none of them is able to calculate the states in the odd-odd intermediate nuclei, or the  $2\nu\beta\beta$ -decay NME  $M^{2\nu}$ . Therefore, nothing can be said about the ability of the approaches to describe these ample nuclear data relevant for  $\beta\beta$  decay.

The PHFB method [16] uses a simple pairing plus quadrupole many-body Hamiltonian, and  $M^{0\nu}$  is calculated from a Hartree-Fock-Bogoliubov wave function with angular momentum projection after variation. With a real Bogoliubov transformation without parity mixing, one can only describe neutron pairs with angular momenta and parity  $0^+, 2^+, 4^+, \dots$  changing into pairs of protons (see Ref. [15] for a detailed discussion). Note, that within the QRPA and the NSM the pairs coupled to any angular momentum and parity are allowed. The corresponding restriction for the IBM-2 [17] is even more severe, since it takes into account only  $0^+$  and  $2^+$  neutron pairs (“S” and “D” pairs) changing into proton pairs. Thus, an apparent agreement of the IBM results with the QRPA ones as seen in Fig. 1 should be considered accidental.

The GCM+PNAMP approach is an extended version of the PHFB one, using a more realistic Hamiltonian and having the possibility to mix the states with different static deformation for description of nuclear ground states. However, the aforementioned PHFB restriction on the structure of the nucleon pairs present in the nuclear wave functions also persists within the GCM+PNAMP framework.

An important issue in the calculation of the NME  $M^{0\nu}$  is the treatment of the nucleon-nucleon short range correlations (SRC). For the first time, the SRC in the context of  $0\nu\beta\beta$ -decay were described consistently by the coupled cluster method with realistic CD-Bonn and Argonne V18 interactions in Ref. [6]. Thereby, a consistent calculation of the NME  $M^{0\nu}$  was performed in which pairing, ground-state correlations and the SRC originate from the same realistic NN interaction, namely from the CD-Bonn and Argonne V18 potentials. The results obtained by the consistent treatment of the SRC are slightly, by few per cents, larger than the ones obtained with the UCOM SRC. The parametrization of the SRC proposed in Ref. [6] has been used in other nuclear structure models to calculate  $M^{0\nu}$  [16, 14], with a similar conclusion on its relative importance.

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